Superfluid Heat Conduction in the Neutron Star Crust

Sanjay Reddy
Los Alamos National Lab

Collaborators :
Deborah Aguilera
Vincenzo Cirigliano
Jose Pons
Rishi Sharma

arXiv:0807.4754

ASU Seminar April 16, 2009

Thermal Conduction 101:

$$Q = \kappa \frac{\partial T}{\partial x}$$

$$\kappa \simeq \frac{1}{3} C_V \, v \, \lambda$$

Specific heat of carrier

velocity of carrier

mean free path

Thermal Conductivity - Down to Earth

material	κ (ergs/cm·s·K)
air	0.00025
bronze	1.10
copper	4.01
diamond	8.95
Earth (dry)	0.015
Freon	0.00073
graphite	19.5
helium (II)	>1000
ice cream (powder)	0.0005

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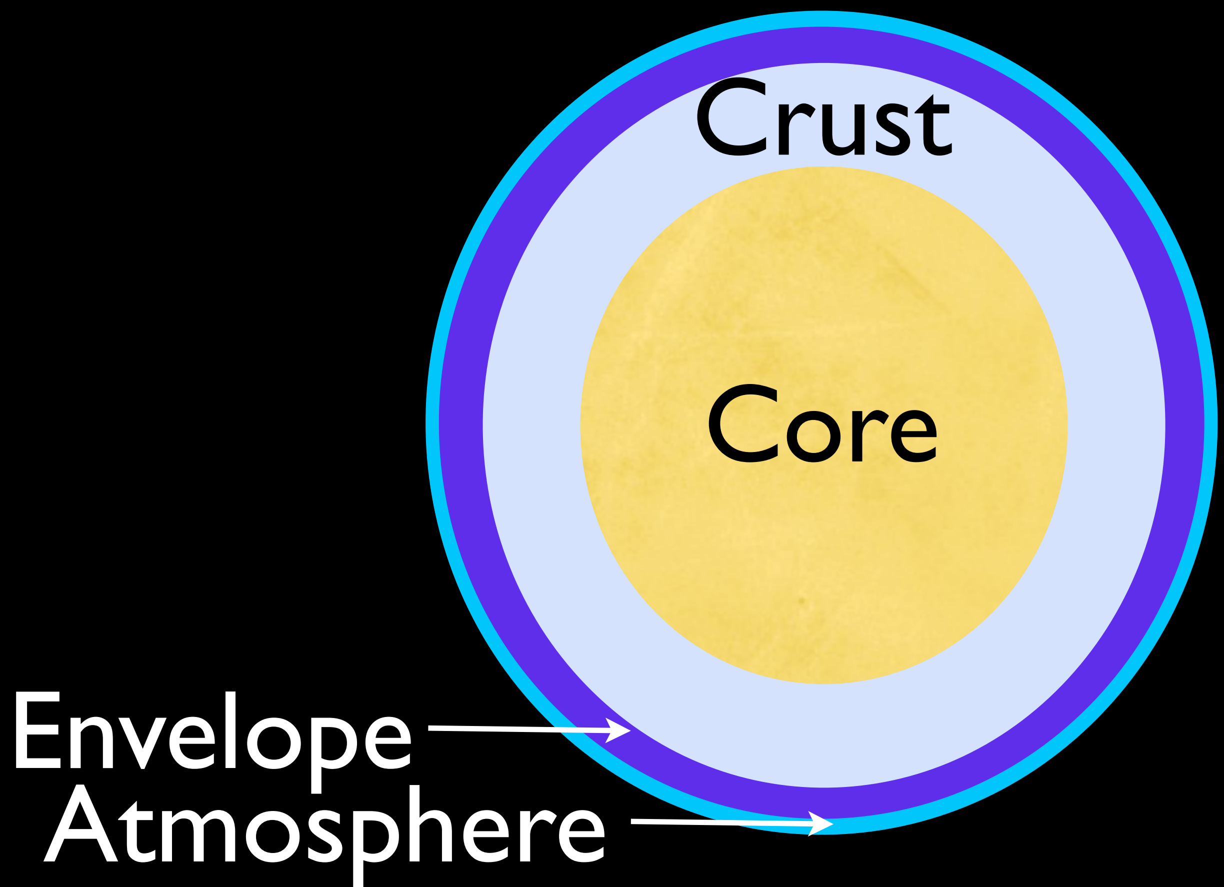
Neutron Star Thermal Evolution

- Long term cooling of isolated neutron stars.
- Thermal profiles of accreting neutron stars.
- Long term cooling of magnetars.
- Early thermal evolution.



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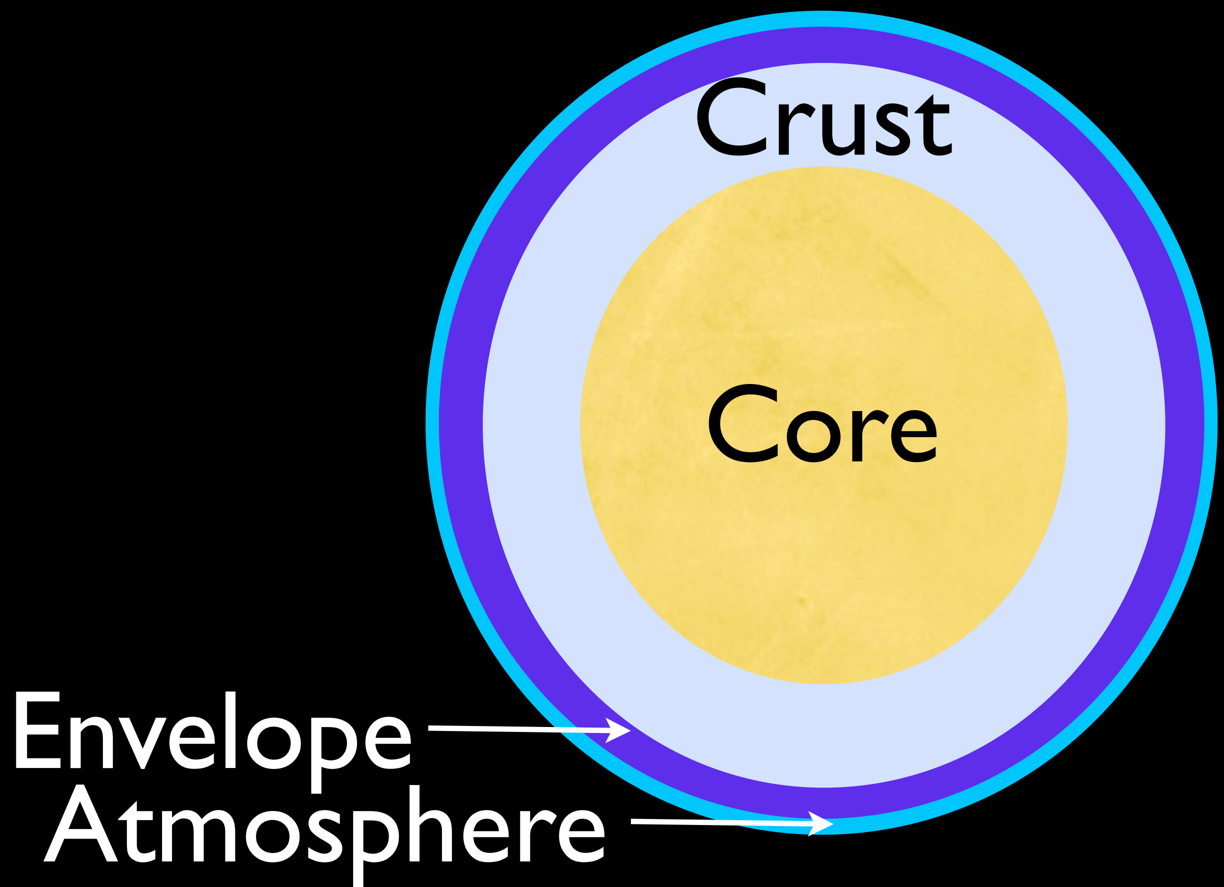
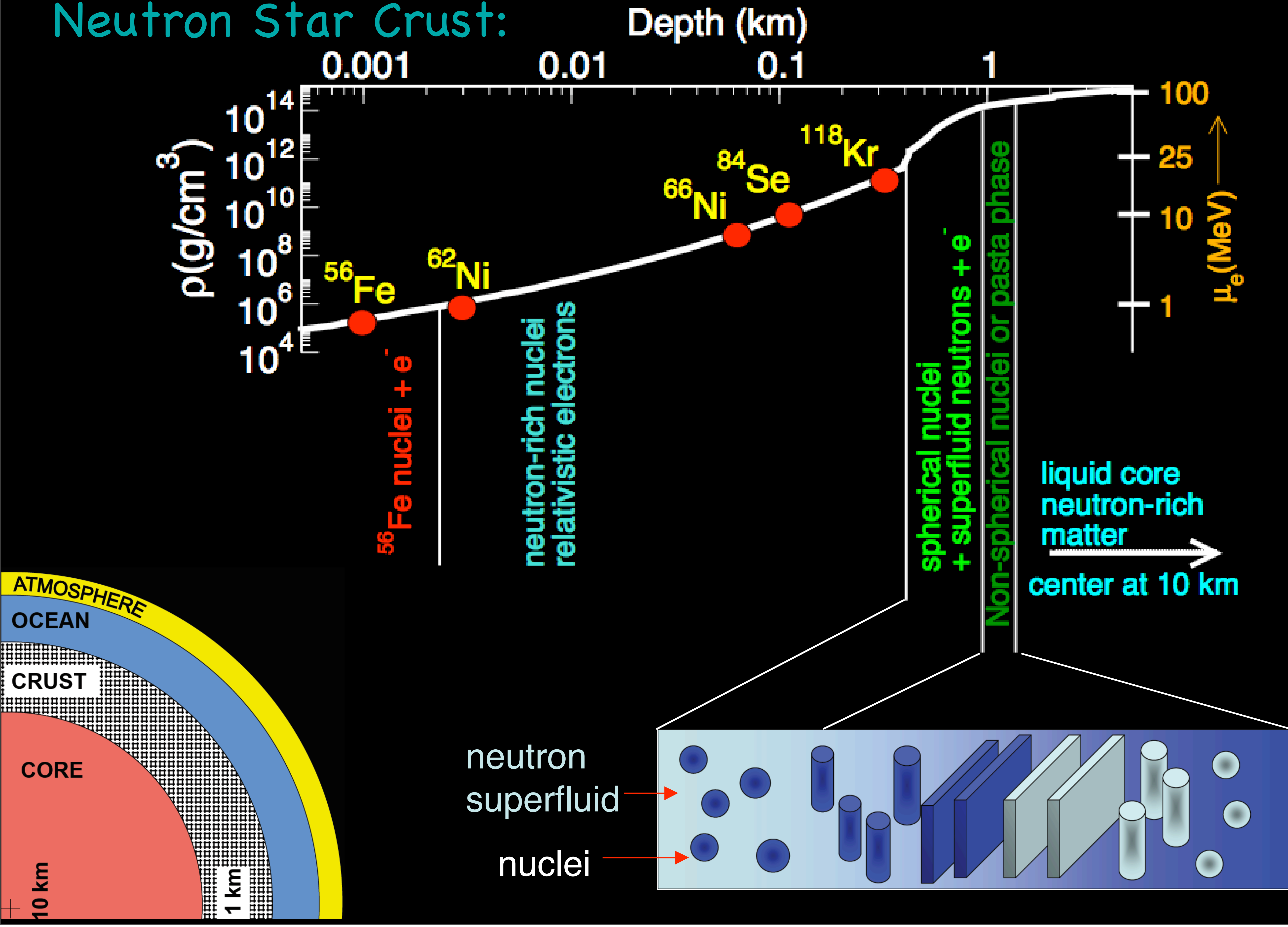


Temperature gradient in the inner crust plays a role.

Neutron Star Crust:



Thermal Conduction in the Crust

Outer Crust:

- Liquid Phase: Electrons & Ions
- Solid Phase: Electrons & Phonons

- Electronic specific heat is large
- Electron mean free path is small
- Electron momentum is large $k_F > 1/a$
- Phonon specific heat is small
- Phonon mean free path is large
- Phonon momentum is small

Electrons or Phonons ?

$$\frac{\kappa_{\rm el}}{\kappa_{\rm lPh}} = \frac{C_{\rm el}}{C_{\rm lPh}} \frac{1}{c} \frac{\lambda_{\rm e}}{\lambda_{\rm lPh}}$$

Typically electrons dominate
- unless there is a large magnetic field.

Magnetic field suppresses transverse conduction

$$\kappa_\perp = \frac{\kappa_\parallel}{1 + (\omega_g \ \tau_e)^2}$$

$$\kappa_\parallel = \kappa_{\rm el}(B=0)$$

$$\omega_g = \frac{eB}{\mu_e}$$ =Gyrofrequency

$\tau_{\rm e}$ = Collision time

Canuto and Ventura (1977)
Uripin & Yakovlev (1980)

Electrons or Phonons ?

$$\frac{\kappa_{\rm el}}{\kappa_{\rm lPh}} = \frac{C_{\rm el}}{C_{\rm lPh}} \frac{1}{c} \frac{\lambda_{\rm e}}{\lambda_{\rm lPh}} \simeq \frac{\mu_e^2}{T^2} \frac{1}{c} \frac{\lambda_{\rm e}}{\lambda_{\rm lPh}} \gg 1$$

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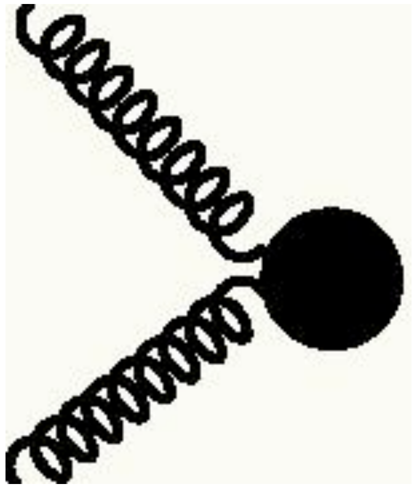
Phonon Conduction in the Outer Crust

Lattice Phonons have large mean free paths.

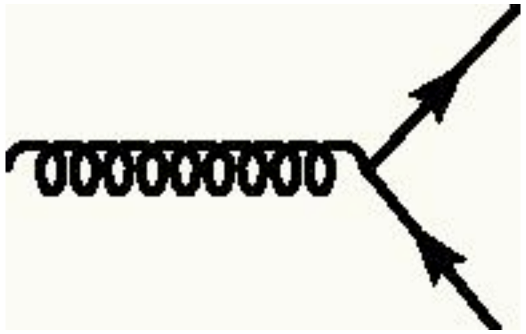
$$\lambda_{\mathrm{lPh}} \gg \lambda_{\mathrm{e}}$$

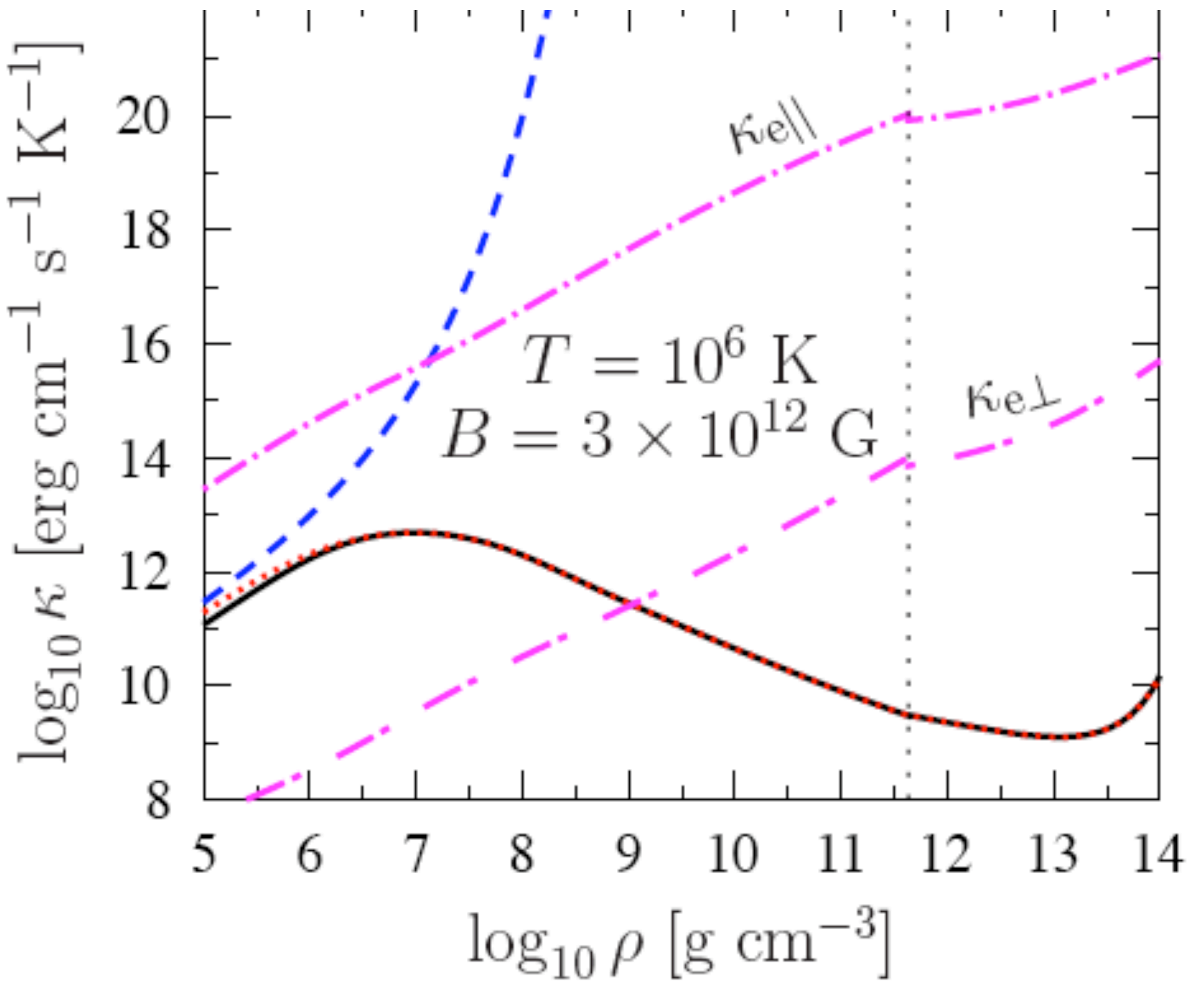
Mean free path set by:

