

# On recent progress in color superconductivity

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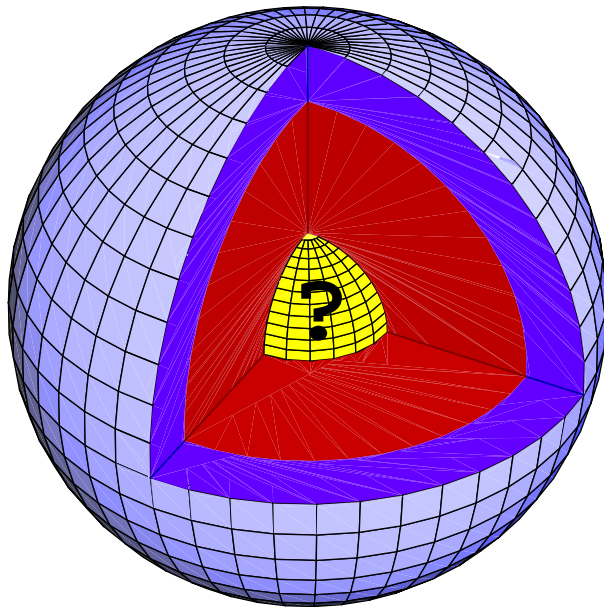
## Matter at high density

Why should one be interested in studying dense matter?

