Phases of high baryon density QCD

ABC of color superconductivity

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Outline

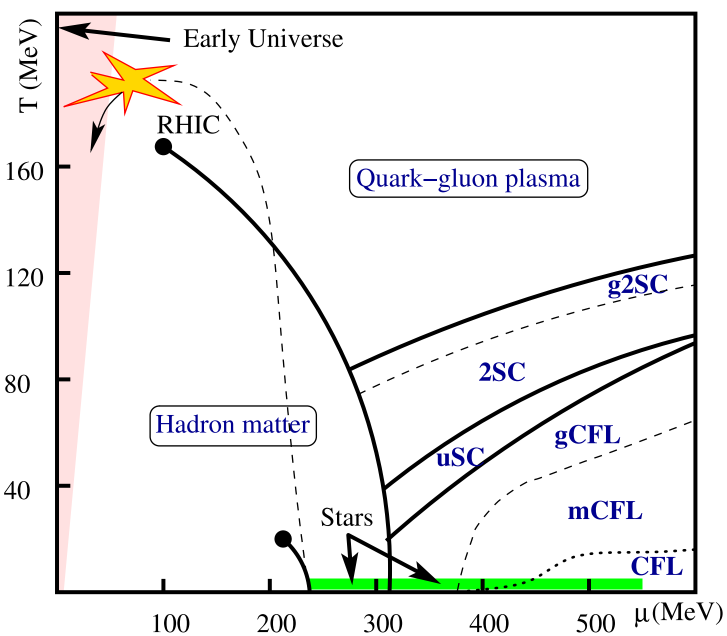
- General introduction to color superconductivity
  - Dense baryon matter in nature
  - Cooper instability and ground state of dense matter
- $N_f = 1$, $N_f = 2$ and $N_f = 3$ color superconductivity
  - Symmetry of ground state
  - General properties
- Charge neutrality and β-equilibrium in quark matter
  - Gapless color superconductivity
  - Effects of non-zero strange quark mass
  - $T - \mu$ phase diagram of dense QCD
- Concluding remarks

Matter at high density

We study this because we need to understand

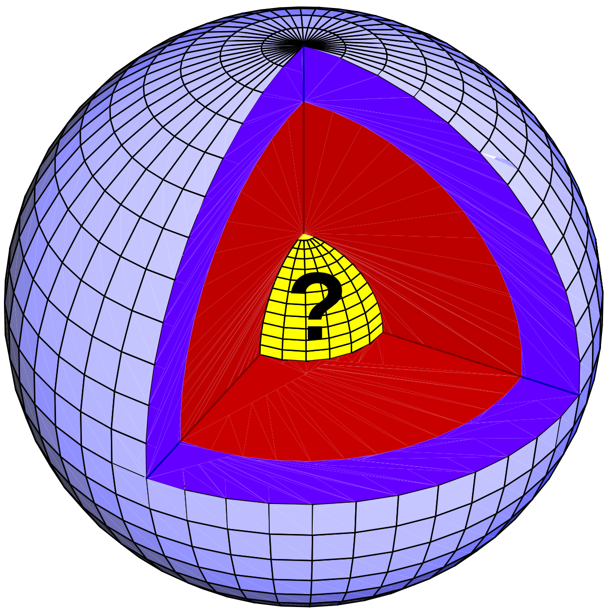
(i) Phase diagram/fundamental properties of QCD

($\mu_q \gtrsim \Lambda_{QCD}$: no lattice results)

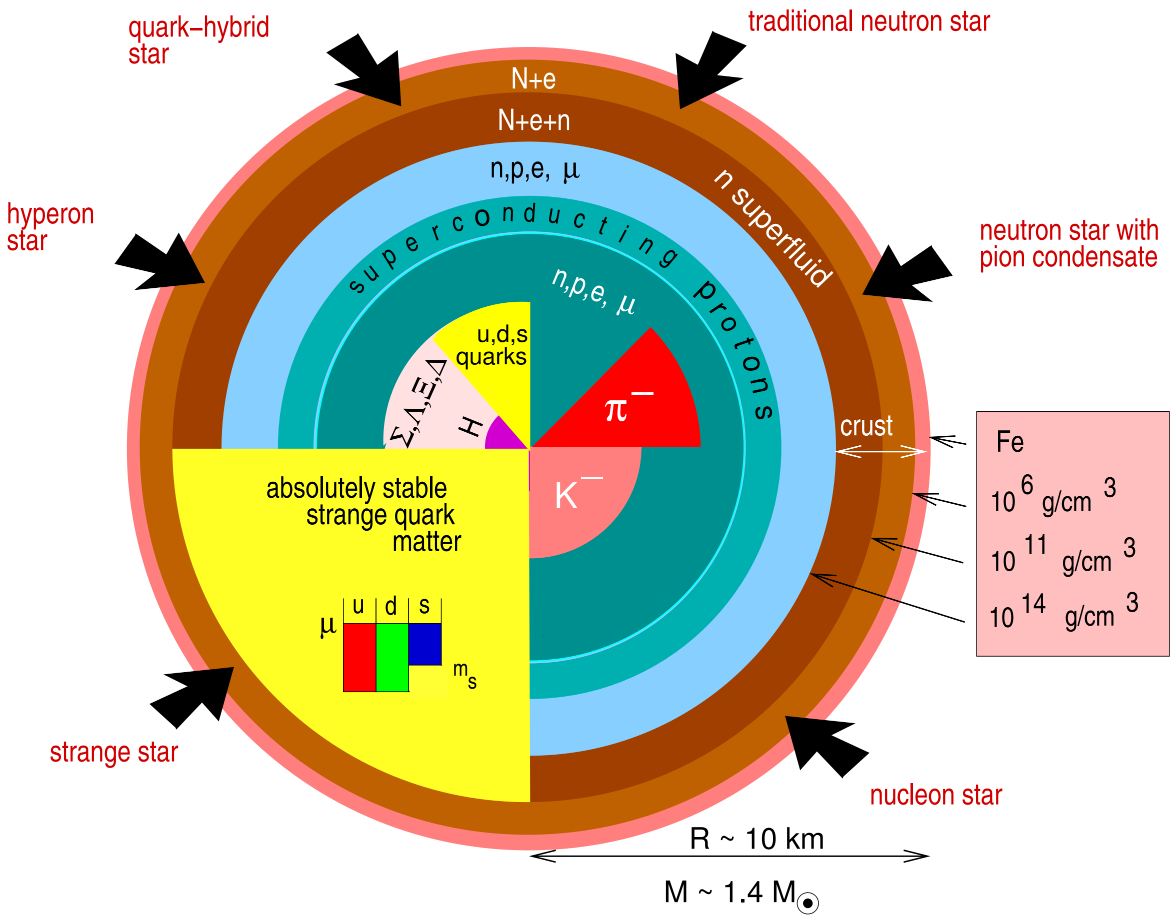


(ii) properties of dense matter that exists in the Universe

(densities in stars $\rho_c \gtrsim 5\rho_0$)



