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EXISTENCE AND MULTIPLICITY OF POSITIVE SOLUTIONS FOR ELLIPTIC SYSTEMS

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1. INTRODUCTION

In this paper we consider the existence and multiplicity of positive radial solutions for elliptic systems of the form

$$\begin{cases} \Delta u + \lambda k_1(|x|)f(u, v) = 0 \\ \Delta v + \mu k_2(|x|)g(u, v) = 0 \\ u|_{\partial\Omega} = v|_{\partial\Omega} = 0 \end{cases} \tag{1}$$

where $(u, v) \in C^2(\bar{\Omega}) \times C^2(\bar{\Omega})$, with $\Omega = \{x \in \mathbb{R}^N : R_1 < |x| < R_2, R_1, R_2 > 0\}$ an annulus with boundary $\partial\Omega$.

The following conditions will be assumed throughout:

$$(A_1) \qquad \begin{cases} f, g\colon [0, \infty) \times [0, \infty) \to [0, \infty) \text{ are continuous.} \\ \lambda \text{ and } \mu \text{ are positive parameters.} \\ k_1, k_2\colon [R_1, R_2] \to [0, \infty) \text{ are continuous and do not} \\ \quad \text{vanish identically on any subinterval of } [R_1, R_2]. \end{cases}$$

By a positive solution of (1) we understand a solution (u, v) with $u \geq 0$, $v \geq 0$ and either $u \not\equiv 0$ or $v \not\equiv 0$. By the maximum principle, each nontrivial component of (u, v) is thus positive in Ω.

In recent years it has been proved that for a single equation, superlinearity or sublinearity of the nonlinearity at both ends (zero and infinity) can guarantee the existence of a positive solution on an annulus. See [1-14], for instance. On the other hand, as was shown in [7, 8, 11], superlinearity at one end and sublinearity at the other end can imply the existence of at least two positive solutions. We also refer to [2, 11, 14] for further results in this direction.

In our previous work [6], we showed the existence and multiplicity of positive solutions for the system (1) when $f(u, v) = f(v)$ and $g(u, v) = g(u)$ are functions of one variable. Our purpose here is to deal with more general f and g. It should be noted that the existence of positive solutions for elliptic systems on a ball has been studied in [15]. In this paper we use a fixed point theorem of cone expansion/compression type which allows us to establish not only existence, but also multiplicity.

2. PRELIMINARIES

We shall seek criteria for the existence of positive radial solutions $u = u(r)$, $v = v(r)$ of (1) which then satisfy

$$\begin{cases} u'' + \dfrac{n-1}{r} u' + \lambda k_1(r) f(u, v) = 0 \\ v'' + \dfrac{n-1}{r} v' + \mu k_2(r) g(u, v) = 0 \\ u(R_1) = u(R_2) = v(R_1) = v(R_2) = 0. \end{cases} \tag{2}$$

By several changes of variables (see e.g. [14]) the system (2) can be brought into the form

$$\begin{cases} u''(t) + \lambda h_1(t) f(u, v) = 0 \\ v''(t) + \mu h_2(t) g(u, v) = 0 \\ u(0) = u(1) = v(0) = v(1) = 0 \end{cases} \tag{3}$$

where $h_1(t)$, $h_2(t)$ are continuous and do not vanish identically on any subinterval of $[0, 1]$. The system (3), in turn, is equivalent to the system of integral equations

$$\begin{cases} u(t) = \lambda \displaystyle\int_0^1 k(t, s) h_1(s) f(u(s), v(s))\, ds \\ v(t) = \mu \displaystyle\int_0^1 k(t, s) h_2(s) g(u(s), v(s))\, ds \end{cases} \tag{4}$$

where $k(t, s)$ is the Green's function

$$k(t, s) = \begin{cases} t(1 - s), & t \le s \\ s(1 - t), & t > s. \end{cases}$$

Let

$$A(u, v)(t) = \lambda \int_0^1 k(t, s) h_1(s) f(u(s), v(s))\, ds$$

$$B(u, v)(t) = \mu \int_0^1 k(t, s) h_2(s) g(u(s), v(s))\, ds$$

$$F(u, v)(t) = (A(u, v)(t), B(u, v)(t)).$$

Then (4) is equivalent to the fixed point equation

$$F(u, v) = (u, v)$$

in the Banach space $X = C([0, 1]^2)$. The following Fixed-Point Theorem of cone expansion/compression type will be crucial in the arguments that follow.

