Gapless superconductivity —
from quark matter to atomic gases

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References

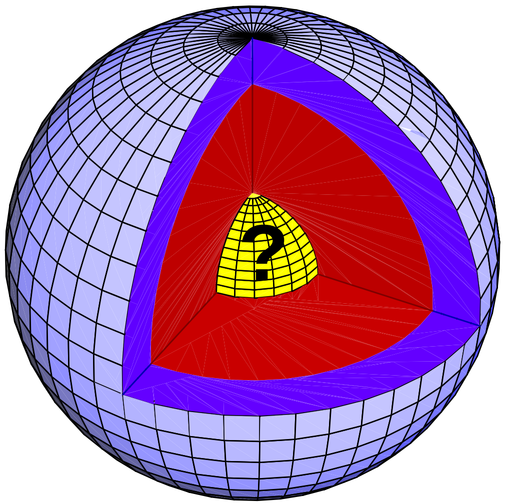
- I. Shovkovy and M. Huang, Phys. Lett. B **564** (2003) 205, `hep-ph/0302142`
- M. Huang and I. Shovkovy, Nucl. Phys. A **729** (2003) 835, `hep-ph/0307273`

Matter at high density

We study this because we need to understand

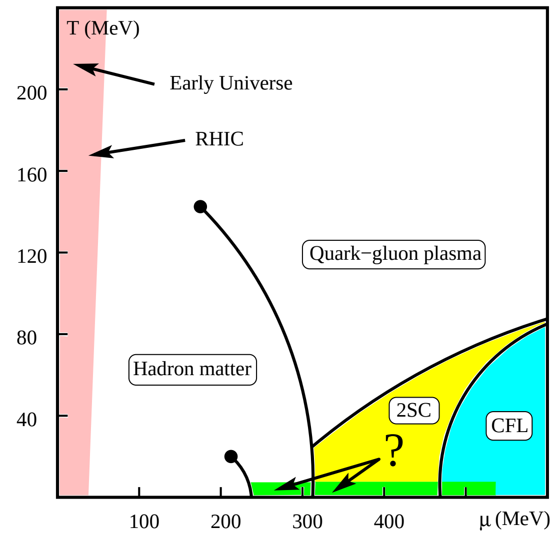
(i) properties of dense matter that exists in the Universe

(densities in stars $\rho_c \gtrsim 5\rho_0$)



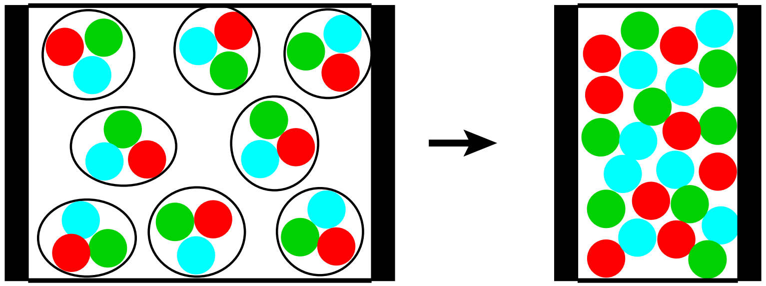
(ii) fundamental properties of QCD

($\mu_q \gg \Lambda_{QCD}$: no lattice results)



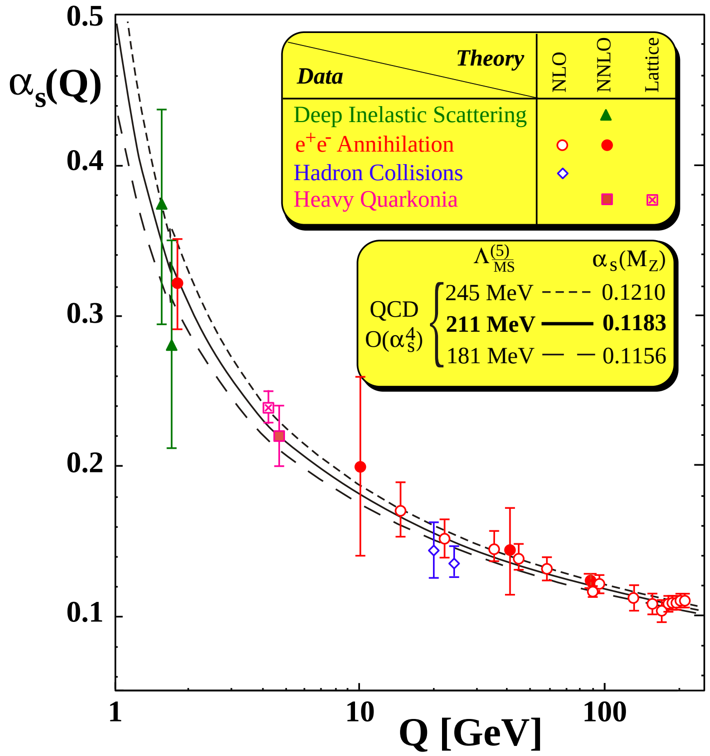
Dense quark matter

- "Squeezing" baryonic matter hard should produce quark matter:



- Very dense quark matter is **weakly** interacting [Collins&Perry,'75]
- Asymptotic freedom: $\alpha_s(\mu) \ll 1$

  $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,'73]



Unfortunately, realistic densities in stars are not very large:

$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3}$$

Ground state of dense quark matter

Educated guess:

(i) Quarks are fermions ($s = \frac{1}{2}$)
(ii) Interaction is weak ($\alpha_s \ll 1$)
$\Rightarrow$ Fermi liquid (?)

