Thermal rates for baryon and anti-baryon production

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References:

- nucl-th/0209075

Introduction

⋆ What are the rates of baryon-antibaryon production (annihilation) in hadronic phase of high energy heavy ion collisions?

⋆ What are the relevant equilibration times?

Why is this of interest?

- SPS energies

Q.: *Why chemical equilibrium ($T_{ch} \approx$ 170 MeV)?* or, in other words,

Why so many $\bar{p}$'s (in view of $\tau_{ann} \approx$ 3 fm/c)?

A.: *"Oversaturation" of pions and $n\pi \to p\bar{p}$*
[Rapp&Shuryak, Phys. Rev. Lett. **86** (2001) 2980]

- RHIC energies

Q.: *Why chemical equilibrium ($T_{ch} \approx$ 170 MeV)?*

Q.: *What is the role of hadronic phase?*

Q.: *How important are multi-meson inelastic collisions?*

Q.: *What about the production of hyperons?*

Method.

- *Difficulties of existing methods:*

(i) no reliable experimental data on the annihilation of hyperons

(ii) no practical method to implement reaction with more than two (finite size) particles in cascade computer codes

(iii) the very concept of localized vacuum interactions is no longer applicable

(iv) other approaches (using, for example, DCC) are more qualitative than quantitative

- *Our method:*

(i) quantitative in nature

(ii) multi-meson processes appear almost on equal footing with two-particle processes

(iii) non-equilibrium — not a problem

(iv) nucleons as well as hyperons are included

(v) in-equilibrium, rates are given by simple analytical formulas

Lagrangian density

Baryon and antibaryon pairs appear from two or more mesons interacting inelastically.

Interaction Lagrangian density (for simplicity, only nucleons are included here):

$$\mathcal{L}_{int} = -\overline{\psi}\gamma^{\mu}\frac{\tau^a}{2}\psi V^a_{\mu} + g_A\overline{\psi}\gamma^{\mu}\gamma^5\frac{\tau^a}{2}\psi A^a_{\mu}$$

where $g_A \approx 1.26$, while V^a_{μ} and A^a_{μ} are not specified yet, but could include vector and axial-vector mesons and/or (multi-)pion fields

Assumptions and limitations:

(i) no interaction to (pseudo-)scalar fields

justification:

— in accordance with non-linear σ-model

— this avoids overcounting of pions

— no clear experimental evidence of scalar component, plus arguments of symmetry

(ii) no interaction to tensor fields

— this assumed to be a good approximation in the time-like region

Derivation of rates

Rate of process $i \to f +$ baryon $+$ antibaryon

$$R_{fi} = \frac{|S_{fi}|^2}{TV}$$

where, e.g., in the axial-vector channel,

$$S_{fi} = g_A \langle f| \int d^4x A^a_\mu(x) J^{5\mu}_a(x) |i\rangle$$

and

$$J^{5\mu}_a = \frac{e^{-ix\cdot(p_1+p_2)}}{V} \sqrt{\frac{m_N^2}{E_1 E_2}} \bar{u}_{p_1,s_1} \gamma^5 \gamma^\mu \frac{\tau_a}{2} v_{p_2,s_2}$$

The corresponding differential rate is:

$$E_1 E_2 \frac{dR_A}{d^3p_1 d^3p_2} = -\frac{g_A^2}{2(2\pi)^6} A^{(-)aa}_{\mu\nu}(p_1+p_2)$$
$$\times \left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} \left(p_1 \cdot p_2 - m_N^2 \right) \right]$$

where the correlator $A^{(-)ab}_{\mu\nu}$ reads

$$A^{(-)ab}_{\mu\nu}(p) = -\sum_i w_i \sum_f (2\pi)^4 \delta^4(p-k)$$
$$\times \langle f| A^a_\mu(0) |i\rangle \langle i| A^b_\nu(0) |f\rangle,$$