THEOREM 2.1 [16]. Let X be a Banach space and let $K \subset X$ be a cone in X. Assume Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1$, $\bar{\Omega}_1 \subset \Omega_2$ and let

$$F: K \cap (\bar{\Omega}_2 \backslash \Omega_1) \to K$$

be a completely continuous operator such that either

(i) $\|Fu\| \leq \|u\|, \quad u \in K \cap \partial\Omega_1$
and
$\|Fu\| \geq \|u\|, \quad u \in K \cap \partial\Omega_2$

or

(ii) $\|Fu\| \geq \|u\|, \quad u \in K \cap \partial\Omega_1$
and
$\|Fu\| \leq \|u\|, \quad u \in K \cap \partial\Omega_2$.

Then F has a fixed point in $K \cap (\bar{\Omega}_2 \backslash \Omega_1)$.

In order to apply Theorem 2.1, we will let K be the cone defined by

$$K = \left\{(u, v) : (u, v) \in X, u, v \geq 0, \min_{1/4 \leq t \leq 3/4} (u(t) + v(t)) \geq \frac{1}{4}(\|u\| + \|v\|)\right\}$$

where $\|u\| = \sup_{t \in [0,1]} |u(t)|$. This choice of a cone is motivated by the concavity of nonnegative solutions of (1). In what follows we set $\|(u, v)\| = \|u\| + \|v\|$.

LEMMA 2.2. $F: X \to X$ is completely continuous and $F(K) \subset K$.

Proof. The complete continuity of F is obvious. To prove $F(K) \subset K$, choose $(u, v) \in K$. Since $k(t, s) \leq s(1 - s)$ for $0 \leq s \leq 1$, and $k(t, s) \geq s(1 - s)/4$ for $1/4 \leq t \leq 3/4$, $0 \leq s \leq 1$, we have

$$\min_{1/4 \leq t \leq 3/4} A(u, v)(t) \geq \frac{\lambda}{4} \int_0^1 s(1 - s)h_1(s)f(u(s), v(s))\, ds \geq \frac{1}{4}\|A(u, v)\|$$

and similarly

$$\min_{1/4 \leq t \leq 3/4} B(u, v)(t) \geq \frac{1}{4}\|B(u, v)\|.$$

Thus

$$\begin{aligned}\min_{1/4 \leq t \leq 3/4} (A(u, v)(t) + B(u, v)(t)) &\geq \min_{1/4 \leq t \leq 3/4} A(u, v)(t) + \min_{1/4 \leq t \leq 3/4} B(u, v)(t) \\ &\geq \frac{1}{4}(\|A(u, v)\| + \|B(u, v)\|) \\ &= \frac{1}{4}\|(A(u, v), B(u, v))\|.\end{aligned}$$

Since $k(t, s) > 0$ for $0 < t, s < 1$ and (A_1) holds, we conclude that $FK \subset K$. ∎

3. EXISTENCE THEOREMS

We introduce the following notation:

$$f_0 = \lim_{(u,v)\to 0} \frac{f(u, v)}{u + v}, \qquad g_0 = \lim_{(u,v)\to 0} \frac{g(u, v)}{u + v},$$

$$f_\infty = \lim_{(u,v)\to \infty} \frac{f(u, v)}{u + v}, \qquad g_\infty = \lim_{(u,v)\to \infty} \frac{g(u, v)}{u + v}.$$

THEOREM 3.1. Assume (A_1) holds. Then for all $\lambda > 0$, $\mu > 0$, the system (1) has at least one positive radial solution in the following cases:

(a) $f_0 = g_0 = 0$, and either $f_\infty = \infty$ or $g_\infty = \infty$ (superlinear)

(b) $f_\infty = g_\infty = 0$, and either $f_0 = \infty$ or $g_0 = \infty$ (sublinear).

Proof. (a) Since $f_0 = g_0 = 0$, we may choose $H_1 > 0$ so that $f(u, v) \le \varepsilon(u + v)$ and $g(u, v) \le \varepsilon(u + v)$ for $0 < u, v \le H_1$, where the constant $\varepsilon > 0$ satisfies

$$2\varepsilon\lambda \int_0^1 s(1 - s)h_1(s)\, ds \le 1, \qquad 2\varepsilon\mu \int_0^1 s(1 - s)h_2(s)\, ds \le 1.$$

Set

$$\Omega_1 = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_1\}.$$

If $(u,v) \in K \cap \partial\Omega_1$, we have

$$\begin{aligned} A(u, v)(t) &\le \lambda \int_0^1 s(1 - s)h_1(s)f(u(s), v(s))\, ds \\ &\le \varepsilon\lambda \int_0^1 s(1 - s)h_1(s)(u(s) + v(s))\, ds \\ &\le \varepsilon\lambda(\|u\| + \|v\|) \int_0^1 s(1 - s)h_1(s)\, ds \\ &\le \frac{\|(u, v)\|}{2} \end{aligned}$$

and similarly,

$$B(u, v)(t) \le \frac{\|(u, v)\|}{2}.$$

Hence

$$\|F(u, v)\| = \|A(u, v)\| + \|B(u, v)\| \le \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_1$.

