Microscopic approach to color superconductivity of dense quark matter

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References:

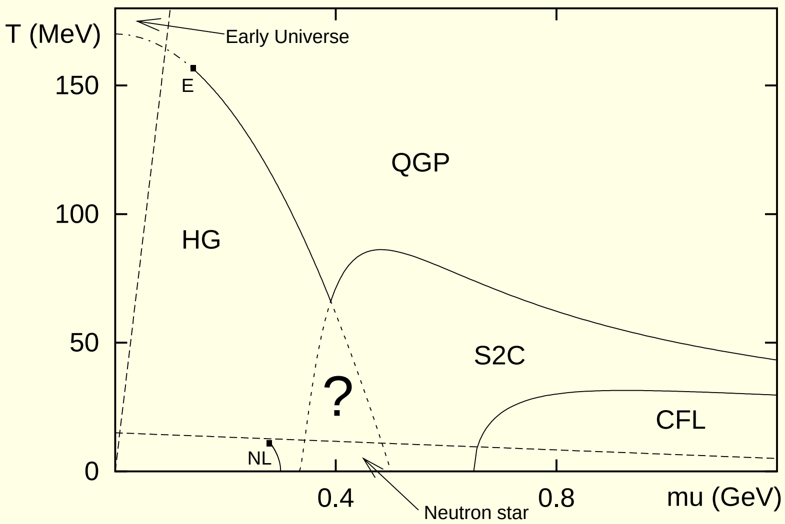
- hep-ph/0104194
- hep-ph/0103269
- hep-ph/0103227, Phys.Rev.D**64**(2001)025005
- hep-ph/0009173, Phys.Rev.D**63**(2001)056005

Outline

Introduction

- Phase diagram of QCD

  “It [Holy Grail] promises mystery, secrecy, adventure and the obtaining of a prize or knowledge available to all but found by few”

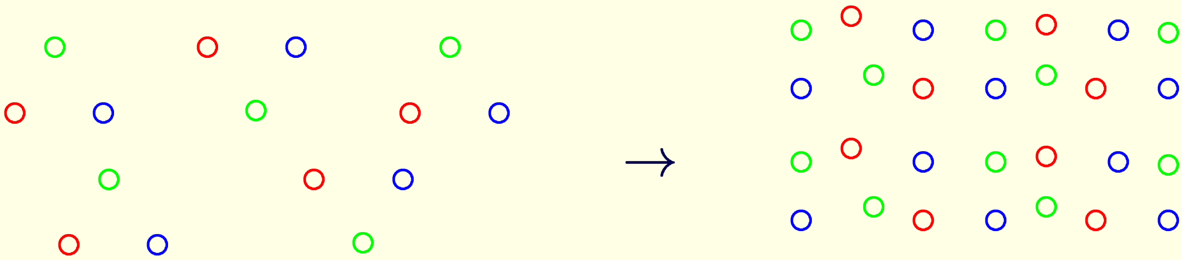


- Interior of neutron stars (neutrino emmision, cooling rate, magnetic field, thermodynamics, R-mode instability, glitches, γ-ray bursts, etc.)

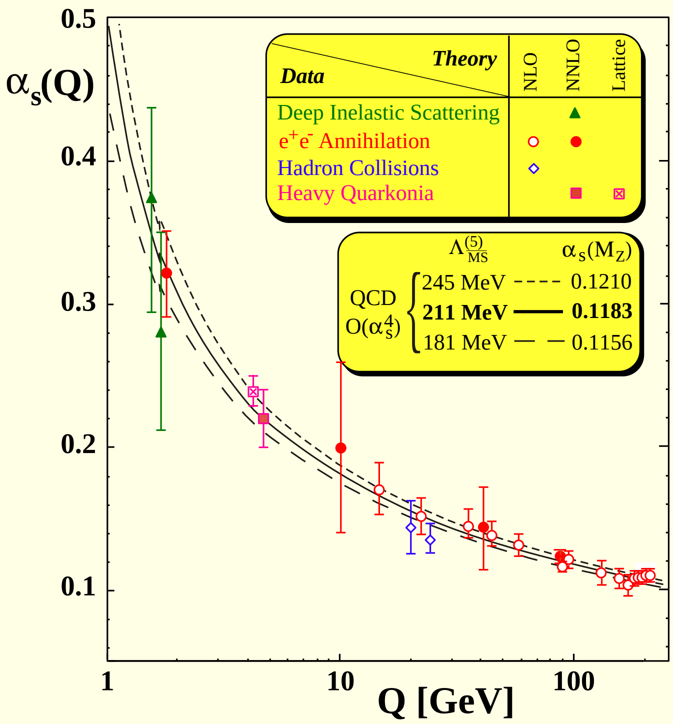
- Heavy ion collisions (?)

