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SURPRISES IN DENSE RELATIVISTIC MATTER IN A MAGNETIC FIELD

*E.V. Gorbar, V.A. Miransky, I.S., arXiv:0904.2164 [hep-ph]

Motivation

- Dynamics of Quantum Hall Effect in graphene (2+1 dimensions)
  - Parity and time-reversal odd Dirac mass

$$\Delta \quad \sim \quad \langle \bar{\Psi}\gamma^3\gamma^5\Psi\rangle$$

[Gorbar, Gusynin, Miransky, I.S., PRB **78**, 085437 (2008)]

- Topological current in relativistic matter in a magnetic field (3+1 dimensions)
  - $\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = \frac{eB}{2\pi^2}\mu$ (free theory!)

[Metlitski, Zhitnitsky, PRD **72**, 045011 (2005)]

Model

- Lagrangian density:

$$\mathcal{L} = \bar{\psi}\left(iD_\nu + \mu_0\delta_\nu^0\right)\gamma^\nu\psi + \frac{G_{\text{int}}}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

- The dimensionless coupling is

$$g \equiv G_{\text{int}}\Lambda^2/(4\pi^2) \ll 1$$

- Magnetic field is inside $D_\nu = \partial_\nu - ieA_\nu$

  where $A_\nu = xB\delta_\nu^2$

Approximation

- Gap equation (mean-field approximation & weak coupling):

$$G^{-1}(u,u') = S^{-1}(u,u') + iG_{\rm int}\left\{G(u,u) - \gamma^5 G(u,u)\gamma^5 - {\rm tr}[G(u,u)] + \gamma^5\,{\rm tr}[\gamma^5 G(u,u)]\right\}\delta^4(u-u'),$$

where

$$iG^{-1}(u,u') = \Big[(i\partial_t+\mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3 + i\tilde{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m\Big]\delta^4(u-u')$$

and

$$iS^{-1}(u,u') = \left[(i\partial_t+\mu_0)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3\right]\delta^4(u-u')$$

Vacuum state

- Magnetic catalysis (spontaneous generation of a nonzero Dirac mass even at $g \ll 1$):

$$m_0^2 = \frac{1}{\pi l^2} \exp\left(-\frac{\Lambda^2 l^2}{g}\right) \quad \text{where} \quad l = 1/\sqrt{|eB|}$$

(along with $\mu = \mu_0$)

[Gusynin, Miransky, I.S., PRL **73**, 3499 (1994); PLB **349**, 477 (1995)]

- The solution exists for $\mu_0 < m_0$, although it will be less stable than the normal state ($m = 0$) already for $\mu_0 \gtrsim m_0/\sqrt{2}$ [Clogston, PRL **9**, 266 (1962)]

"Normal" ground state

- Gap equation allows another solution,

$$\mu \simeq \mu_0 \text{ and } \Delta \simeq -g\mu_0 eB/\Lambda^2$$

- This solution is almost independent of temperature when $T \ll \mu$
- This is the normal ground state since its symmetry is same as in the Lagrangian
- Besides, there is no solution with Δ=0…

Change of ground state

- The free energy in the state with $m \neq 0$ (broken symmetry)

$$\Omega_m \simeq -\frac{m_0^2}{2(2\pi l)^2}\left(1 + (m_0 l)^2 \ln |\Lambda l|\right)$$

- The free energy in the normal state, $\Delta \neq 0$

$$\Omega_\Delta \simeq -\frac{\mu_0^2}{(2\pi l)^2}\left(1 + g\frac{|eB|}{\Lambda^2}\ln\frac{\Lambda^2}{\mu_0^2}\right)$$

- So, indeed symmetry is restored for $\mu > \mu_c$,

$$\mu_c \simeq m_0/\sqrt{2}$$

Physical meaning of Δ

- Consider the dispersion relations of the quasiparticles:

$$\omega_{n,\sigma} = -\mu \pm \sqrt{[k_3 + \sigma\Delta]^2 + 2n|eB|}$$

  where $\sigma = \pm 1$ is the chirality
- So, the distribution of opposite chirality quasiparticles are *shifted* with respect to the longitudinal momentum (!)
- All Landau levels ($n \geq 0$) are affected by Δ

Induced axial current

- The axial current in the ground state is

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = \frac{eB}{2\pi^2}\mu - \frac{|eB|}{2\pi^2}\Delta - \frac{|eB|}{\pi^2}\Delta\sum_{n=1}^{\infty}\kappa(\sqrt{2n|eB|},\Lambda)$$

- In addition to topological contribution, $\frac{eB}{2\pi^2}\mu$ there are dynamical ones $\propto \Delta$

- The cutoff function:

$$\kappa(x,\Lambda) \simeq \begin{cases} 1 & , x \ll \Lambda \\ 0 & , x \gg \Lambda \end{cases}$$

Potential implications

- Physical properties to be affected
  - transport
  - emission

    (if sensitive to anisotropy and/or CP violation)
- Specific physical systems
  - Compact stars
    - Quark stars (quarks)
    - Hybrid stars (quarks, electrons)
    - Neutron stars (electrons)
    - White dwarfs (electrons)
  - Heavy ion collisions [Kharzeev & Zhitnitsky, NPA **797**, 67(2007), Kharzeev, McLerran & Warringa, NPA 803, 227 (2008), …]

Pulsar kicks

- The dynamical chiral shift parameter is driven by chemical potential ($T \ll \mu$)

$$\Delta \simeq -g\mu_0 eB/\Lambda^2$$

  and almost independent of temperature
- This creates strong anisotropy in the distribution of left quarks/electrons
- The anisotropy is transferred to neutrinos by elastic scattering even at large T
- Pulsar gets a kick when neutrinos escape

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Facilitated supernova explosions

- Because of the robustness of the axial currents at finite temperatures, even supernova explosions may be affected

- A small early-time neutrino asymmetry may facilitate explosions and give a kick at the same time, e.g., see

  [Fryer & Kusenko, Astrophys. J. Supp. **163**, 335 (2006)]

Summary

- $\mu < \mu_c$: Chiral symmetry is broken in the ground state (magnetic catalysis)
- $\mu > \mu_c$: Normal ground state of dense relativistic matter in a magnetic field is characterized by
  - Axial current along the field (topological and dynamical contributions)
  - Chiral shift parameter
- No solution with vanishing Δ exists

Outlook

- Detailed analysis of normal ground state in models with explicitly broken chiral symmetry (work in progress)
- Calculation of neutrino emission/diffusion in the state with axial currents
- Transport properties of the normal state with nonzero axial currents
- Modification of the “chiral magnetic effect” due to Δ in heavy ion collisions
  [Fukushima, Kharzeev & Warringa, PRD **78**, 074033 (2008)]

THANK YOU