



Effect of color superconductivity on cooling rate of quark stars

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Reference:

- [hep-ph/0204132](#)

Symmetry of ground state

$$N_f = 3 \text{ (CFL phase)}$$

$$\begin{aligned} SU(3)_L \times SU(3)_R \times SU(3)_c &\rightarrow SU(3)_{L+R+c} \\ U(1)_B &- \text{broken} \\ \text{approx. } U(1)_A &- \text{broken} \end{aligned}$$

- Modified electromagnetism $U(1)_{e\tilde{m}}$ survives
- Parity is preserved

Low energy dynamics

Chiral limit:

9 NG bosons ($\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$, and ϕ)
[Alford,Rajagopal,Wilczek'99]

$\oplus \eta'$ pseudo-NG boson, $m_{\eta'} \sim \Delta \exp\left(-\frac{\pi}{\alpha}\right)$
[Son,Stephanov,Zhitnitsky'01]

Massive quarks ($m_u, m_d, m_s \ll \mu$):

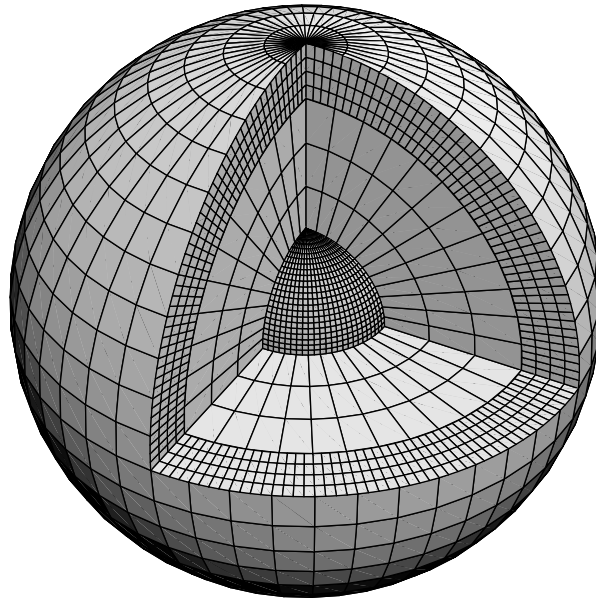
$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0$, and η

get nonzero masses (inverse hierarchy)

Low energy dynamics is dominated by massless ϕ related to breaking of baryon number

A compact star with a quark core

The aftermath of a supernova explosion:



Neutrino trapping \rightarrow deleptonization process by diffusion \rightarrow “initial” stage ($T \lesssim 40$ MeV)

Cooling of NS is dominated by neutrino emission during the first 10^5 years ($T \gtrsim 10$ keV)

Neutrino and photon emissivities of the CFL quark matter are strongly suppressed

[Jaikumar,Prakash,Schäfer, astro-ph/0203088]

Would the core remain hot for 10^{24} years?

Thermal conductivity

Thermal conductivity tends to wash away a temperature gradient, i.e.,

$$u_i = -\kappa \partial_i T$$

Kinetic properties are dominated by low-energy degrees of freedom.

Let us estimate the partial contribution of a (pseudo-) NG boson, described by

$$L = \frac{1}{2} \left(\partial_0 \varphi \partial_0 \varphi - v^2 \partial_i \varphi \partial_i \varphi - m^2 \varphi^2 \right) + \dots$$

The corresponding heat current is

$$u_i = \frac{\partial L}{\partial (\partial^i \varphi)} \partial_0 \varphi = v^2 \partial_i \varphi \partial_0 \varphi$$

By using the linear response theory, we derive

$$\kappa_{ij} = -\frac{i}{2T} \lim_{\Omega \rightarrow 0} \frac{1}{\Omega} \left[\Pi_{ij}^R(\Omega + i\epsilon) - \Pi_{ij}^A(\Omega - i\epsilon) \right]$$

where $\Pi_{ij}(\Omega)$ is the heat current correlator.

Rotational invariance implies that $\kappa_{ij} \equiv \kappa \delta_{ij}$.

At $T \lesssim m$, the direct calculation gives

$$\begin{aligned} \kappa &= \frac{m^5}{24\pi^2 v \Gamma T^2} \int_0^\infty \frac{x^4 dx}{\sinh^2 \frac{m\sqrt{1+x^2}}{2T}} \\ &\simeq \frac{m^{5/2} \sqrt{T}}{2\sqrt{2}\pi^{3/2} v \Gamma} e^{-m/T}, \end{aligned}$$

where Γ is the decay width.

Γ is a phenomenological parameter in the propagator of the (pseudo-) NG boson,

$$S(\omega, \vec{k}) = \frac{1}{(\omega + i\Gamma/2)^2 - v^2 \vec{k}^2 - m^2}$$

At $T \lesssim \tilde{T} \ll m_{lpNG}$, dominant contribution to κ comes from the massless NG boson:

$$\kappa_\phi = \frac{4T^3}{3\pi^2 v \Gamma_\phi} \int_0^\infty \frac{x^4 dx}{\sinh^2 x} = \frac{2\pi^2 T^3}{45 v \Gamma_\phi}$$

(In agreement with classical relation $\kappa = \frac{1}{3} \bar{v} l c_v$)

Mean free path of the NG boson

Trade width Γ for mean free path: $\ell = \frac{\bar{v}}{\Gamma}$

NG boson is a composite particle, $\phi \rightarrow qq$:

[Gusynin, Shovkovy, Nucl. Phys. A700(2002)577]

$$\Gamma_{\phi \rightarrow qq}(k) \sim vk \exp\left(-\sqrt{\frac{3\Delta}{2T}}\right)$$

Then, the mean free path would be

$$\ell_{\phi \rightarrow qq} \sim \frac{v}{T} \exp\left(\sqrt{\frac{3\Delta}{2T}}\right)$$

i.e., $\ell \gtrsim 30$ km for $T \lesssim \Delta/33$ (for $\Delta \simeq 50$ MeV).