Impurity scattering



Electron absorption





Perez-Azorin (2006) Chugunov and Haensel (2007)

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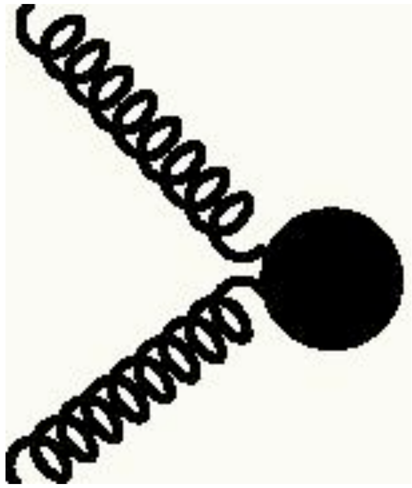

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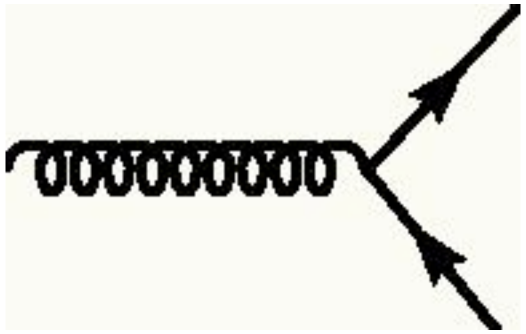

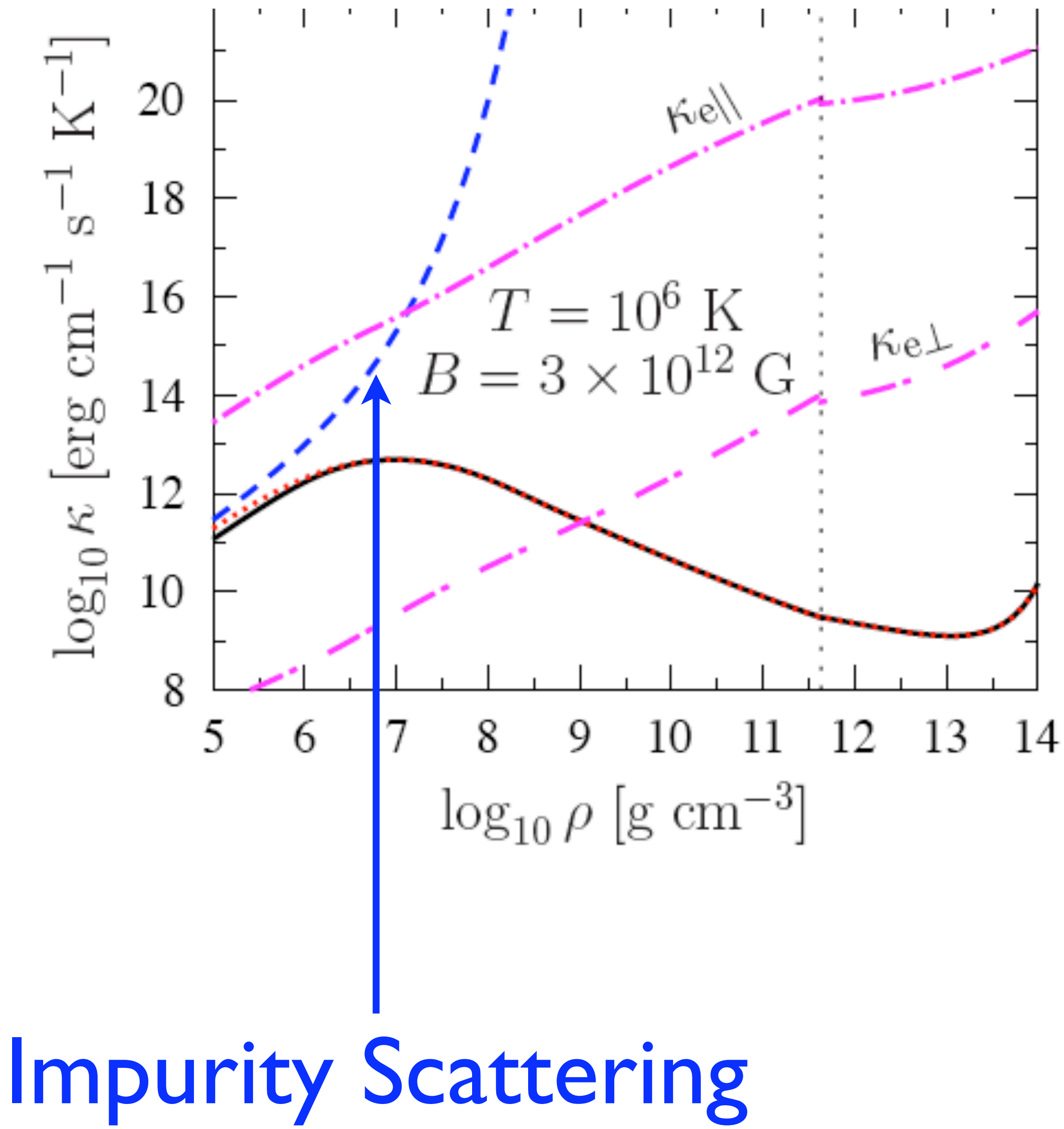

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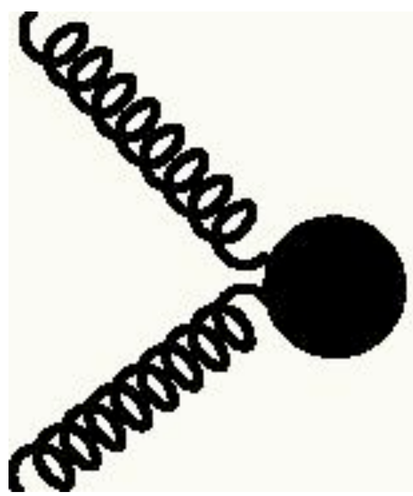

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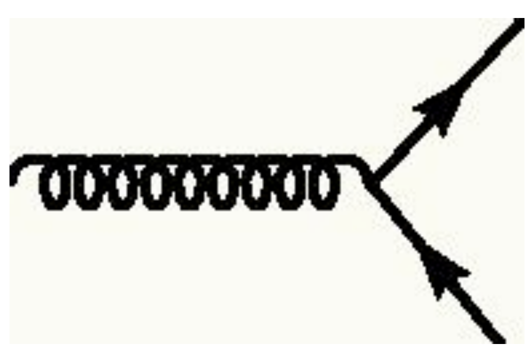
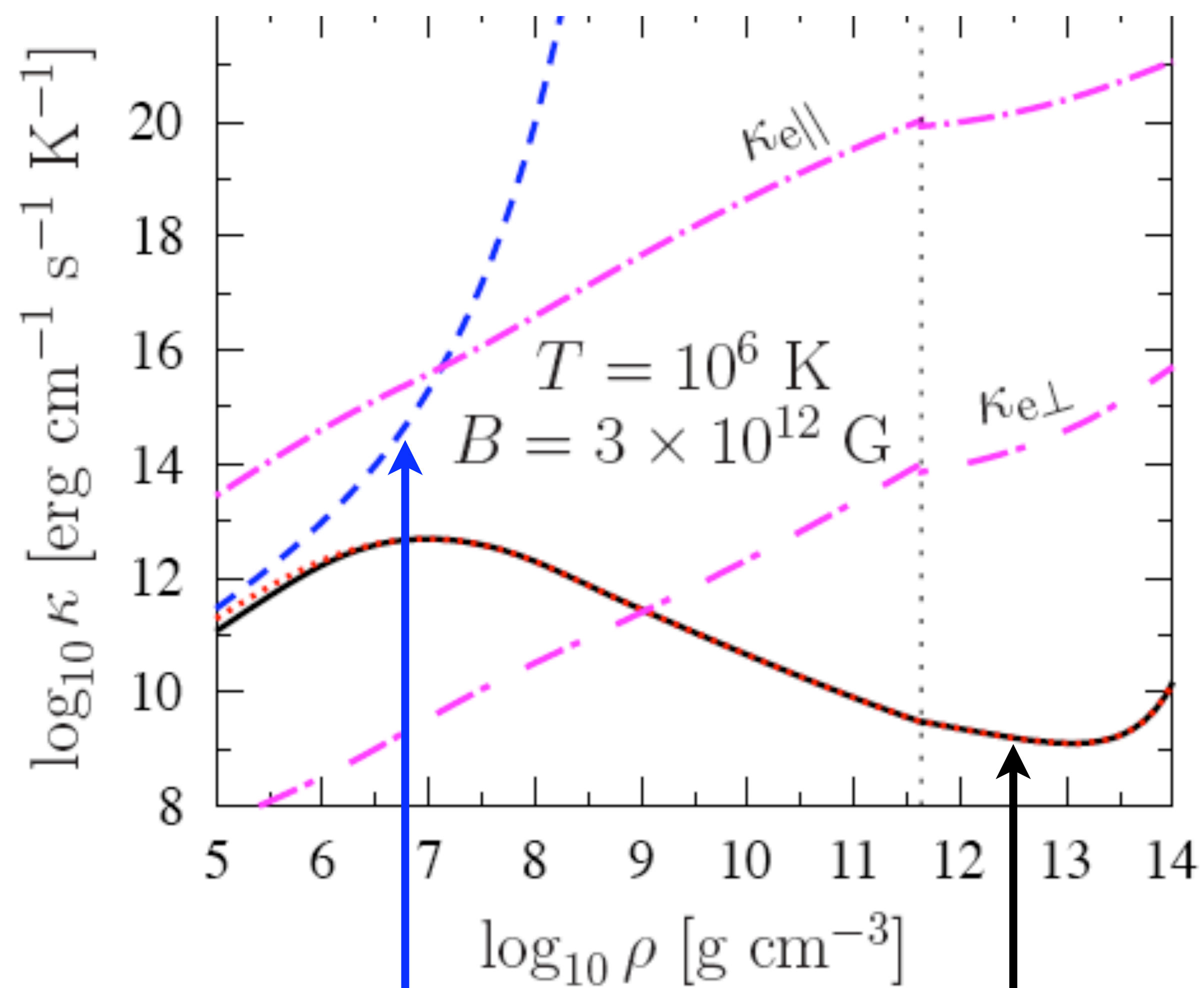




Impurity Scattering

Electron Absorption

Perez-Azorin (2006) Chugunov and Haensel (2007)

Phonon Conduction in Insulators

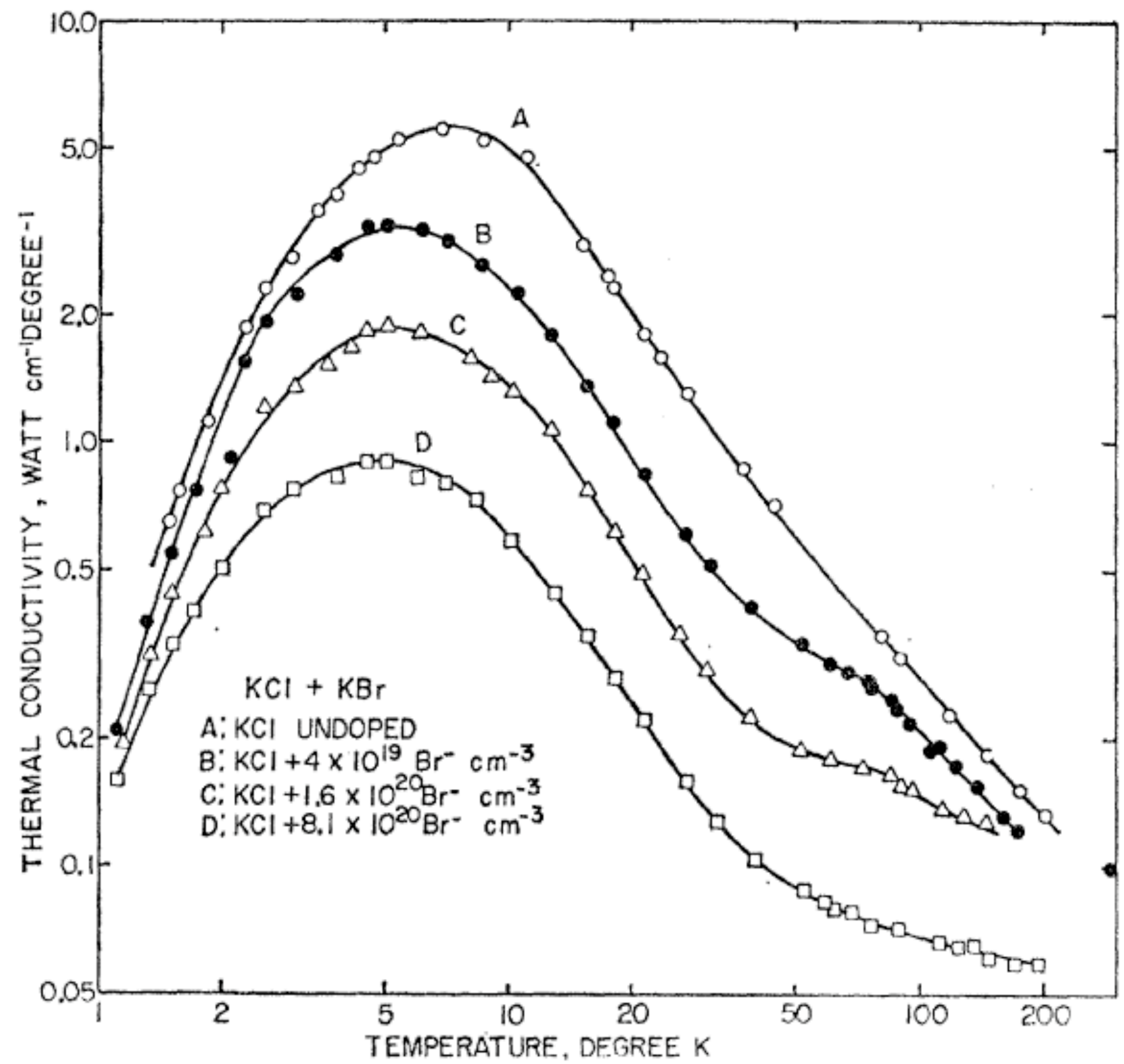
At low temperature phonons have very large mean free path.