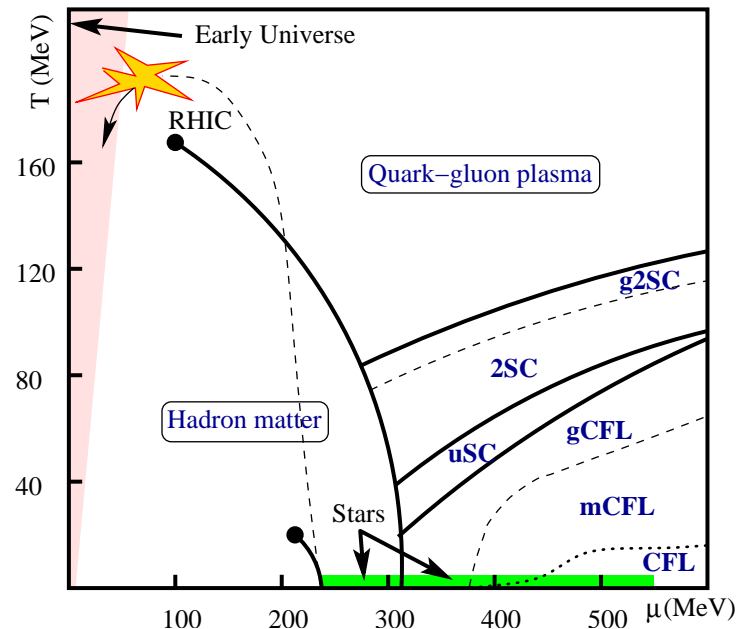
(i) Dense matter exists in the Universe

(ii) Fundamental properties of QCD

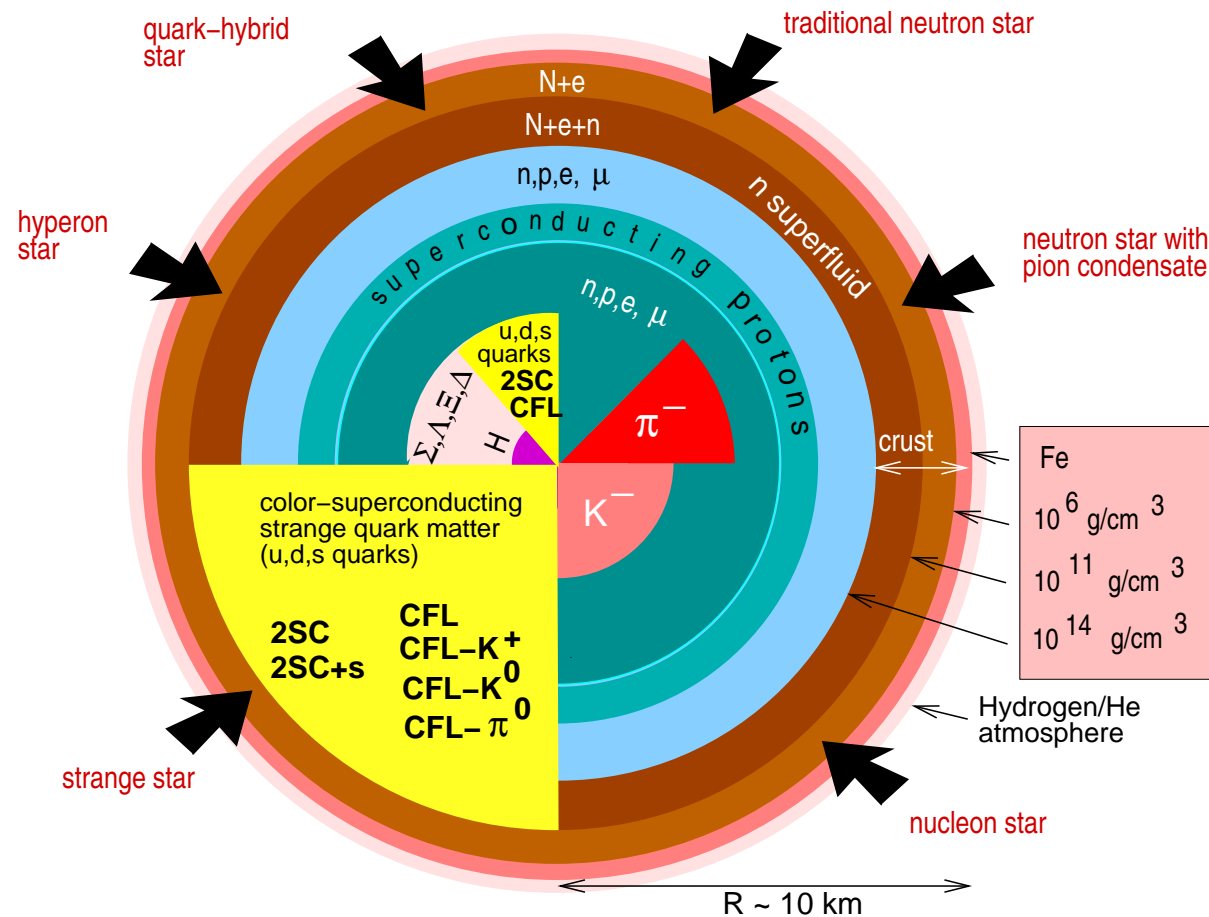
(densities in stars  $\rho_c \gtrsim 5\rho_0$ )



( $\mu \gtrsim \Lambda_{QCD}$ : no lattice results)



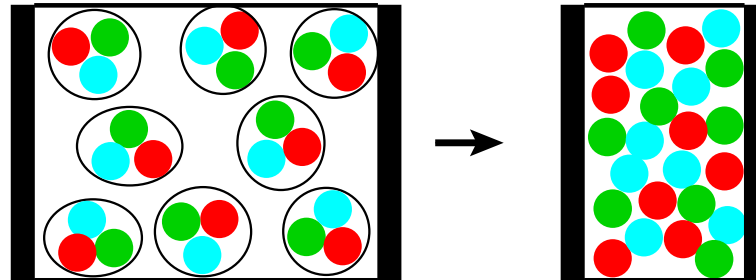
# Phases of matter inside compact stars



[figure from F. Weber, astro-ph/0407155]

## Dense matter might be deconfined

- “Squeezing” baryonic matter hard should produce quark matter:

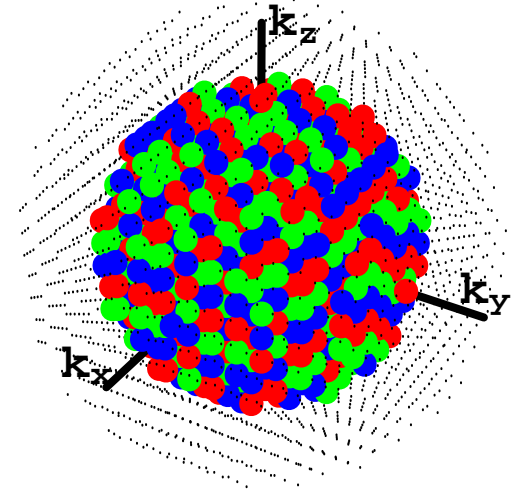


- Conjecture: quark matter may exist in stars  
[Ivanenko&Kurdgelaidze,'65], [Itoh'70], [Collins&Perry,'75]
- What is the ground state of quark matter?
- What is the effect of charge neutrality and  $\beta$ -equilibrium?