with $\quad w_i = \dfrac{e^{-\beta E_i}}{Z}$ $\qquad$ in equilibrium

Different interpretations

- *Pion fluctuations:*

$$A_\mu^a \equiv \frac{i}{2}\mathrm{Tr}\left[\tau_a\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right)\right]$$
$$= \frac{1}{f_\pi}\partial_\mu\pi^a + \cdots$$
$$V_\mu^a \equiv -\frac{i}{2}\mathrm{Tr}\left[\tau_a\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right)\right]$$
$$= \frac{1}{f_\pi^2}\varepsilon^{abc}\pi^b\partial_\mu\pi^c + \cdots$$

where

$$\xi = \exp\left(\frac{i}{2f_\pi}\pi^a\tau_a\right)$$

- *Vector meson fluctuations:*

$$V_\mu^a = -g_{\rho NN}\rho_\mu^a$$
$$A_\mu^a = \frac{g_{a_1NN}}{g_A}a_{1\mu}^a + \frac{g_{\pi NN}}{g_A m_N}\partial_\mu\pi^a$$

- *Is it possible to include everything made of quarks?*

Quark interpretation

- *VMD hypothesis*

$$\bar{q}\gamma_\mu \frac{\tau^a}{2} q = \frac{m_\rho^2}{g_{\rho\pi\pi}} \rho_\mu^a$$

- *Goldberger-Trieman relation*

$$\bar{q}\gamma_\mu \gamma^5 \frac{\tau^a}{2} q = \frac{m_{a_1}^2}{g_{a_1}} a_{1\mu}^a + \text{pions}$$

Therefore, in terms of quark fields,

$$V_\mu^a = -\frac{g_{\rho NN} g_{\rho\pi\pi}}{m_\rho^2} \bar{q}\gamma_\mu \frac{\tau^a}{2} q$$

$$A_\mu^a = \frac{g_{a_1 NN} g_{a_1}}{m_{a_1}^2} \bar{q}\gamma_\mu \gamma^5 \frac{\tau^a}{2} q$$

Then, in equilibrium,

$$V_{\mu\nu}^{(-)ab}(k) = -\frac{6\pi}{1 - e^{\beta k^0}} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \rho_V(s = k^2)$$

and, by using OPE [Shifman,Vainshtein&Zakharov,1979], we derive

$$\rho_V(s) = \frac{s}{32 f_\pi^4 \pi^2} \left(1 + \frac{\alpha_s(s)}{\pi} + \cdots \right)$$

for $s > 4m_N^2$

Similar expressions are valid for axial-vector correlator, and $\rho_A(s) \approx \rho_V(s)$

Formal expression for rates

Differential rate becomes

$$E_1 E_2 \frac{dR}{d^3p_1 d^3p_2} = \frac{3(1+\alpha_s/\pi)}{8(2\pi)^7 f_\pi^4} \frac{s(s-m_N^2)}{e^{\beta(E_1+E_2)}-1}$$

where $k = p_1 + p_2$.

The total rate, then, reads

$$R(\bar{N}N) \simeq \frac{9(1+\alpha_s/\pi) m_N^4 T^4}{(2\pi)^5 f_\pi^4} \left[z_N^2 K_1^2(z_N) \right.$$
$$\left. + 4 z_N K_1(z_N) K_2(z_N) + \left(8 + z_N^2\right) K_2^2(z_N) \right]$$

where $z_N \equiv m_N/T$

• Nucleons are composite and, thus, one should include the appropriate form factors, $F(s)$

Then, the rate is modified as follows:

$$R(\bar{N}N) \;\; \rightarrow \;\; F(s)^2 R(\bar{N}N)$$

with $F(s)^2 \ll 1$

• One needs to know form factor $F(s)$ in the *time-like* region

Form factors

Annihilation rate in kinetic theory:

$$E_1 E_2 \frac{dR_{\rm ann}(\bar{p}p)}{d^3p_1 d^3p_2} = \frac{2}{(2\pi)^6} \frac{\sqrt{s(s-4m_N^2)}\sigma_{\rm ann}^{\bar{p}p}(s)}{e^{-\beta(E_1+E_2)}}$$