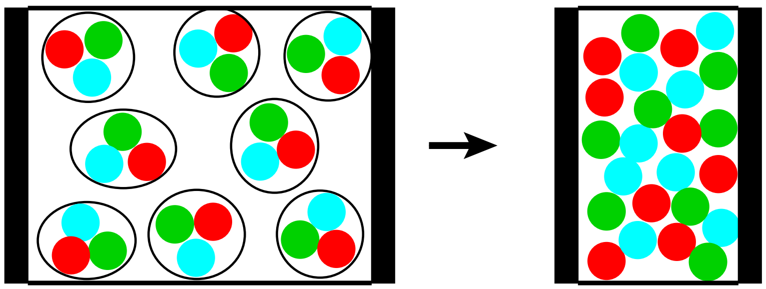
Neutron, quark or hybrid stars



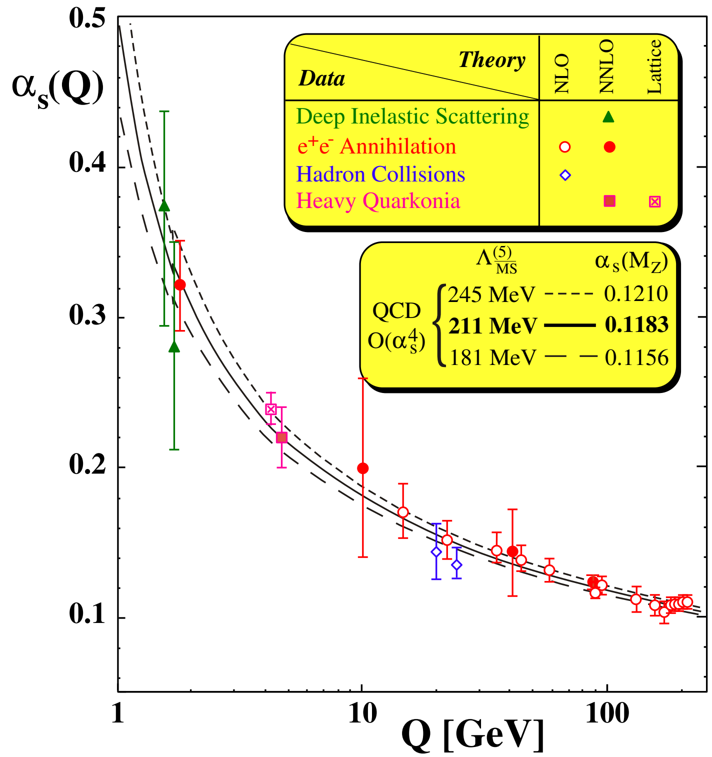
[figure is taken from a talk by F. Weber's]

Dense quark matter

- "Squeezing" baryonic matter hard should produce quark matter:



- Very dense quark matter is **weakly** interacting [Collins&Perry,'75]
- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
  $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,'73]



Unfortunately, realistic densities in stars are not very large:

$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3}$$

What is the ground state of dense quark matter?

Educated guess:

(i) Quarks are fermions ($s = \frac{1}{2}$)
(ii) Interaction is weak ($\alpha_s \ll 1$)

$\Rightarrow$ Fermi liquid (?)

(cf., electron gas in metals/alloys)

Further refinement:

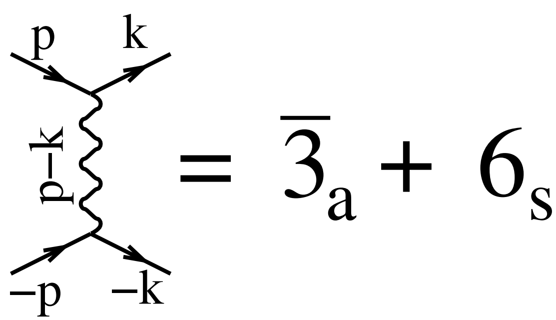
(i) Degenerate Fermi surface
(ii) Interaction is nonzero

$\Rightarrow$ Cooper instability (?)

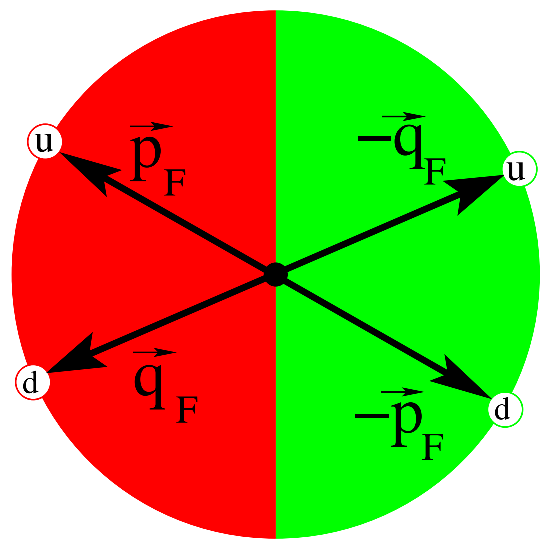
(cf., the Cooper instability in superconducting metals/alloys)

Attractive diquark interaction

- One-gluon exchange ($\mu_q \gg \Lambda_{QCD}$) [Barrois,'78; Bailin&Love,'84]:



$$\langle u_p\, d_{-p} \rangle = - \langle u_q\, d_{-q} \rangle \neq 0$$



- 't Hooft (instanton-induced) interaction ($\mu_q \ll \Lambda_{QCD}$) [Alford et al, hep-ph/9711395] & [Rapp et al, hep-ph/9711396]:

$$\mathcal{L}_{\text{eff}} \sim \frac{1}{N_c^2(N_c-1)} \left(\psi^T C \tau_2 \lambda_A^a \psi\right) \left(\bar{\psi} \tau_2 \lambda_A^a C \bar{\psi}^T\right) + \cdots$$

Color superconductivity, $N_f = 1$

- Wave function of a Cooper-pair: $\left(|\textcolor{red}{\bullet}\textcolor{green}{\bullet}\rangle - |\textcolor{green}{\bullet}\textcolor{red}{\bullet}\rangle\right)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$
  - antisymmetric in color (attractive diquark $\bar{3}_c$ channel)
  - Pauli principle: symmetric in spin, i.e., spin-1 channel
- Diquark condensate (gap $\sim$ 10 - 100 keV):

$$\langle \left(\bar{\Psi}^C\right)^a \gamma_5 \Psi^b \rangle \simeq \varepsilon^{abc} \Delta_{c\delta} \left(\alpha\, \hat{\mathbf{k}}^\delta + \beta\, \gamma_\perp^\delta(\vec{\mathbf{k}})\right)$$

- Many possibilities, see Ph.D. thesis [A.Schmitt, nucl-th/0405076]:
  - Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
  - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta}$
  - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
  - A-phase: $\Delta_{c\delta} = \delta_{c3}\left(\delta_{\delta 1} + i\delta_{\delta 2}\right) \rightarrow$ largest pressure
- Meissner effect $\oplus$ type I superconductor $\rightarrow$ affect star properties

Color superconductivity, $N_f = 2$

- Wave function of a Cooper-pair:
  Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|\mathrm{u,d}\rangle - |\mathrm{d,u}\rangle)_1$
- Diquark condensate (spin-0 gap $\sim$ 10 - 100 MeV):

$$\langle \left(\bar{\Psi}^C\right)_i^a \gamma_5 \Psi_j^b \rangle \simeq \varepsilon^{ab3} \epsilon_{ij} \Delta$$