(cf., electron gas in metals/alloys)

Further refinement:

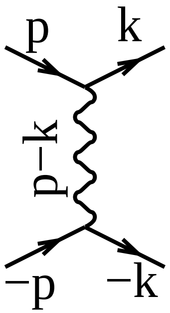
(i) Degenerate Fermi surface
(ii) Interaction is nonzero
$\Rightarrow$ Cooper instability (?)

(cf., the Cooper instability in superconducting metals/alloys)

Color superconductivity

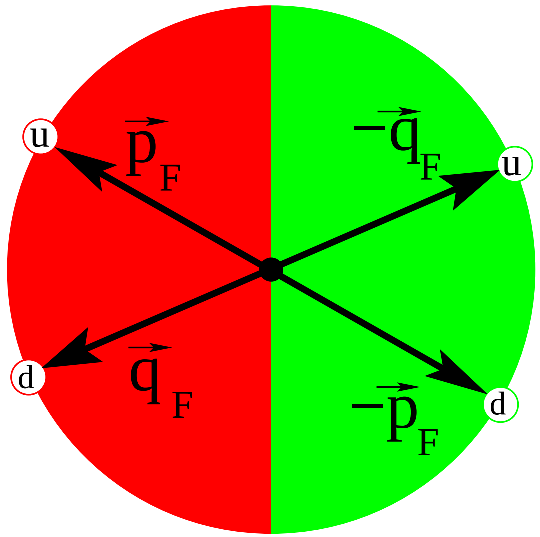
Simplest case, 2SC [Barrois,'78; Bailin&Love,'84]

- $N_f = 2$: "up" and "down" quarks
- $N_c = 3$: "red", "green" and "blue"
- $p_F^{\rm up} = p_F^{\rm down} = \mu_q$
- Quark-quark interaction:



$$= \bar{3}_a + 6_s$$

$$\langle u_p\, d_{-p} \rangle = -\langle u_q\, d_{-q} \rangle \neq 0$$



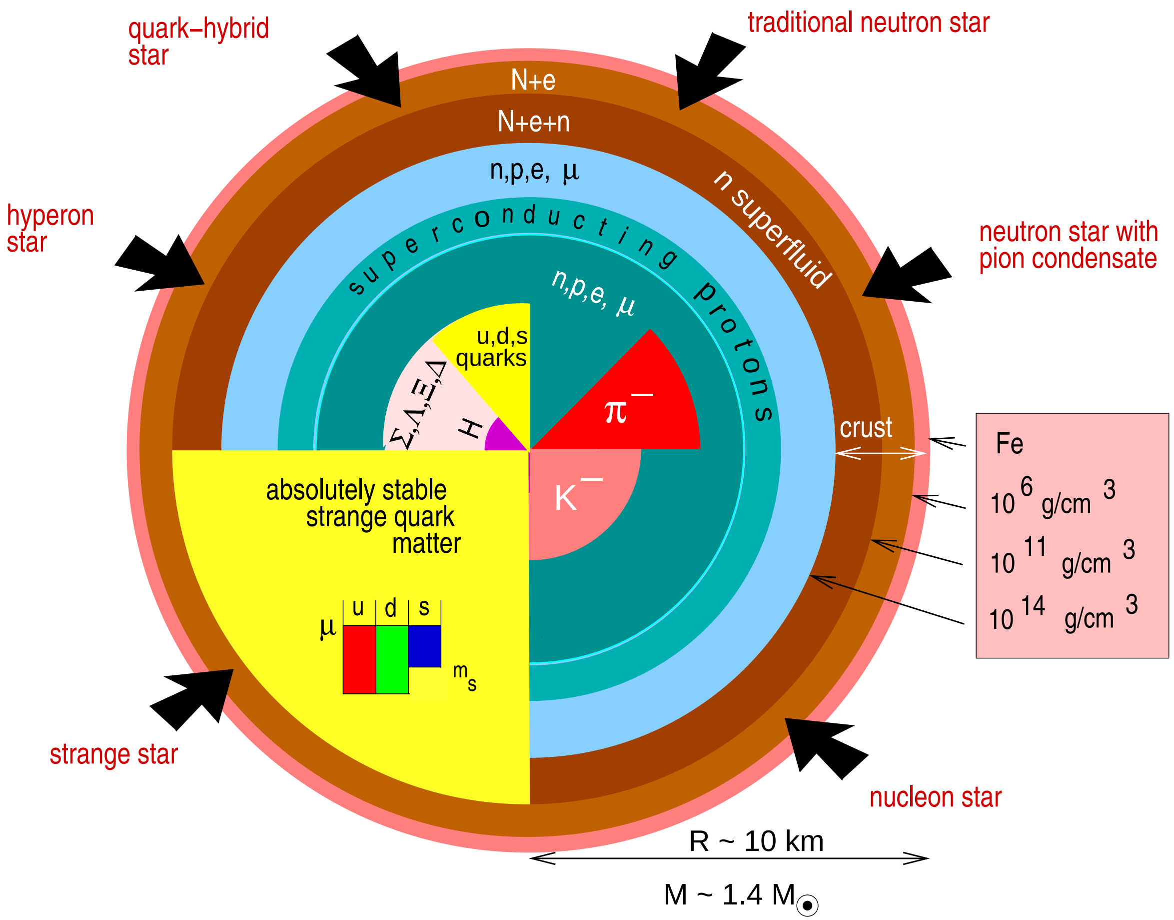
Cooper instability → color superconductivity

$$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle)$$

Properties of 2SC ground state

- Chiral symmetry $SU(2)_L \times SU(2)_R$ is intact
- Color symmetry is broken (by Anderson-Higgs mechanism): $SU(3)_c \to SU(2)_c$
  - color Meissner effect (for 5 gluons)
  - low energy $SU(2)_c$ gluodynamics (3 gluons, decoupled)
- Modified electromagnetic $U(1)_{\widetilde{\rm em}}$ and modified $U(1)_{\widetilde{B}}$ survive
  - no electromagnetic Meissner effect
  - no superfluidity
- Two ("blue-up" and "blue-down") quarks remain gapless
- Approximate $U(1)_A$ is broken $\to$ light pseudo-NG boson
- Parity is preserved

Neutron, quark or hybrid stars



[figure is taken from a talk by F. Weber's]

Is there SC inside stars?

