Color superconductivity and compact stars

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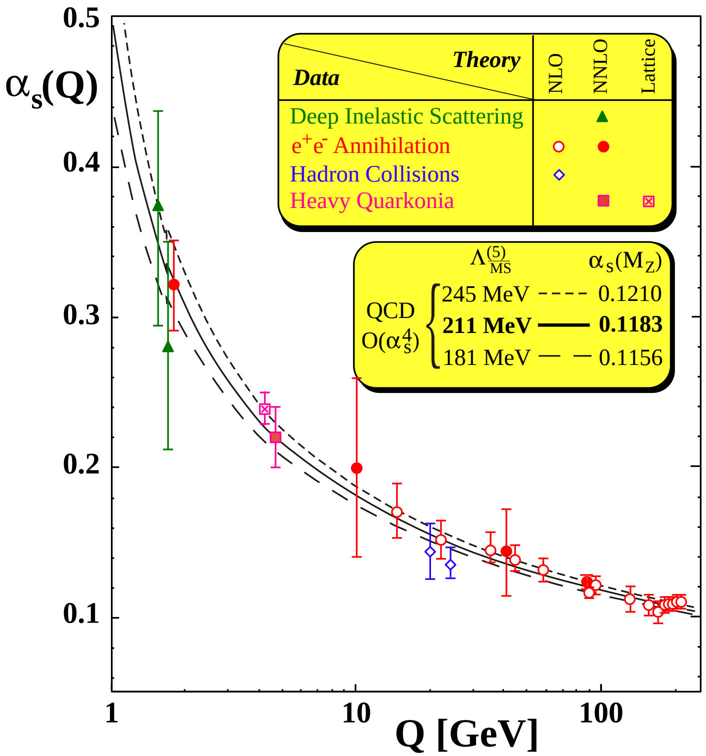
References

- I. Shovkovy, M. Hanauske and M. Huang, Phys. Rev. D **67** (2003) 103004, `hep-ph/0303027`
- I. Shovkovy and M. Huang, Phys. Lett. B **564** (2003) 205, `hep-ph/0302142`

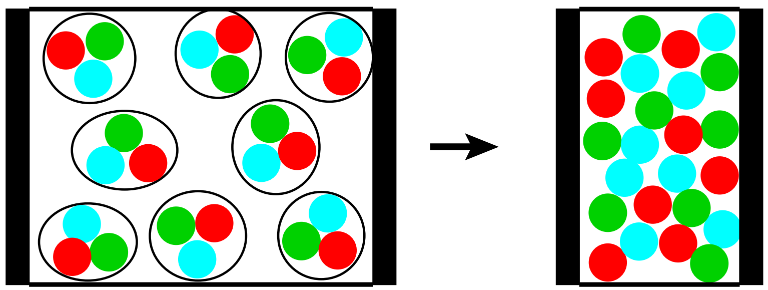
Weakly interacting quark matter

The property of asymptotic freedom: $\alpha_s(\mu) \ll 1$ for $\mu \gg \Lambda_{QCD}$

[Gross & Wilczek,73], [Politzer,73]



- Dense quark matter is weakly interacting [Collins & Perry,75]
- "Squeezing" quark matter