- Symmetry of ground state (2SC phase):
  - chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ — intact
  - baryon number $\mathrm{U}(1)_B \to \tilde{\mathrm{U}}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}} T_8$
  - approximate axial $\mathrm{U}(1)_A$ is broken $\to$ 1 pseudo-NG boson
  - Color gauge symmetry $\mathrm{SU}(3)_c \to \mathrm{SU}(2)_c$ by Anderson-Higgs mechanism $\to$ 5 massive gluons
  - Gauge symmetry $\mathrm{U}(1)_{\mathrm{em}} \to \tilde{\mathrm{U}}(1)_{\mathrm{em}}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$

Physical properties of 2SC phase

- Pressure/equation of state:

$$P \simeq \frac{\mu^4}{2\pi^2} - B + \boxed{\frac{\mu^2 \Delta^2}{\pi^2}} \qquad \text{may be important}$$

- Transport/specific heat is dominated by
  - Unpaired "blue-up" and "blue-down" quarks
  - 1 pseudo-NG boson that results from breaking $U(1)_A$
  - 3 gluons of unbroken $\mathrm{SU}(2)_c$ (decoupled from blue quarks)
  - low energy photon of $\tilde{\mathrm{U}}(1)_{\mathrm{em}}$
- No superfluidity $\rightarrow$ no rotational vorticies
- No electromagnetic Meissner effect $\rightarrow$ no magnetic flux tubes
- Neutrino emissivity/cooling rate is large (direct URCA)

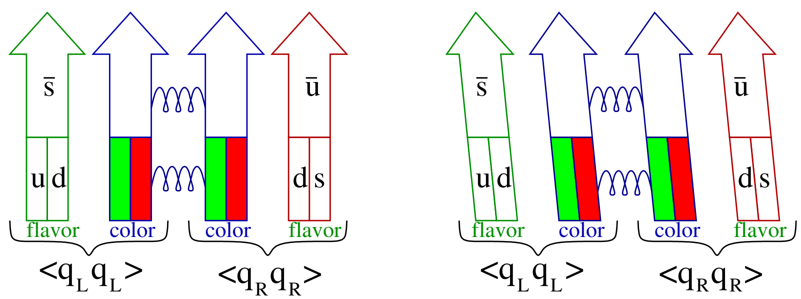
Color superconductivity, $N_f = 3$

- Wave function of a Cooper-pair:
  Pauli principle: $(|\color{red}\bullet\color{green}\bullet\rangle - |\color{green}\bullet\color{red}\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|\mathrm{u,d}\rangle - |\mathrm{d,u}\rangle)_{\bar{3}}$

- Diquark condensate (spin-0 gap $\sim$ 10 - 100 MeV):

$$\langle \left(\bar{\Psi}_L^C\right)_i^a \left(\Psi_L\right)_j^b \rangle = \langle \left(\bar{\Psi}_R^C\right)_i^a \left(\Psi_R\right)_j^b \rangle \simeq \sum_I \varepsilon^{abI} \epsilon_{ijI} \Delta$$

- Color-flavor locking [Alford, Rajagopal, Wilczek, hep-ph/9804403]



There are no $\langle q_L q_R \rangle$ condensates, but SU(3)$_L \times$ SU(3)$_R$ chiral symmetry is broken through locking with color!

Color superconductivity, $N_f = 3$

- Symmetry of ground state (CFL phase):
  - chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$ $\rightarrow$ 8 (pseudo-)NG bosons, i.e., π^0, $\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$, η
  - baryon number $U(1)_B$ is broken $\rightarrow$ 1 NG boson (ϕ)
  - approximate axial $U(1)_A$ is broken $\rightarrow$ 1 pseudo-NG boson (η')
  - Color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism $\rightarrow$ 8 massive gluons
  - Gauge symmetry $U(1)_{\rm em} \rightarrow \tilde{U}(1)_{\rm em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
- Quasiparticle spectrum (9 quark quasiparticles):
  - octet under $SU(3)_{L+R+c}$ with gap Δ
  - singlet under $SU(3)_{L+R+c}$ with gap 2Δ

CFL: low-energy effective action

[Casalbuoni & Gatto, 1999], [Son & Stephanov, 2000], [Schäfer & Bedaque, 2001]

$$\mathcal{L}_{eff} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\nabla_0\Sigma\nabla_0\Sigma^\dagger - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger\right] + \frac{1}{2}\left[(\partial_0\eta')^2 - v_{\eta'}^2(\partial_i\eta')^2\right]$$

$$+\frac{1}{2}\left[(\partial_0\phi)^2 - v_\phi^2(\partial_i\phi)^2\right] + 2A\left[\det(M)\mathrm{Tr}\left(M^{-1}\Sigma e^{\sqrt{\frac{2}{3}}\frac{i}{f_{\eta'}}\mathcal{I}\eta'}\right) + h.c.\right]$$

where $\quad \Sigma(\pi) \equiv \exp\left(i\pi_A\lambda^A\right), \qquad M = \mathrm{diag}(m_u, m_d, m_s),$

and $\quad \nabla_0\Sigma = \partial_0\Sigma + i\frac{MM^\dagger}{2p_F}\Sigma - i\Sigma\frac{M^\dagger M}{2p_F}$

- Effective chemical potential of strangeness: $\quad \mu_s^{\mathrm{eff}} \simeq m_s^2/2\mu_q$
- Kaon condensation when $m_s \gtrsim (\Delta^2 m_u)^{1/3}$: $\quad \langle\Sigma\rangle = \exp\left(i\alpha\lambda^6\right)$, with $\quad \cos\alpha \sim \frac{\Delta^2 m_u}{m_s^3} < 1$
- Abnormal number (2) of NG bosons after $\mathrm{SU(2)}\times\mathrm{U(1)}_Y \to \tilde{\mathrm{U}}(1)$

  [Miransky & Shovkovy, hep-ph/0108178], [Schäfer et al, hep/ph-0108210]

Physical properties of CFL phase

- Pressure/equation of state:

$$P \simeq \frac{3\mu^4}{4\pi^2} - B + \boxed{\frac{3\mu^2\Delta^2}{\pi^2}} \qquad \text{may be important}$$

- Transport/specific heat is dominated by [Shovkovy & Ellis, 2002]
  - 1 NG boson (ϕ) that results from breaking $U(1)_B$
  - low energy photon of $\tilde{\mathrm{U}}(1)_{\mathrm{em}}$
  - 1 pseudo-NG boson (η') that results from breaking $U(1)_A$
  - 8 ($\times 3$ polarizations) light plasmons with mass $\sim \Delta$ (?)