Weakly interacting dense quark matter

- Densities $\gtrsim 3n_0$, where $n_0 \approx 0.17\text{fm}^{-3}$
  - "Squeezing" quark matter



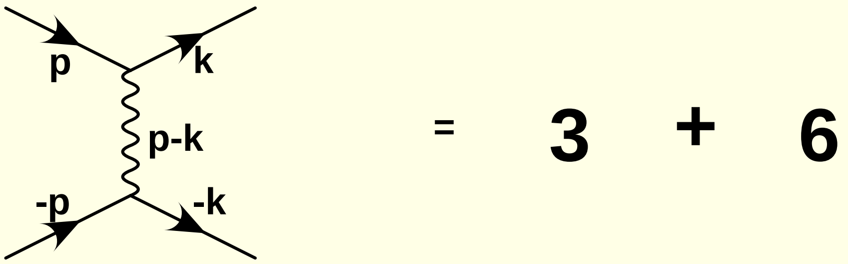
  - Asymptotic freedom
    [Gross,Wilczek,Politzer,'73]



  - Weakly interacting quarks
    [Collins and Perry,'75]

- Coupling constant: $\alpha(\mu) \ll 1$
  (where $\mu \gg \Lambda_{QCD}$)

- One-gluon exchange is the dominant interaction between quarks [Bailin & Love,'84]



- Antisymmetric $\bar{3}$ channel is **attractive** (!)

- Cooper instability around the Fermi surface

- Diquark condensate

$$N_f = 2: \qquad \varepsilon_{ij}\varepsilon^{3ab}\langle(\psi_a^i)^T C\gamma^5\psi_b^j\rangle \neq 0$$

$$N_f = 3: \qquad \sum_{I=1}^{3}\varepsilon_{ijI}\varepsilon^{abI}\langle(\psi_a^i)^T C\gamma^5\psi_b^j\rangle \neq 0$$

- Symmetries *(note, parity is preserved)*

$$N_f = 2: \qquad SU(2)_L \times SU(2)_R \times SU(2)_c$$
$$N_f = 3: \qquad SU(3)_{L+R+c}$$

The value of the gap

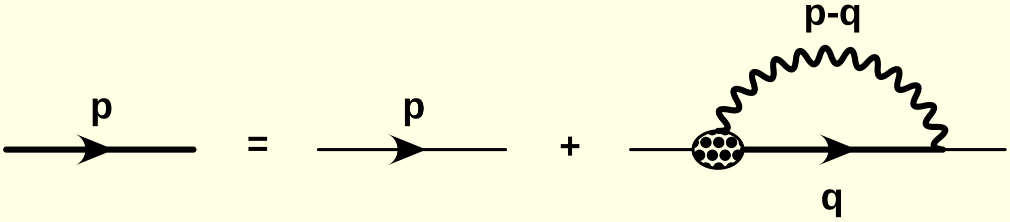
- **If** the gluon interaction were **screened** at the scale of $m_D \sim \sqrt{\alpha}\mu$ [Bailin & Love,'84], then

$$\Delta \sim \mu \exp\left(-\frac{C}{\alpha}\right) \;\rightarrow\; \Delta \ll 1 \text{ MeV}$$

- Phenomenological 4-fermion models [Alford,Rajagopal&Wilczek; Rapp,Schäfer,Shuryak&Velkovsky,'88] give

$$\boxed{\Delta \sim 50 \text{ to } 100 \text{ MeV}}$$

- Proper perturbative QCD analysis:
  - There is only **dynamical screening** for magnetic gluons [Son,'99]
  - Method of Schwinger-Dyson equation [Hong,Miransky,Shovkovy&Wijewardhana,'00]:



$$\Delta \simeq \frac{(4\pi)^{3/2} e\mu}{\alpha^{5/2}_{(\mu)}} e^{-\frac{C}{\sqrt{\alpha_{(\mu)}}}} \sim 20 \text{ to } 70 \text{ MeV}$$

Gluon propagator (HDL)