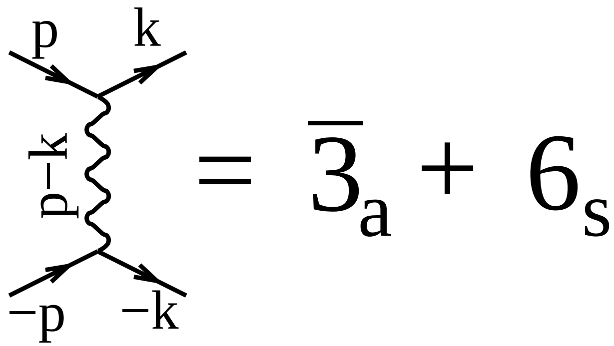


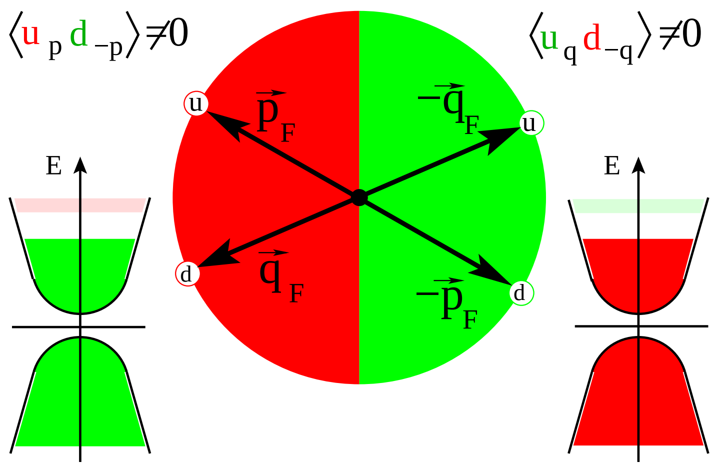
- Realistic densities in compact stars: $\rho \lesssim 10\rho_0$, where $\rho_0 \approx 0.15$ fm^{-3}, (corresponding coupling $\alpha_s \sim 1$)

Basics of color superconductivity

Asymptotic density ($\mu \gg \Lambda_{QCD}$):

- $\alpha_s(\mu) \ll 1$ (weak coupling)
- One-gluon interaction is dominant
- Color $\bar{3}_a$ channel is **attractive** (!)





- BCS mechanism for quarks leads to color superconductivity
- By using Pauli principle ($s = 0$):

$$N_f = 2 : \varepsilon_{ij}\varepsilon^{3ab}\langle(\psi_a^i)^T C\gamma^5\psi_b^j\rangle \neq 0$$

$$N_f = 3 : \sum_{I=1}^{3}\varepsilon_{ijI}\varepsilon^{abI}\langle(\psi_a^i)^T C\gamma^5\psi_b^j\rangle \neq 0$$

[Son], [Schafer et al.], [Hong et al.], [Pisarski et al.], [Shovkovy et al.] (1999)

Properties of 2SC ground state

(**up & down** quarks only)

- Chiral symmetry $SU(2)_L \times SU(2)_R$ is intact
- Color symmetry is broken (by Anderson-Higgs mechanism): $SU(3)_c \to SU(2)_c$
  - color Meissner effect (for 5 gluons)
  - low energy $SU(2)_c$ gluodynamics (decoupled)
- Modified electromagnetic $U(1)_{\widetilde{em}}$ and modified $U(1)_{\widetilde{B}}$ survive
  - no electromagnetic Meissner effect
  - no superfluidity
- Approximate $U(1)_A$ is broken $\to$ light pseudo-NG boson
- Parity is preserved

Signatures of CSC in compact stars

Color superconductivity → **gap** in quasiparticle spectrum

- Thermodynamic properties (equation of state)
  - mass-radius relation [Alford&Reddy,02], [Lugones&Horvath,02]
  - internal star structure [Baldo et al.,02], [Shovkovy et al.,03]
- Transport properties (conductivities, viscosities, mean free paths)
  - cooling rate [Page et al.,02], [Shovkovy&Ellis,02]
  - r-mode instability [Madsen,99]
  - glitches (crystalline phase) [Alford et al.,00]
- Other properties
  - magnetic field generation/penetration [Alford et al.,00]
  - rotational vortices [Iida&Baym,02]

Neutrality vs. color superconductivity

- The "best" 2SC phase appears when $n_d \approx n_u$,
- but neutral matter appears when $n_d \approx 2n_u$
- Electrons do not help (!):