Rayleigh scattering off impurities dominates at long-wavelength

$$\sigma_R = \pi r_0^6 \, q^4 \simeq \frac{A}{v^4} \, T^4$$

$$\lambda = \frac{1}{n_I \, \sigma_R}$$

$$\kappa \simeq \frac{1}{3} C_V \, v \, \lambda \simeq B \, \frac{v^2}{n_I \, T}$$



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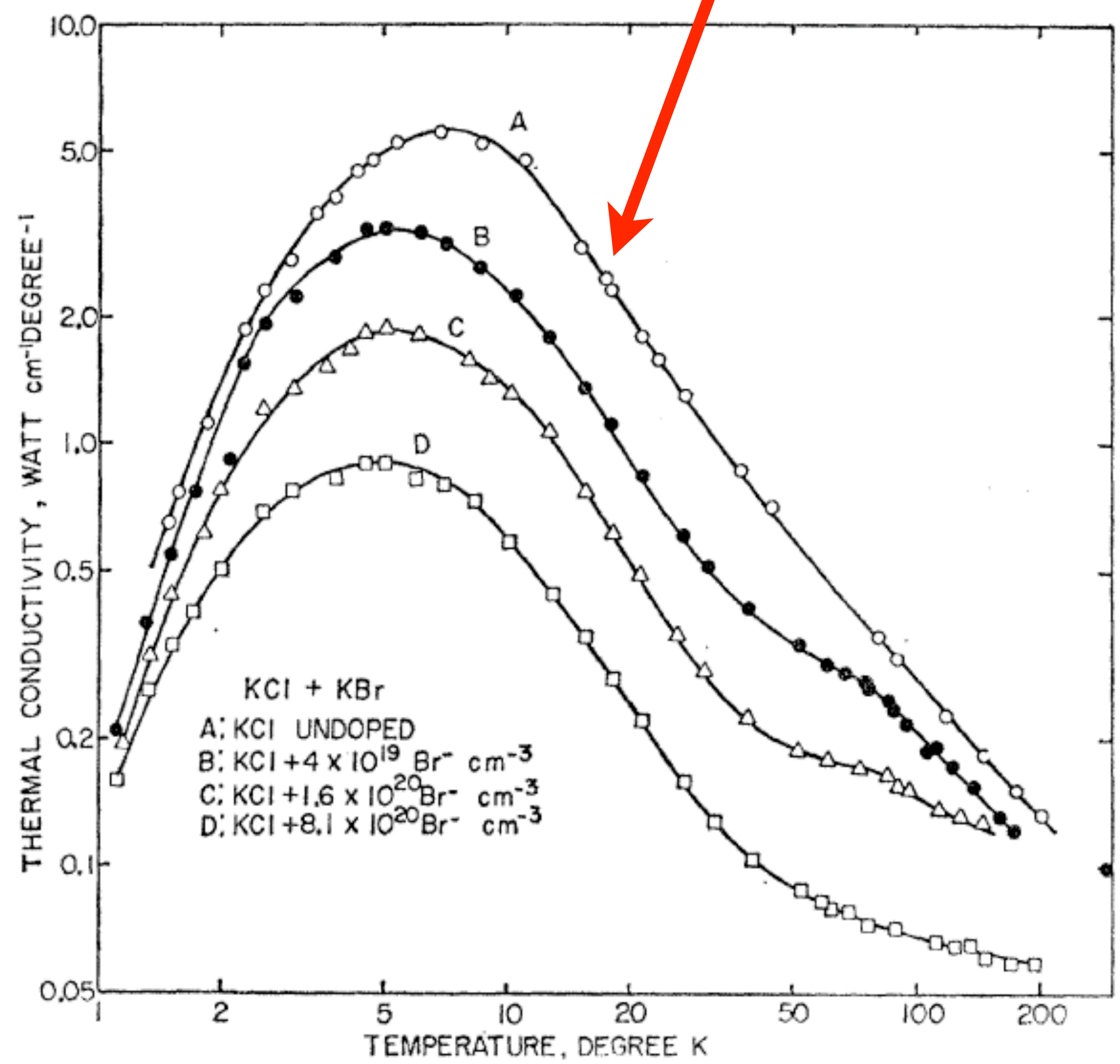
Rayleigh Scattering



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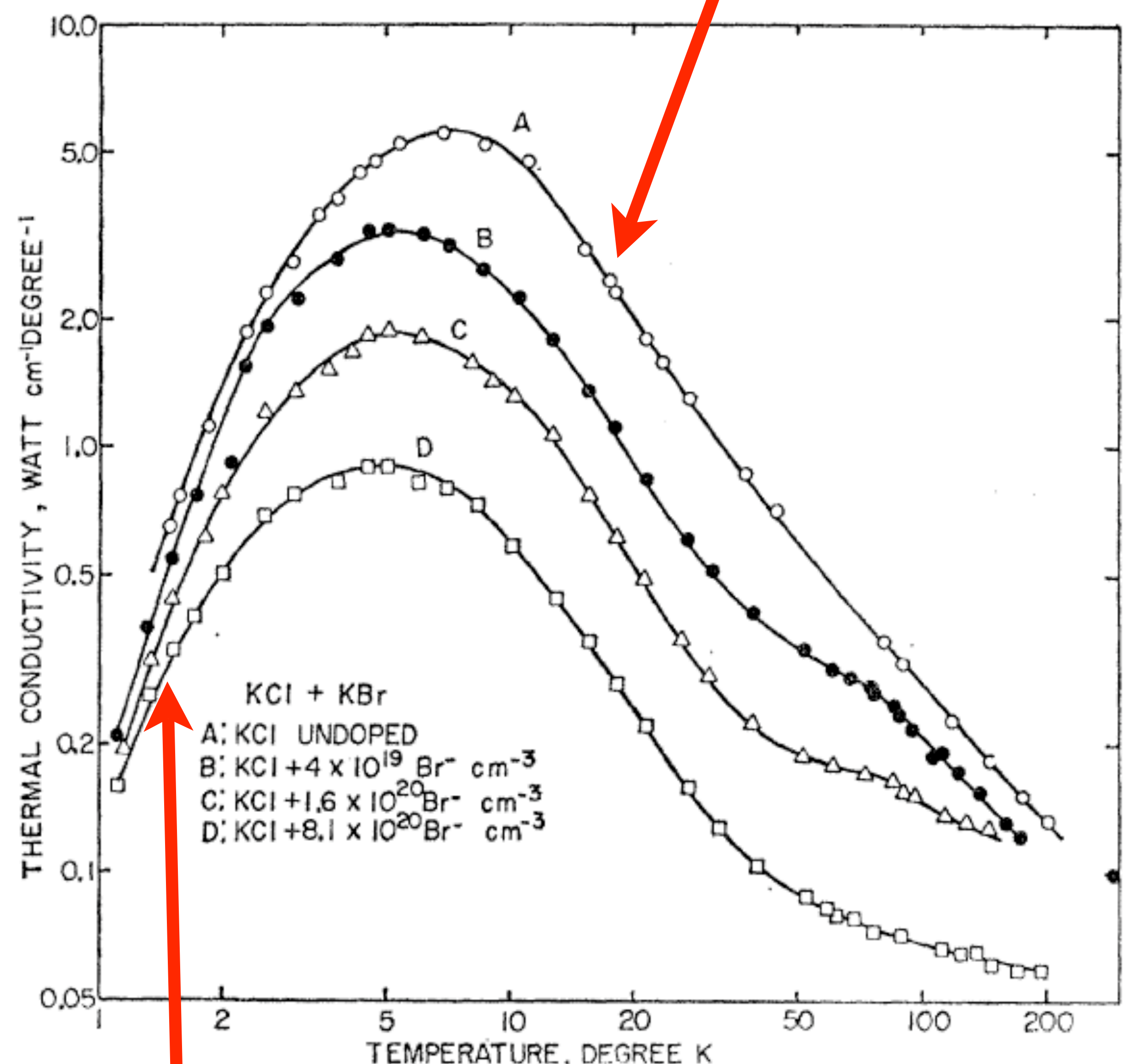
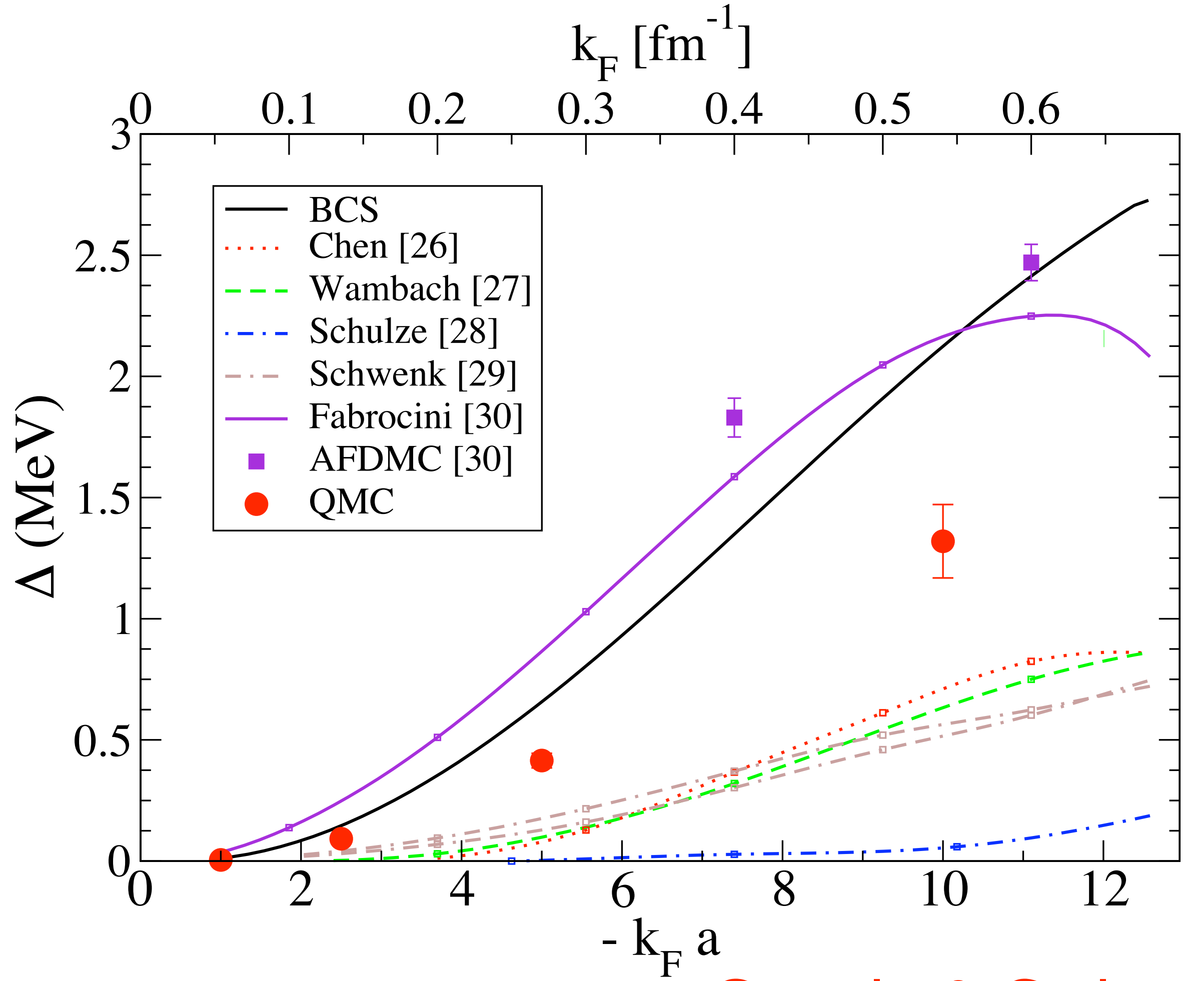


Boundary Scattering

Baumann & Pohl (1967)
Ziman (1960)

Heat Transport in the Inner Crust

- Neutron matter in the crust is superfluid.
- Neutron particle-hole excitations are gapped



Gezerlis & Carlson (2008)

Low energy degrees of freedom:

1. Electrons
2. Lattice Phonons (1 long. + 2 Trans.)
3. Superfluid Phonons

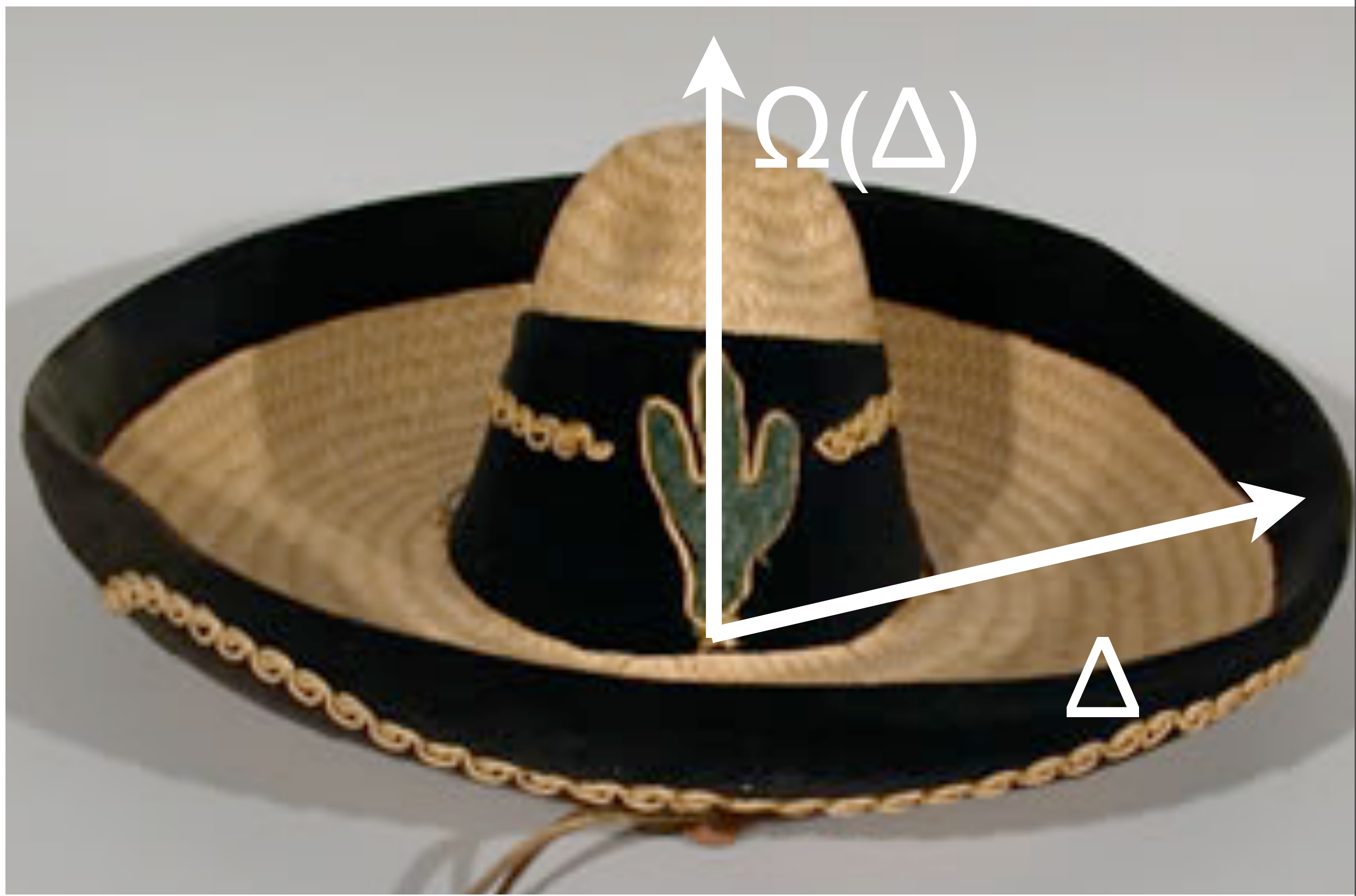
Pairing in neutron matter

Attractive interactions destabilize the Fermi surface:

$$H = \sum_{k,s=\uparrow,\downarrow} (\frac{k^2}{2m} - \mu)\ a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s}$$

$$\Delta = g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle$$

Cooper pairs leads to superfluidity



Energy gap for fermions:

$$E(p) = \sqrt{(\frac{p^2}{2M} - \mu)^2 + \Delta^2}$$

New collective mode:
Superfluid Phonon

$$\omega(k) = v_s\ k$$

Superfluidity in the Crust Enhances Heat Conduction:

$$\kappa_{\rm sPh} = 1.5{\times}10^{22} \left(\frac{T}{10^8 \text{ K}}\right)^3 \left(\frac{0.1}{v_s}\right)^2 \left(\frac{\lambda_{\rm sPh}}{\rm cm}\right) \frac{\rm erg}{\rm cm\ s\ K}$$

Conventional Wisdom: Electrons dominate conduction

At neutron drip and T=10^8 K $\kappa_{\rm e} \simeq 10^{18} \frac{\rm ergs}{\rm cm\ s\ K}$