- Region of dominant interaction:

$$|\Delta| \ll |k_4| \ll |\vec{k}| \ll \mu$$

- Dynamical screening (magnetic modes)

$$i\mathcal{D}_{\mu\nu} \simeq -\frac{|\vec{k}|O^{(mag)}_{\mu\nu}}{|\vec{k}|^3 + \pi M_D^2 |k_4|/2} - \frac{O^{(el)}_{\mu\nu}}{k_4^2 + |\vec{k}|^2 + 2M_D^2},$$

where $M_D^2 = \alpha_s N_f \mu/\pi$, and

$$O^{(mag)}_{\mu\nu}(q) = g_{\mu\nu} - u_\mu u_\nu + \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2},$$

$$O^{(el)}_{\mu\nu}(q) = u_\mu u_\nu - \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2} - \frac{q_\mu q_\nu}{q^2},$$

where $u_\mu = (1,0,0,0)$ and $\vec{q}_\mu = q_\mu - (u \cdot q)u_\mu$.

- Meissner effect ($|k_4|, |\vec{k}| \lesssim |\Delta|$) is irrelevant

- No strong coupling effects for $\mu \gg \Lambda_{QCD}$

$$\Lambda_{QCD} \ll |\Delta| \sim (\ln\mu)^{-5/2} e^{\ln\mu - C\sqrt{b\ln\mu}}$$

- Note: no decoupling of strong IR dynamics in lower dimensions (d=2+1 or d=1+1)

S2C: Pseudo-NG bosons

- Approximate symmetries of dense QCD:
  - U(1)$_A$ [in addition to the exact U(1)$_V$]
  - SU(3)$_{cL}\times$SU(3)$_{cR}$ [replaces SU(3)$_c$]

- Explanation:
  - Anomaly is negligible at large μ ($\gg \Lambda_{QCD}$)
  - Leading order kernel of BS equation does not mix L- and R-handed quarks

- Approximate symmetry breaking partern:
  - SU(3)$_{cL}\times$SU(3)$_{cR} \to$ SU(2)$_{cL}\times$SU(2)$_{cR}$
  - $U(1)_A$ is broken [mixed with axial color]
  - $\tilde{U}(1)_V$ is intact [generator $\sim \frac{1}{3}I - \frac{2}{\sqrt{3}}T^8$]

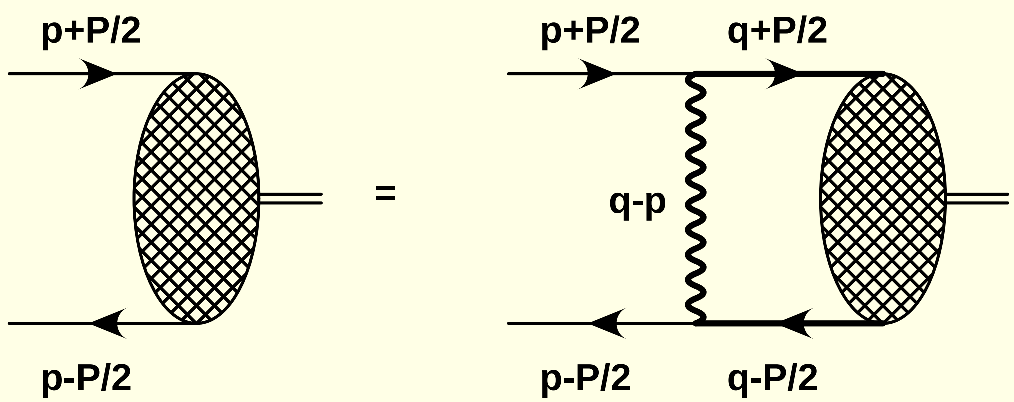
- Total number of pNGBs?
  ~~5~~ (scalars) $\oplus$ 5 (pseudoscalars)
  [taking Higgs mechanism into account]

Quantum numbers & properties of pNGBs

- $U(1)_A$ generator: $\gamma^5 I$ [or $\frac{2}{3}I + \frac{2}{\sqrt{3}}T^8$]
  1 pseudoscalar: SU(2)$_c$ singlet

- Generators: $\gamma^5 T^A$, $A = 4, 5, 6, 7$
  4 pseudoscalars: SU(2)$_c$ doublet and anti-doublet [generators $\sim \gamma^5(T^4 \pm T^5)$ & $\gamma^5(T^6 \pm T^7)$]

- Bethe-Salpeter equation:



- Decay constants and velocities:

$$F = \frac{\mu}{2\sqrt{2}\pi}, \qquad v = \frac{1}{\sqrt{3}}$$

Masses of the pNGBs

- Add axial color phases to the gap function

$$\Delta \to \mathcal{P}_+ U^\dagger \Delta U^* + \mathcal{P}_- U \Delta U^T,$$

where $U = \exp\left(i\gamma^5 \omega^A T^A\right)$

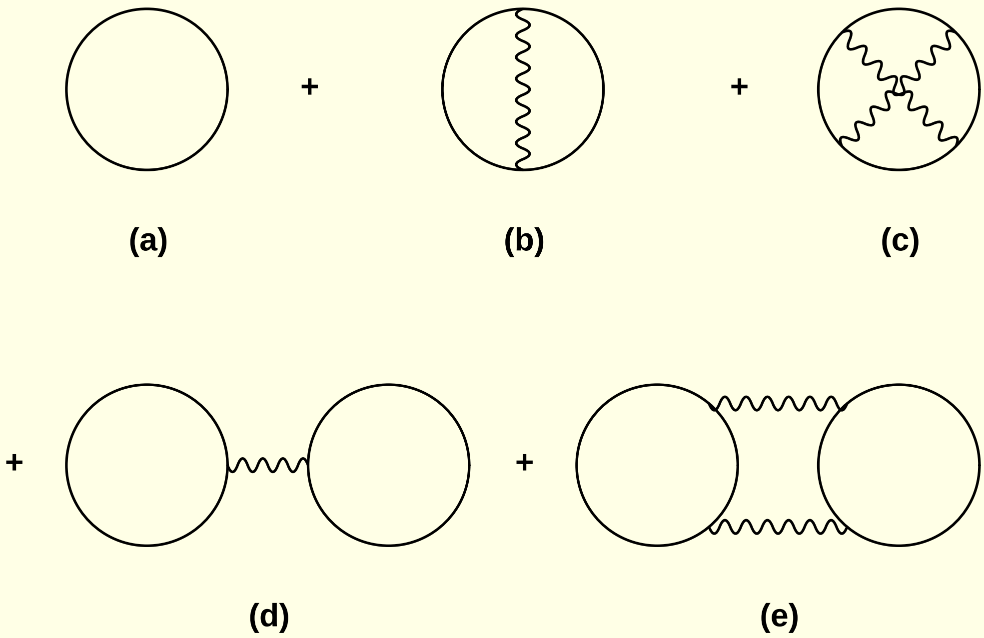
- Effective potential becomes a function of the U field

$$V(\Delta) \;\to\; V(\Delta, U)$$

- Masses of the pNGBs:

$$M_x^2 \sim \frac{1}{F^2} \frac{\partial^2 V(\Delta, U)}{\partial \omega_x^2}\bigg|_{\omega^A=0}$$

- Vacuum energy diagrams:



Results

- Quadratic term of the vacuum energy:

$$\delta_\omega E_{vac} \simeq \frac{3i}{4}\sum_{A=4}^{7}(\omega^A)^2 \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(q) D_{\kappa\lambda}(q)$$
$$\times\Big\{ \left[\Pi_4^{\mu\kappa}(q) - \Pi_1^{\mu\kappa}(q)\right]\left[\Pi_4^{\nu\lambda}(q) - \Pi_1^{\nu\lambda}(q)\right] + 2\tilde{\Pi}_4^{\mu\kappa}(q)\tilde{\Pi}_4^{\nu\lambda}(q)$$
$$+ \left[\Pi_4^{\mu\kappa}(q) - \Pi_8^{\mu\kappa}(q)\right]\left[\Pi_4^{\nu\lambda}(q) - \Pi_8^{\nu\lambda}(q)\right]\Big\},$$

where $\Pi_A^{\nu\lambda}(q)$ $(A = 1, 4, 8)$ are different components of the gluon polarization tensor.

- Masses of the doublet and antidoublet:

$$M_{\lambda,\chi}^2 \simeq C_\lambda \sqrt{\alpha}|\Delta|^2, \quad \text{where} \quad C_\lambda \simeq 7.5$$

- Mass of the singlet

$$M_\eta^2 = 0 \qquad \text{(perturbatively)},$$

- Non-perturbatively (screened instantons) [Son, Stephanov & Zhitnitsky,'00],

$$M_\eta^2 \simeq \frac{C_\eta}{\alpha^7}|\Delta^-|^2 \exp\left(-\frac{2\pi}{\alpha}\right),$$

where $C_\eta \simeq 10^5$

Binding of (anti-) doublets

- SU(2)$_c$ is confined at scale [Rischke, Son & Stephanov,'00]

  $$\Lambda'_{QCD} \simeq |\Delta| \exp\left[-C_0 \alpha^2 \mathrm{e}^{\frac{C}{\sqrt{\alpha}}}\right] \ll M_{\lambda,\chi},$$

  where $C_0 \simeq 10^{-3}$ and $C = 3\left(\frac{\pi}{2}\right)^{3/2}$.