The answer is: **we do not know yet**

Arguments in favor:

- Relatively high densities in stars, $\rho_c \gtrsim 5\rho_0$, suggest that quarks may be deconfined
- An attractive diquark channel is likely to exist
- Temperatures are quite low, $T \lesssim 50$ keV, to allow pairing

Arguments against:

- Strongly coupled dynamics is not under control
- Matter may not necessarily be deconfined at existing densities
- Specific conditions inside stars (e.g., β-equilibrium) may not favor color superconductivity

The natural approach: **To give predictions and to test them**

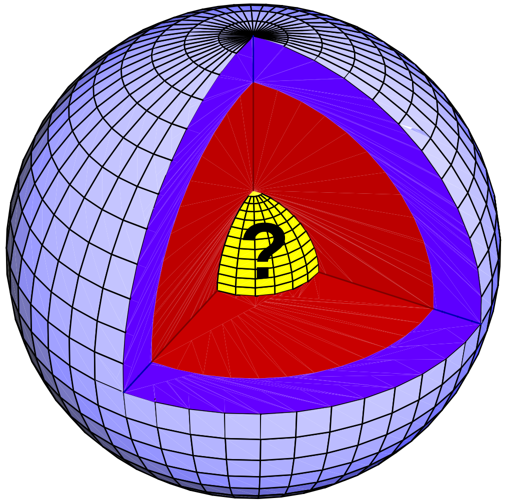
Specific conditions inside stars

Matter in the bulk of a star is

(i) β-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:
$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$

Otherwise, a star would **not** be stable!



- Coulomb energy (when $n_Q \neq 0$)

$$E_{\rm Coulomb} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/{\rm fm}^3}\right)^2 \left(\frac{R}{1\ {\rm km}}\right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/{\rm fm}^3 \quad \Rightarrow \quad E_{\rm Coulomb}^{\rm 2SC} \gg M_\odot c^2$

Neutrality vs. color superconductivity

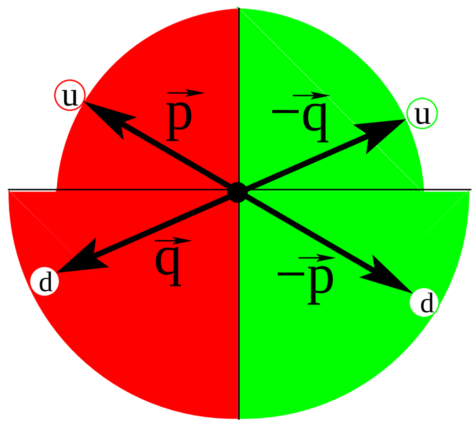
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in β-equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \quad \Rightarrow \quad \mu_d \approx 2^{1/3}\mu_u \;\Rightarrow\; \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{i.e.,} \qquad n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

The "best" Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Mismatch vs. coupling strength

Mismatch parameter μ_e is **not** a free model parameter,

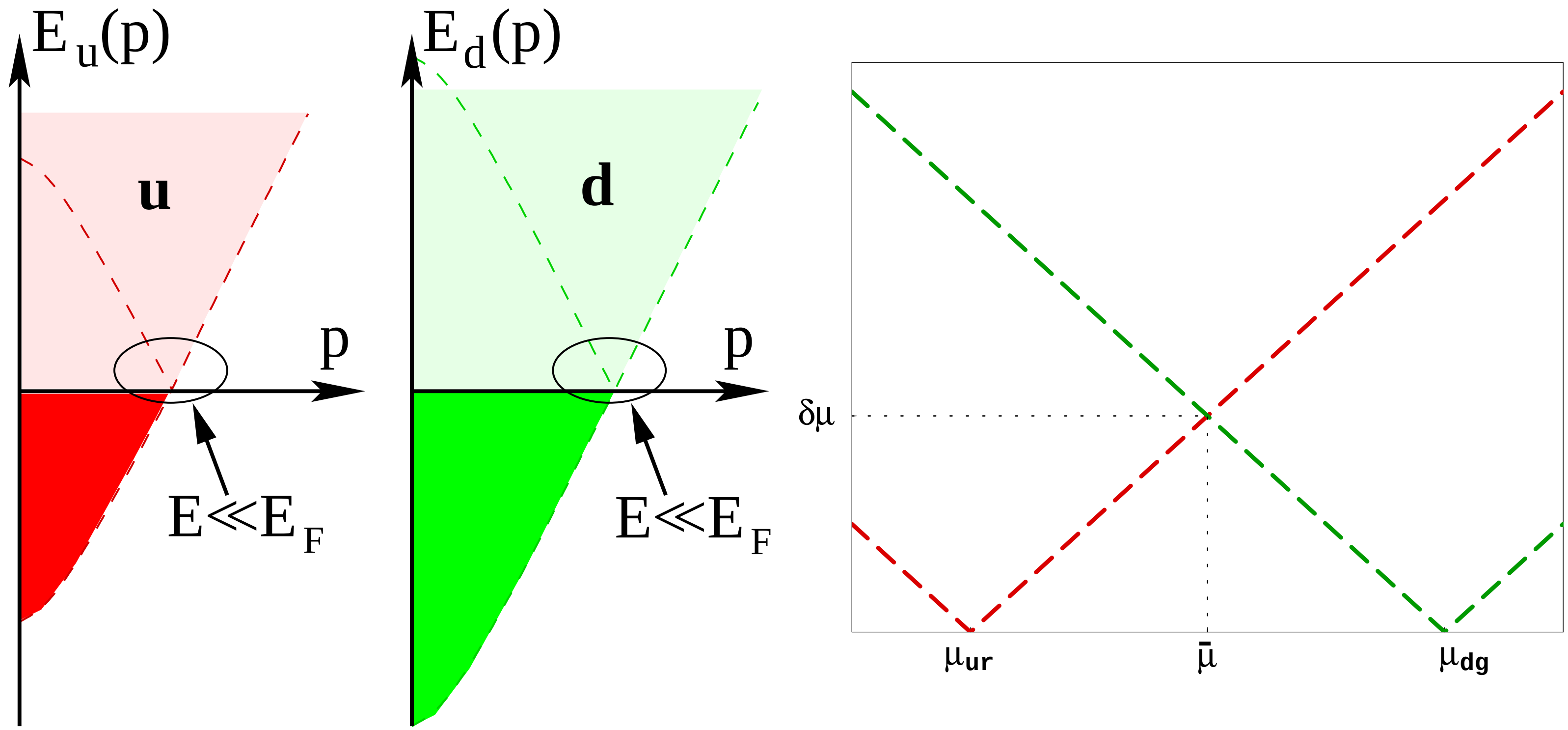
$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \qquad \Rightarrow \qquad \mu_e = \mu_e(\bar{\mu}_q, \Delta)$$