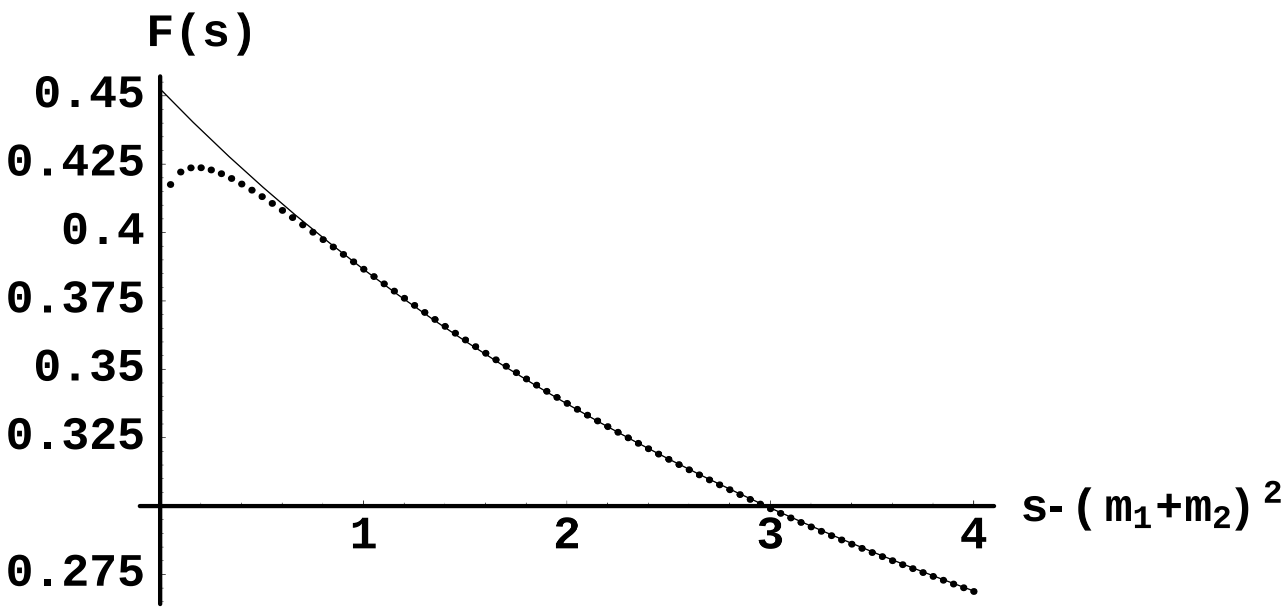
By comparing with production rate, we derive

$$F^2(s) = \frac{2^7 \pi f_\pi^4}{3(1+\alpha_s/\pi)} \sqrt{1 - \frac{4m_N^2}{s}} \frac{\sigma_{\rm ann}^{\bar{p}p}(s)}{s}$$

By using experimental data for $\sigma_{\rm ann}^{\bar{p}p}(s)$ from [Cugnon&Vandermeulen, Ann. Phys. (France) **14** (1989) 49], the following fit is derived:

$$F(s) = \frac{1}{2.21 + (s - 4m_N^2)/\Lambda^2}$$

where $\Lambda \approx 1.63$ GeV



SU(3) vs. SU(2) fluctuations

Interaction Lagrangian density:

$$\frac{g_{\rho NN}}{\sqrt{2}}\Big[(1-\alpha_V)\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\left[\mathcal{V}_\mu,\mathcal{B}\right]\right)+\beta_V\,\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\mathcal{B}\right)\mathrm{Tr}\left(\mathcal{V}_\mu\right)$$
$$+\alpha_V\,\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\left\{\mathcal{V}_\mu,\mathcal{B}\right\}\right)+g_A\alpha_A\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\gamma^5\left\{\mathcal{A}_\mu,\mathcal{B}\right\}\right)$$
$$+g_A(1-\alpha_A)\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\gamma^5\left[\mathcal{A}_\mu,\mathcal{B}\right]\right)+g_A\beta_A\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\gamma^5\mathcal{B}\right)\mathrm{Tr}\left(\mathcal{A}_\mu\right)\Big]$$

where the parameters are

$\alpha_V = (1-\beta_V)/2 = 0$	from OZI rule
$\beta_V = 1$	to satisfy $g_{\omega NN} = 3g_{\rho NN}$
$\alpha_A = 2/3$	≈ 0.68 (experimental)
$\beta_A = 1 - 2\alpha_A = -1/3$	from OZI rule

baryon octet:

$$\mathcal{B}=\begin{pmatrix}\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p\\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & n\\ \Xi^- & \Xi^0 & \frac{-2\Lambda^0}{\sqrt{6}}\end{pmatrix}$$

vector nonet:

$$\mathcal{V}=\begin{pmatrix}\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+}\\ \rho^- & -\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & -\phi\end{pmatrix}$$