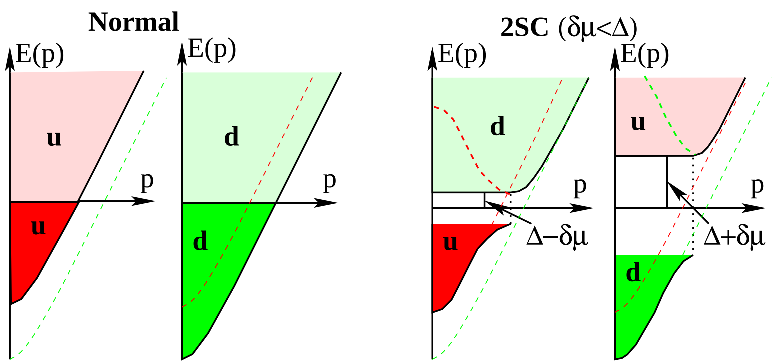
$$n_d \approx 2n_u \;\Rightarrow\; \mu_d \approx 2^{1/3}\mu_u \;\Rightarrow\; \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{Thus, } n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

- Cooper pairing with a mismatch between Fermi surfaces of pairing quarks:

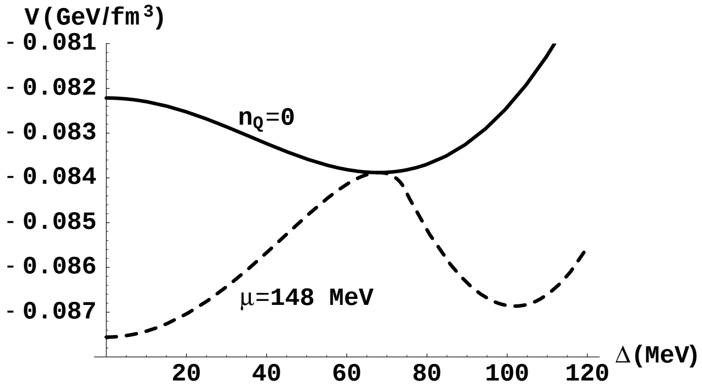
$$\mu_d - \mu_u = \mu_e$$

Gaps: $(\Delta + \mu_e/2)$ and $(\Delta - \mu_e/2)$

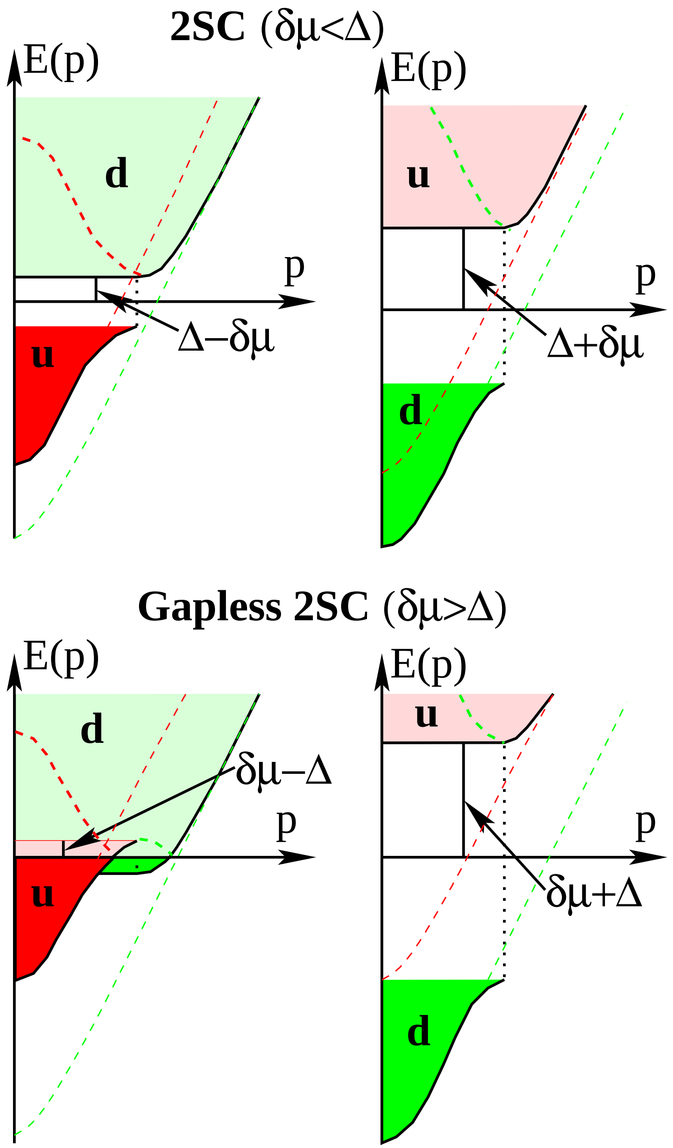


2SC vs. gapless 2SC phase

- Either $\delta\mu < \Delta$, or $\delta\mu > \Delta$
- Gapless 2SC is a *stable* phase of neutral matter in β-equilibrium [I.A.S.&M.Huang,hep-ph/0302142]