But the diquark coupling strength (η) **is** a model parameter:

1. $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing: Normal phase is the ground state
2. $\eta \gtrsim 0.8$ — strong coupling wins over a mismatch between the Fermi surfaces: 2SC is the ground state
3. $0.7 \lesssim \eta \lesssim 0.8$ — regime of intermediate coupling. The ground state is a new gapless color superconducting phase.

Quasiparticle spectrum in normal phase

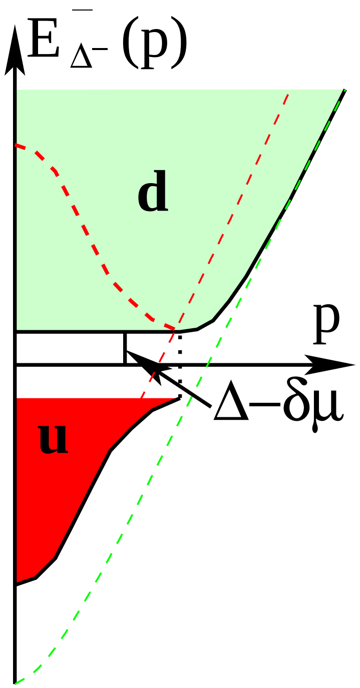
"Weak" coupling (normal phase)

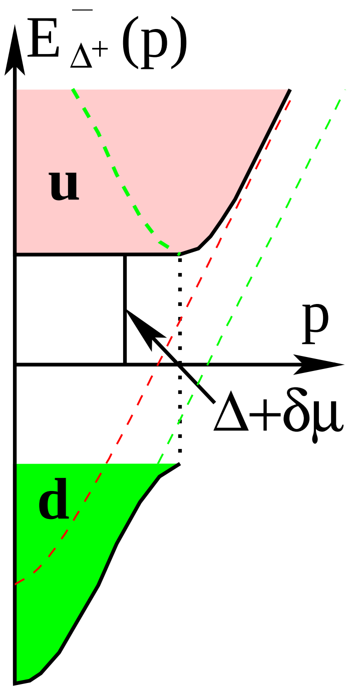


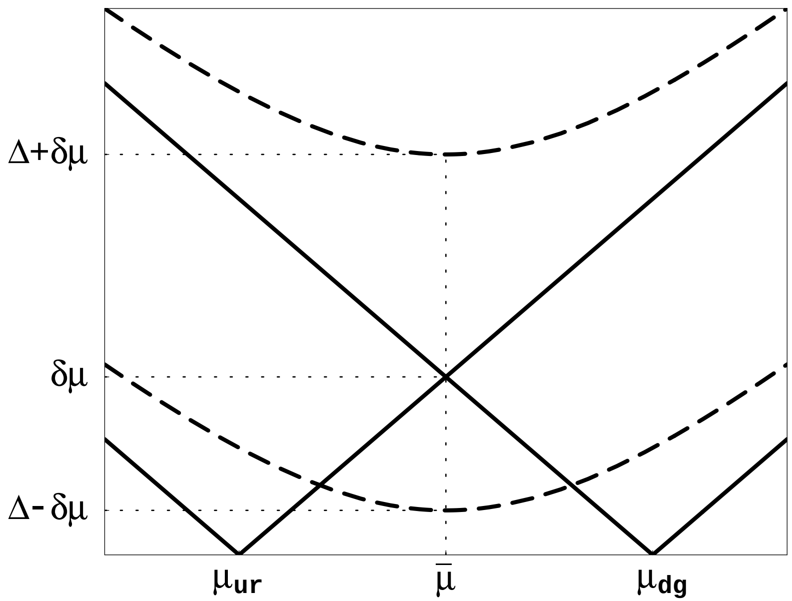
How does this spectrum change when Cooper pairs are formed?