(ideal mixing), and axial-vector nonet:

$$\mathcal{A}=\begin{pmatrix}\frac{a_1^0}{\sqrt{2}}+\frac{f_1(1285)}{\sqrt{2}} & a_1^+ & K_1^+\\ a_1^- & -\frac{a_1^0}{\sqrt{2}}+\frac{f_1(1285)}{\sqrt{2}} & K_1^0\\ K_1^- & \bar{K}_1^0 & -f_1(1420)\end{pmatrix}$$

Table I: Relative strength of vector couplings for $\alpha_V = 0$ and $\beta_V = 1$

	p	n	Λ	Σ^0	Σ^+	Σ^-	Ξ^0	Ξ^-
$\overline{p}$	ρ^0 3ω	$\sqrt{2}\rho^+$	$-\sqrt{3}K^+_*$	$-K^+_*$	$-\sqrt{2}K^0_*$	0	0	0
$\overline{n}$	$\sqrt{2}\rho^-$	$-\rho^0$ 3ω	$-\sqrt{3}K^0_*$	K^0_*	0	$-\sqrt{2}K^+_*$	0	0
$\overline{\Lambda}$	$-\sqrt{3}K^-_*$	$-\sqrt{3}\overline{K}^0_*$	2ω $-\sqrt{2}\phi$	0	0	0	$\sqrt{3}K^0_*$	$\sqrt{3}K^+_*$
$\overline{\Sigma}^0$	$-K^-_*$	$\overline{K}^0_*$	0	2ω $-\sqrt{2}\phi$	$-2\rho^-$	$2\rho^+$	$-K^0_*$	K^+_*
$\overline{\Sigma}^+$	$-\sqrt{2}\overline{K}^0_*$	0	0	$-2\rho^+$	$2\rho^0$ 2ω $-\sqrt{2}\phi$	0	$\sqrt{2}K^+_*$	0
$\overline{\Sigma}^-$	0	$-\sqrt{2}K^-_*$	0	$2\rho^-$	0	$2\rho^0$ 2ω $-\sqrt{2}\phi$	0	$-\sqrt{2}K^0_*$
$\overline{\Xi}^0$	0	0	$\sqrt{3}\overline{K}^0_*$	$-\overline{K}^0_*$	$\sqrt{2}K^-_*$	0	ρ^0 ω $-2\sqrt{2}\phi$	$-\sqrt{2}\rho^+$
$\overline{\Xi}^-$	0	0	$\sqrt{3}K^-_*$	K^-_*	0	$\sqrt{2}\overline{K}^0_*$	$-\sqrt{2}\rho^-$	$-\rho^0$ ω $-2\sqrt{2}\phi$

Table II: Relative strength of axial-vector couplings for $\alpha_A = 2/3$ and $\beta_A = -1/3$. Notation: $\tilde{f}_1 \equiv f_1(1420)$