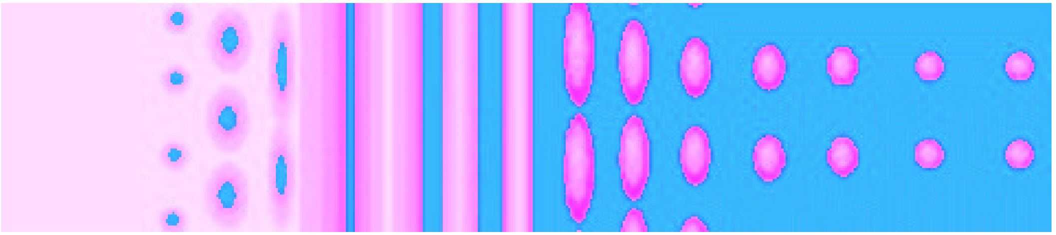


- Extra 2 gapless quasiparticles
- [Gubankova et al, hep-ph/0304016]



Neutral quark phases

- Locally neutral phases:
  - Normal quark matter
  - gapless 2SC matter [Shovkovy&Huang,03]
- Globally neutral mixed phases [Glendenning,92], e.g., 2SC+NQ (?)



$$\rho_e^{(MP)} = \chi_B^A \rho_e^{(A)}(\mu, \mu_e) + (1 - \chi_B^A)\rho_e^{(B)}(\mu, \mu_e) = 0$$

where

$$\chi_B^A \equiv \frac{V^{(A)}}{V^{(A)} + V^{(B)}} \text{ is the volume fraction of phase A}$$

Gibbs construction (2SC+NQ)

- Mechanical equilibrium:

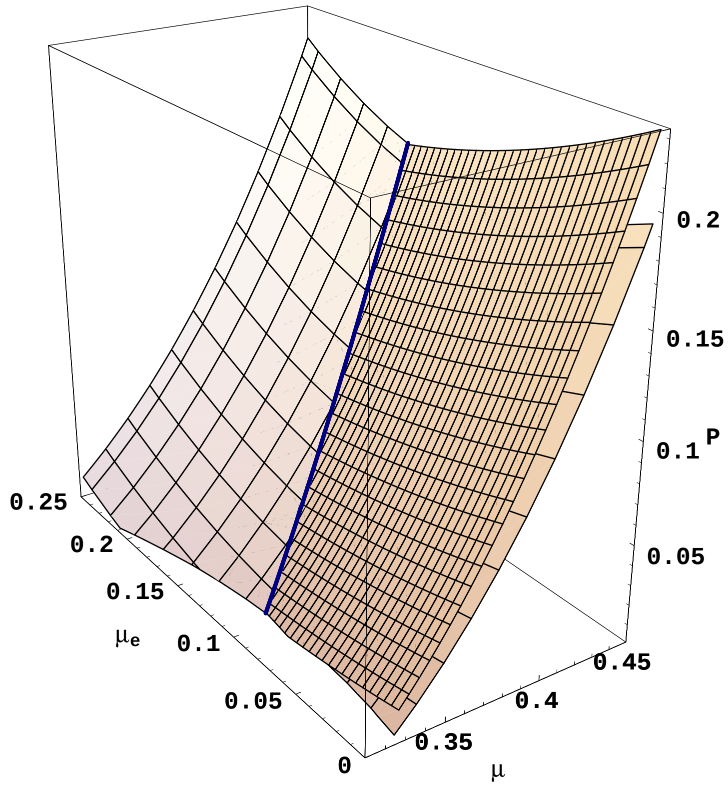
$$P^{(2SC)}(\mu,\mu_e) = P^{(NQ)}(\mu,\mu_e)$$