Quasiparticle spectrum in 2SC phase

“Strong” coupling (2SC phase)



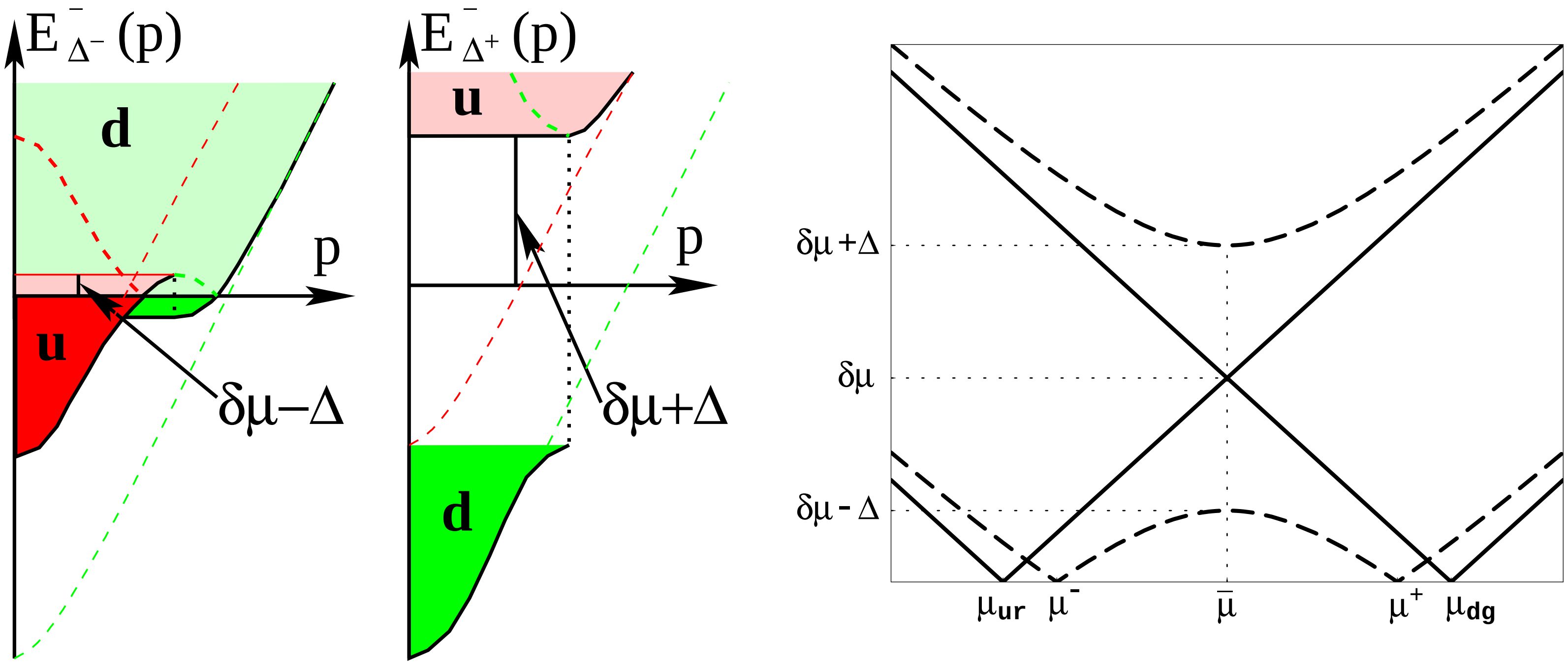




The energy gaps in the quasiparticle spectra are $\boxed{\Delta - \delta\mu}$ & $\boxed{\Delta + \delta\mu}$

Quasiparticle spectrum in g2SC phase

"Intermediate" coupling (gapless phase)



The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Sarma phase in condensed matter

Type II superconductors in a constant magnetic field:

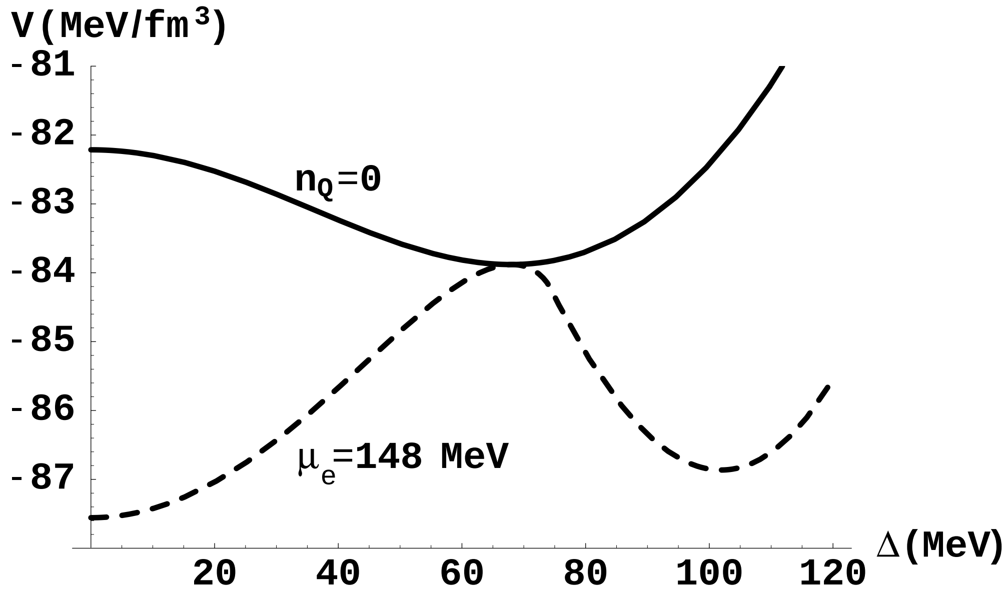
[G. Sarma, J. Phys. Chem. Solids **24** (1963) 1029.]