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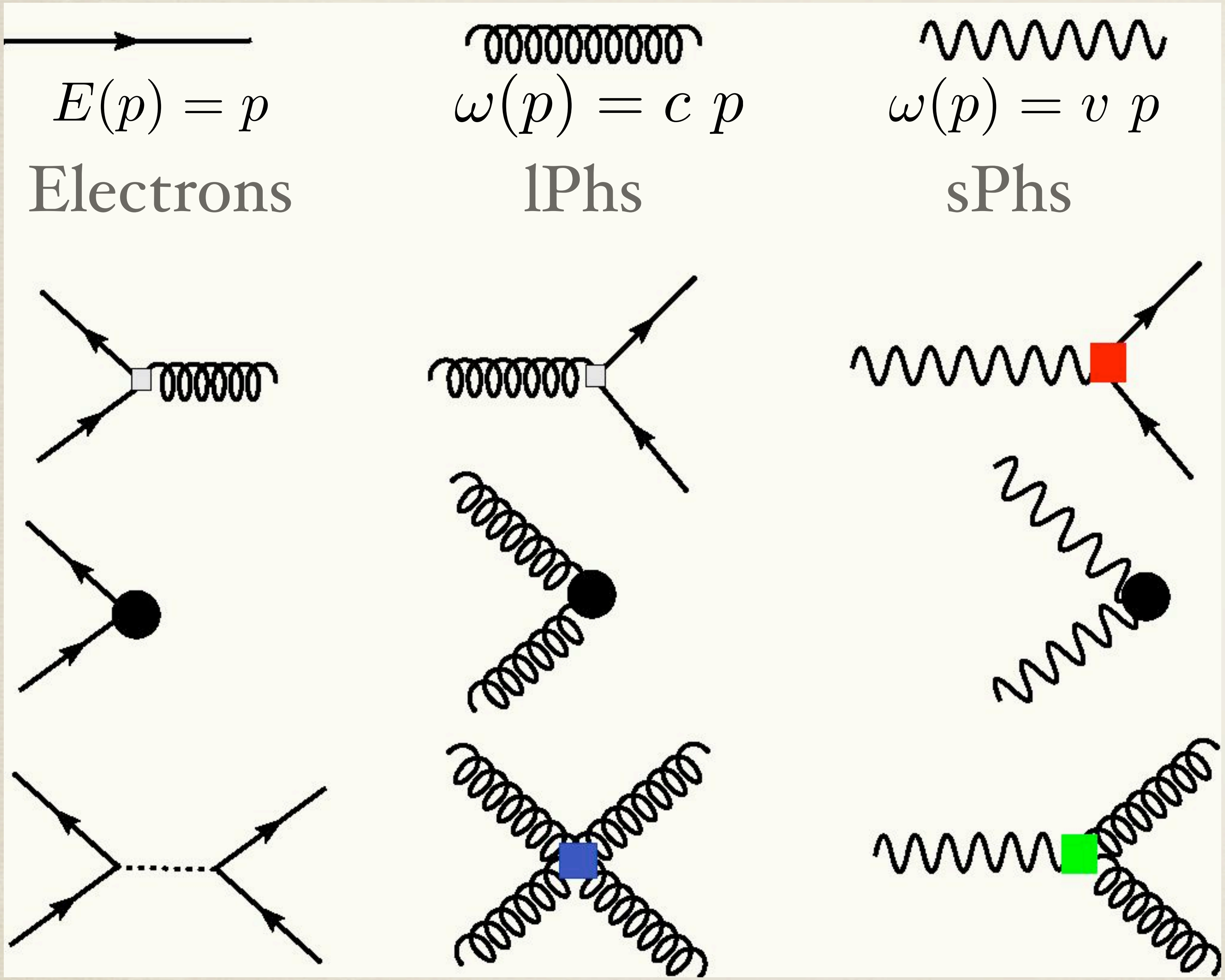
Microscopic Processes in the Crust



$$c \simeq 0.01 - 0.1$$
$$v \simeq 0.01 - 0.1$$

Electron-phonon processes

Impurity (Rayleigh) scattering

Multi electron and phonon processes

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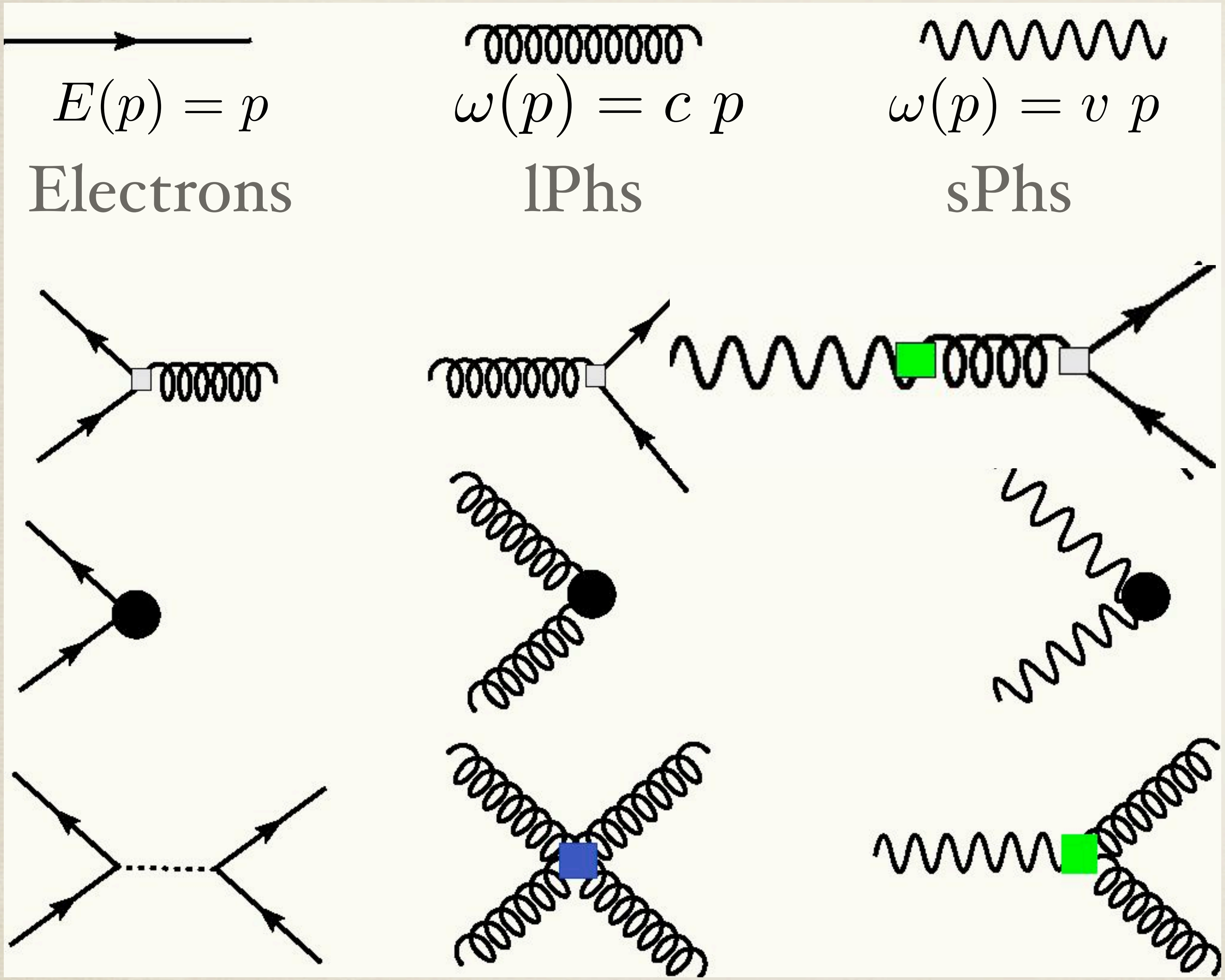

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Electron-phonon processes

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Multi electron and phonon processes

Elastic process

Rayleigh Scattering $\sigma_R = \pi\ r_0^2 \left(\frac{q^4 r_0^4}{1 + q^4 r_0^4} \right)$

r_o =Typical nuclear radii
q = sPh momentum

$q\ r_0 \simeq 10^{-3} \left(\frac{T}{10^7 \mathrm{K}} \right) \ll 1$

Scattering dominated by impurities: $\lambda_{\mathrm{R}} = \frac{1}{n_I\ \sigma_R} = \frac{v_s^4}{81\ \pi\ n_I\ r_0^6\ T^4}$

Very large mean free path!

$$\lambda_{\mathrm{Ray}} = 450\ \left(\frac{v_s}{0.1}\right)^4 \left(\frac{x}{10}\right)^3 \left(\frac{10\ \mathrm{fm}}{r_0}\right)^3\ T_7^{-4}\ \mathrm{cm}$$

If only impurity scattering is relevant: $\kappa_{\mathrm{sPh}}(T = 10^8 \mathrm{K}) \simeq 10^{21} \frac{\mathrm{ergs}}{\mathrm{cm\ s\ K}}$

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EFT}}^{\mathrm{sPh}} &= \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\,\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots \\
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kinetic terms

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self-coupling

$$\mathcal{L}_{\rm sPh-lPh} = g_{\rm mix}\,\partial_o\phi\,\partial_i\xi_i + \frac{1}{\Lambda^2}\,\partial_o\phi\,\partial_i\xi^i\partial_i\xi^i + \cdots$$

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lPh-sPh mixing

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kinetic terms — coupling to Fermions — self-coupling

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lPh-sPh mixing — sPh→2 lPh

Electron-Phonon Coupling

Fetter & Walecka

$$\mathcal{H}_{\rm el-Ion} = \int d^3x \int d^3y \, V(x-y) \, \psi^\dagger(x)\psi(x) \, \boxed{\Psi^\dagger(y)\Psi(y)}$$

$$V(x-y) = \frac{4\pi \, Z \, e^2}{q_{\rm TF}^2} \, \delta(x-y) \quad \text{for } q \ll q_{\rm TF}$$

$$\Psi^\dagger(y)\Psi(y) = n_{\rm Ion} + \delta\rho(y)$$

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Fluctuation in density due to displacement $\vec{d}(y)$

$$\delta\rho(y) = -n_{\mathrm{Ion}}\ \nabla\cdot\vec{d}(y) + \cdots$$

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Canonically normalized lattice phonon field

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Fetter & Walecka

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$$\vec{d}(y) = \frac{1}{\sqrt{n_{\mathrm{Ion}} \; AM}} \boxed{\vec{\xi}(x)}$$

Canonically normalized lattice phonon field

$$\mathcal{H}_{\mathrm{el-lPh}} = \boxed{\frac{4\pi Z e^2}{q_{\mathrm{TF}}^2} \; \frac{n_{\mathrm{Ion}}}{\sqrt{n_{\mathrm{Ion}} AM}}} \int d^3x \; \psi^\dagger(x)\psi(x) \; \nabla \cdot \vec{\xi}(x)$$

$$\mathcal{H}_{\mathrm{el-lPh}} = \boxed{\frac{1}{f_{\mathrm{el-ph}}}} \int d^3x \; \psi^\dagger(x)\psi(x) \; \nabla \cdot \vec{\xi}(x)$$

Neutron-IPh Interaction

$$\mathcal{H}_{\mathrm{n-Ion}} = \int \; d^3x \int d^3y \; V_{\mathrm{n-A}}(x-y) \; \psi_n^\dagger(x)\psi_n(x) \; \Psi^\dagger(y)\Psi(y)$$

$$V_{\mathrm{n-A}} = \frac{2\pi \; a_{\mathrm{n-A}}}{A \; M} \; \delta^3(x)$$

Low-energy neutron-nucleus potential (Fermi Potential)

Neutron-IPh Interaction

$$\mathcal{H}_{\rm n-Ion} = \int d^3x \int d^3y \; V_{\rm n-A}(x-y) \; \psi_n^\dagger(x)\psi_n(x) \; \Psi^\dagger(y)\Psi(y)$$

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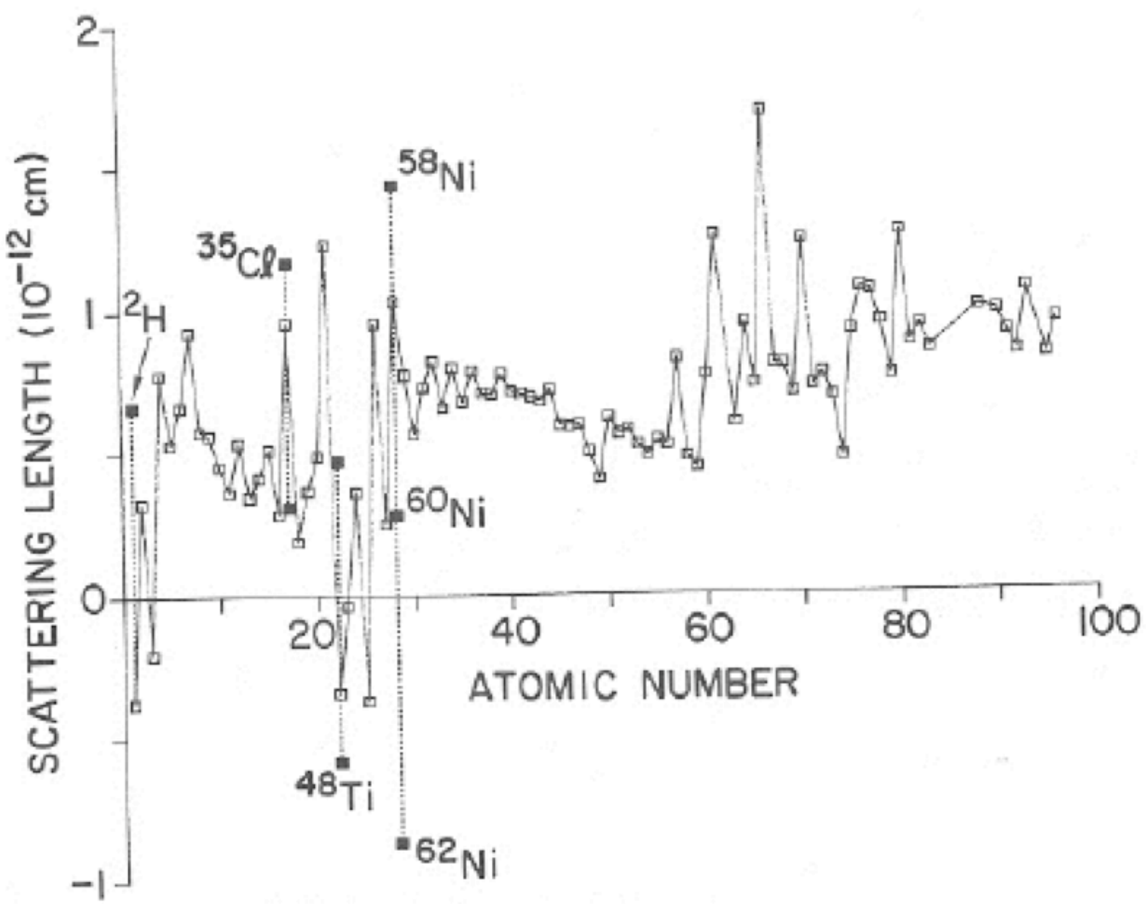

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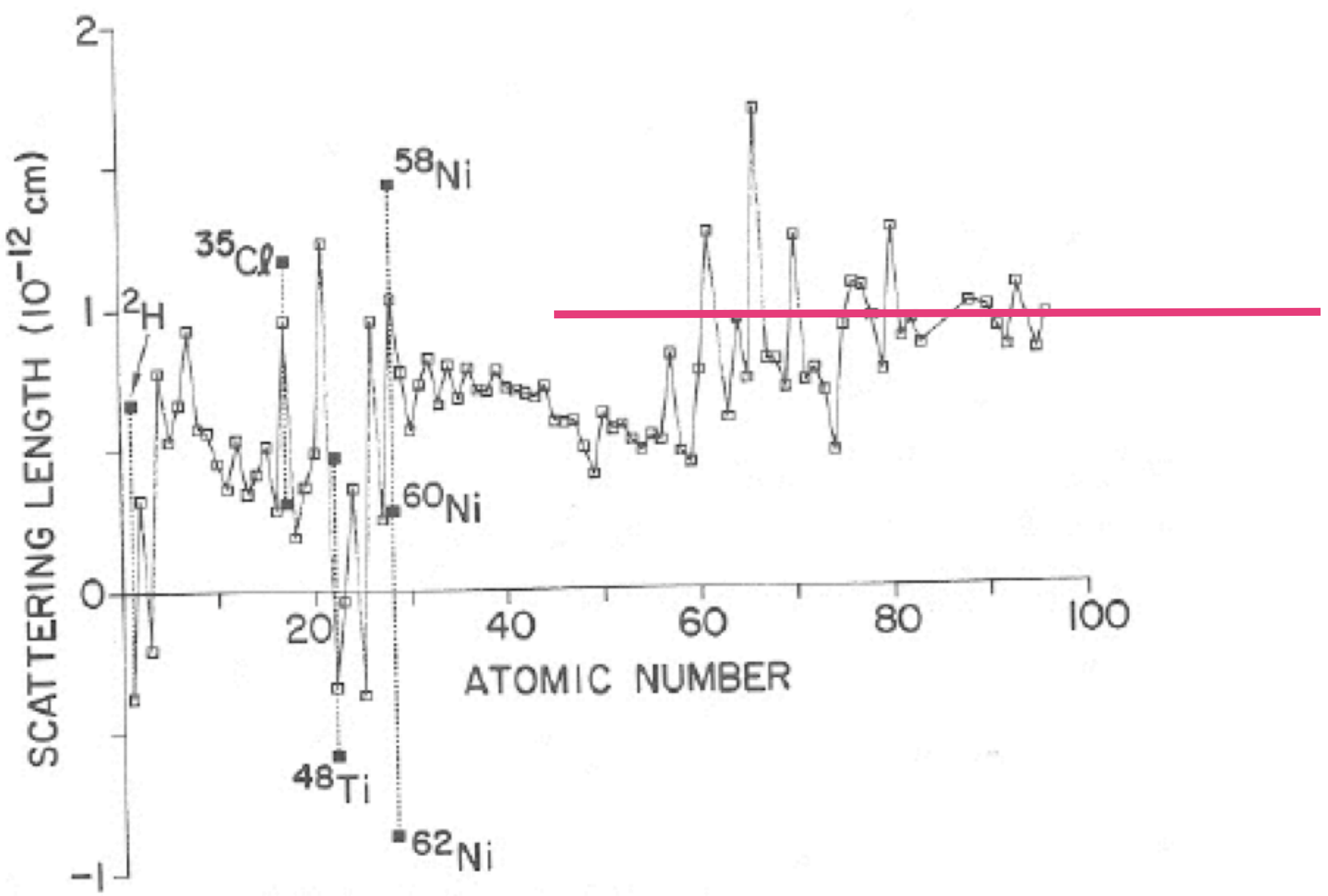
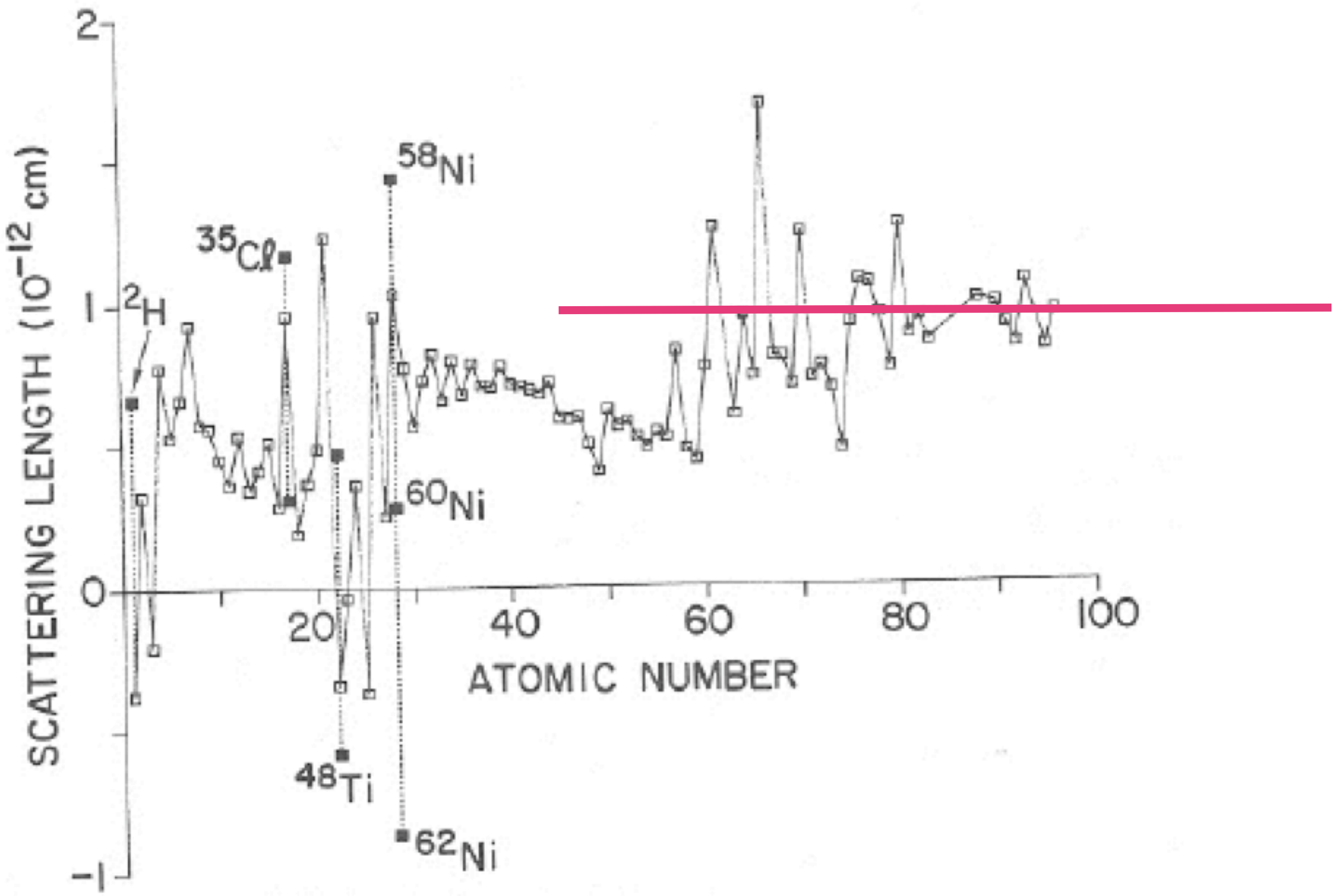