- Chemical equilibrium:

$$\mu = \mu^{(2SC)} = \mu^{(NQ)},$$
$$\mu_e = \mu_e^{(2SC)} = \mu_e^{(NQ)}$$

- From the condition of neutrality

$$\chi_{NQ}^{2SC} = \frac{\rho_e^{(NQ)}}{\rho_e^{(NQ)} - \rho_e^{(2SC)}},$$



Energy density: $\varepsilon^{(MP)} = \chi_{NQ}^{2SC}\varepsilon^{(2SC)}(\mu,\mu_e) + (1-\chi_{NQ}^{2SC})\varepsilon^{(NQ)}(\mu,\mu_e)$

Coulomb forces and surface tension

- Extra surface and Coulomb energies per unit volume [Heiselberg et al.,93], [Glendenning & Pei,95]

$$\mathcal{E}_S \simeq C_S^{(\text{geom})} \frac{\sigma}{R}, \qquad \mathcal{E}_C \simeq C_C^{(\text{geom})} \left(\rho_e^{(A)} - \rho_e^{(B)}\right)^2 R^2$$

- Minimizing the sum with respect to R, one gets

$$\mathcal{E}_{C+S} \simeq (8 \text{ MeV fm}^{-3}) \left(\frac{\sigma}{\sigma_0} \frac{\rho_e^{(A)} - \rho_e^{(B)}}{\rho_e^{(0)}}\right)^{2/3}, \qquad (\text{“slabs”})$$

where $\sigma_0 = 50$ MeV fm^{-2} and $\rho_e^{(0)} = 0.4e$ fm^{-3}

- Thickness of “slabs”

$$a \simeq (9.4 \text{ fm}) \left(\frac{\sigma}{\sigma_0}\right)^{1/3} \left(\frac{\rho_e^{(0)}}{\rho_e^{(A)} - \rho_e^{(B)}}\right)^{2/3},$$

Coulomb and surface effects in 2SC+NQ matter

Coulomb effects are easy to estimate, while surface tension is usually **not** well known

There are three possible cases:

- Low surface tension ($\sigma \lesssim 20$ MeV fm^{-2}):
  little effect; mixed phase survives ("slabs" with $a \simeq 10$ fm)
- Intermediate values of surface tension ($20 \lesssim \sigma \lesssim 50$ MeV fm^{-2}):
  phase transition occurs at higher densities, $3\rho_0 \lesssim \rho_B \lesssim 5\rho_0$
- Large values of surface tension ($\sigma \gtrsim 50$ MeV fm^{-2}):
  homogeneous phase is more favorable than mixed phase