- Doublet-antidoublet interaction is

  $$V(r) \simeq -\frac{4\pi\alpha}{\epsilon r}, \quad \text{for} \quad \frac{1}{|\Delta^-|} \ll r \ll \frac{1}{\Lambda'_{QCD}},$$

  where $\epsilon \approx \frac{2\alpha\mu^2}{9\pi|\Delta^-|^2}$ is the dielectric constant

- **Colorless** bound states (λ_0=udud) with binding energy ($\Lambda'_{QCD} \ll E_n \ll M_{\lambda,\chi}$)

  $$E_n \simeq -\frac{M_{\lambda,\chi}}{n^2}\left(\frac{3\pi|\Delta|}{\mu}\right)^4 \sim -\frac{\alpha^{-39/4}}{n^2}|\Delta| \mathrm{e}^{-\frac{4C}{\sqrt{\alpha}}}$$

  are formed

CFL: Collective modes

- Current-current correlation function:

$$\langle j_\mu j_\nu \rangle = \frac{q^2 \Pi_1 O_{\mu\nu}^{(mag)}}{q^2 + \Pi_1} + \frac{q^2 \left[\Pi_2 \Pi_3 + (\Pi_4)^2\right] O_{\mu\nu}^{(el)}}{(q^2 + \Pi_2)\Pi_3 + (\Pi_4)^2}$$

where the polarization tensor reads

$$\Pi_{\mu\nu} = \Pi_1 O_{\mu\nu}^{(mag)} + \Pi_2 O_{\mu\nu}^{(el)} + \Pi_3 O_{\mu\nu}^{(\|)} + \Pi_4 O_{\mu\lambda}^{(mix)}$$

- NG bosons (related to chiral symmetry)

$$\omega_{ng} \sim q \quad \text{(for all } T < T_c\text{)}$$

- Plasmon with mass

$$M_{gl}^{(1)} \simeq \omega_p \equiv \frac{g_s \mu}{\sqrt{2}\pi}$$

- "Light" plasmon with mass

$$1.36|\Delta| < M_{gl}^{(2)} < 2|\Delta| \quad \text{(for all } T < T_c\text{)}$$

- Gapless CG modes (scalars)

— exist in neacritical region, $T \in [T^*, T_c]$

— look like "revived" NG bosons

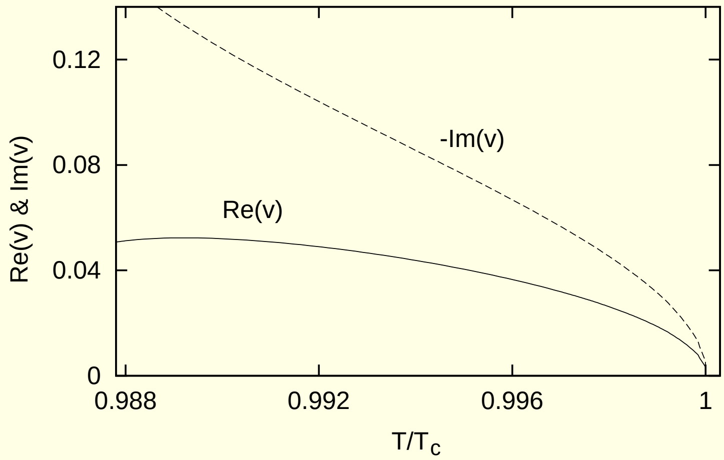
— made of correlated oscilation of super-conducting and normal components

Gapless Carlson-Goldman modes

- Gapless modes, discovered experimentally in dirty superconductors [Carlson & Goldman,'75]
- Landau damping vs. dirt [Ohashi & Takada,'97]
- Nearcritical region $T \to T_c - \epsilon$, clean limit,

$$v^2 + i\frac{45\pi|\Delta|}{224T}v - \frac{9\zeta(3)|\Delta|^2}{8\pi^2T^2} = 0, \qquad \frac{|\Delta|}{T} \ll 1$$

$$v \equiv \frac{q_0}{|\vec{q}|} = \frac{|\Delta|}{T}(\pm x^* - iy^*), \qquad \begin{array}{l} x^* \approx 0.19 \\ y^* \approx 0.32 \end{array}$$