If we further assume $f_\infty = \infty$, then there is an $\hat{H} > 0$ such that $f(u, v) \geq \eta(u + v)$ for $u + v \geq \hat{H}$, where $\eta > 0$ is chosen so that

$$\frac{\eta\lambda}{4} \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right) h_1(s)\, ds \geq 1.$$

Let $H_2 = \max\{2H_1, 4\hat{H}\}$ and set

$$\Omega_2 = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_2\}.$$

If $(u, v) \in K \cap \partial\Omega_2$, we have

$$\min_{1/4 \leq t \leq 3/4} (u(t) + v(t)) \geq \frac{1}{4}(\|u, v)\|) \geq \hat{H}$$

and

$$\begin{aligned} A(u, v)\left(\frac{1}{2}\right) &= \lambda \int_0^1 k\left(\frac{1}{2}, s\right) h_1(s) f(u(s), v(s))\, ds \\ &\geq \lambda \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right) h_1(s) f(u(s), v(s))\, ds \\ &\geq \eta\lambda \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right) h_1(s)(u(s) + v(s))\, ds \\ &\geq \frac{\eta\lambda}{4}(\|u\| + \|v\|) \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right) h_1(s)\, ds \\ &\geq \|(u, v)\|. \end{aligned}$$

Therefore

$$\|F(u, v)\| \geq A(u, v)\left(\frac{1}{2}\right) \geq \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_2$. An analogous estimate holds if $g_\infty = \infty$.

Now by Theorem 2.1, F has a fixed point $(u, v) \in K \cap (\bar{\Omega}_2 \backslash \Omega_1)$ such that $H_1 \leq \|(u, v)\| \leq H_2$, and so (1) has a positive radial solution.

(b) If $f_0 = \infty$, we now choose $H_1 > 0$ so that $f(u, v) \geq \hat{\eta}(u + v)$ for $0 \leq u, v \leq H_1$, where $\hat{\eta}$ satisfies

$$\frac{\hat{\eta}\lambda}{4} \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right) h_1(s)\, ds \geq 1.$$

Set

$$\Omega_1 = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_1\}.$$

If $(u, v) \in K \cap \partial\Omega_1$, we have

$$\begin{aligned} A(u, v)\left(\frac{1}{2}\right) &= \lambda \int_0^1 k\left(\frac{1}{2}, s\right)h_1(s)f(u(s), v(s))\,ds \\ &\geq \lambda \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right)h_1(s)f(u(s), v(s))\,ds \\ &\geq \hat{\eta}\lambda \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right)h_1(s)(u(s) + v(s))\,ds \\ &\geq \frac{\hat{\eta}\lambda}{4}(\|u\| + \|v\|) \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right)h_1(s)\,ds \\ &\geq \|(u, v)\|. \end{aligned}$$

Therefore

$$\|F(u, v)\| \geq A(u, v)\left(\frac{1}{2}\right) \geq \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_1$. An analogous estimate holds if $g_0 = \infty$.

We now determine Ω_2. Let us define two new functions $f^*(t) = \max_{0 \leq u+v \leq t} f(u, v)$ and $g^*(t) = \max_{0 \leq u+v \leq t} g(u, v)$. Note that $f^*(t)$ and $g^*(t)$ are nondecreasing in their respective arguments. Moreover, from $f_\infty = g_\infty = 0$, it follows that

$$\lim_{t \to \infty} \frac{f^*(t)}{t} = 0, \qquad \lim_{t \to \infty} \frac{g^*(t)}{t} = 0.$$

Therefore, there is an $H_2 > 2H_1$ such that $f^*(t) \leq \varepsilon t$, $g^*(t) \leq \varepsilon t$ for $t \geq H_2$, where the constant $\varepsilon > 0$ satisfies

$$\varepsilon\lambda \int_0^1 s(1 - s)h_1(s)\,ds \leq \frac{1}{2}, \qquad \varepsilon\mu \int_0^1 s(1 - s)h_2(s)\,ds \leq \frac{1}{2}.$$