    [Gusynin & Shovkovy, hep-ph/0108175]

- Superfluidity $\rightarrow$ rotational vorticies
- No electromagnetic Meissner effect $\rightarrow$ no magnetic flux tubes
- Neutrino emissivity/cooling rate is suppressed ($\sim e^{-\Delta/T}$)

Potential signatures of CSC in compact stars

Color superconductivity → **gap** in quasiparticle spectrum

- Thermodynamic properties (equation of state)
  - mass-radius relation [Alford&Reddy,02], [Lugones&Horvath,02]
  - internal star structure [Baldo et al.,02], [Shovkovy et al.,03]
- Transport properties (conductivities, viscosities, mean free paths)
  - cooling rate [Page et al.,02], [Shovkovy&Ellis,02]
  - r-mode instability [Madsen,99], [Manuel et al, hep-ph/0406058]
  - glitches (crystalline phase) [Alford et al.,00]
- Other properties
  - magnetic field generation/penetration [Alford et al.,00]
  - rotational vortices [Iida&Baym,02]

Example: cooling of a hybrid star (with CFL core)

Numerically, thermal energy is [Shovkovy&Ellis,hep-ph/0204132]

$$E_{CFL}(T) \approx 2.1 \times 10^{42} R_{0,km}^{3} T_{MeV}^{4} \text{ erg}$$

Thermal energy of the nuclear layer:

$$E_{NM}(T) \simeq 8.1 \times 10^{49} \frac{M - M_0}{M_\odot} \left(\frac{\rho_0}{\rho}\right)^{2/3} T_{MeV}^{2} \text{ erg}$$

Note that $E_{CFL}(T) \ll E_{NM}(T)$ for $T \lesssim \tilde{T}$

- Thermal energy of core is drained by efficient heat conductivity
- CFL core has a very little slowing effect on the cooling rate
- Cooling would not differ much from that of ordinary neutron star

Is there CSC inside compact stars?

The answer is: **we do not know yet**

Arguments in favor:

- Relatively high densities in stars, $\rho_c \gtrsim 5\rho_0$, suggest that quarks may be deconfined
- An attractive diquark channel is likely to exist
- Temperatures are quite low, $T \lesssim 50$ keV, to allow pairing

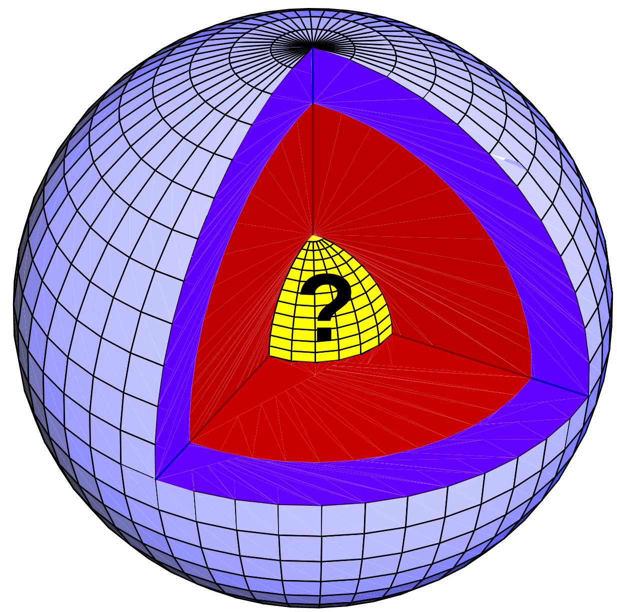
Arguments against:

- Strongly coupled dynamics is not under control
- Matter may not necessarily be deconfined at existing densities
- Specific conditions inside stars (e.g., β-equilibrium) may not favor color superconductivity

The natural approach: **To give predictions and to test them**

Specific conditions inside stars

<u>Matter in the bulk of a star is</u>



(i) β-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:

$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$

Otherwise, a star would **not** be stable!

- Coulomb energy (when $n_Q \neq 0$)

$$E_{\rm Coulomb} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/{\rm fm}^3}\right)^2 \left(\frac{R}{1~{\rm km}}\right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/{\rm fm}^3 \quad \Rightarrow \quad E_{\rm Coulomb}^{\rm 2SC} \gg M_\odot c^2$

Neutrality vs. color superconductivity

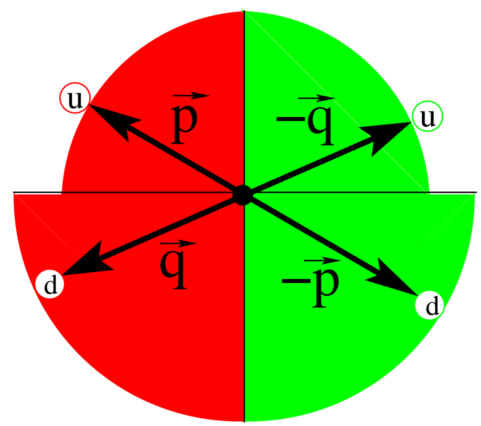
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in β-equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \quad \Rightarrow \quad \mu_d \approx 2^{1/3}\mu_u \ \Rightarrow \ \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{i.e.,} \qquad n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

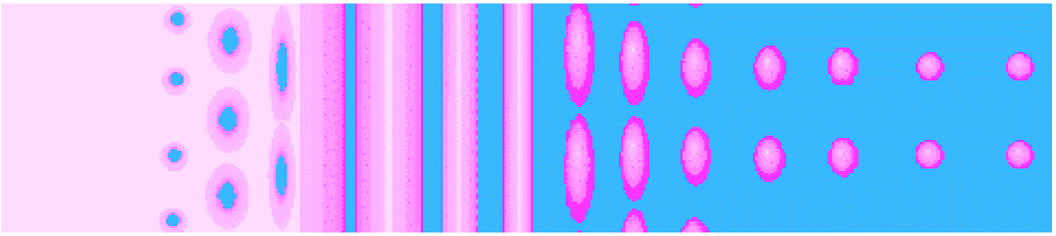
The "best" Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Globally neutral matter (mixed phases)

- Gibbs construction of mixed phase [Glendenning,92], e.g., 2SC+NQ [Shovkovy,Hanauske,Huang,hep-ph/0303027]



$$\rho_e^{(MP)} = \chi_B^A \rho_e^{(A)}(\mu,\mu_e) + (1-\chi_B^A)\rho_e^{(B)}(\mu,\mu_e) = 0$$

where

$$\chi_B^A \equiv \frac{V^{(A)}}{V^{(A)}+V^{(B)}}$$ is the volume fraction of phase A