## Ground state of dense quark matter

Educated guess:

- (i) Quarks are fermions ( $s = \frac{1}{2}$ )
  - (ii) Interaction is weak ( $\alpha_s \ll 1$ )
- }  $\Rightarrow$  Fermi liquid (?)
- (cf., electron gas in metals/alloys)



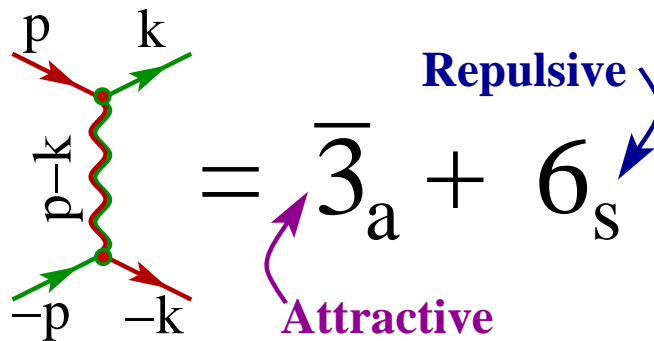
Further refinement:

- (i) Degenerate Fermi surface
  - (ii) Attractive interaction (?)
- }  $\Rightarrow$  Cooper instability
- (cf., the Cooper instability in superconducting metals/alloys)

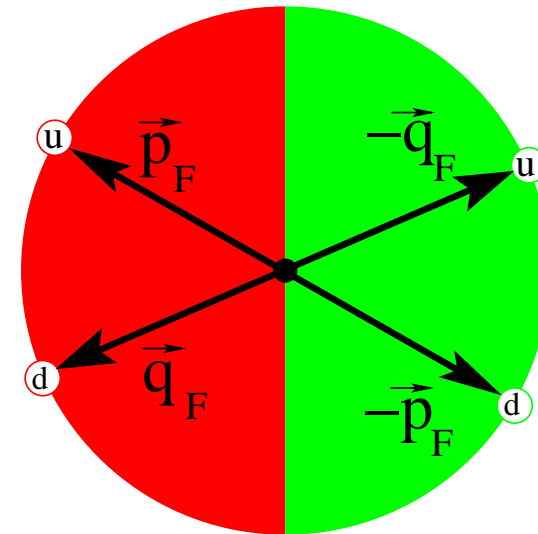
## Color superconductivity in dense QCD

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84; Son,'99]

- $N_f = 2$ : “up” and “down” quarks
- $N_c = 3$ : “red”, “green” and “blue”
- $p_F^{\text{up}} = p_F^{\text{down}} = \mu$
- Quark-quark interaction:



$$\langle \mathbf{u}_p \mathbf{d}_{-p} \rangle = - \langle \mathbf{u}_q \mathbf{d}_{-q} \rangle \neq 0$$



Cooper instability  $\rightarrow$  color superconductivity

$$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle) \quad (\Leftarrow \text{Pauli principle})$$

## 2SC ground state properties

- Wave function of a Cooper-pair:

Pauli principle:  $(|\color{red}\bullet\color{green}\bullet\rangle - |\color{green}\bullet\color{red}\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_1$

- Diquark condensate (spin-0 gap  $\sim 10 - 100$  MeV):

$$\langle (\bar{\Psi}^C)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

- Symmetry of ground state (2SC phase):
  - chiral  $SU(2)_L \times SU(2)_R$  — intact
  - baryon number  $U(1)_B \rightarrow \tilde{U}(1)_B$  with  $\tilde{B} = B - \frac{2}{\sqrt{3}}T_8$
  - approximate axial  $U(1)_A$  is broken  $\rightarrow$  1 pseudo-NG boson
  - Color gauge symmetry  $SU(3)_c \rightarrow SU(2)_c$  by Anderson-Higgs mechanism  $\rightarrow$  5 massive gluons
  - Gauge symmetry  $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$  with  $\tilde{Q} = Q - \frac{1}{\sqrt{3}}T_8$

## Physical properties of 2SC phase

- Pressure/equation of state:

$$P \simeq \frac{\mu^4}{2\pi^2} - B + \boxed{\frac{\mu^2 \Delta^2}{\pi^2}} \quad \text{may be (un-)important}$$

- Transport/specific heat is dominated by
  - Unpaired “blue-up” and “blue-down” quarks
  - 1 pseudo-NG boson that results from breaking  $U(1)_A$
  - 3 gluons of unbroken  $SU(2)_c$  (decoupled from blue quarks)
  - low energy photon of  $\tilde{U}(1)_{\text{em}}$
- No superfluidity  $\rightarrow$  no rotational vortices
- No electromagnetic Meissner effect  $\rightarrow$  no magnetic flux tubes
- Neutrino emissivity/cooling rate is large (direct URCA)