	p	n	Λ	Σ^0	Σ^+	Σ^-	Ξ^0	Ξ^-
$\overline{p}$	a_1^0 $\frac{1}{3}f_1$	$\sqrt{2}a_1^+$	$-\frac{5}{3\sqrt{3}}K_1^+$	$\frac{1}{3}K_1^+$	$\frac{\sqrt{2}}{3}K_1^0$	0	0	0
$\overline{n}$	$\sqrt{2}a_1^-$	$-a_1^0$ $\frac{1}{3}f_1$	$-\frac{5}{3\sqrt{3}}K_1^0$	$-\frac{1}{3}K_1^0$	0	$\frac{\sqrt{2}}{3}K_1^+$	0	0
$\overline{\Lambda}$	$-\frac{5}{3\sqrt{3}}K_1^-$	$-\frac{5}{3\sqrt{3}}\overline{K}_1^0$	$-\frac{2}{9}f_1$ $-\frac{5\sqrt{2}}{9}\tilde{f}_1$	$\frac{4}{3\sqrt{3}}a_1^0$	$\frac{4}{3\sqrt{3}}a_1^-$	$\frac{4}{3\sqrt{3}}a_1^+$	$\frac{1}{3\sqrt{3}}K_1^0$	$\frac{1}{3\sqrt{3}}K_1^+$
$\overline{\Sigma}^0$	$\frac{1}{3}K_1^-$	$-\frac{1}{3}\overline{K}_1^0$	$\frac{4}{3\sqrt{3}}a_1^0$	$\frac{2}{3}f_1$ $-\frac{\sqrt{2}}{3}\tilde{f}_1$	$-\frac{2}{3}a_1^-$	$\frac{2}{3}a_1^+$	$-K_1^0$	K_1^+
$\overline{\Sigma}^+$	$\frac{\sqrt{2}}{3}\overline{K}_1^0$	0	$\frac{4}{3\sqrt{3}}a_1^+$	$-\frac{2}{3}a_1^+$	$\frac{2}{3}a_1^0$ $\frac{2}{3}f_1$ $\frac{\sqrt{2}}{3}\tilde{f}_1$	0	$\sqrt{2}K_1^+$	0
$\overline{\Sigma}^-$	0	$\frac{\sqrt{2}}{3}K_1^-$	$\frac{4}{3\sqrt{3}}a_1^-$	$\frac{2}{3}a_1^-$	0	$-\frac{2}{3}a_1^0$ $\frac{2}{3}f_1$ $\frac{\sqrt{2}}{3}\tilde{f}_1$	0	$\sqrt{2}K_1^0$
$\overline{\Xi}^0$	0	0	$\frac{1}{3\sqrt{3}}\overline{K}_1^0$	$-\overline{K}_1^0$	$\sqrt{2}K_1^-$	0	$-\frac{1}{3}a_1^0$ $-\frac{1}{3}f_1$ $-\frac{2\sqrt{2}}{3}\tilde{f}_1$	$\frac{\sqrt{2}}{3}a_1^+$
$\overline{\Xi}^-$	0	0	$\frac{1}{3\sqrt{3}}K_1^-$	K_1^-	0	$\sqrt{2}\overline{K}_1^0$	$\frac{\sqrt{2}}{3}a_1^-$	$\frac{1}{3}a_1^0$ $-\frac{1}{3}f_1$ $-\frac{2\sqrt{2}}{3}\tilde{f}_1$

Rates with SU(3) fluctuations

Baryon-antibaryon production rates read

$$
\begin{aligned}
R(n\bar{p}) &= 2R_{+}^{(m_N,m_N)} + 2R_{-}^{(m_N,m_N)} \\
R(p\bar{p}) &= 2R_{+}^{(m_N,m_N)} + \frac{82}{81}R_{-}^{(m_N,m_N)} \\
R(\Lambda\bar{p}) &= 3R_{+}^{(m_\Lambda,m_N)} + \frac{25}{27}R_{-}^{(m_\Lambda,m_N)} \\
R(\Xi^{-}\bar{\Lambda}) &= 3R_{+}^{(m_\Lambda,m_\Xi)} + \frac{1}{27}R_{-}^{(m_\Lambda,m_\Xi)}
\end{aligned}
$$

plus 42 other combinations

In general,

$$R(b_1,\bar{b}_2) = N_V^{(b_1,\bar{b}_2)}R_{+}^{(m_1,m_2)} + N_A^{(b_1,\bar{b}_2)}R_{-}^{(m_1,m_2)}$$

where

$R_{\pm}^{(m_1,m_2)}$ are defined as follows:

$$\frac{3F^2(\bar{s})(1+\frac{\alpha_s}{\pi})T^8}{8(2\pi)^5 f_\pi^4} z_1^2 z_2^2 \Big[4z_1K_1(z_1)K_2(z_2) + 4z_2K_1(z_2)K_2(z_1)$$
$$\pm(z_1 \pm z_2)^2 K_1(z_1)K_1(z_2) + [16 + (z_1 \pm z_2)^2]K_2(z_1)K_2(z_2)\Big]$$