Neutron-lPh Interaction

$$\mathcal{H}_{\rm n-Ion} = \int d^3x \int d^3y \; V_{\rm n-A}(x-y) \; \psi_n^\dagger(x)\psi_n(x) \; \Psi^\dagger(y)\Psi(y)$$

$$V_{\rm n-A} = \frac{2\pi \; a_{\rm n-A}}{A \; M} \; \delta^3(x)$$

Low-energy neutron-nucleus potential (Fermi Potential)



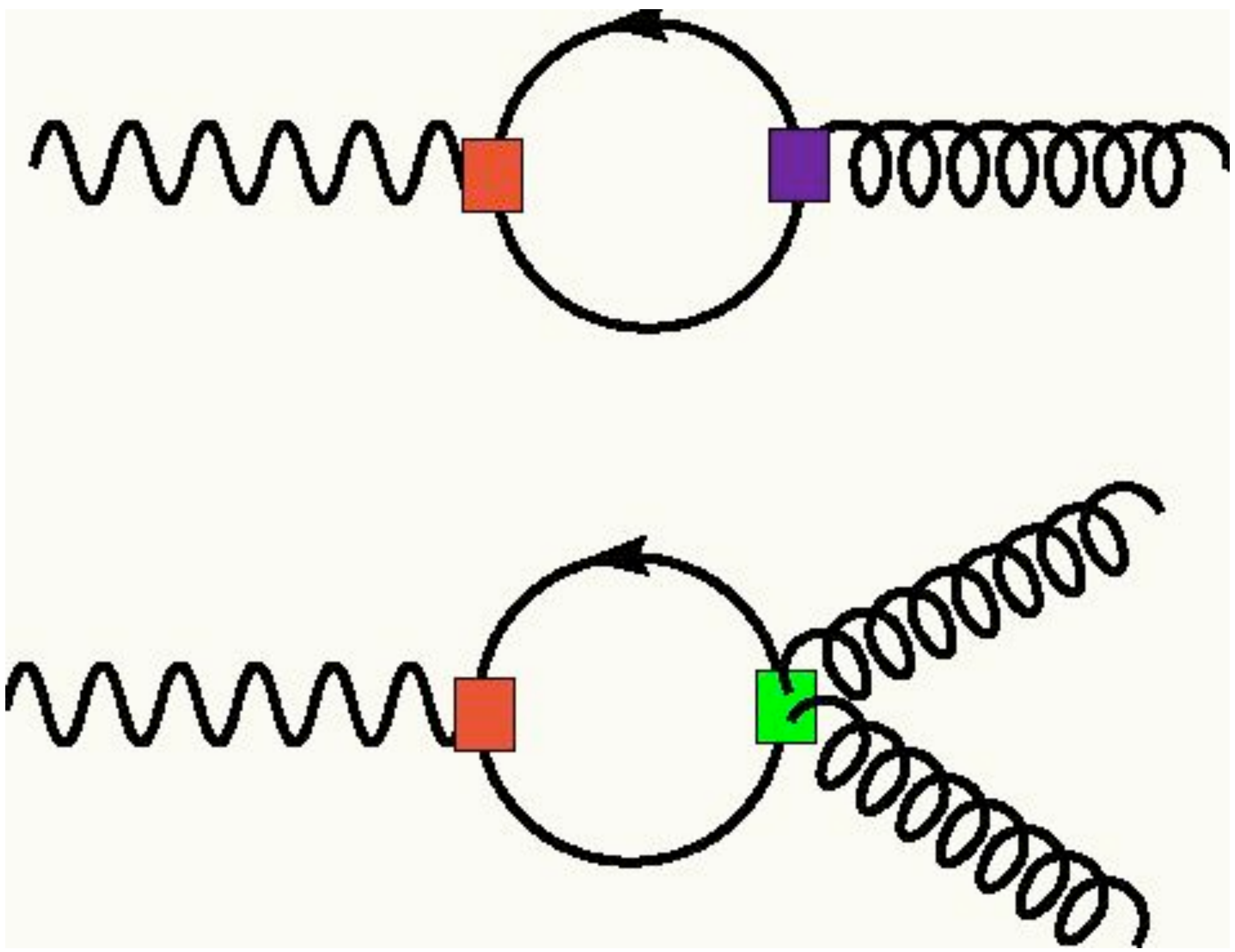
$$\mathcal{H}_{\rm n-lPh} = \frac{1}{f_{\rm n-lPh}} \int d^3x \; \psi_n^\dagger(x)\psi_n(x) \; \partial_i\xi_i(x)$$

$$\frac{1}{f_{\rm n-lPh}} = -2\pi a_{\rm n-A} \sqrt{\frac{n_{\rm Ion}}{A \; M^3}}$$

sPh-lPh Interactions

$$\mathcal{L}_{\mathrm{sPh-lPh}} = g_{\mathrm{mix}}\ \partial_o\phi\ \partial_i\xi_i + \frac{g_{\mathrm{mix}}}{\Lambda^2}\ \partial_o\phi\ \partial_i\xi^i\partial_i\xi^i + \cdots$$

"Integrate-out" neutron and ion degree of freedom



$$g_{\mathrm{mix}} = 2a_{\mathrm{n-Ion}}\ \sqrt{\frac{n_{\mathrm{Ion}}\ k_{\mathrm{Fn}}}{A\ M^2}}$$

$$\Lambda^2 = \sqrt{n_{\mathrm{Ion}}\ A\ M}$$

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$$\mathcal{L}_{\rm sPh-lPh} = g_{\rm mix}\ \partial_o\phi\ \partial_i\xi_i + \frac{g_{\rm mix}}{\Lambda^2}\ \partial_o\phi\ \partial_i\xi^i\partial_i\xi^i + \cdots$$

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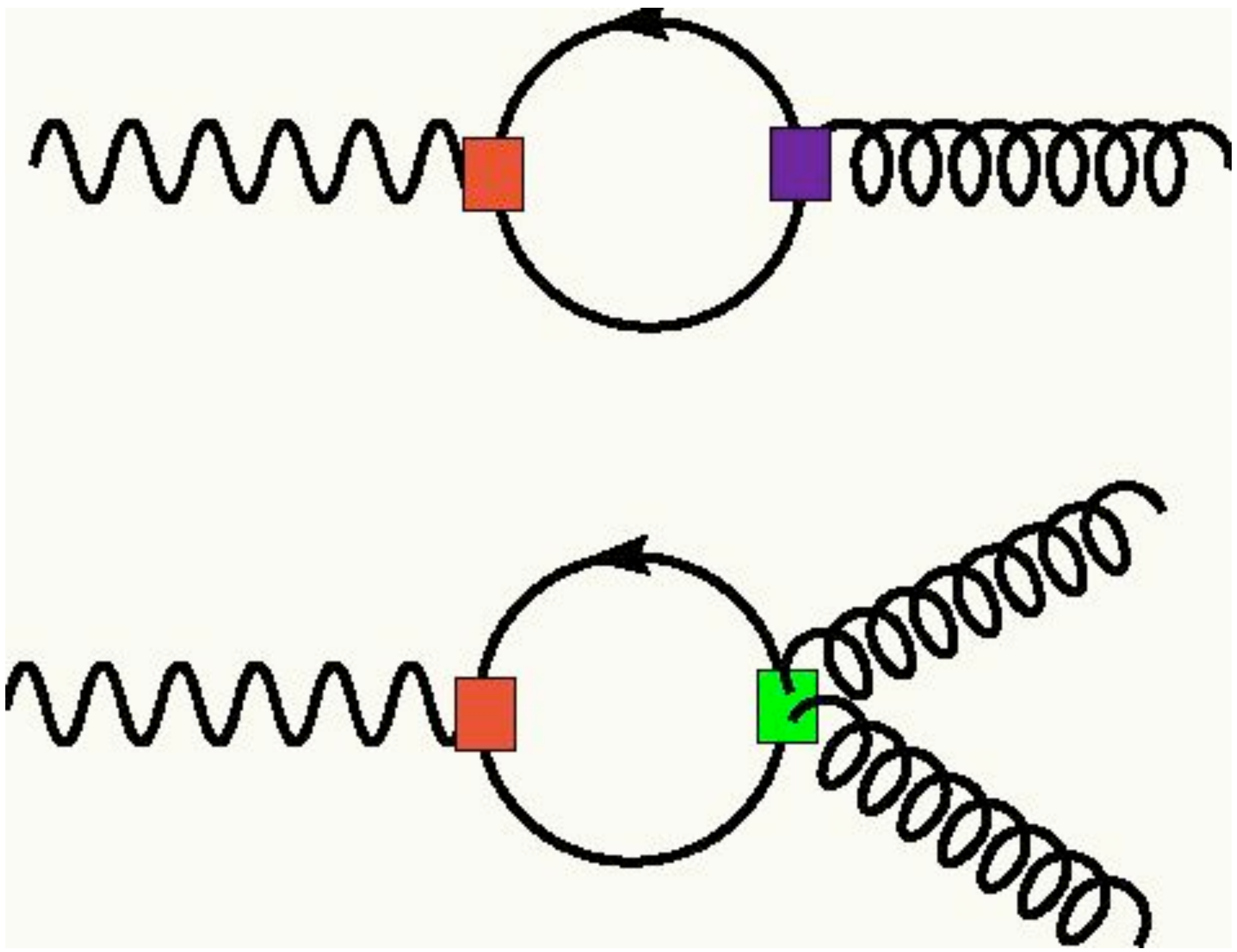

$$g_{\rm mix} = 2a_{\rm n-Ion}\ \sqrt{\frac{n_{\rm Ion}\ k_{\rm Fn}}{A\ M^2}}$$
$$\Lambda^2 = \sqrt{n_{\rm Ion}\ A\ M}$$

In the neutron star crust:

$$g_{\rm mix} \simeq 10^{-3}$$
$$\Lambda \simeq 50\ {\rm MeV}$$

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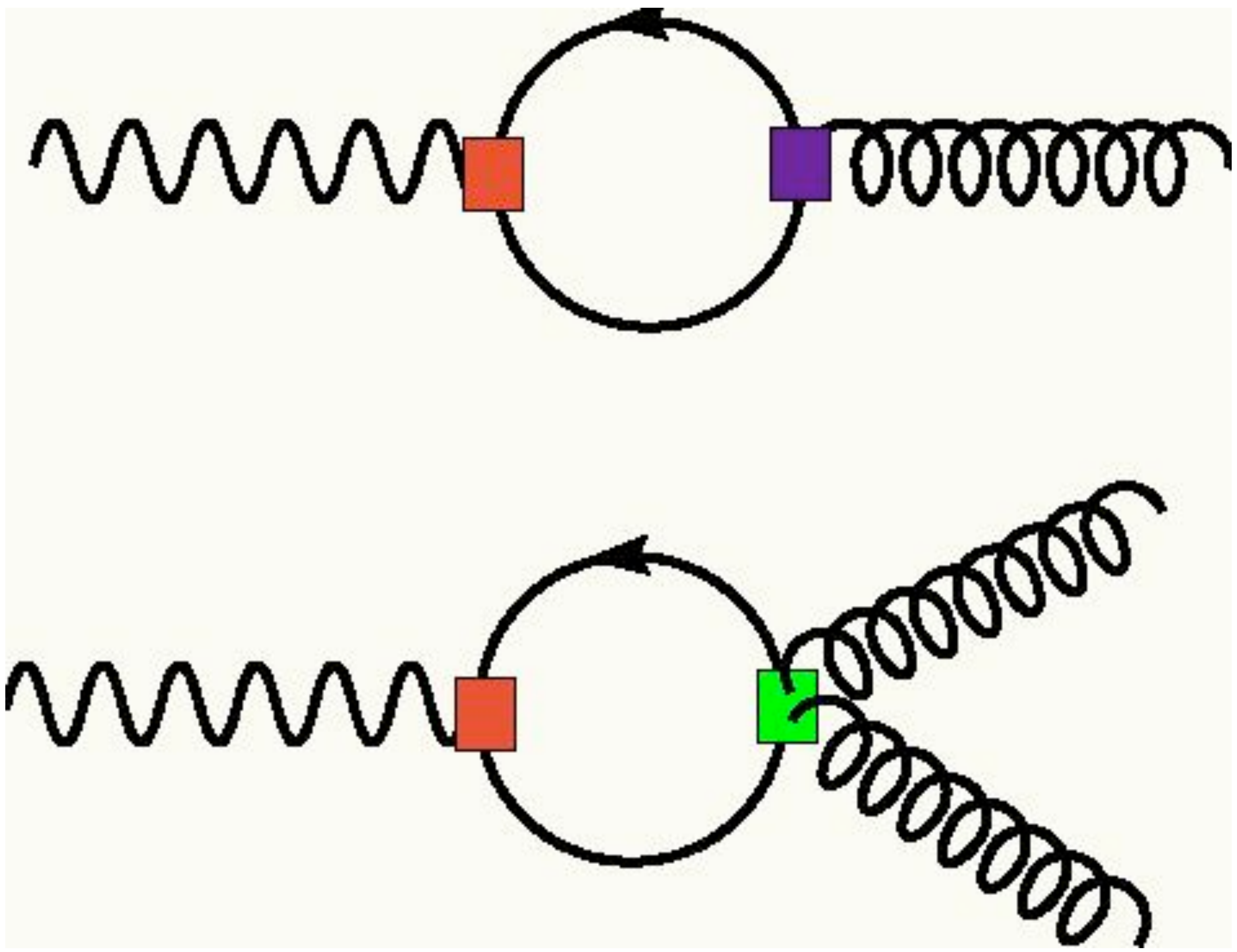