## Color superconductivity, $N_f = 3$

- Wave function of a Cooper-pair:

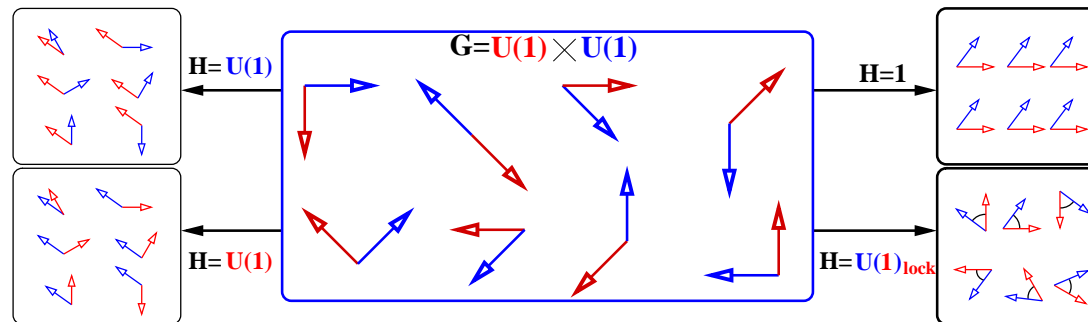
Pauli principle:  $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_{\bar{3}}$

- Diquark condensate (spin-0 gap  $\sim 10 - 100$  MeV):

$$\langle (\bar{\Psi}_L^C)_i^\alpha (\Psi_L)_j^\beta \rangle = \langle (\bar{\Psi}_R^C)_i^\alpha (\Psi_R)_j^\beta \rangle \simeq \sum \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- Color-flavor locking:  $SU(3)_L \times SU(3)_R \times SU(3)_c \rightarrow SU(3)_{L+R+c}$

[Alford et al. hep-ph/9804403]



There are no  $\langle q_L q_R \rangle$  condensates, but  $SU(3)_L \times SU(3)_R$  chiral symmetry is broken down to  $SU(3)_V$  through locking with color!

## Color superconductivity, $N_f = 3$

- Symmetry of ground state (CFL phase):
  - chiral  $SU(3)_L \times SU(3)_R$  is broken down to  $SU(3)_{L+R+c}$   
 $\rightarrow$  8 (pseudo-)NG bosons, i.e.,  $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$
  - baryon number  $U(1)_B$  is broken  $\rightarrow$  1 NG boson ( $\phi$ )
  - approximate axial  $U(1)_A$  is broken  $\rightarrow$  1 pseudo-NG boson ( $\eta'$ )
  - Color gauge symmetry  $SU(3)_c$  is broken by Anderson-Higgs mechanism  $\rightarrow$  8 massive gluons
  - Gauge symmetry  $U(1)_{\text{em}} \rightarrow \tilde{U}(1)_{\text{em}}$  with  $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$
- Quasiparticle spectrum (9 quark quasiparticles):
  - octet under  $SU(3)_{L+R+c}$  with gap  $\Delta$
  - singlet under  $SU(3)_{L+R+c}$  with gap  $2\Delta$

## Physical properties of CFL phase

- Pressure/equation of state:

$$P \simeq \frac{3\mu^4}{4\pi^2} - B + \boxed{\frac{3\mu^2\Delta^2}{\pi^2}} \quad \text{may be (un-)important}$$

- Transport/specific heat is dominated by [Shovkovy & Ellis, 2002]
  - 1 NG boson ( $\phi$ ) that results from breaking  $U(1)_B$
  - low energy photon of  $\tilde{U}(1)_{\text{em}}$
  - 1 pseudo-NG boson ( $\eta'$ ) that results from breaking  $U(1)_A$
  - 8 ( $\times 3$  polarizations) light plasmons with mass  $\sim \Delta$  (?)