- There is a price to pay to overcome surface tension

Locally neutral matter

Mismatch parameter μ_e is **not** a free model parameter,

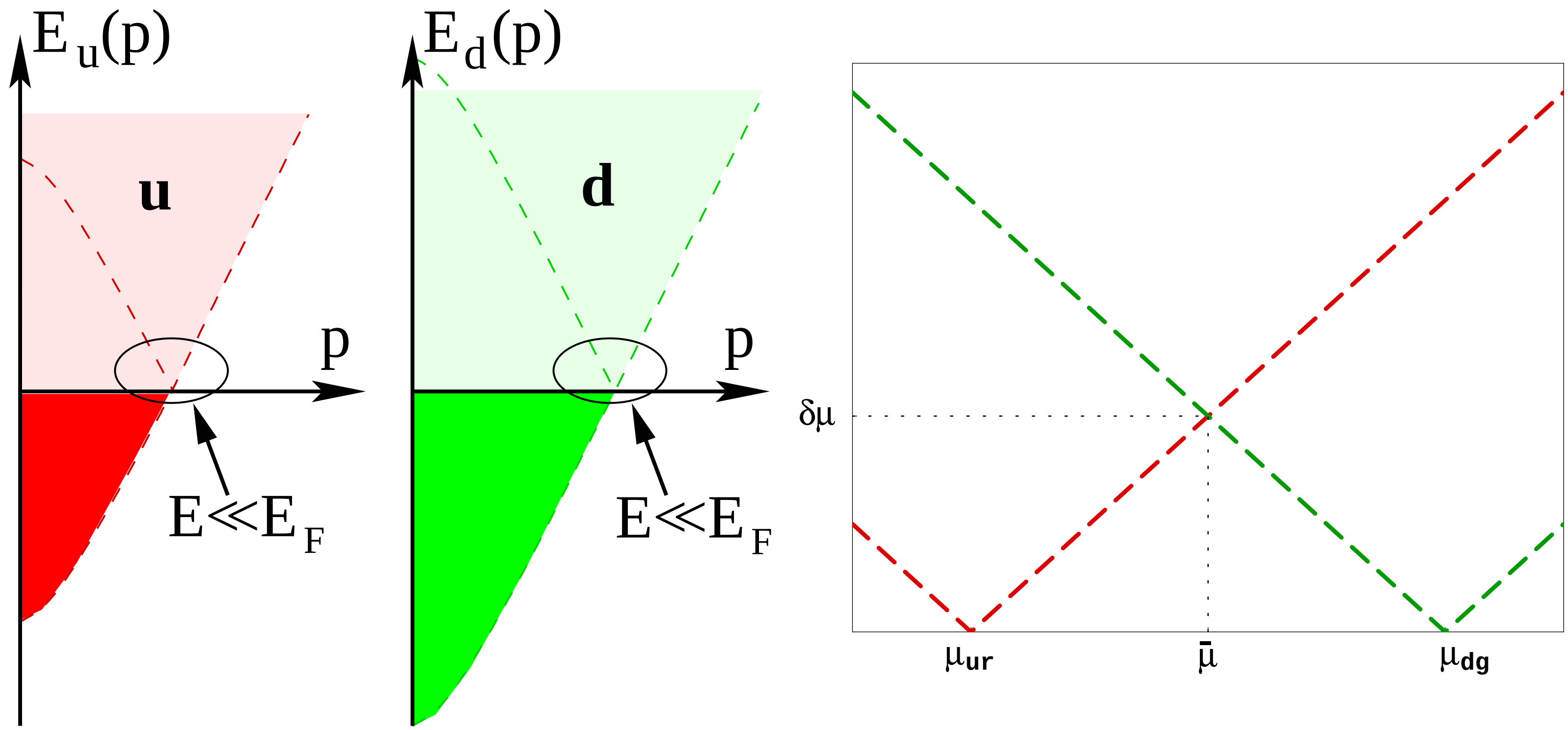
$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \qquad \Rightarrow \qquad \mu_e = \mu_e(\bar{\mu}_q, \Delta)$$

But the diquark coupling strength (η) **is** a model parameter:

1. $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing: Normal phase is the ground state
2. $\eta \gtrsim 0.8$ — strong coupling wins over a mismatch between the Fermi surfaces: 2SC is the ground state
3. $0.7 \lesssim \eta \lesssim 0.8$ — regime of intermediate coupling. The ground state is a new gapless color superconducting phase.

Quasiparticle spectrum in normal phase

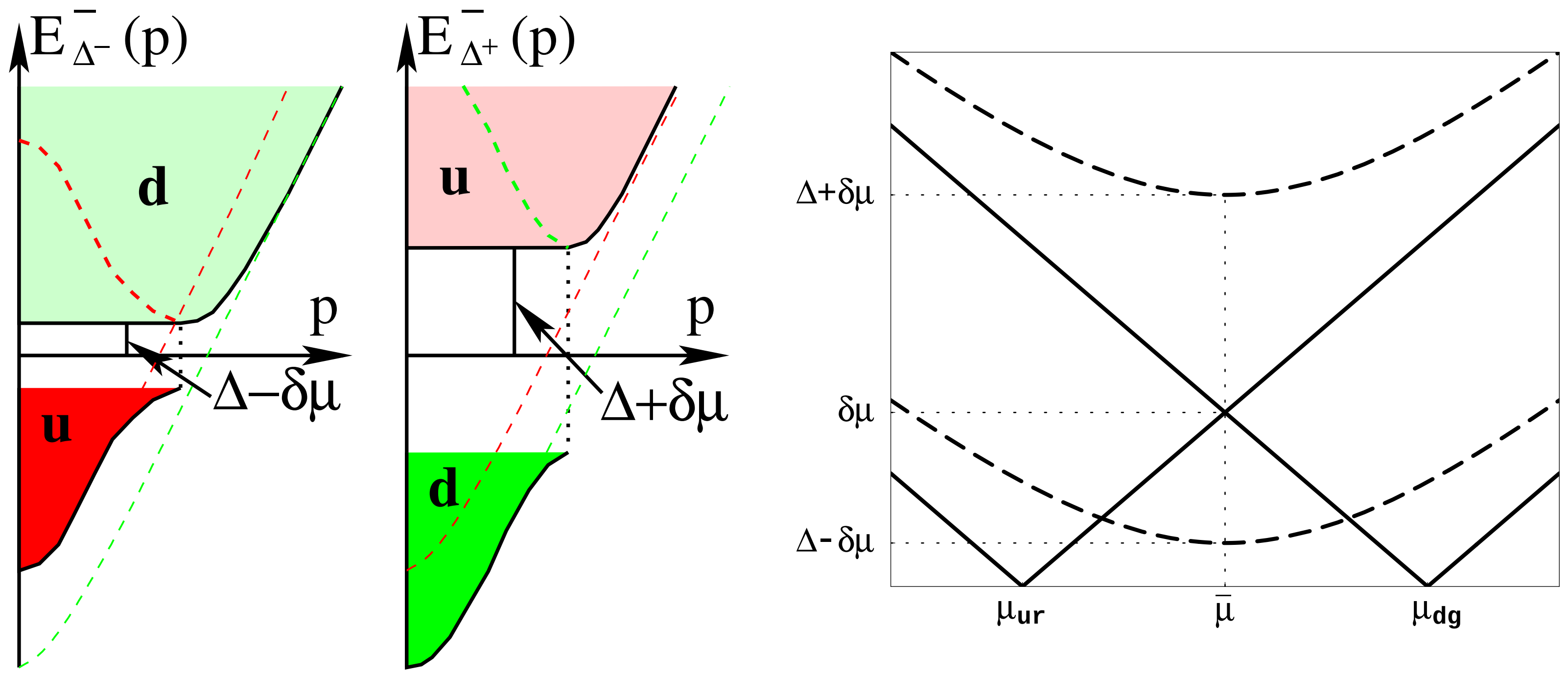
"Weak" coupling (normal phase)



How does this spectrum change when Cooper pairs are formed?