Similar estimates are valid for hadron-quark mixed phases

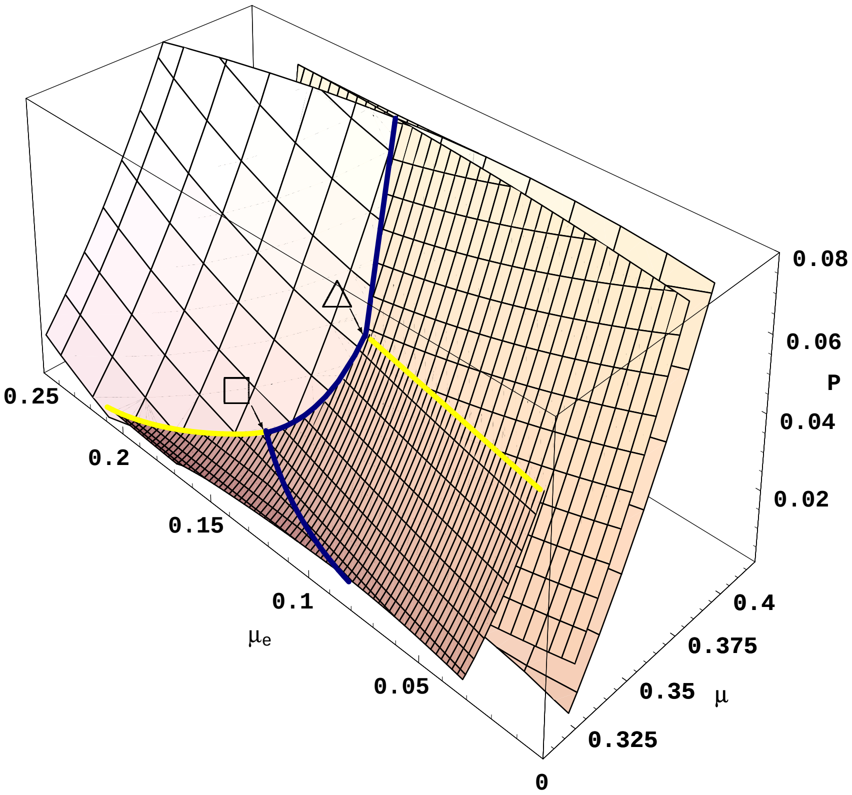
[Heiselberg et al.,01], [Alford et al.,01]

Hadronic matter

- At low densities $\sim \rho_0$ quarks are confined
- Some hadronic description is required
- We use hadronic chiral $SU(3)_L \times SU(3)_R$ model [Papazoglou,98], [Papazoglou,99], [Hanauske,00]
  - nonlinear realization of $SU(3)_L \times SU(3)_R$
  - spontaneous symmetry breaking
  - small explicite symmetry breaking
  - QCD motivated dilaton field is included
- Model describes well hadronic masses, finite nuclei, hypernuclei and neutron star properties

Hybrid matter

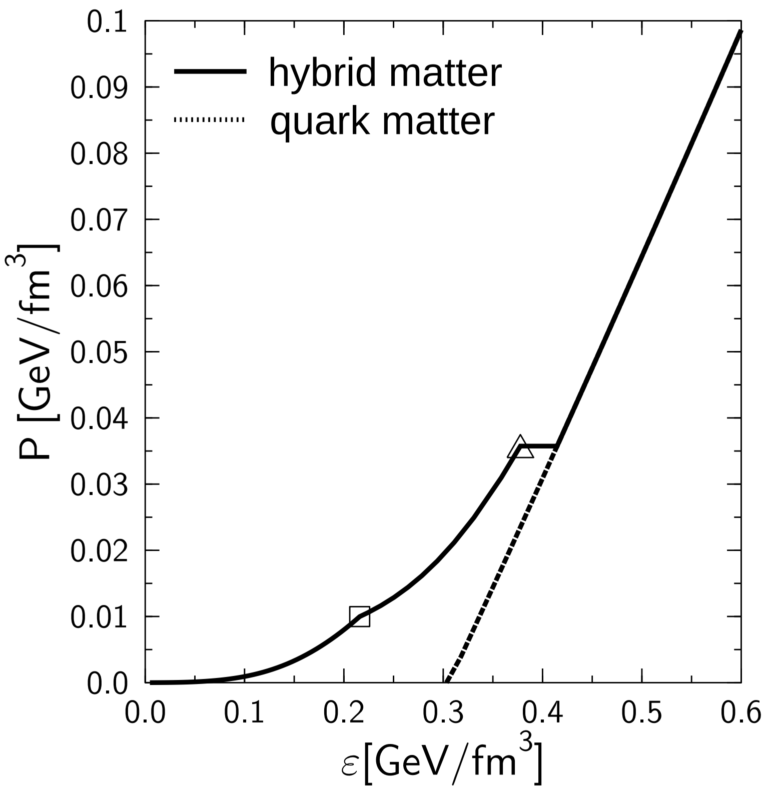
Hadronic phase → Hadron-quark MP → 2SC+NQ quark MP



- Star crust matter:
  $\rho_B \leq 0.08$ fm^{-3} [Baym et al., 71], [Negele & Vautherin, 73]
- Hadronic matter:
  $0.08 \leq \rho_B \leq 1.49$ fm^{-3}
- Hadron-quark mixed phase:
  $1.49 \leq \rho_B \leq 2.56$ fm^{-3}
- 2SC+NQ quark mixed phase:
  $\rho_B \geq 2.75$ fm^{-3}

$\triangle$-point is a triple point (!)