[Gusynin & Shovkovy, hep-ph/0108175]

- Superfluidity  $\rightarrow$  rotational vortices
- No electromagnetic Meissner effect  $\rightarrow$  no magnetic flux tubes
- Neutrino emissivity/cooling rate is suppressed ( $\sim e^{-\Delta/T}$ )

## Color superconductivity, $N_f = 1$

- Wave function of a Cooper-pair:  $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$ 
  - antisymmetric in color (attractive diquark  $\bar{3}_c$  channel)
  - Pauli principle: symmetric in spin, i.e., spin-1 channel
- Diquark condensate (gap  $\sim 10 - 100$  keV):
 
$$\langle (\bar{\Psi}^C)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left( \hat{\mathbf{k}}^\delta \sin \theta + \gamma_\perp^\delta(\vec{\mathbf{k}}) \cos \theta \right)$$
- Many possibilities, see Ph.D. thesis [A.Schmitt, nucl-th/0405076]:
  - Polar phase:  $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
  - Color-spin-locked phase:  $\Delta_{c\delta} = \delta_{c\delta}$
  - Planar phase:  $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
  - A-phase:  $\Delta_{c\delta} = \delta_{c3}(\delta_{\delta 1} + i\delta_{\delta 2}) \rightarrow$  largest pressure
- Meissner effect  $\oplus$  type I superconductor  $\rightarrow$  affect star properties

## Color superconductivity in stars

Is there CSC matter inside compact stars?

Arguments in favor:

- 😊 Relatively high densities in stars,  $\rho_c \gtrsim 5\rho_0$ , suggest that quarks may be deconfined
- 😊 An attractive diquark channel is likely to exist
- 😊 Temperatures are quite low,  $T \lesssim 50$  keV, to allow pairing

Arguments against:

- 😞 Strongly coupled dynamics is not under control
- 😞 Matter may not necessarily be deconfined at existing densities
- 😞 Specific conditions inside stars (e.g.,  $\beta$ -equilibrium) may not favor color superconductivity

The natural approach: to give model predictions and to test them

## Importance of neutrality inside a star

Matter in the bulk of a star is

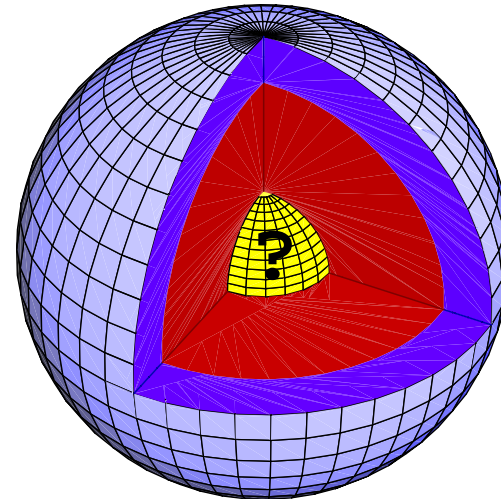
- (i)  $\beta$ -equilibrated:  $\mu_d = \mu_u + \mu_e$
- (ii) electrically and color neutral:  
 $n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$

Otherwise, a star is **not** stable!

- Coulomb energy (when  $n_Q \neq 0$ )

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_{\odot} c^2 \left( \frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left( \frac{R}{1 \text{ km}} \right)^5$$

In 2SC phase,  $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2\text{SC}} \gg M_{\odot} c^2$



## Neutrality vs. color superconductivity

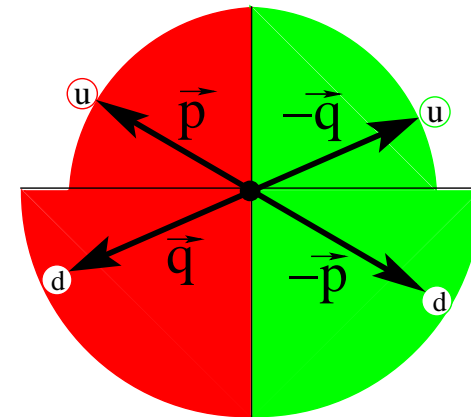
- The “best” 2SC phase appears when  $n_d \approx n_u$
- Neutral matter (in  $\beta$ -equilibrium) appears when  $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u$$

i.e.,  $n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$

The “best” Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



## Appearance of a gapless phase

Mismatch parameter  $\mu_e$  is **not** a free model parameter,

$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \quad \Rightarrow \quad \mu_e = \mu_e(\mu, \Delta)$$