NG bosons scattering amplitude $\sim k^4/\mu^4$

[Son, hep-ph/0204199]:

$$\sigma_{\phi\phi} \simeq \frac{T^6}{\mu^8}$$

So, the mean free path scales as

$$\ell_{\phi\phi} \sim \frac{1}{\sigma_{\phi\phi} n_{\phi}} \sim \frac{\mu^8}{T^9} \approx 8 \times 10^5 \frac{\mu_{500}^8}{T_{MeV}^9} \text{ km}$$

Photon contributions

Photon mean free path \gg star size ($T \lesssim \tilde{T}$)

Photon emission is strongly suppressed

Are there photons inside the quark core?

Nuclear matter is a plasma

The plasma frequency is

$$\Omega_p = \sqrt{\frac{4\pi e^2 Y_e \rho}{m_e m_p}} \simeq 470 \sqrt{\frac{\rho Y_e}{\rho_0}} \text{ MeV},$$

where

$n_e = Y_e \rho / m_p$ is the electron density

ρ is the nuclear matter density

m_p and m_e are the proton and electron masses

$Y_e \simeq 0.1$ is the number of electrons per baryon

$\rho_0 \approx 2.8 \times 10^{14} \text{ g cm}^{-3}$ is equilibrium nuclear matter density.

There is photon trapping at $T \lesssim \tilde{T}$

Thus, photons also contribute to the thermal conductivity

Total thermal conductivity

Mean free path of photons and NG bosons

$$\ell \simeq R_0, \text{ where } R_0 \text{ is the core size}$$

(geometrical restriction)

The expression for the thermal conductivity

$$\kappa_{CFL} = \kappa_\phi + \kappa_\gamma \simeq \frac{2\pi^2}{9} T^3 R_0$$

where $v_\phi = 1/\sqrt{3}$ and $v_\gamma \approx 1$ were used.

Numerically, this yields the value

$$\kappa_{CFL} \simeq 1.2 \times 10^{32} T_{MeV}^3 R_{0,km} \text{ erg cm}^{-1} \text{ sec}^{-1} \text{ K}^{-1}$$

Compare with

$$\kappa_{NM} \lesssim 10^{24} \text{ erg cm}^{-1} \text{ sec}^{-1} \text{ K}^{-1}$$

Typical “relaxation time”:

$$\frac{R_{0,km}}{v} \simeq 6 \times 10^{-4} \text{ sec}$$

Cooling of a star

Thermal energy vs. emissivity

Thermal energy of the core:

$$\begin{aligned} E_{CFL}(T) &= \frac{4\pi R_0^3}{3} \int_0^T c_v(T') dT' \\ &= \frac{6(1 + 2v^3)T}{5} \left(\frac{\pi T R_0}{3v} \right)^3, \end{aligned}$$

or numerically

$$E_{CFL}(T) \simeq 2.1 \times 10^{42} R_{0,km}^3 T_{MeV}^4 \text{ erg}$$

Thermal energy of the nuclear layer:

$$E_{NM}(T) \simeq 8.1 \times 10^{49} \frac{M - M_0}{M_\odot} \left(\frac{\rho_0}{\rho} \right)^{2/3} T_{MeV}^2 \text{ erg}$$

Note that $E_{CFL}(T) \ll E_{NM}(T)$ for $T \lesssim \tilde{T}$

Thus, CFL quark core has a very little slowing effect on the star cooling rate

Other kinetic properties

Shear viscosity: massless NG bosons and photons should also dominate

Electrical conductivity: the lightest *charged* pseudo-NG boson (*i.e.*, the K^+) dominates:

$$\sigma_{el} \sim \frac{m_{K^+}^{5/2}}{\sqrt{T}} \exp\left(-\frac{m_{K^+}}{T}\right)$$

which is suppressed as $T \rightarrow 0$ (neutrality)

S2C phase ($N_f = 2$): many kinetic properties are dominated by 2 gapless quarks

Separate issue – bare CFL quark stars:

- Photon free ($\ell \gg R$)
- Emissivities are suppressed – very dim
- NG bosons scatter off boundary
- candidates of dark matter (?)

Conclusion and Outlook

- The low energy dynamics of CFL quark matter is determined by symmetry
- Cooling of CFL quark matter by neutrino and photon emission is strongly suppressed
- Transport properties (conductivities, viscosities, etc.) are dominated by massless NG bosons and photons
- Cooling of compact stars with quark cores does not differ much from that of neutron stars
- Observational signatures of quark stars(?)
More work is needed
- Motivation for further studies:
color superconductivity is likely to exist in the cores of some compact stars