Set

$$\Omega_2 = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_2\}.$$

If $(u, v) \in K \cap \partial\Omega$, we have

$$\begin{aligned} A(u, v)(t) &\leq \lambda \int_0^1 s(1 - s)h_1(s)f(u(s), v(s))\,ds \\ &\leq \lambda \int_0^1 s(1 - s)h_1(s)f^*(H_2)\,ds \\ &\leq \varepsilon\lambda H_2 \int_0^1 s(1 - s)h_1(s)\,ds \\ &\leq \frac{H_2}{2} = \frac{\|(u, v)\|}{2}. \end{aligned}$$

Similarly, $B(u, v)(t) \le \|(u, v)\|/2$, and so

$$\|F(u, v)\| \le \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_2$. Applying Theorem 2.1, we obtain the existence of a positive radial solution (u, v) for (1). ■

4. MULTIPLICITY THEOREMS

In this section we consider the multiplicity of solutions. The idea is as follows: we construct sets $\Omega_3 \subset \Omega_4$, such that $\Omega_1 \subset \Omega_3 \subset \Omega_4 \subset \Omega_2$, where Ω_1, Ω_2 are constructed in Theorem 3.1. This will allow us to apply the fixed point theorem twice. In this direction we shall need:

$$(\mathrm{A}_2) \qquad f(u, v), g(u, v) > 0 \qquad \text{for } u, v > 0.$$

THEOREM 4.1. Assume (A_1), (A_2) hold.

(a) If $f_0 = g_0 = f_\infty = g_\infty = 0$, then there is a positive constant σ_1 such that (1) has at least two positive radial solutions for all $\lambda, \mu \ge \sigma_1$.

(b) If either $f_0 = \infty$ or $g_0 = \infty$, and either $f_\infty = \infty$ or $g_\infty = \infty$, then there is a positive constant σ_2 such that (1) has at least two positive radial solutions for all $\lambda, \mu \le \sigma_2$.

Proof. (a) For $(u, v) \in K$ and $\|(u, v)\| = q$, let

$$m(q) = \min\left\{\int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right)h_1(s)f(u(s), v(s))\,\mathrm{d}s,\ \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right)h_2(s)g(u(s), v(s))\,\mathrm{d}s\right\}.$$

It follows from (A_1), (A_2) that $m(q) > 0$ for $q > 0$.

Choose two numbers $0 < H_3 < H_4$, let $\sigma_1 = \max\{H_3/(2m(H_3)), H_4/(2m(H_4))\}$ and set

$$\Omega_i = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_i\} \qquad (i = 3, 4).$$

Then, for $\lambda, \mu \ge \sigma_1$ and $(u, v) \in K \cap \partial\Omega_i$ $(i = 3, 4)$, we have

$$A(u, v)\left(\frac{1}{2}\right) \ge \lambda \int_{1/4}^{3/4} k\left(\frac{1}{2}, s\right)h_1(s)f(u(s), v(s))\,\mathrm{d}s \ge \lambda m(H_i) \ge \frac{H_i}{2}, \qquad (i = 3, 4)$$

and similarly, $B(u, v)(1/2) \ge H_i/2$, $(i = 3, 4)$, which implies

$$\|F(u, v)\| \ge H_i = \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_i$ $(i = 3, 4)$. Since $f_0 = g_0 = 0$ and $f_\infty = g_\infty = 0$, it follows from the proof of Theorem 3.1(a) and (b), respectively, that we can choose $H_1 < H_3/2$ and $H_2 > 2H_4$ such that

$$\|F(u, v)\| \le \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_i$, $(i = 1, 2)$, where

$$\Omega_i = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_i\}, \qquad (i = 1, 2).$$

Applying Theorem 2.1 to Ω_1, Ω_3 and Ω_4, Ω_2 we get a positive radial solution (u_1, v_1) such that $H_1 \le \|(u_1, v_1)\| \le H_3$ and another positive radial solution (u_2, v_2) such that $H_4 \le \|(u_2, v_2)\| \le H_2$. Since $H_3 < H_4$, the two solutions are distinct.

(b) For $(u, v) \in K$ and $\|(u, v)\| = q$, let

$$M(q) = \max\left\{\int_0^1 s(1-s)h_1(s)f(u(s), v(s))\,ds,\ \int_0^1 s(1-s)h_2(s)g(u(s), v(s))\,ds\right\}.$$

Then as above, $M(q) > 0$ for $q > 0$.