Equation of state



Star structure is determined by metric that satisfies Tolman-Oppenheimer-Volkoff equations

Input: equation of state $P(\varepsilon)$

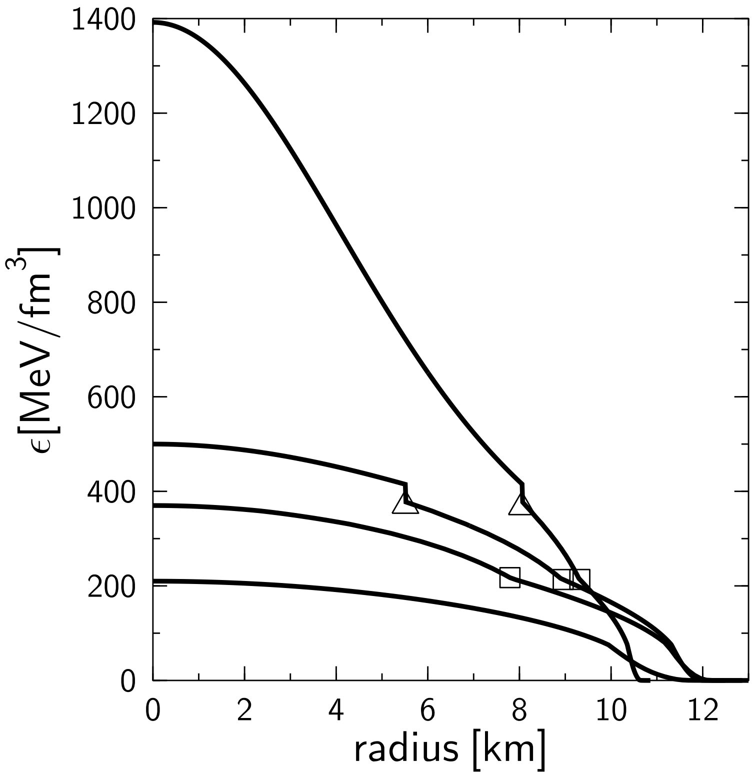
$\square$ – beginning of hadron-quark MP