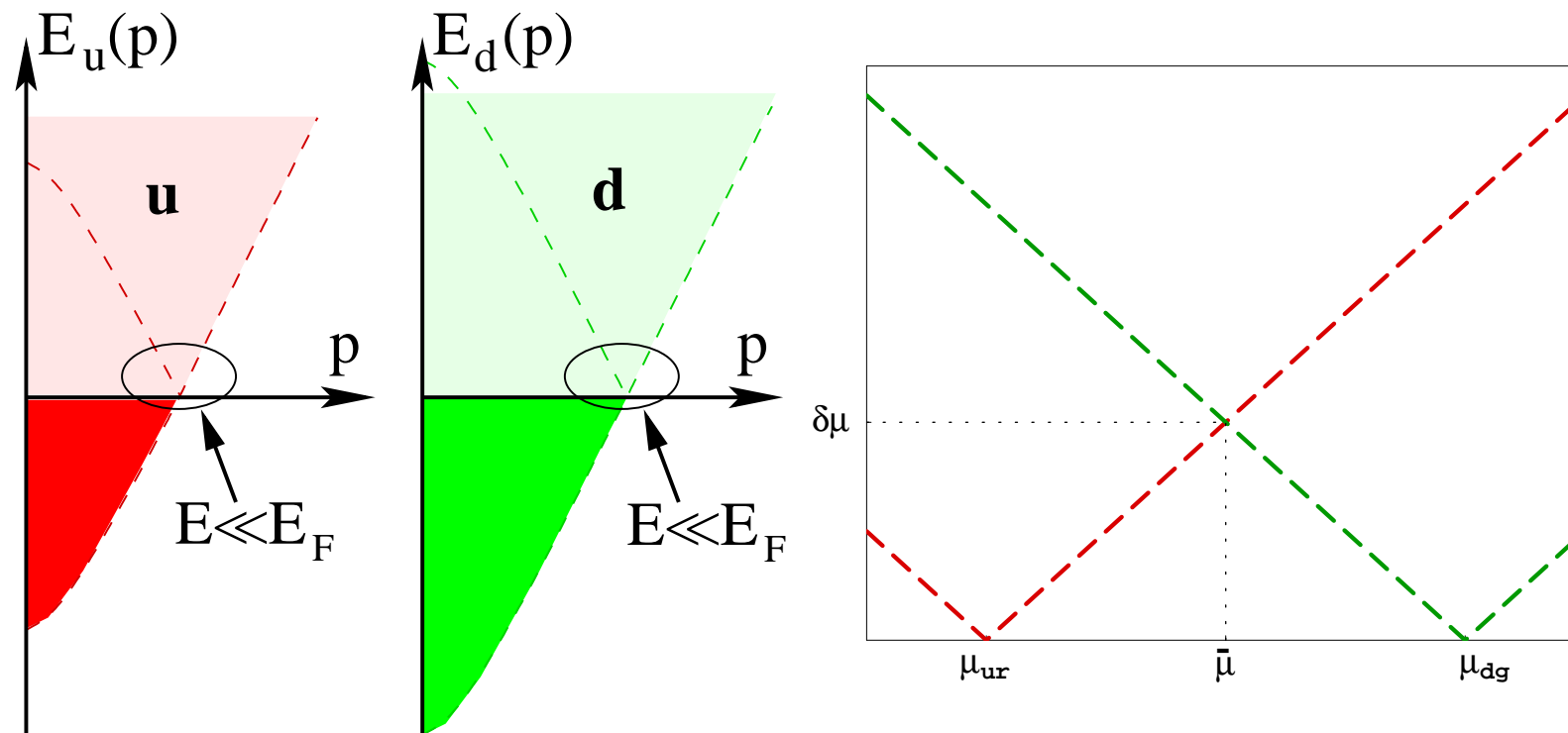
Three dynamical regimes determined by coupling strength  $\eta$ :

1.  $\eta \lesssim 0.7$  — the mismatch does not allow Cooper pairing:  
normal phase is the ground state
2.  $\eta \gtrsim 0.8$  — strong coupling wins over the mismatch between the  
Fermi surfaces: 2SC is the ground state
3.  $0.7 \lesssim \eta \lesssim 0.8$  — regime of intermediate coupling strength:  
the ground state is the gapless 2SC phase [hep-ph/0302142]