Quasiparticle spectrum in 2SC phase

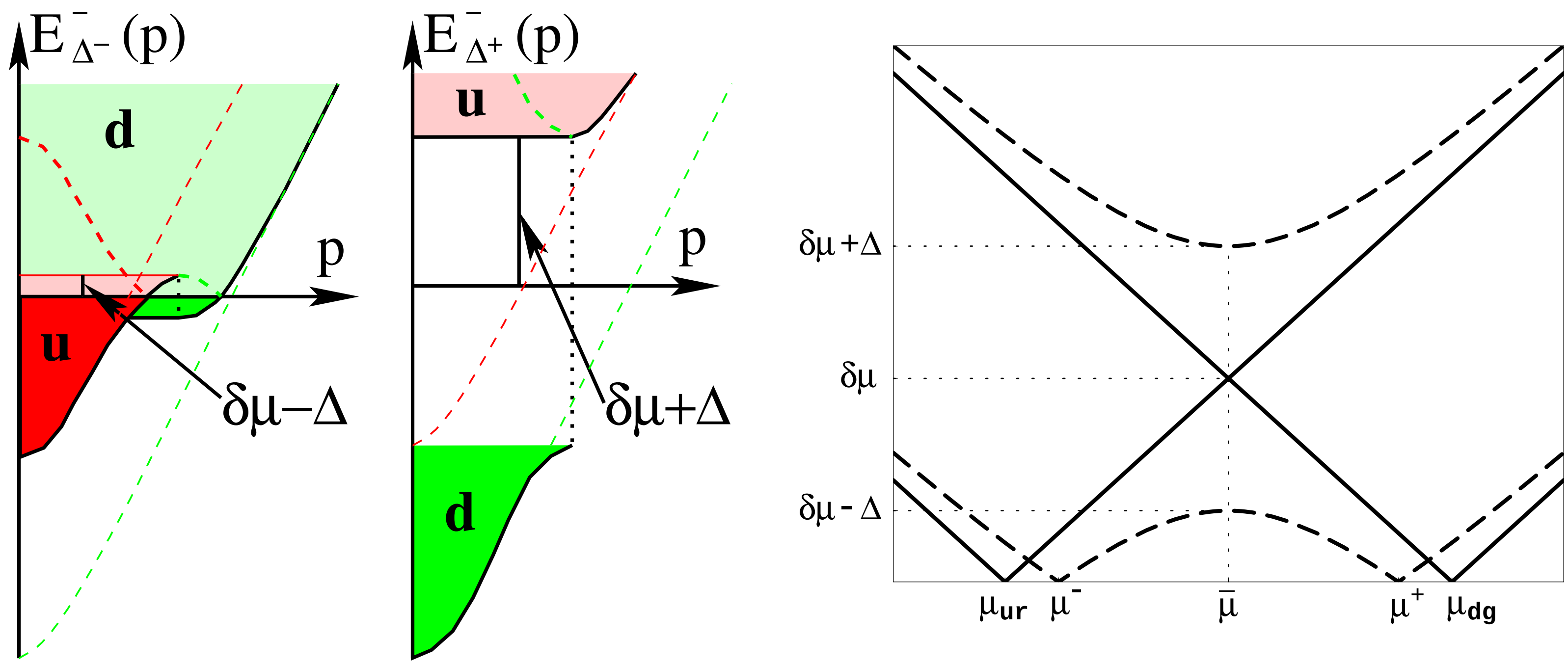
"Strong" coupling (2SC phase)



The energy gaps in the quasiparticle spectra are $\Delta - \delta\mu$ & $\Delta + \delta\mu$

Quasiparticle spectrum in g2SC phase

"Intermediate" coupling (gapless phase)



The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Sarma phase in condensed matter

Type II superconductors in a constant magnetic field:

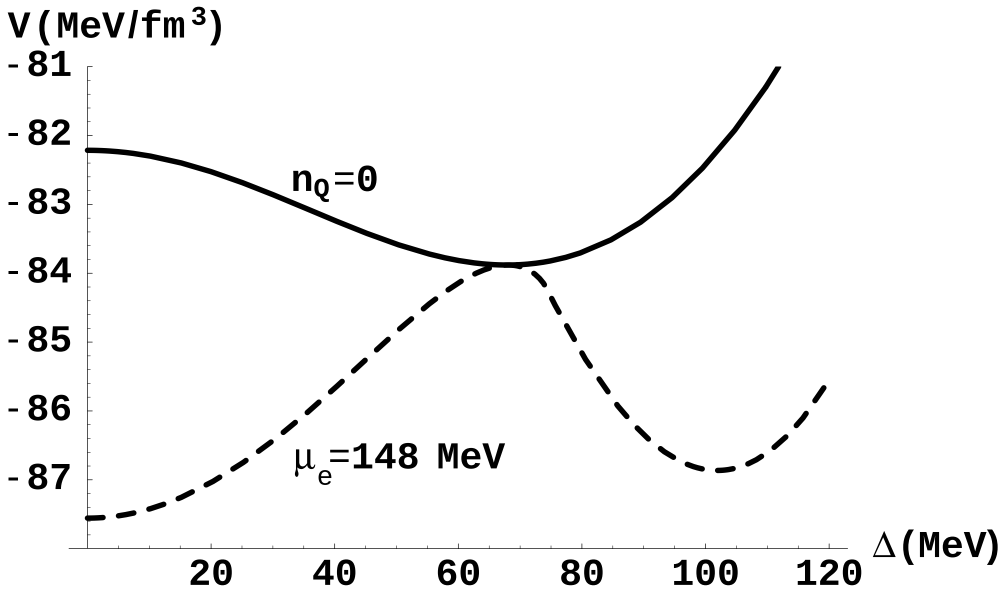
[G. Sarma, J. Phys. Chem. Solids **24** (1963) 1029.]

- Magnetic field originates from ferromagnetic order of impurities in $La_{1-x}Gd_x$ and $Y_{1-x}Gd_xOs_2$ [B.Matthias,H.Suhl & Corenzwit, Phys. Rev. Lett. **1** (1958) 92], [N.Phillips,B.Matthias, Phys. Rev. **121** (1961) 105]
- Pairing happens between spin-$\uparrow$ and spin-$\downarrow$ holes/electrons
- Fermi momenta of $\uparrow$- and $\downarrow$-quasiparticles are different
- The mismatch parameter $\delta\mu \sim H \sim n_{\text{impurity}}$

The gapless "Sarma" phase is **unstable**!

Stability of g2SC phase

Effective potential at $T = 0$ [I.S. & M.Huang, Phys. Lett. B564 (2003) 205]:

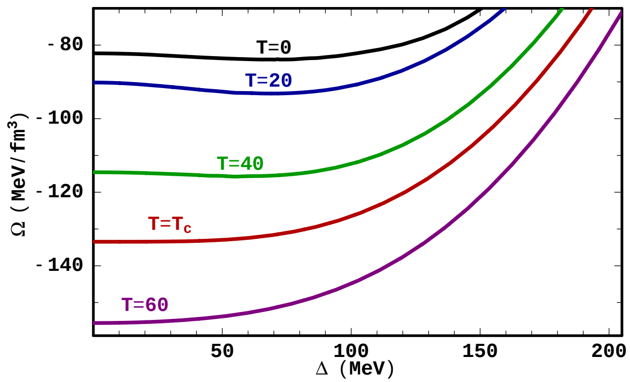


(Q.: Mixed phase? → A.: Unlikely if $\sigma \gtrsim 20$ MeV/fm^2 [I.S., Hanauske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054])

No Sarma instability in g2SC phase if $n_Q = 0$ is enforced *locally*!