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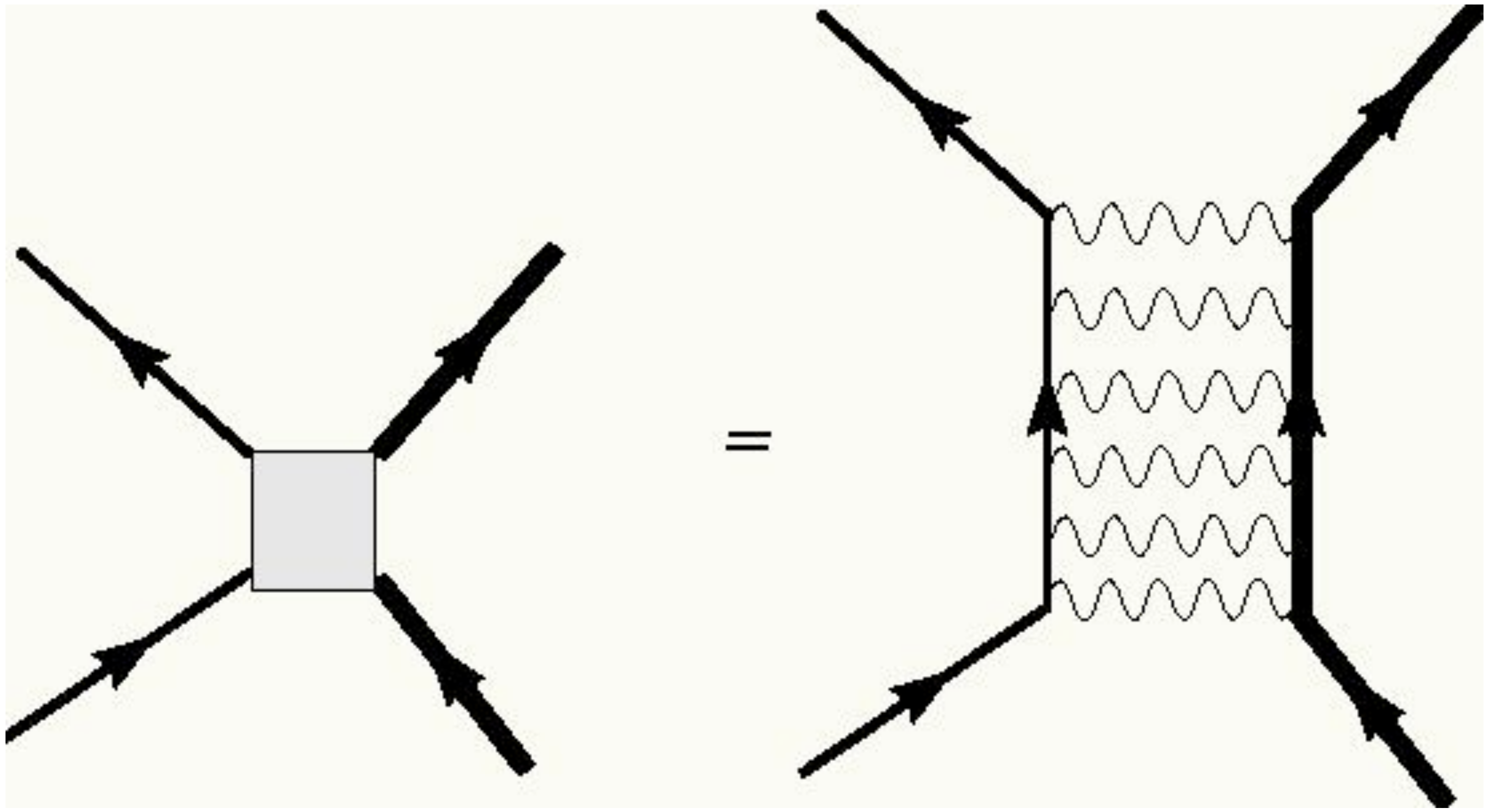
In the neutron star crust:

$$g_{\rm mix} \simeq 10^{-3}$$

$$\Lambda \simeq 50\ {\rm MeV}$$

This is a systematic expansion (EFT)

Neutron-Nucleus Interaction



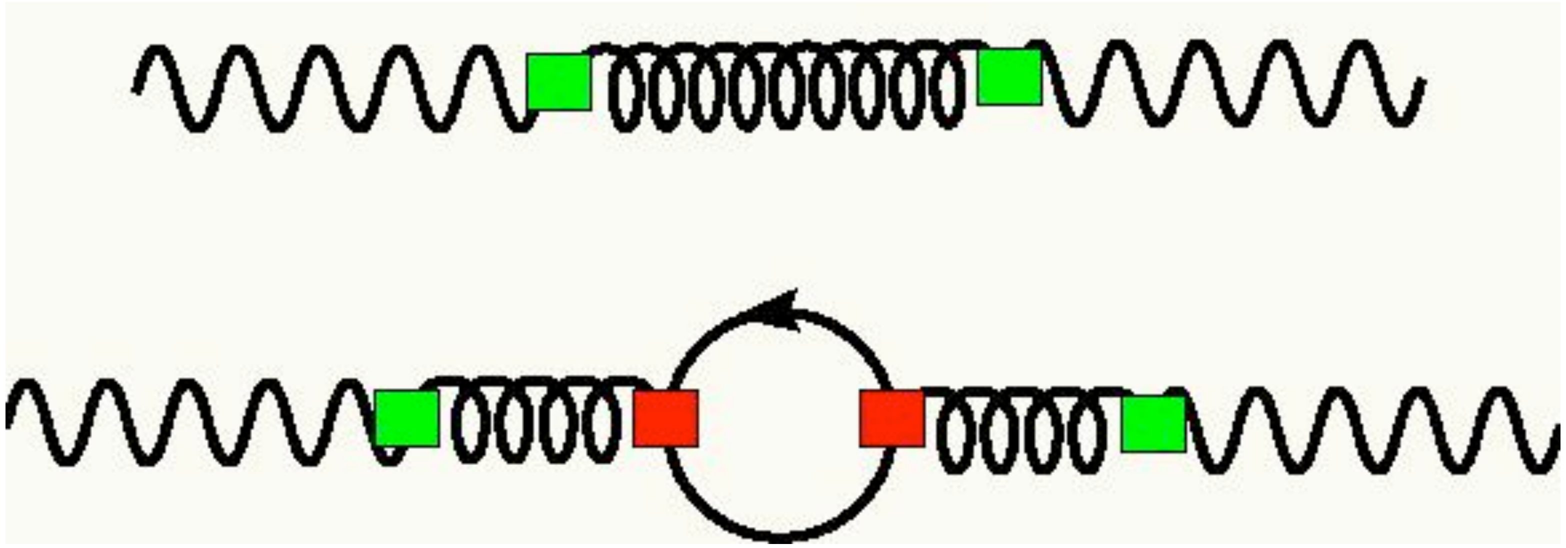
$$V_{\rm eff} \simeq \frac{2\pi}{m}\ a_{\rm nI}$$

$$V_{\rm eff} \simeq \frac{2\pi}{m}\ \frac{1}{\frac{1}{a} - \frac{1}{2} r\ k^2}$$

In medium Pauli blocking and effective range corrections can suppress the interaction.

$$V_{\rm eff} \simeq \frac{2\pi}{m}\ \frac{1}{\frac{1}{a} - A\ k_F\ - B\ r\ k_F^2}$$

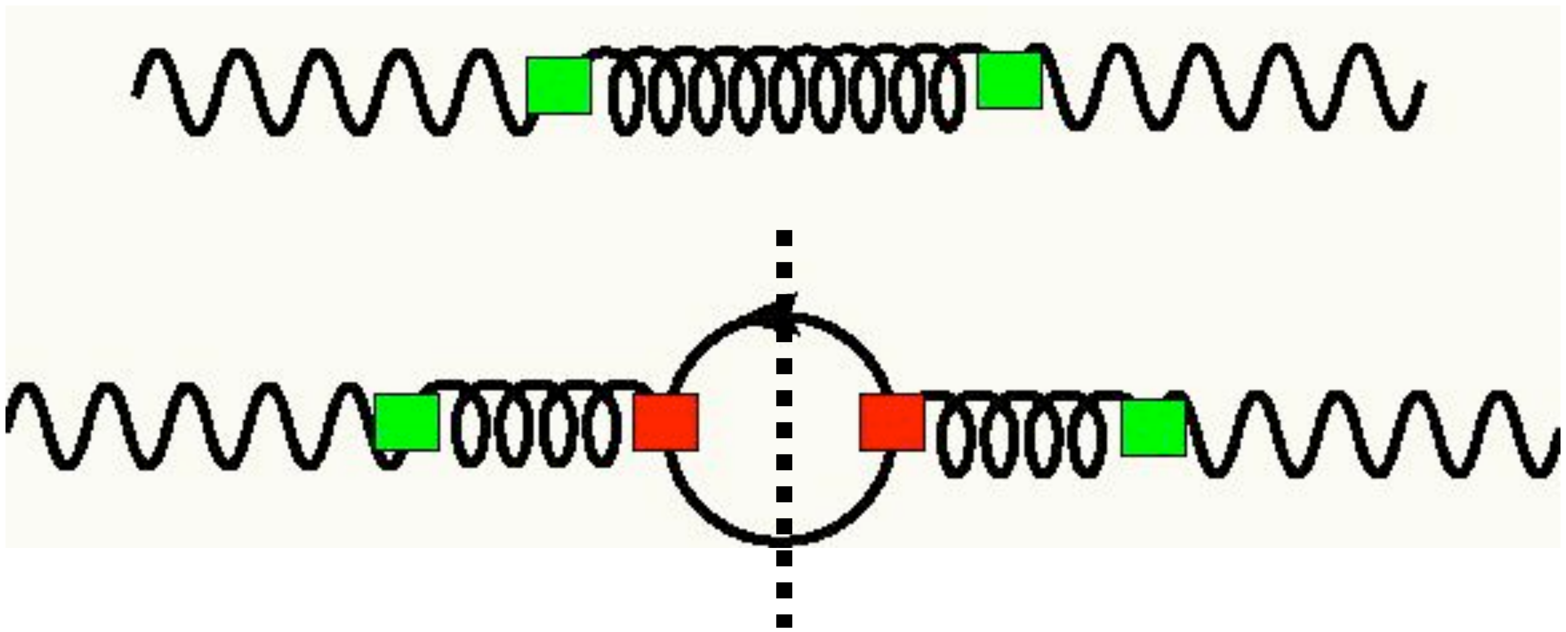
Mixing and Dissipation



Mixing leads to oscillations

Dissipation of lPh leads to dissipation of sPh

Mixing and Dissipation



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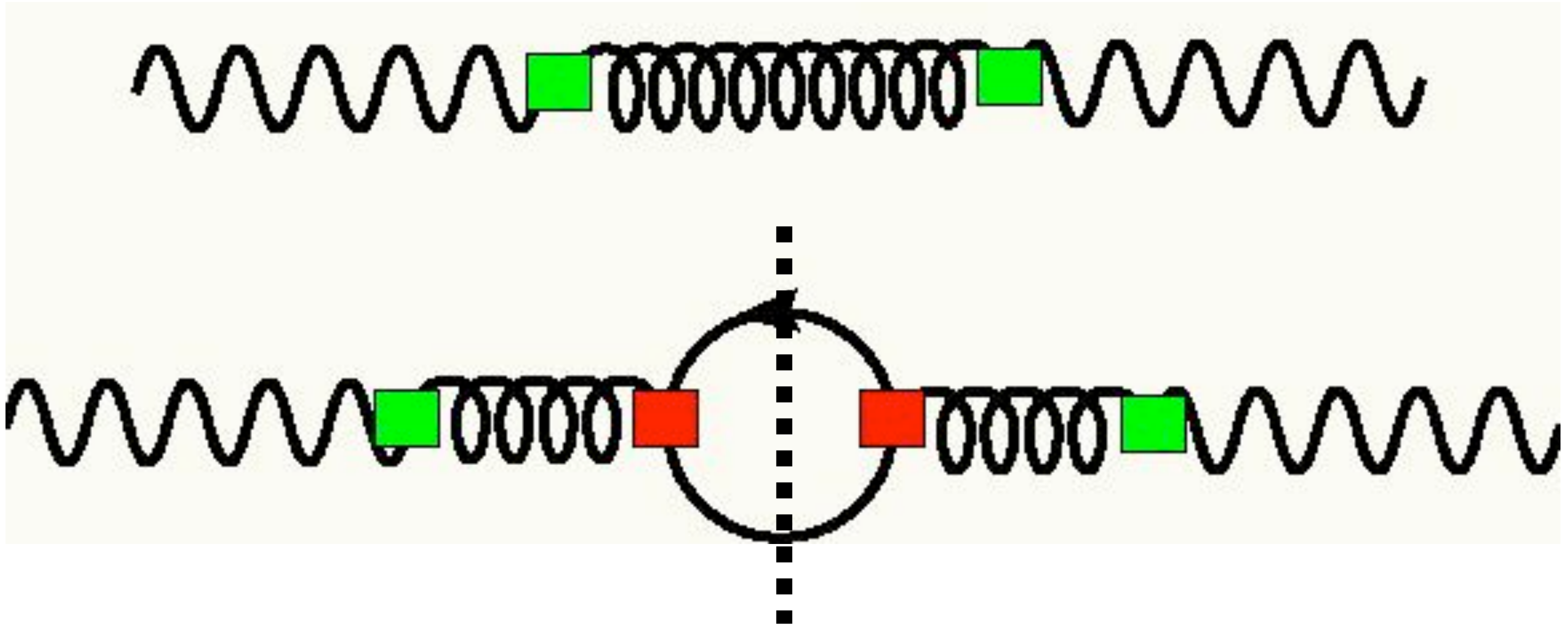


Mixing leads to oscillations

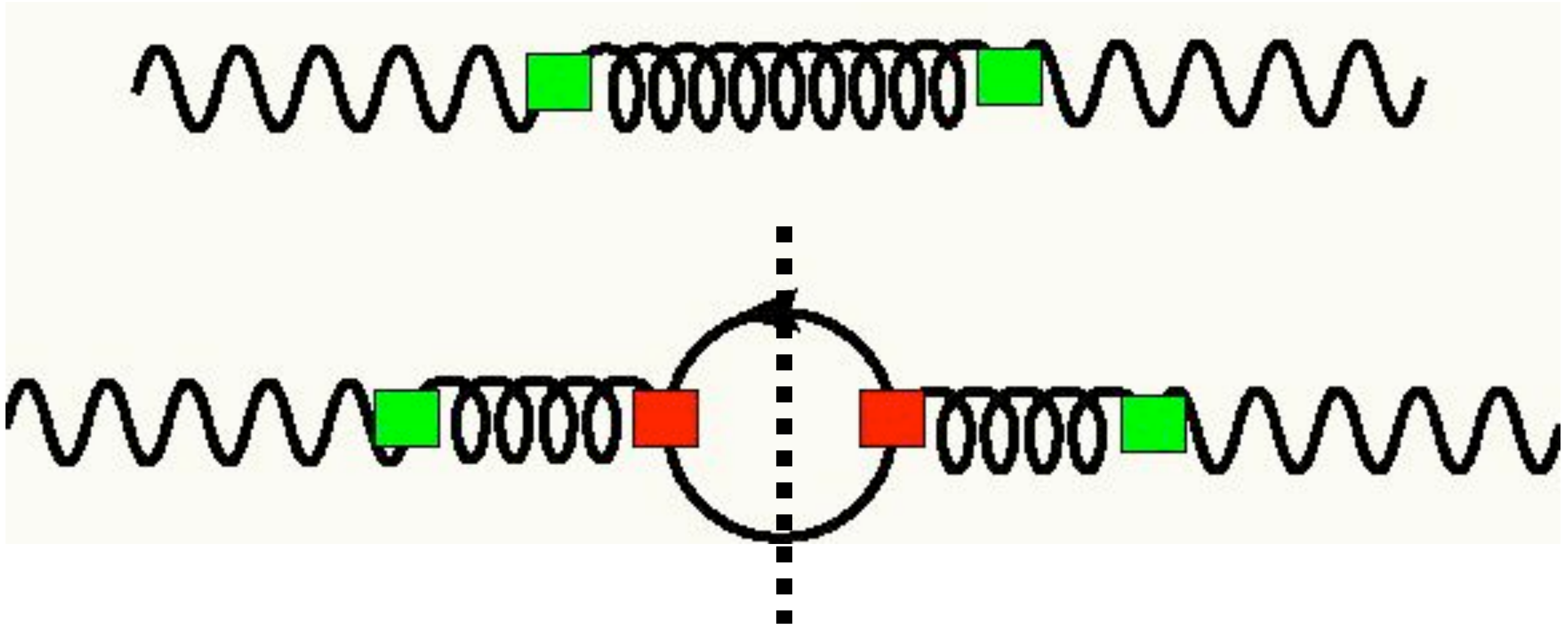
Dissipation of lPh leads to dissipation of sPh

$$\lambda_{\rm abs}(\omega) = \frac{v_s^2}{g_{\rm mix}^2} \, \frac{1 + (1-\alpha^2)^2 \, (\omega \, \tau_{\rm lPh})^2}{\alpha \, (\omega \, \tau_{\rm lPh})^2} \, \lambda_{\rm lPh}(\omega)$$