with

$z_i = m_i/T$

$\bar{s} \equiv \langle s \rangle_T = (m_1+m_2)^2 + 3(m_1+m_2)T$

Tables III and IV give multipliers N_V and N_A

Table III: Numerical vector channel multipliers in the expressions for the rates ($\alpha_V = 0$ and $\beta_V = 1$)

	p	n	Λ	Σ^0	Σ^+	Σ^-	Ξ^0	Ξ^-
$\overline{p}$	2	2	3	1	2	0	0	0
$\overline{n}$	2	2	3	1	0	2	0	0
$\overline{\Lambda}$	3	3	$\frac{8}{9}$	0	0	0	3	3
$\overline{\Sigma}^0$	1	1	0	$\frac{8}{9}$	4	4	1	1
$\overline{\Sigma}^+$	2	0	0	4	$\frac{44}{9}$	0	2	0
$\overline{\Sigma}^-$	0	2	0	4	0	$\frac{44}{9}$	0	2
$\overline{\Xi}^0$	0	0	3	1	2	0	$\frac{26}{9}$	2
$\overline{\Xi}^-$	0	0	3	1	0	2	2	$\frac{26}{9}$

Table IV: Numerical axial-vector channel multipliers in the expressions for the rates ($\alpha_A = 2/3$ and $\beta_A = -1/3$)

	p	n	Λ	Σ^0	Σ^+	Σ^-	Ξ^0	Ξ^-
$\overline{p}$	$\frac{82}{81}$	2	$\frac{25}{27}$	$\frac{1}{9}$	$\frac{2}{9}$	0	0	0
$\overline{n}$	2	$\frac{82}{81}$	$\frac{25}{27}$	$\frac{1}{9}$	0	$\frac{2}{9}$	0	0
$\overline{\Lambda}$	$\frac{25}{27}$	$\frac{25}{27}$	$\frac{104}{729}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{1}{27}$	$\frac{1}{27}$
$\overline{\Sigma}^0$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{16}{27}$	$\frac{8}{81}$	$\frac{4}{9}$	$\frac{4}{9}$	1	1
$\overline{\Sigma}^+$	$\frac{2}{9}$	0	$\frac{16}{27}$	$\frac{4}{9}$	$\frac{44}{81}$	0	2	0
$\overline{\Sigma}^-$	0	$\frac{2}{9}$	$\frac{16}{27}$	$\frac{4}{9}$	0	$\frac{44}{81}$	0	2
$\overline{\Xi}^0$	0	0	$\frac{1}{27}$	1	2	0	$\frac{26}{81}$	$\frac{2}{9}$
$\overline{\Xi}^-$	0	0	$\frac{1}{27}$	1	0	2	$\frac{2}{9}$	$\frac{26}{81}$

Equilibration times

Let us consider a system out of equilibrium (i.e., too many or too few baryons, $\mu_B = 0$)

Density of antibaryons $\bar{b}$ satisfies

$$\frac{dn_{\bar{b}}}{dt} = \sum_{b'} R(b'\bar{b}) \left[1 - \frac{n_{b'} n_{\bar{b}}}{n_{b'}^{\text{equil}} n_{\bar{b}}^{\text{equil}}} \right]$$

Thus, time scale for reaching equilibrium is

$$\tau_{\bar{b}} = \frac{n_{\bar{b}}^{\text{equil}}}{\sum_{b'} R(b'\bar{b})}$$

Some values:

$$\begin{aligned}
\tau_{\bar{p}}(170\ \text{MeV}) &\approx 11\ \text{fm}/c \\
\tau_{\bar{\Lambda}}(170\ \text{MeV}) &\approx 8\ \text{fm}/c \\
\tau_{\bar{\Sigma}}(170\ \text{MeV}) &\approx 11\ \text{fm}/c \\
\tau_{\bar{\Xi}}(170\ \text{MeV}) &\approx 16\ \text{fm}/c
\end{aligned}$$

$$\begin{aligned}
\tau_{\bar{p}}(120\ \text{MeV}) &\approx 227\ \text{fm}/c \\
\tau_{\bar{\Lambda}}(120\ \text{MeV}) &\approx 163\ \text{fm}/c \\
\tau_{\bar{\Sigma}}(120\ \text{MeV}) &\approx 266\ \text{fm}/c \\
\tau_{\bar{\Xi}}(120\ \text{MeV}) &\approx 467\ \text{fm}/c
\end{aligned}$$