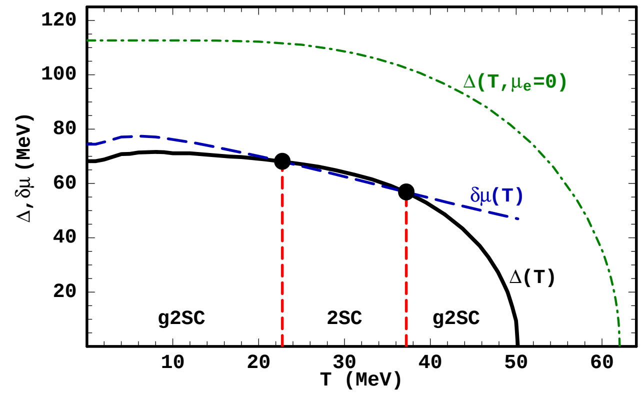
Finite temperature properties

1 Effective potential at $T \neq 0$:



i.e., 2nd order phase transition

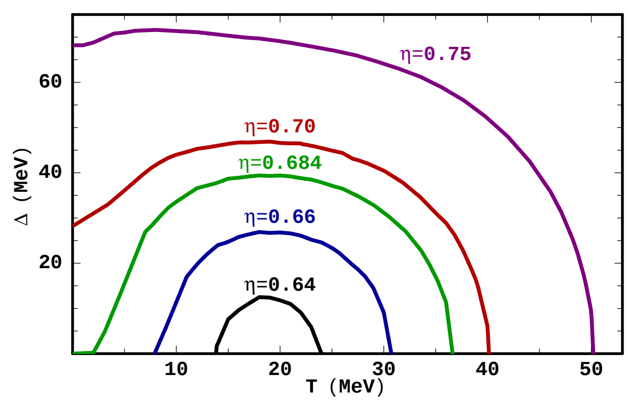
2 *Nonmonotonic* $\Delta(T)$ dependence:



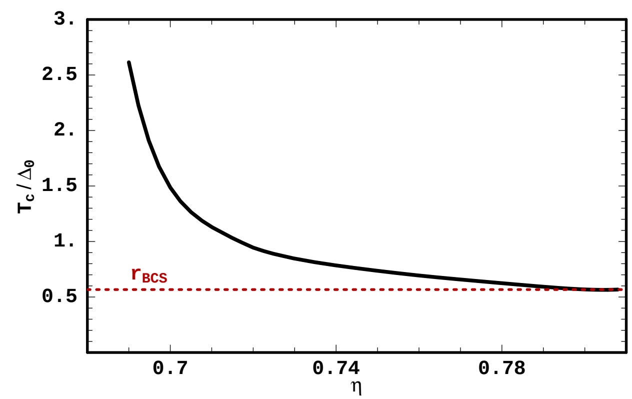
Note: g2SC → 2SC → g2SC → NQ

[M.Huang & I.S., Nucl. Phys. A 729 (2003) 835]

3 *Extreme nonmonotonic* temperature dependence at weaker couplings:



4 T_c/Δ_0 is *not universal* (unlike in BCS), and it can be arbitrarily large!



Strange quark matter, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$p_{s,F} \simeq \mu_q - \frac{m_s^2}{2\mu_q}$$

In the ground state, one may have

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystaline pairing (nonzero momentum pairing, LOFF)

Recently, other possibilities were proposed as well ...

Gapless $N_f = 3$ quark matter

- Mismatch parameter $\delta\mu \equiv m_s^2/2\mu_q = -\mu_8$

$$\mu_{bd}^{\text{eff}} = \mu_q - \frac{2}{3}\mu_8, \qquad \text{and} \qquad \mu_{gs}^{\text{eff}} = \mu_q + \frac{1}{3}\mu_8 - \frac{m_s^2}{2\mu_q}$$

- Gapless modes appear when $|\delta\mu| \equiv \frac{m_s^2}{2\mu_q} > \Delta_0$

  $T = 0$: [Alford, Kouvaris & Rajagopal, hep-ph/0311286]

- More possibilities at $T \neq 0$

  [Rüster, Shovkovy & Rischke, hep-ph/0405170]

- Many different color-flavor pairing channels (3 or 9 gap functions)

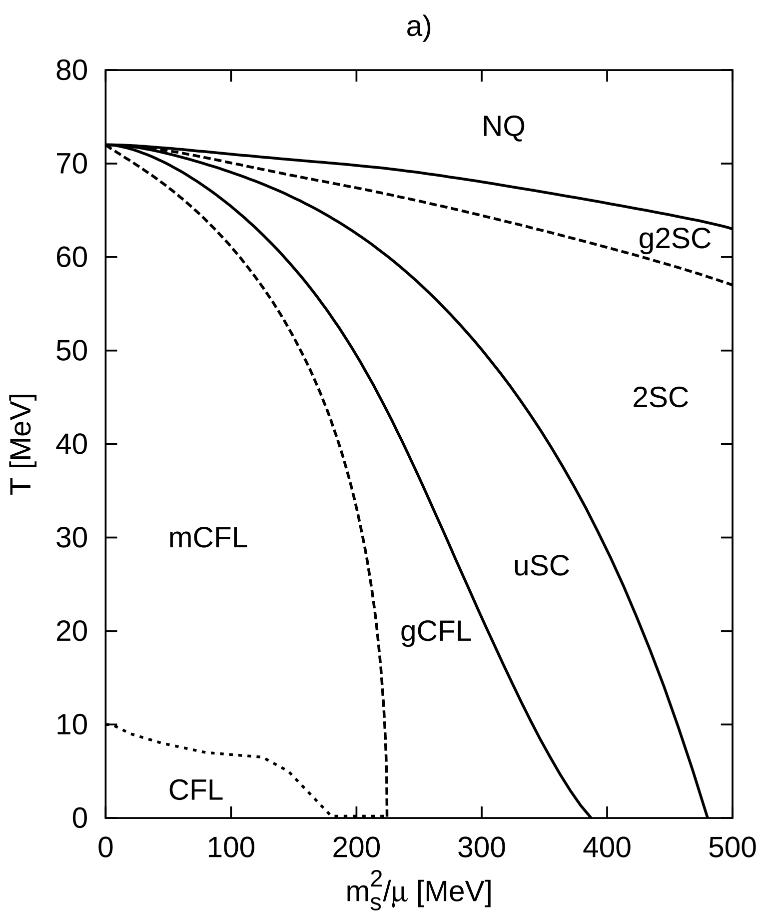
$$\phi_{ij}^{ab} \simeq \phi_1\ \varepsilon_{1ij}\varepsilon^{1ab} + \phi_2\ \varepsilon_{2ij}\varepsilon^{2ab} + \phi_3\ \varepsilon_{3ij}\varepsilon^{3ab} + \ldots$$