sPh mean free path

lPh mean free path

Mixing and Dissipation



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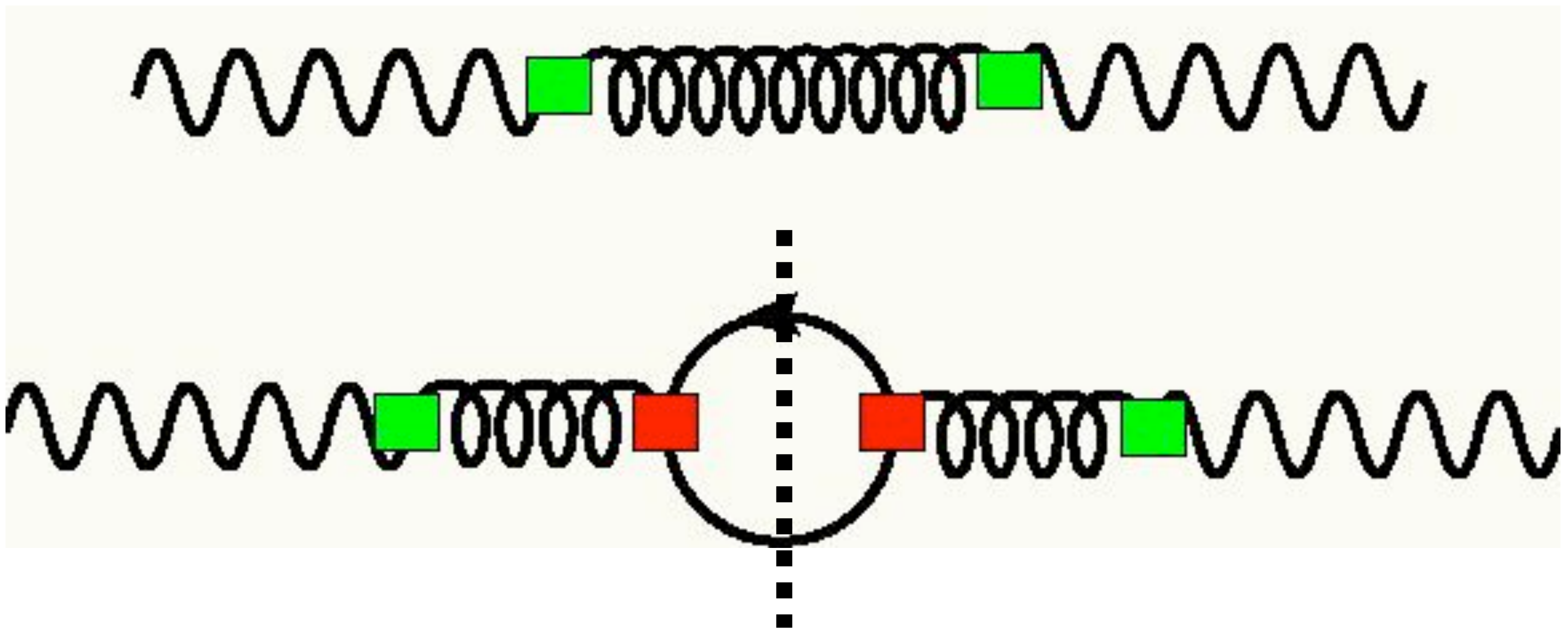
sPh mean free path

$$\alpha = \frac{c_s}{v_s}$$

$$\lambda_{\rm lPh} = c_s\ \tau_{\rm lPh}$$

lPh mean free path

Mixing and Dissipation



Mixing leads to oscillations

Dissipation of lPh leads to dissipation of sPh

$$\lambda_{\rm abs}(\omega) = \frac{v_s^2}{g_{\rm mix}^2} \frac{1+(1-\alpha^2)^2\ (\omega\ \tau_{\rm lPh})^2}{\alpha\ (\omega\ \tau_{\rm lPh})^2}\ \lambda_{\rm lPh}(\omega)$$

sPh mean free path

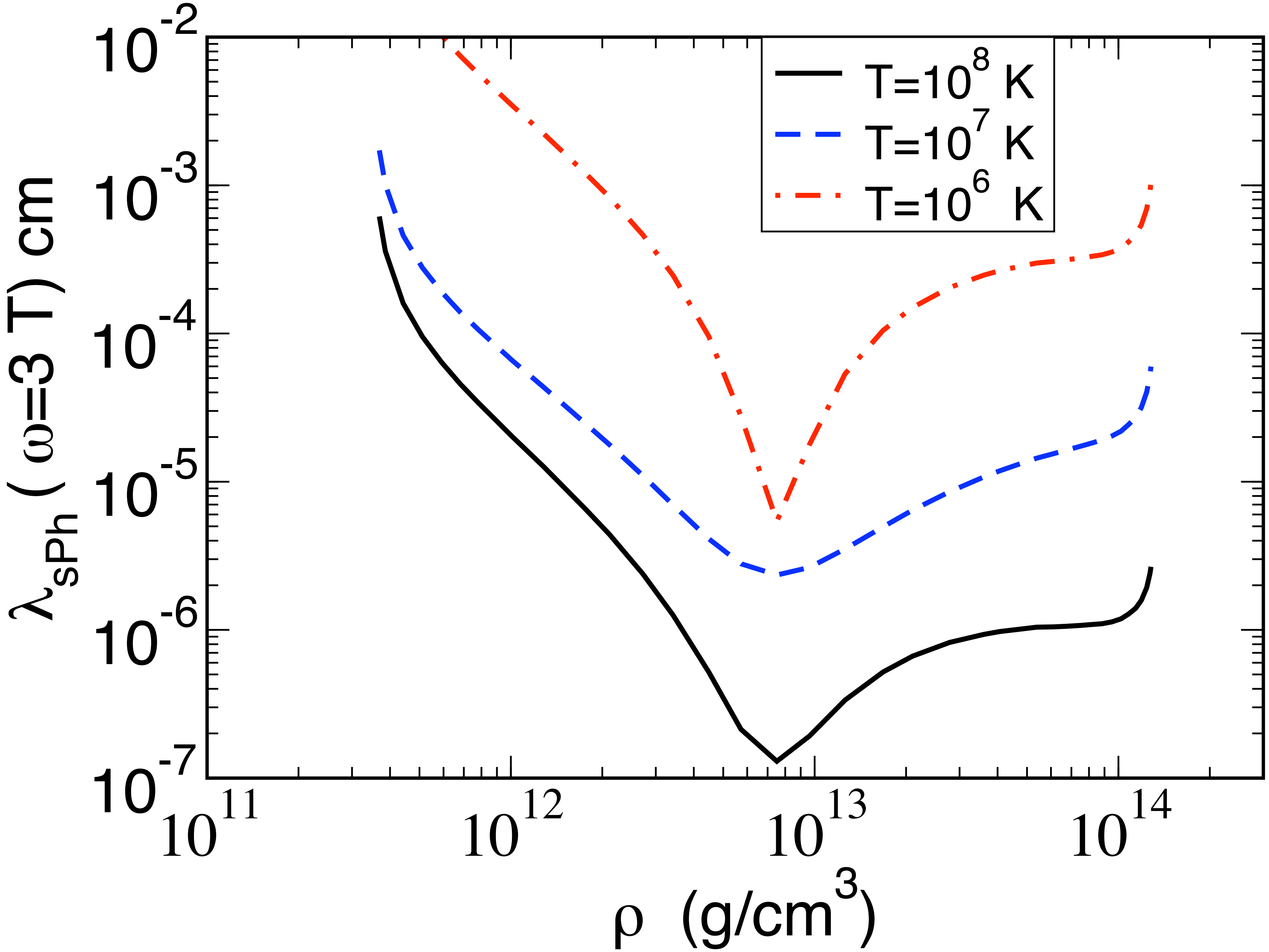
lPh mean free path

$$\alpha = \frac{c_s}{v_s}$$

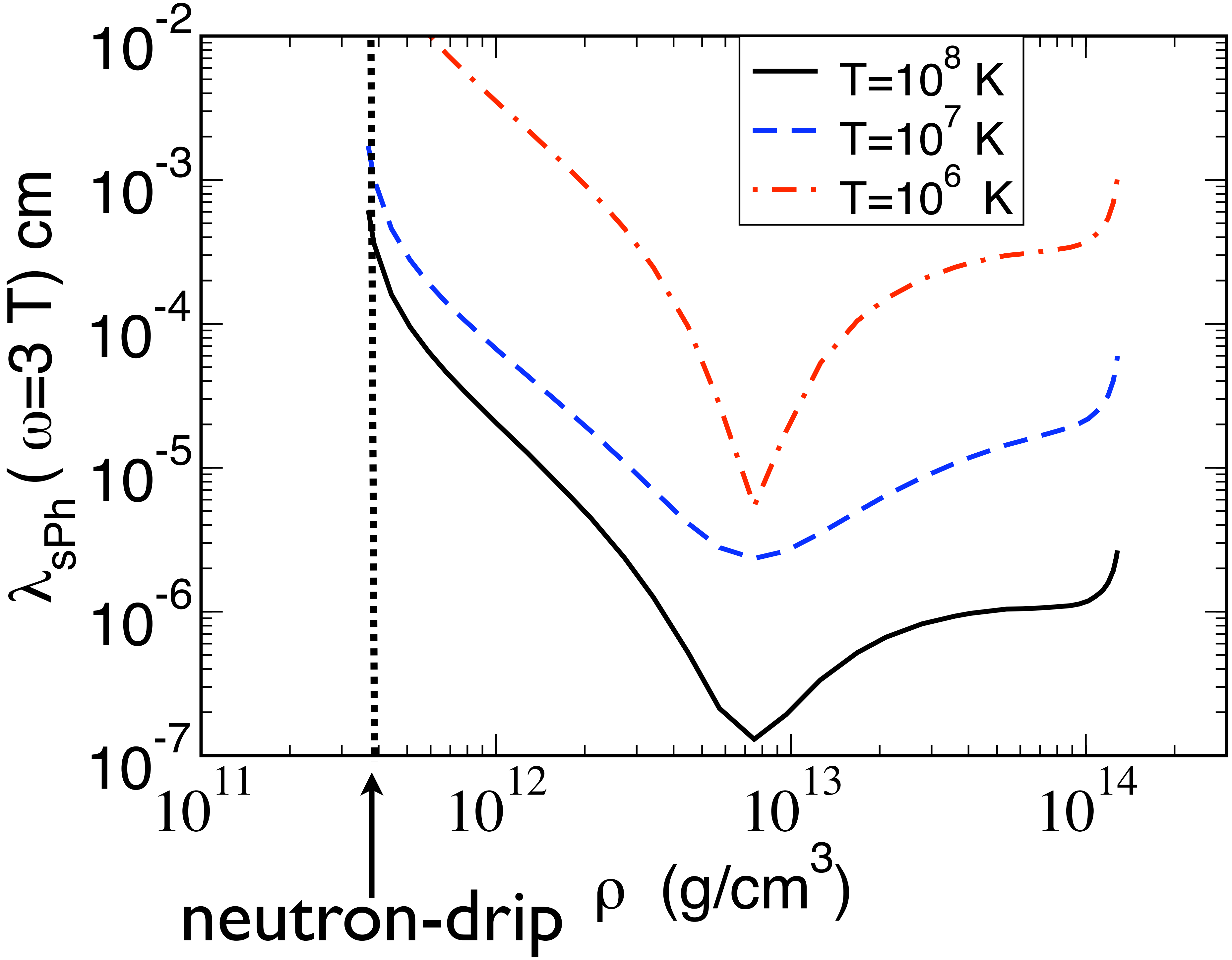
$$\lambda_{\rm lPh} = c_s\ \tau_{\rm lPh}$$