## Quasiparticle spectrum in normal phase

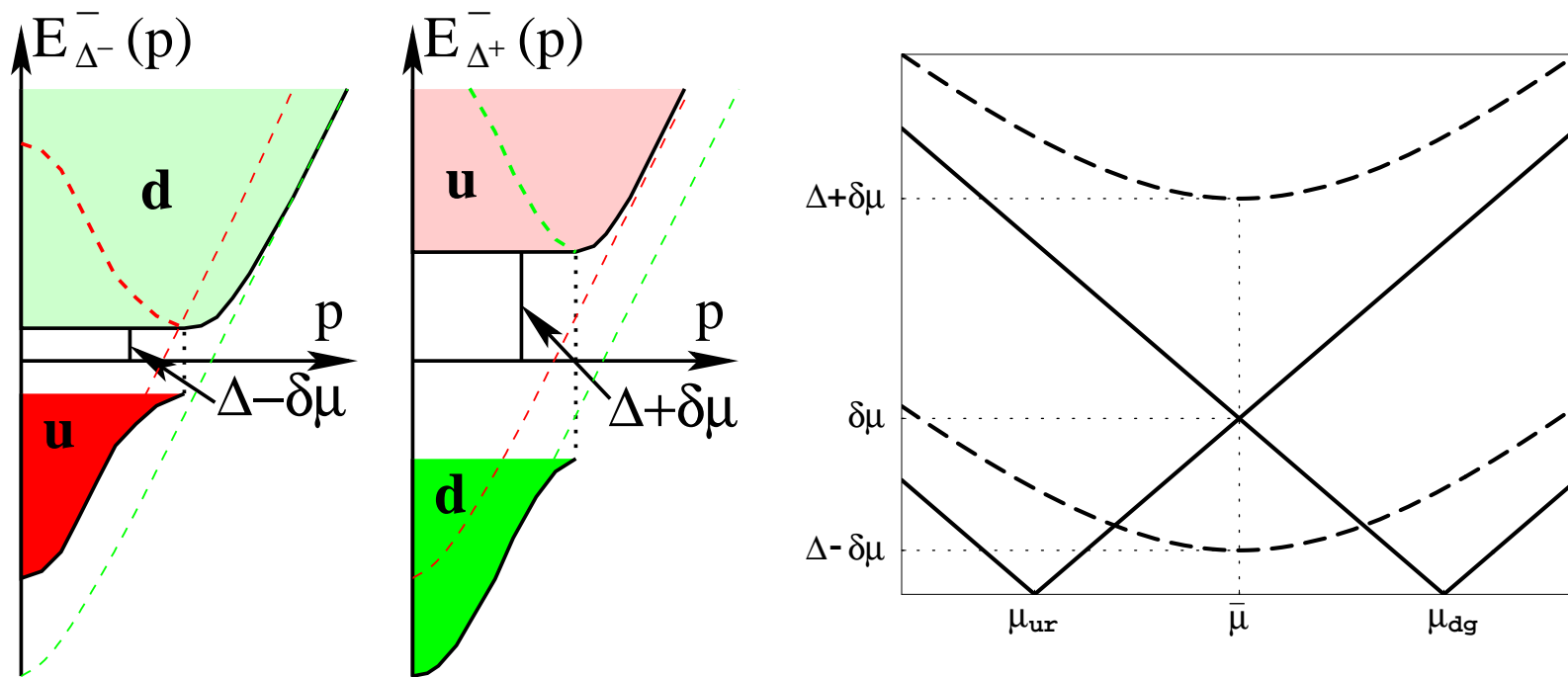
“Weak” coupling (normal phase)



How does this spectrum change when Cooper pairs are formed?