Results. SU(2)

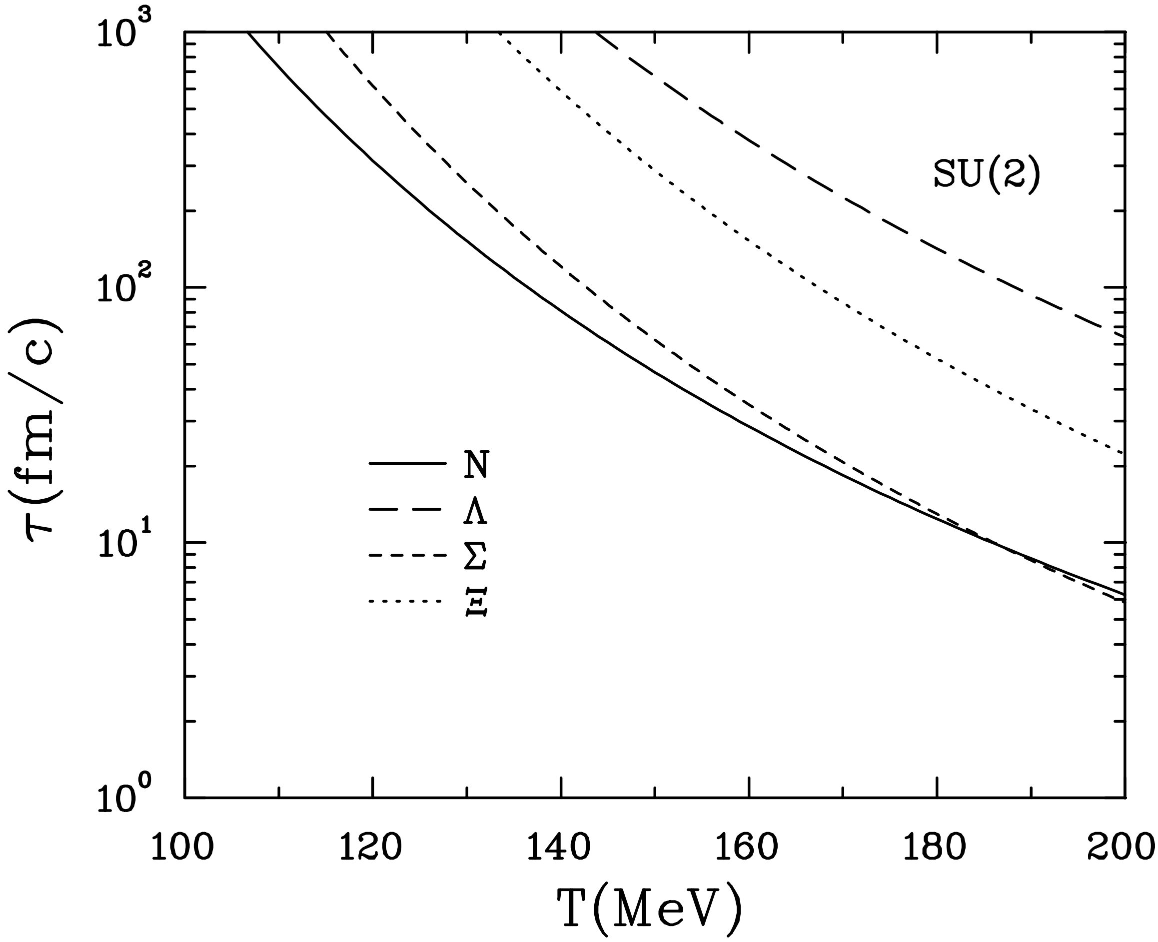


FIGURE 1. Numerical results for equilibration times in the case when only the fluctuations in the $SU(2)$ meson sector are taken into account.