Away from resonance $\lambda_{\rm sPh} \simeq 10^5\ \lambda_{\rm lPh}$

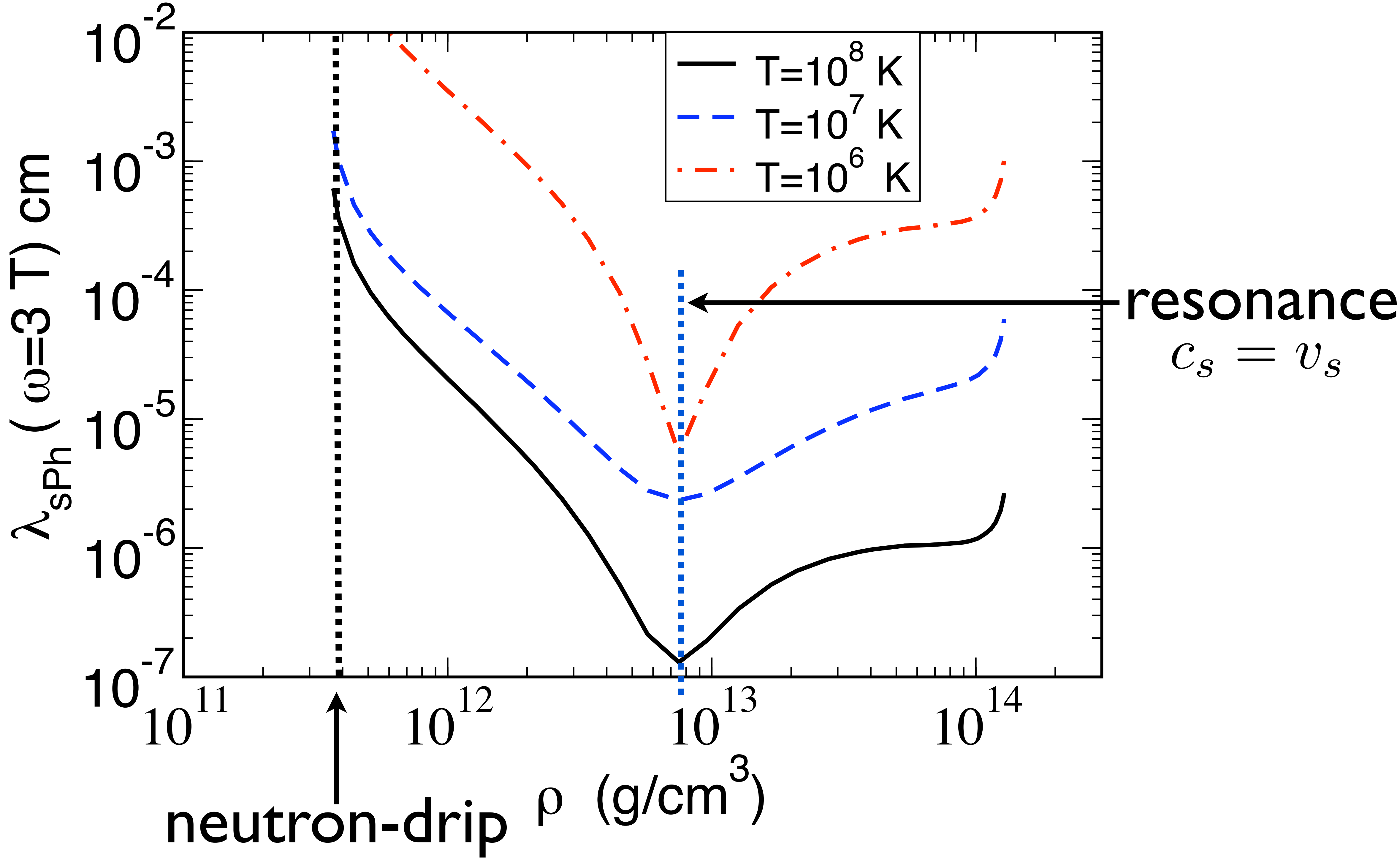
Superfluid Phonon Mean Free Path



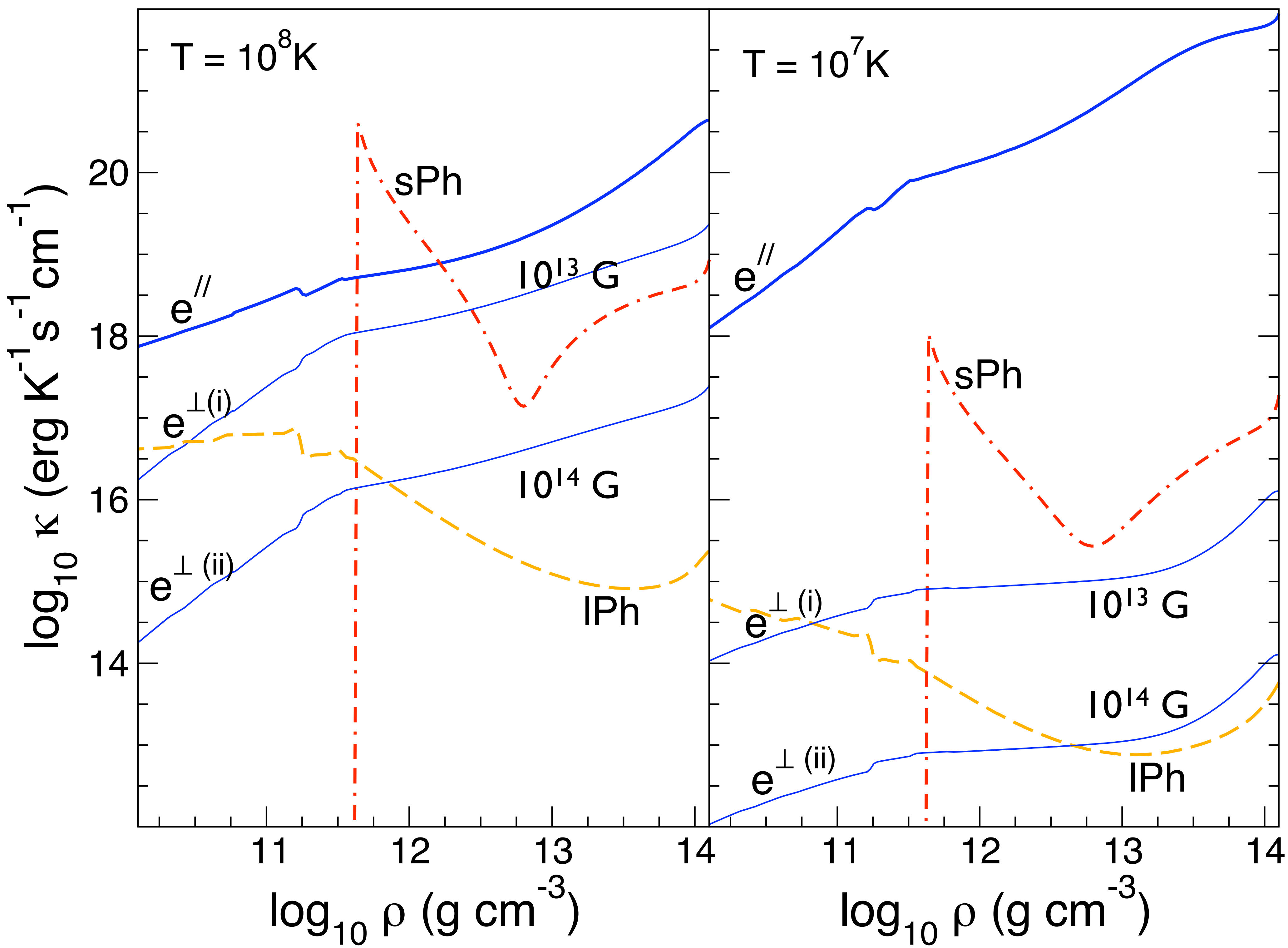
Superfluid Phonon Mean Free Path



Superfluid Phonon Mean Free Path



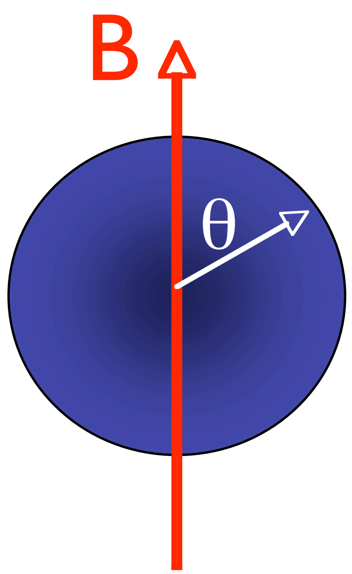
Thermal Conductivity

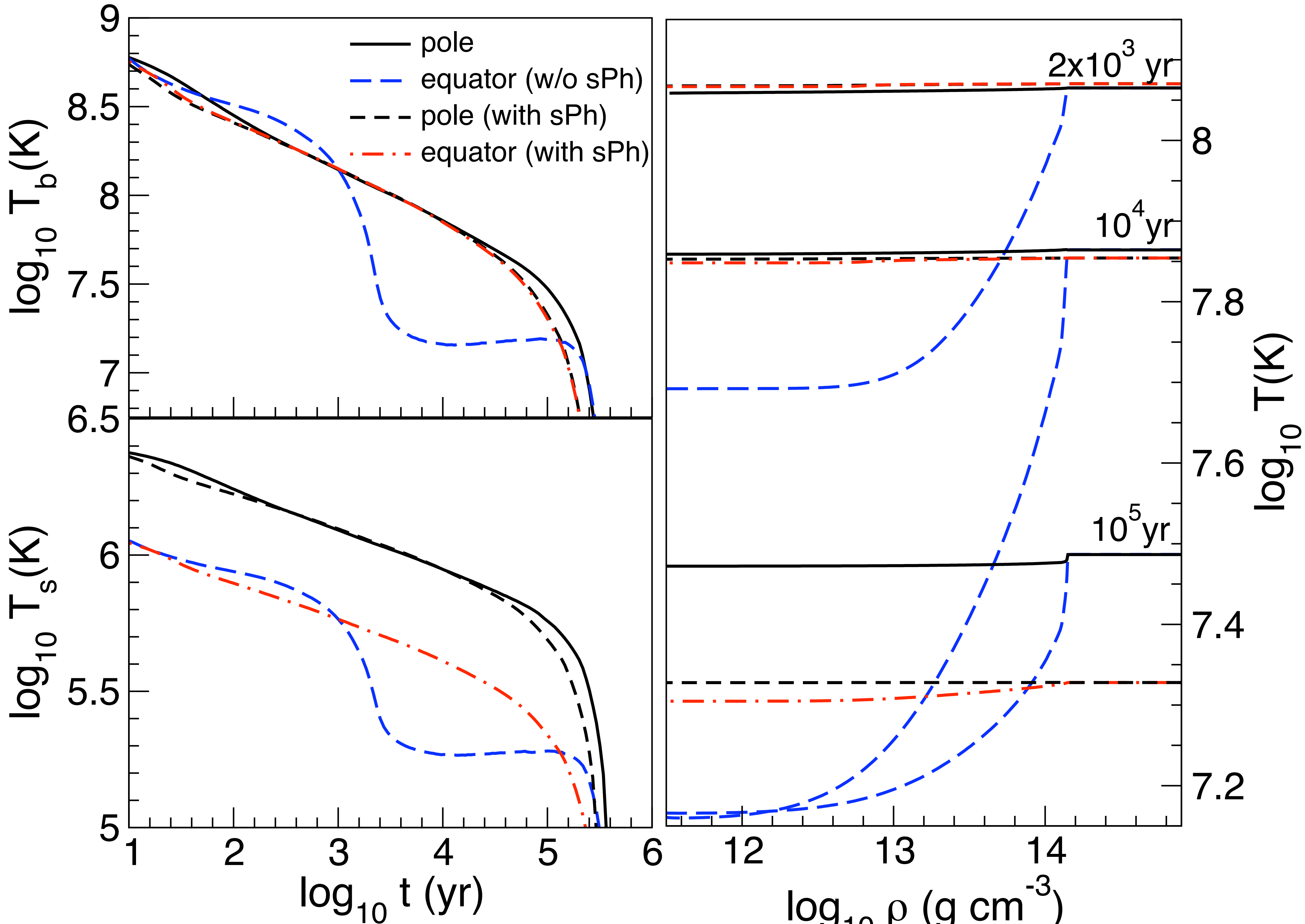


Consequences for Magnetar Cooling

Temperature anisotropy due anisotropic conduction.

$$T^4(\theta) = T_{\rm eff}^4 \left(\cos^2\theta + \frac{\kappa^\perp}{\kappa^\parallel}\sin^2\theta\right)$$

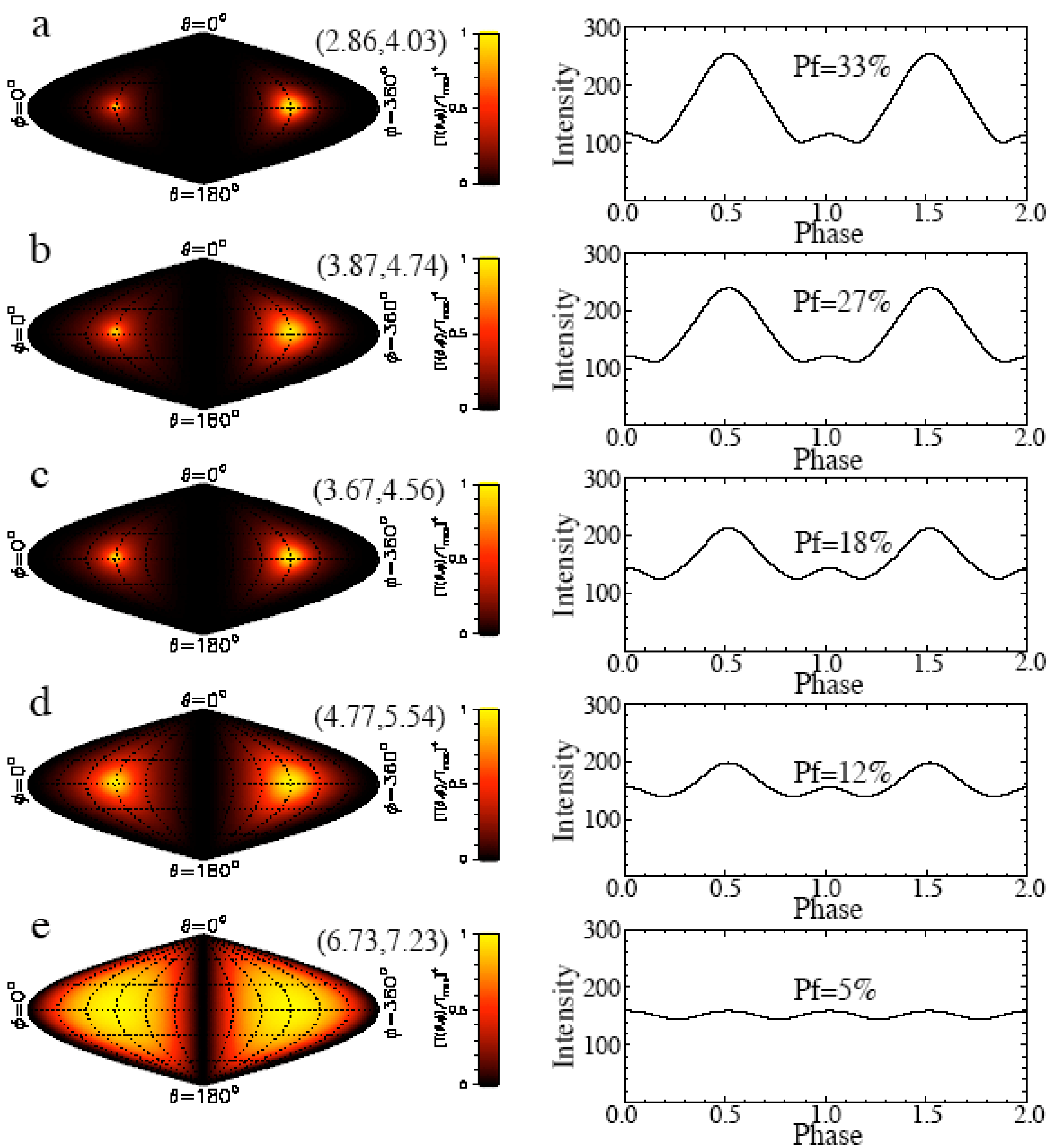


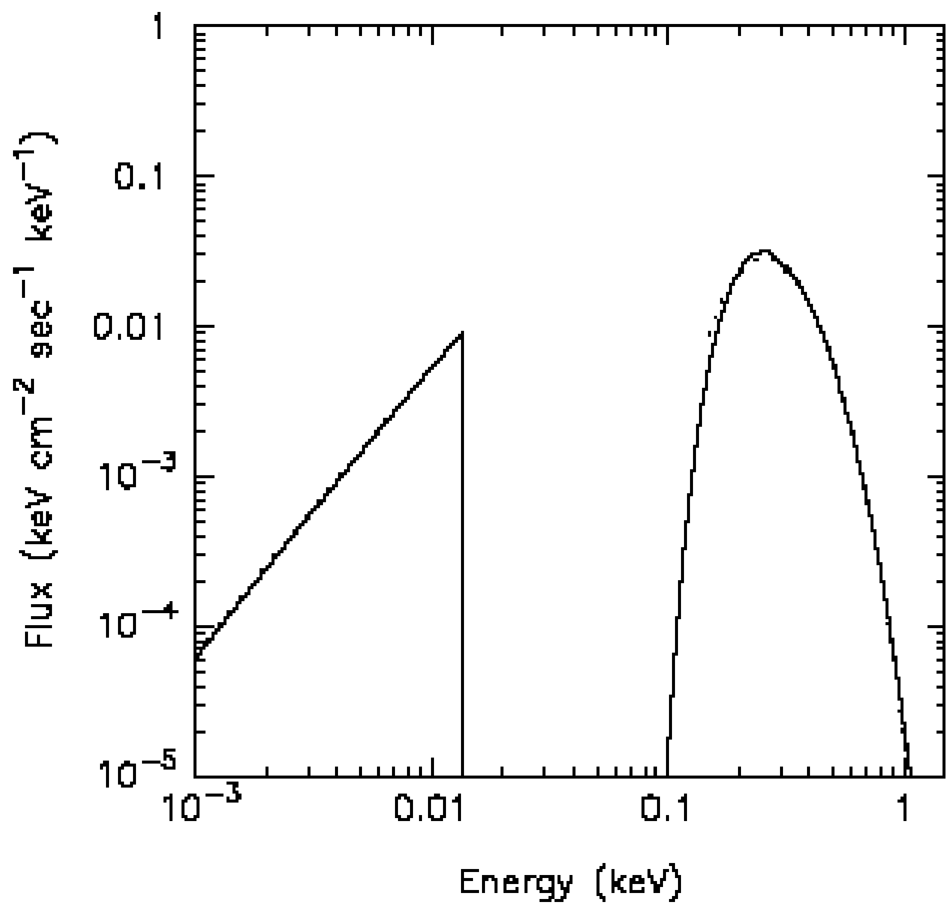


sPh can limit the anisotropy

Observational Consequences

Hot spots on the surface will lead to pulsations in the thermal emission.





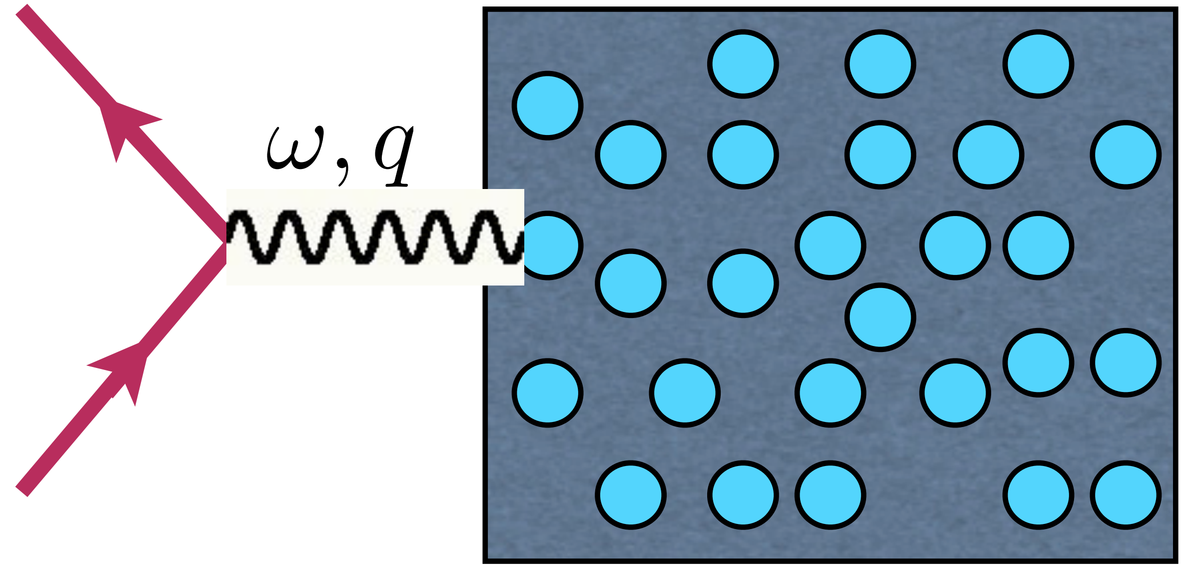
Temperature anisotropy will affect the spectrum.
2 Black-body fit to RXJ 1856 works well.

Geppert, Kueker & Page (2006)

Conclusions

- New mode for heat conduction in the inner crust.
- Low energy EFT for sPhs, lPhs and electrons.
- sPh conduction is likely to be important for thermal evolution of magnetars.
- Could also play a role in accreting neutron stars and first 100 years of NS evolution.

Electron Scattering and the Dynamic Structure Factor



$$\frac{1}{\lambda_e} = \nu_e = \frac{4\pi Z^2 \, e^4 \, n_{\rm ions}}{k_{Fe}^2 \, v_{Fe}} \, \Lambda_\kappa$$

$$\Lambda_\kappa = \int_0^{2k_{\rm Fe}} dq \; q^3 \; V_{\rm eff}(q)^2 \; S_\kappa(q) \; \left[1 - \frac{v_{\rm Fe}^2 q^2}{4k_{\rm Fe}^2}\right]$$

Flowers & Itoh (1976)
Yakovlev & Urpin (1980)
Potekhin et al. (1999)

Coulomb Logarithm

$$\Lambda_\kappa = \int d\omega \; S(q,\omega) \; \left[\frac{z + f(q) \; z^3}{1 - e^{-z}}\right] \qquad z = \frac{\omega}{T}$$

Dynamic Structure Factor

$$S(q,\omega) = \int dt \; e^{i\omega t} \; \langle \rho(q,t)\rho(-q,0)\rangle$$

Potekhin (1999)

Plasma physics of the outer crust:

$$\Gamma = \frac{Z^2 \ e^2}{a \ kT} \quad \Gamma_c \simeq 175$$

$$a \simeq 125 \left(\frac{A}{50} \ \frac{1}{\rho_{10}} \right)^{1/3} \ \text{fm}$$

$$kT = 4.4 \ 10^{-5} \frac{T}{10^8 \ K} \ \text{fm}^{-1}$$

$$\omega_{\text{plasmon}} = \sqrt{\frac{4\pi e^2 Z^2 n_I}{AM}}$$

$$q_{\text{TFe}} = \sqrt{\frac{4e^2}{\pi}} \ k_{\text{Fe}}$$