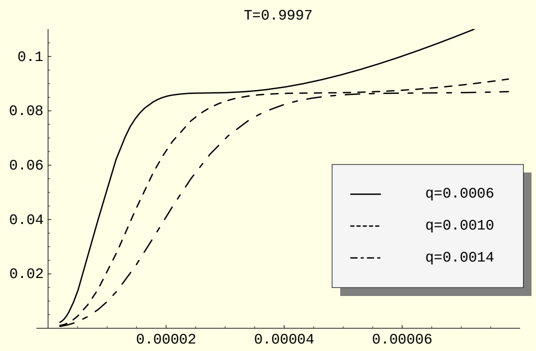
- Compare with NG bosons (pseudoscalars)

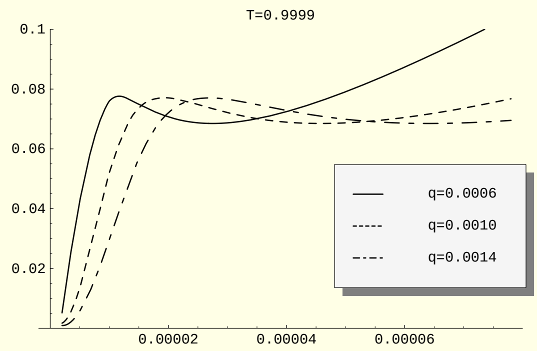
$$v_{ng}^2 + i\frac{5\pi|\Delta|}{32T}v_{ng} - \frac{7\zeta(3)|\Delta|^2}{8\pi^2T^2} = 0 \qquad \frac{|\Delta|}{T} \ll 1$$

$$v_{ng} \equiv \frac{q_0}{|\vec{q}|} = \frac{|\Delta|}{T}(\pm x^* - iy^*), \qquad \begin{array}{l} x^* \approx 0.22 \\ y^* \approx 0.25 \end{array}$$

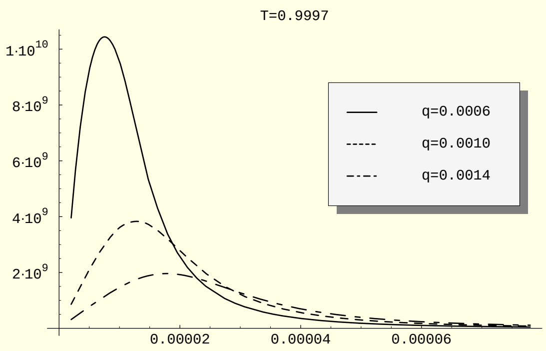
Numerical results (clean limit)

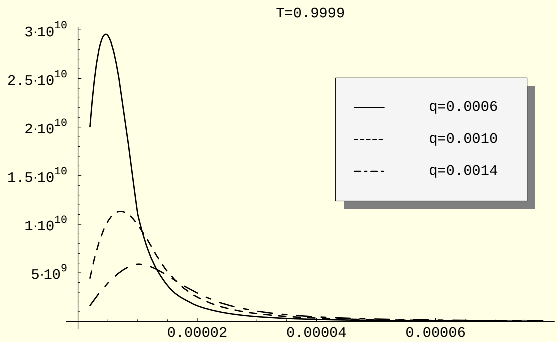
- Spectral densities of electric gluon quasi-particles at $T = 0.9997T_c$ and $T = 0.9999T_c$ (peaks correspond to CG modes, $\omega \sim |\Delta| q$)





- Spectral densities of NG bosons at $T = 0.9997T_c$ and $T = 0.9999T_c$ ($\omega_{ng} \sim |\Delta| q$)





Conclusion and Outlook

- Studies of dense quark matter are under theoretical controle, assuming $\mu \gg \Lambda_{QCD}$

- Many properties of color superconductors have already been revealed:
  - phases: S2C, CFL, crystalline, gapless
  - color-flavor unlocking ($m_s \neq 0$)
  - (pseudo-)NG boson properties (CFL, S2C)
  - confinement of unbroken $SU(2)_c$ (S2C)
  - collective excitations (CFL, S2C)
  - enforced electrical neutrality (CFL, μ_e, m_s)
  - sum rules (CFL)
  - neutrino transponrt (S2C)
  - kaon condensation (CFL, μ_e, m_s)

- Motivation for further studies: color superconductivity is likely to exist in the cores of some compact stars