Results. SU(3)

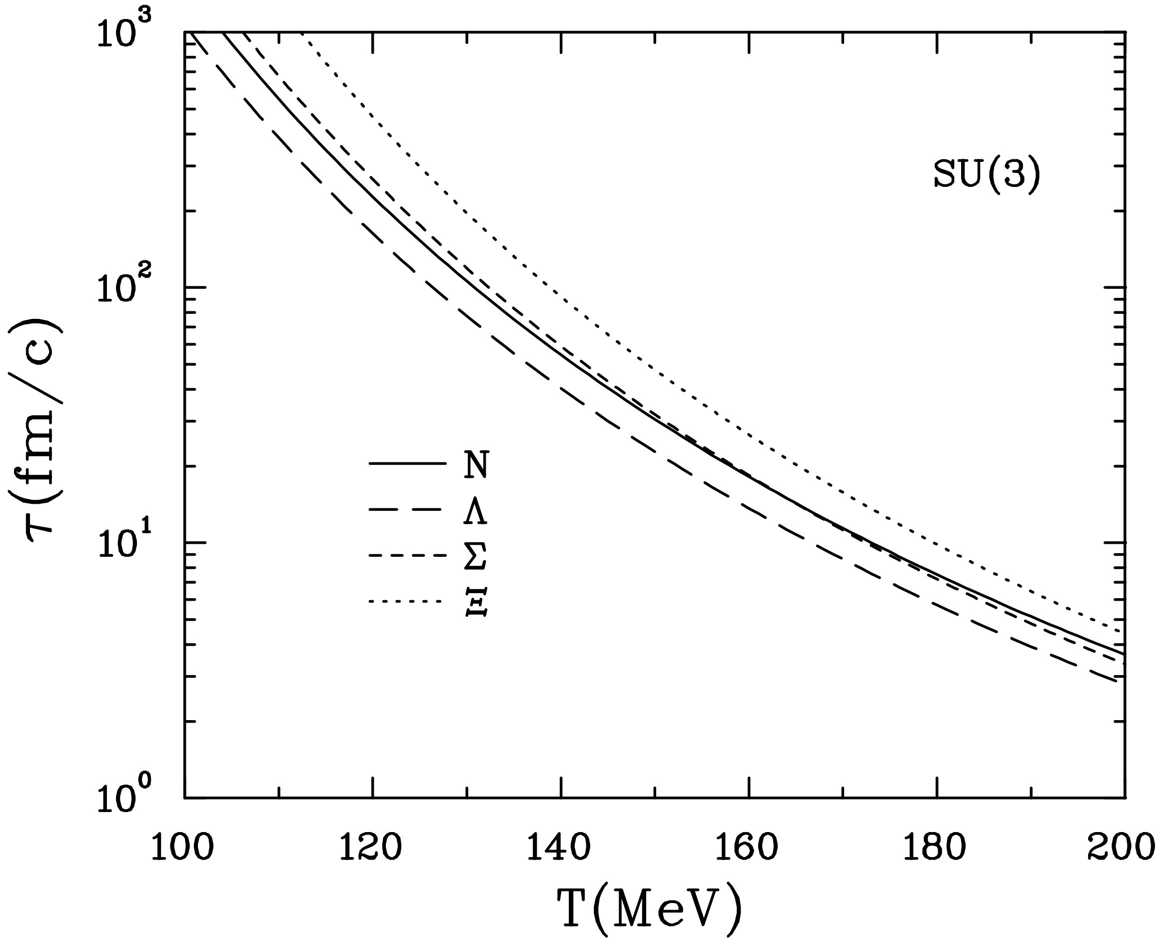


FIGURE 2. Numerical results for equilibration times in the case when all fluctuations in the $SU(3)$ meson sector are taken into account.

Conclusions

- A new *quantitative* method for calculating baryon-antibaryon production rates is developed

- The method is perfectly suitable for *non-equilibrium* systems

- In near-equilibium, the rates are given by simple *analytical* expressions

- By making use of underlying quark picture, essentially *all* strongly interacting inelastic processes are included

- As a by-product, simple parametrizations of corresponding *form factors* are derived

Outlook

- The generalization for finite μ_B has to be done

- Role of scalar- and tensor-like fluctuations should be further clarified

- Calculation for other baryons, such as spin-3/2 decupled, should be carried out in the analogous way

- Study of dynamical model of expanding matter (as in RHIC/LHC) should be done