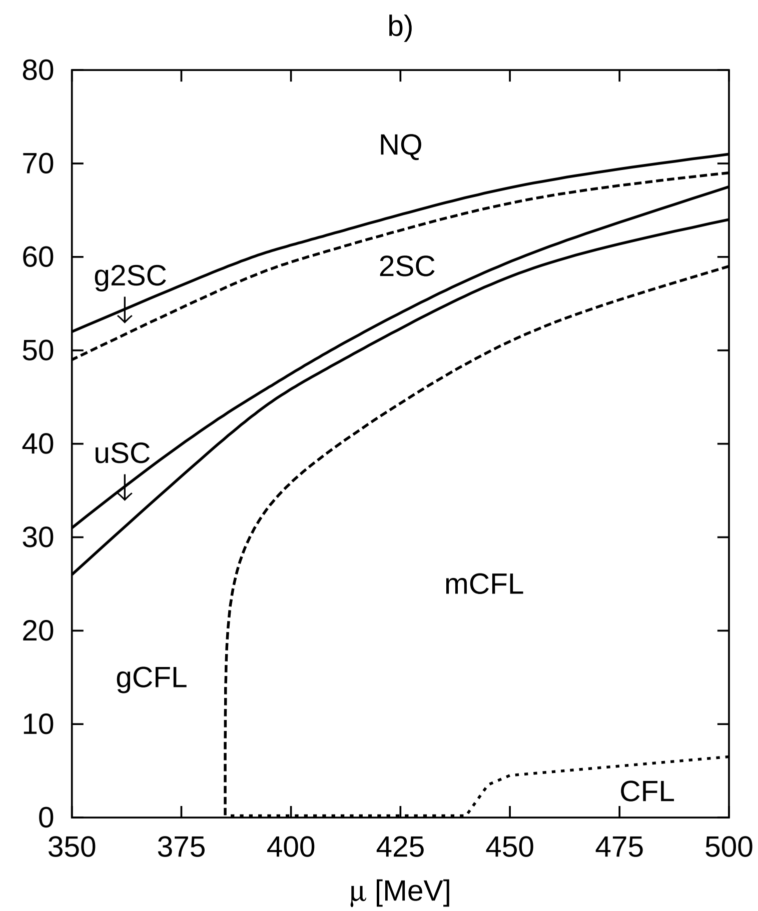
- Zoo of phases: CFL, mCFL, gCFL, uSC, dSC, 2SC, g2SC, NQ

Overview of phases with strangeness

- Color-flavor locked (CFL) phase
  - "Enforced pairing": $n_u = n_d = n_s$ $(T \simeq 0)$
    [Rajagopal, Wilczek, 2001]
  - Natural insulator, $n_{\text{el}} \simeq 0$
  - Little specific heat and low neutrino emissivity
- Metallic CFL phase $(n_{\text{el}} \neq 0)$
  - $T = 0$: gapless CFL phase (no "enforced pairing")
  - $T \neq 0$: thermal effects $\to n_{\text{el}} \neq 0$
  - Large specific heat and high neutrino emissivity
- uSC phase: only ud- & us-pairing (no ds-pairing)
- dSC phase: only du- & ds-pairing (no us-pairing)

Phase diagram



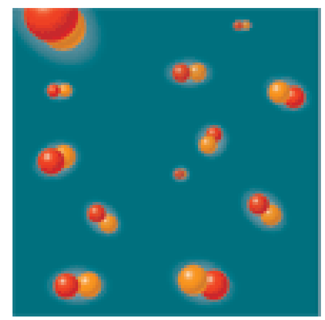


a) $\mu = 500$ MeV   b) $m_s = 250$ MeV

Gapless superfluidity (?)

Neutron stars are very far and very hard to study.

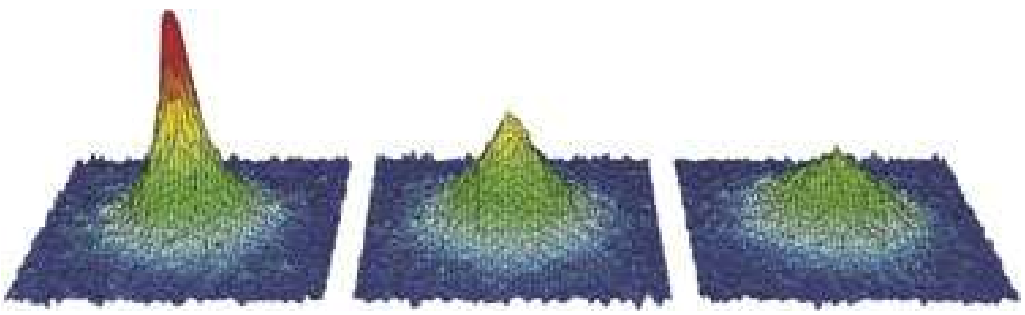
However, one might be able to study **gapless superfluidity** in table-top experiments on ultracold atoms.



> "A much-anticipated atomic soup might lay bare the inner workings of high-temperature superconductors, neutron stars, and primordial matter – and perhaps win its creator a Nobel Prize" ["Ultracold Atoms Spark a Hot Race", Science **301** (2003) 750]

<u>6 teams in the race:</u> (1) D. Jin in Boulder, (2) R. Hulet at Rice U., (3) C. Salomon at École Normale Supérieure, (4) J. Thomas at Duke U., (5) W. Ketterle at MIT, (6) M. Ingucio in Florence U.

First experimental results



"Condensation of fermionic atom pairs near the BEC-BCS crossover regime, where Bose-Einstein condensation changes over into condensation of Cooper pairs."

Jan 30, 2004: [C. Regal, M. Greiner & D. Jin, Phys. Rev. Lett. **92** (2004) 040403]

System: **equal numbers** of $|\frac{9}{2},-\frac{9}{2}\rangle$ and $|\frac{9}{2},-\frac{7}{2}\rangle$ spin states of ^{40}K

Interaction between atoms is tuned by changing a magnetic field in the vicinity of the Feschbach resonance ($B \approx 202$ G)

Gapless superfluidity should appear when $N_{|\frac{9}{2},-\frac{9}{2}\rangle} \neq N_{|\frac{9}{2},-\frac{7}{2}\rangle}$

[Liu,Wilczek,Zoller, cond-mat/0404478]

Toward gapless superfluidity in atomic gases

How to prepare?

By applying an appropriate pulse of a microwave frequency to the trapped mixture of fermion atoms [Liu,Wilczek,Zoller, cond-mat/0404478]

Difficulty: cooling may not be efficient when the setting is such that

$$N_{|\frac{9}{2},-\frac{9}{2}\rangle} \neq N_{|\frac{9}{2},-\frac{7}{2}\rangle}$$

How to detect?

- Time-of-flight expansion images. [K. Davis, et al., PRL **75** (1995) 3969]
- Pairwise projection of fermionic atoms onto molecules. [C. Regal, M. Greiner & D. Jin, PRL **92** (2004) 040403]
- Stimulated scattering of photons (?) [B. Deb, A. Mishra, H. Mishra, P. Panigrahi, cond-mat/0308369]

Concluding remarks

- Sufficiently dense matter is deconfined and color superconducting
- Many different CSC phases were proposed (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K^0, CFL+η, ...)
- The $T - \mu$ and $T - m_s^2/\mu$ phase diagrams are proposed
- It is possible that CSC exists inside compact stars
- Conditions inside stars may favor only some CSC phases
- CSC may affect observables from stars (note though that no unambiguous signatures have been proposed yet)
- Gapless phases may possibly be tested in experiment on trapped cold gases of fermionic atoms (e.g., ^{6}Li and ^{40}K)