- Magnetic field originates from ferromagnetic order of impurities in $La_{1-x}Gd_x$ and $Y_{1-x}Gd_xOs_2$ [B.Matthias,H.Suhl & Corenzwit, Phys. Rev. Lett. **1** (1958) 92], [N.Phillips,B.Matthias, Phys. Rev. **121** (1961) 105]
- Pairing happens between spin-$\uparrow$ and spin-$\downarrow$ holes/electrons
- Fermi momenta of $\uparrow$- and $\downarrow$-quasiparticles are different
- The mismatch parameter $\delta\mu \sim H \sim n_{\text{impurity}}$

The gapless "Sarma" phase is **unstable**!

Stability of g2SC phase

Effective potential at $T = 0$:

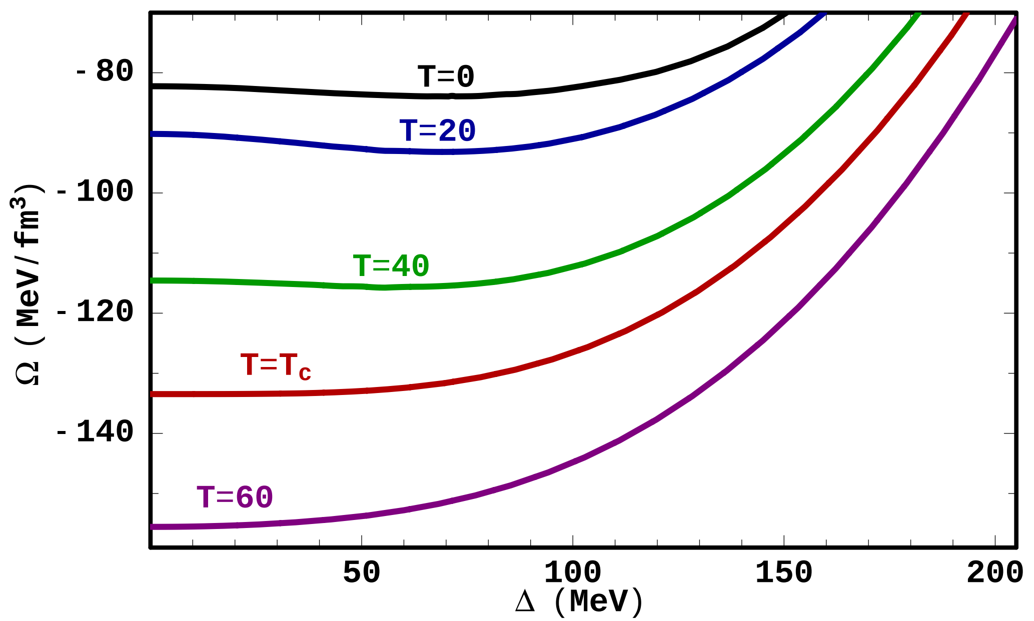


(Mixed phase? — unlikely if $\sigma \gtrsim 20\ \mathrm{MeV/fm^2}$)

Thus, g2SC is stable provided $n_Q = 0$ is enforced *locally*

Finite temperature case

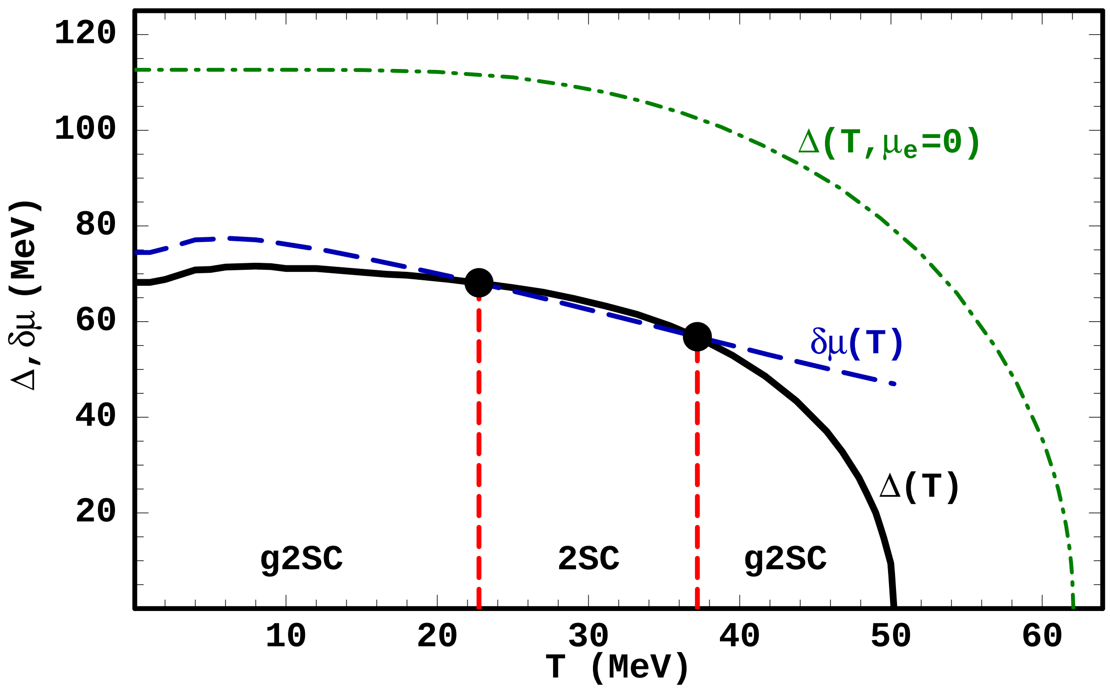
Effective potential at $T \neq 0$:



(Note: 2nd order phase transition)