Choose two numbers $0 < H_3 < H_4$, let $\sigma_2 = \min\{H_3/(2M(H_3)), H_4/(2M(H_4))\}$ and set

$$\Omega_i = \{(u, v) : (u, v) \in X, \|(u, v\| < H_i\}, \qquad (i = 3, 4).$$

Then for $\lambda, \mu \le \sigma_2$ and $(u, v) \in K \cap \partial\Omega_i$, $(i = 3, 4)$, we have

$$A(u, v)(t) \le \lambda M(H_i) \le \sigma_2 M(H_i) \le H_i/2, \qquad (i = 3, 4)$$

and similarly, $B(u, v)(t) \le \sigma_2 M(H_i) \le H_i/2$, $(i = 3, 4)$, which implies

$$\|F(u, v)\| \le H_i = \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_i$ $(i = 3, 4)$. Since either $f_0 = \infty$ or $g_0 = \infty$, and either $f_\infty = \infty$ or $g_\infty = \infty$, it follows from the proof of Theorem 3.1(b) and (a), respectively, that we can choose $H_1 < H_3/2$ and $H_2 > 2H_4$ such that

$$\|F(u, v)\| \ge \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_i$ $(i = 1, 2)$, where

$$\Omega_i = \{(u, v) : (u, v) \in X, \|(u, v)\| < H_i\}, \qquad (i = 1, 2).$$

Once again we obtain the existence of two distinct positive radial solutions. ■

The following existence result is a simple consequence of the preceding theorems.

THEOREM 4.2. Assume (A_1), (A_2) hold.

(a) If $f_0 = g_0 = 0$ or $f_\infty = g_\infty = 0$, then there is a positive constant σ_3 such that (1) has at least one positive radial solution for all $\lambda, \mu \ge \sigma_3$.

(b) If $f_0 = \infty$ or $g_0 = \infty$, or if $f_\infty = \infty$ or $g_\infty = \infty$, then there is a positive constant σ_4 such that (1) has at least one positive radial solution for all $\lambda, \mu \le \sigma_4$.

5. REMARKS

The results of this paper are also valid for the following mixed-type boundary value problems and are handled in an analogous manner.

$$\begin{cases} \Delta u + \lambda k_1(|x|)f(u, v) = 0 \\ \Delta v + \mu k_2(|x|)g(u, v) = 0 \\ u = v = 0 \quad \text{on } |x| = R_1, \qquad \dfrac{\partial u}{\partial r} = \dfrac{\partial v}{\partial r} = 0 \quad \text{on } |x| = R_2 \end{cases} \tag{5}$$

$$\begin{cases} \Delta u + \lambda k_1(|x|)f(u, v) = 0 \\ \Delta u + \mu k_2(|x|)g(u, v) = 0 \\ \dfrac{\partial u}{\partial r} = \dfrac{\partial v}{\partial r} = 0 \quad \text{on } |x| = R_1, \qquad u = v = 0 \quad \text{on } |x| = R_2 \end{cases} \tag{6}$$

where $\partial/\partial r$ denotes differentiation in the radial direction.

It is easy to check that (5) and (6) are equivalent to the integral equations

$$\begin{cases} u(t) = \lambda \displaystyle\int_0^1 k_1(t, s)h_1(s)f(u(s), v(s))\,\mathrm{d}s \\ v(t) = \mu \displaystyle\int_0^1 k_1(t, s)h_2(s)g(u(s), v(s))\,\mathrm{d}s \end{cases} \tag{7}$$

$$\begin{cases} u(t) = \lambda \displaystyle\int_0^1 k_2(t, s)h_1(s)f(u(s), v(s))\,\mathrm{d}s \\ v(t) = \mu \displaystyle\int_0^1 k_2(t, s)h_2(s)g(u(s), v(s))\,\mathrm{d}s \end{cases} \tag{8}$$

respectively, where

$$k_1(t, s) = \begin{cases} t, & t \le s \\ s, & t > s, \end{cases}$$

and

$$k_2(t, s) = \begin{cases} 1 - s, & t \le s \\ 1 - t, & t > s. \end{cases}$$

For problem (7), one considers the cone

$$K_1 = \left\{(u, v) : (u, v) \in X,\ u, v \ge 0,\ \min_{1/2 \le t \le 1} (u(t) + v(t)) \ge \frac{1}{2}(\|u\| + \|v\|)\right\}$$

and for problem (8), one considers the cone

$$K_2 = \left\{(u, v) : (u, v) \in X,\ u, v \ge 0,\ \min_{0 \le t \le 1/2} (u(t) + v(t)) \ge \frac{1}{2}(\|u\| + \|v\|)\right\}.$$

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