$\triangle$ – beginning of 2SC+NQ quark MP

- ε and ρ_B have jumps at the triple point

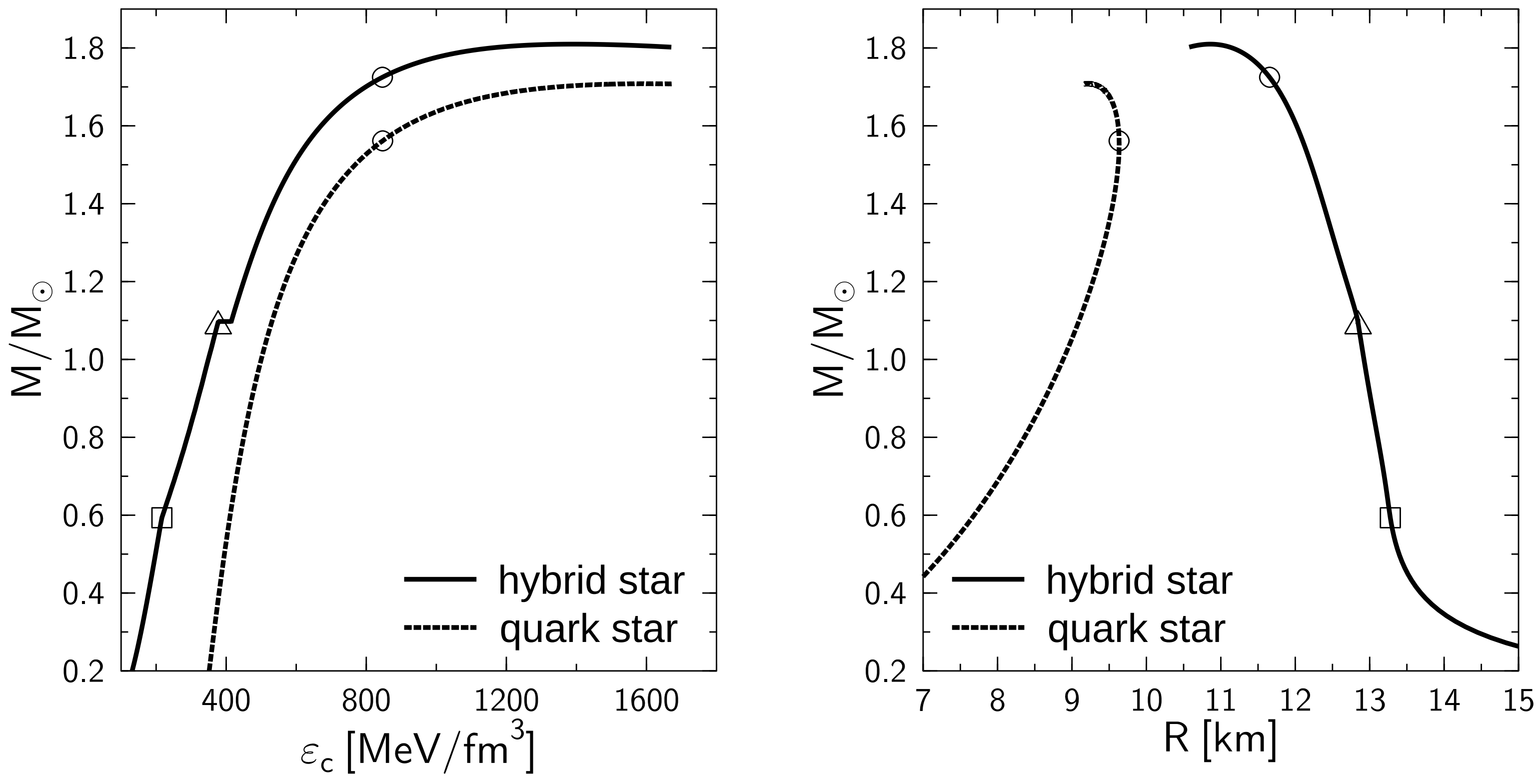
Compact star structure

- $\epsilon_c = 210\ \mathrm{MeV/fm^3}$ – pure hadronic star;
- $\epsilon_c = 370\ \mathrm{MeV/fm^3}$ – hybrid star without quark core;
- $\epsilon_c = 500\ \mathrm{MeV/fm^3}$ – hybrid star with a quark core;
- $\epsilon_c = 1392\ \mathrm{MeV/fm^3}$ – largest mass star with parameters:
  $M_{\max} = 1.81 M_\odot$
  $\rho_c = 7.58\rho_0$
  $R = 10.86$ km



There are no stars with $378 \leq \epsilon_c \leq 415\ \mathrm{MeV/fm^3}$

Mass-radius relation



Stars heavier than "◯-stars" may have strange matter in their cores

Summary

- Realistic EoS of nonstrange hybrid baryon matter is constructed
- Charge neutrality and β-equilibrium are taken into account; they play very important role
- Two-flavor color superconducting matter appears naturally as a component of 2SC+NQ mixed phase
- Construction of 2SC+NQ mixed phase is very stable: volume fractions of components change little with changing density
- The first example of a triple point is obtained and studied
- Sharp interface between the two mixed phases is observed; this is smoothed over distances of about 10 fm

Outlook

- Generalization including strange quarks is needed
- All kinds of hybrid star properties should be studied
  - neutrino emissivity and mean free path
  - cooling rates of mixed phases
  - magnetic properties
- Surface tension effects and screening of Coulomb forces should be addressed
- Studies of color superconductivity in rotating stars are of interest
- Search for signatures of color superconductivity in compact stars should be made systematic