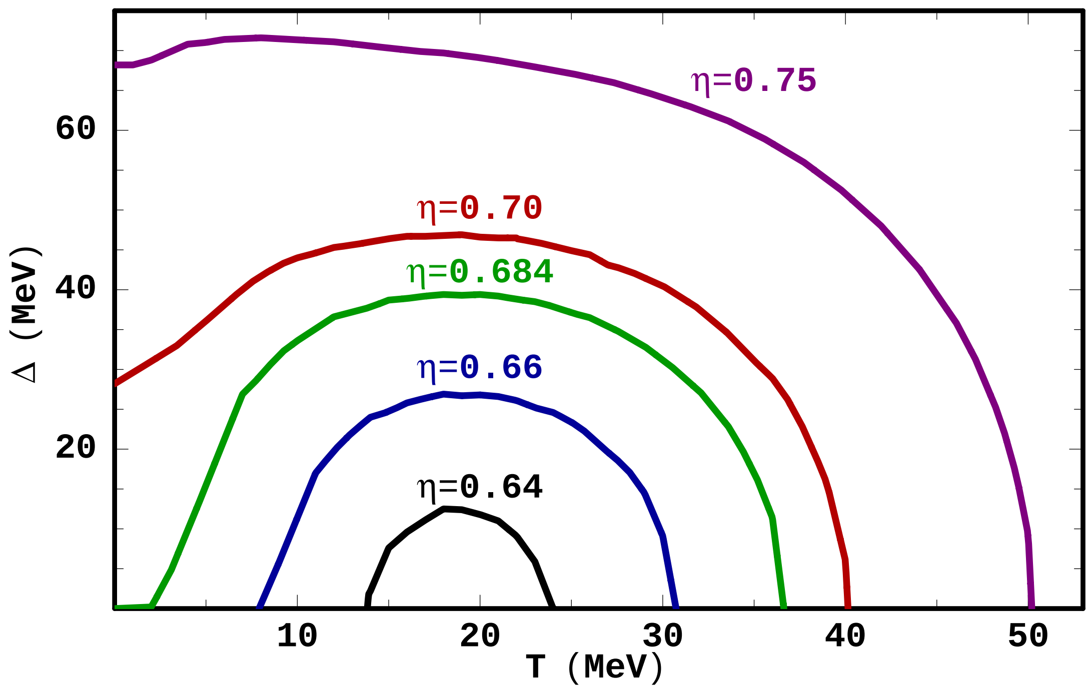
The low energy action is not of the Ginsburg-Landau type

Finite temperature properties



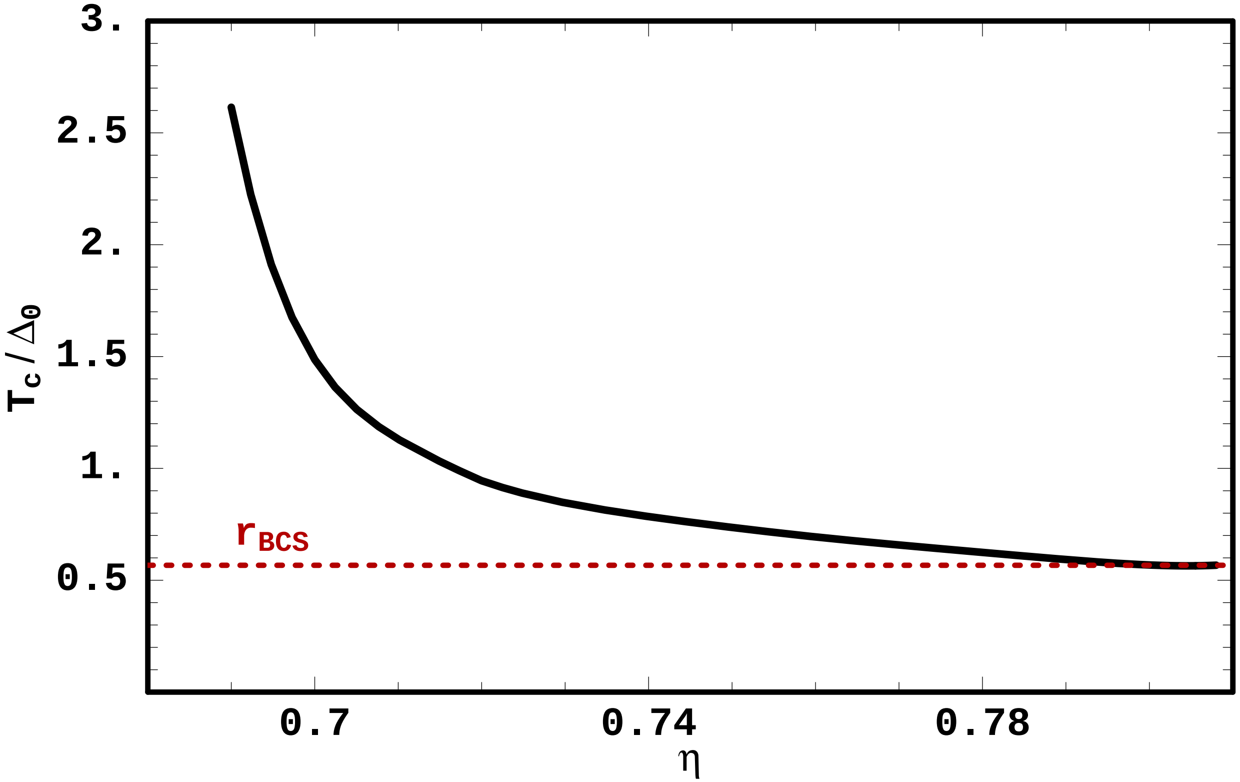
- *Nonmonotonic* temperature dependence
- Transitional behavior: g2SC $\to$ 2SC $\to$ g2SC $\to$ normal phase

Temperature dependence of the gap



- Extreme *nonmonotonic* temperature dependence
- Transitional behavior: normal phase → g2SC → normal phase

Nonuniversal ratio T_c/Δ_0

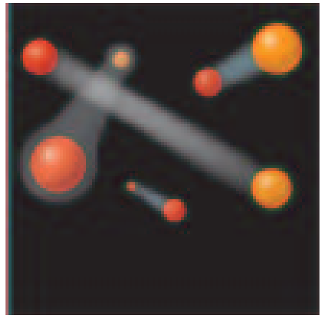


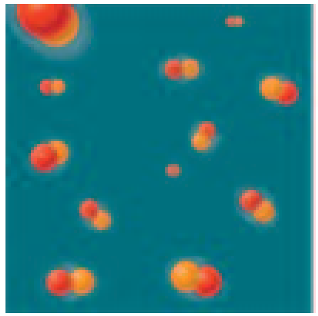
- The ratio is *not universal* (unlike in BCS)
- The value of T_c/Δ_0 can be *arbitrarily* large

Gapless superfluidity (?)

Neutron stars are very far and very hard to study.

However, one might be able to study **gapless superfluidity** in table-top experiments on ultracold atoms.

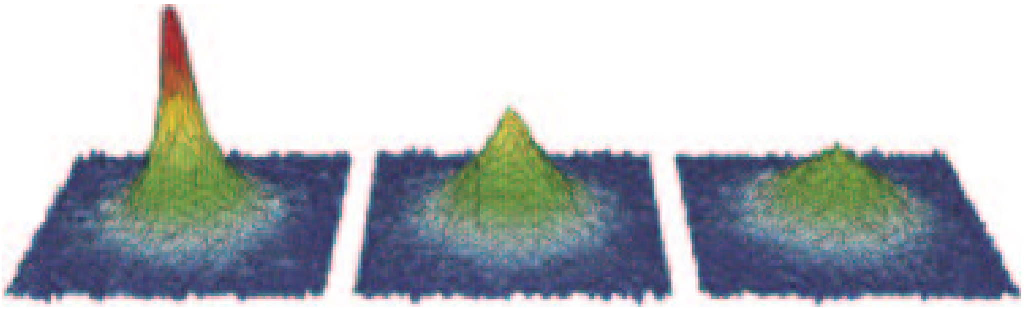




> "A much-anticipated atomic soup might lay bare the inner workings of high-temperature superconductors, neutron stars, and primordial matter – and perhaps win its creator a Nobel Prize" ["Ultracold Atoms Spark a Hot Race", Science **301** (2003) 750]

<u>6 teams in the race:</u> (1) D. Jin in Boulder, (2) R. Hulet at Rice U., (3) C. Salomon at École Normale Supérieure, (4) J. Thomas at Duke U., (5) W. Ketterle at MIT, (6) M. Ingucio in Florence U.

First experimental results



"Condensation of fermionic atom pairs near the BEC-BCS crossover regime, where Bose-Einstein condensation changes over into condensation of Cooper pairs."

Jan 30, 2004: [C. Regal, M. Greiner & D. Jin, Phys. Rev. Lett. **92** (2004) 040403]

System: **equal numbers** of $|\frac{9}{2},-\frac{9}{2}\rangle$ and $|\frac{9}{2},-\frac{7}{2}\rangle$ spin states of ^{40}K

Interaction between atoms is tuned by changing a magnetic field in the vicinity of the Feschbach resonance ($B \approx 202$ G)

Gapless superfluidity should appear when $N_{|\frac{9}{2},-\frac{9}{2}\rangle} \neq N_{|\frac{9}{2},-\frac{7}{2}\rangle}$

[W.Liu & F.Wilczek, Phys. Rev. Lett. **90** (2003) 047002]

Toward gapless superfluidity in atomic gases

How to prepare?

By applying an appropriate pulse of a microwave frequency to the trapped mixture of fermion atoms

Difficulty: cooling may not be efficient when the setting is such that

$$N_{|\frac{9}{2},-\frac{9}{2}\rangle} \neq N_{|\frac{9}{2},-\frac{7}{2}\rangle}$$

How to detect?

- Time-of-flight expansion images. [K. Davis, et al., PRL **75** (1995) 3969]
- Pairwise projection of fermionic atoms onto molecules. [C. Regal, M. Greiner & D. Jin, PRL **92** (2004) 040403]
- Stimulated scattering of photons (?) [B. Deb, A. Mishra, H. Mishra, P. Panigrahi, cond-mat/0308369]

Summary

- The g2SC phase is a new state of matter that may exist in cores of compact stars
- The g2SC phase is stable if the neutrality is enforced locally
- The spectrum of low-energy excitations in the g2SC phase has extra gapless modes (these should affect transport properties)
- Finite temperature properties of the g2SC phase are rather unusual [$\Delta(T)$ is nonmonotonic; T_c/Δ_0 is nonuniversal]
- A gapless phase of $N_f = 3$ quark matter is also possible
- Similar gapless phases may appear in trapped cold gases of fermionic atoms (e.g., ^{6}Li and ^{40}K)

The last word

There is a lot of

new physics out there

to be learned

Additional references

[1] Unstable Sarma phase:

G. Sarma, J. Phys. Chem. Solids **24** (1963) 1029.

[2] Asymmetric nuclear matter:

A. Sedrakian and U. Lombardo, Phys. Rev. Lett. **84** (2000) 602.

[3] Interior gap superfluidity (atomic gases):

W. Liu, F. Wilczek, Phys. Rev. Lett. **90** (2003) 047002.

[4] Breached pairing superfluidity:

E. Gubankova, W. Liu, F. Wilczek, Phys. Rev. Lett. **91** (2003) 032001.

[5] Gapless color flavor locked phase:

M. Alford, C. Kouvaris, K. Rajagopal, hep-ph/0311286.