## Quasiparticle spectrum in 2SC phase

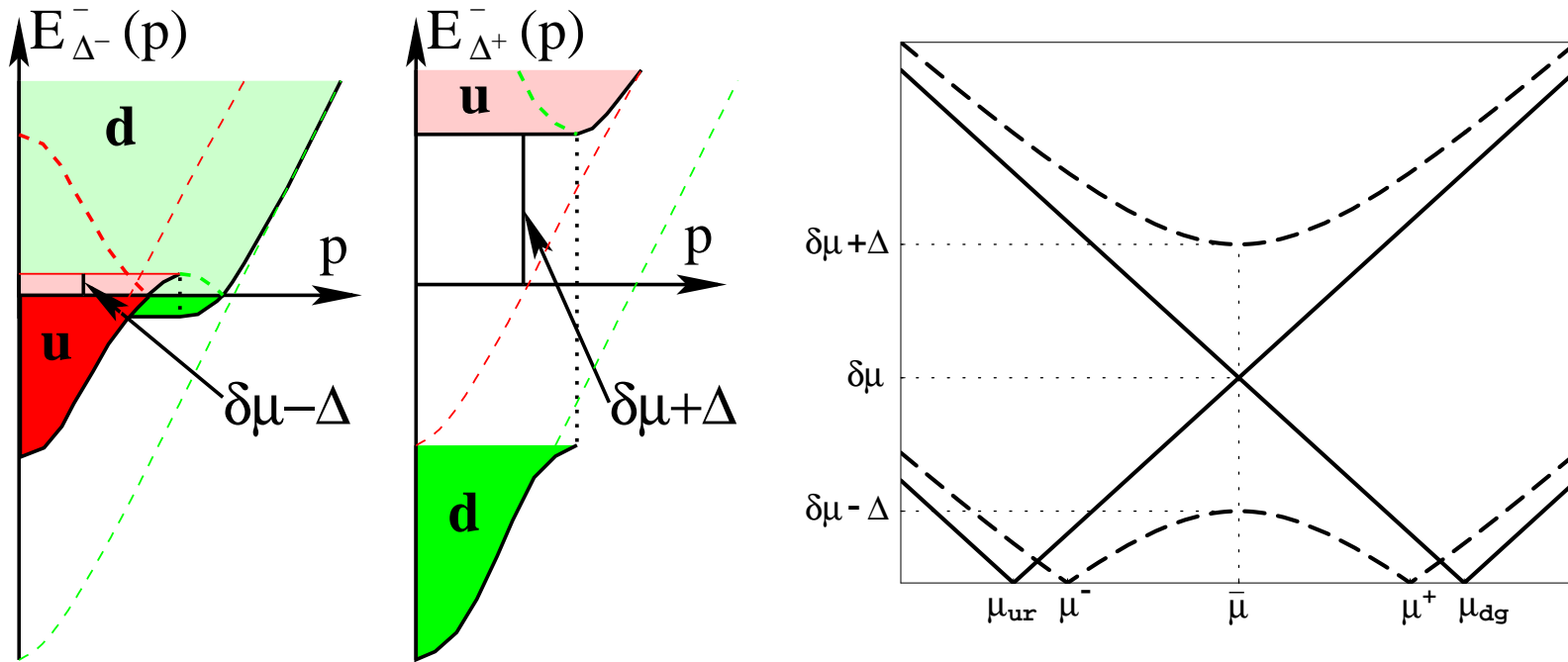
“Strong” coupling (2SC phase)



The energy gaps in the quasiparticle spectra are  $\Delta - \delta\mu$  &  $\Delta + \delta\mu$

# Quasiparticle spectrum in g2SC phase

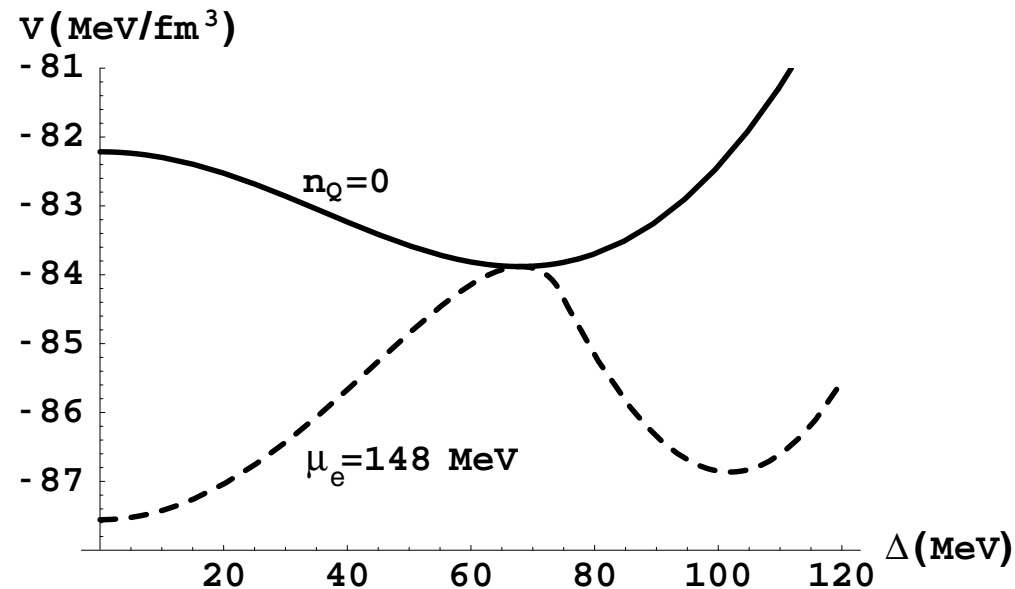
“Intermediate” coupling (gapless phase)



The energy gaps in the quasiparticle spectra are  $0$  &  $\Delta + \delta\mu$

## Stability of g2SC phase

Effective potential at  $T = 0$  [I.S. & M.Huang, Phys. Lett. B564 (2003) 205]:



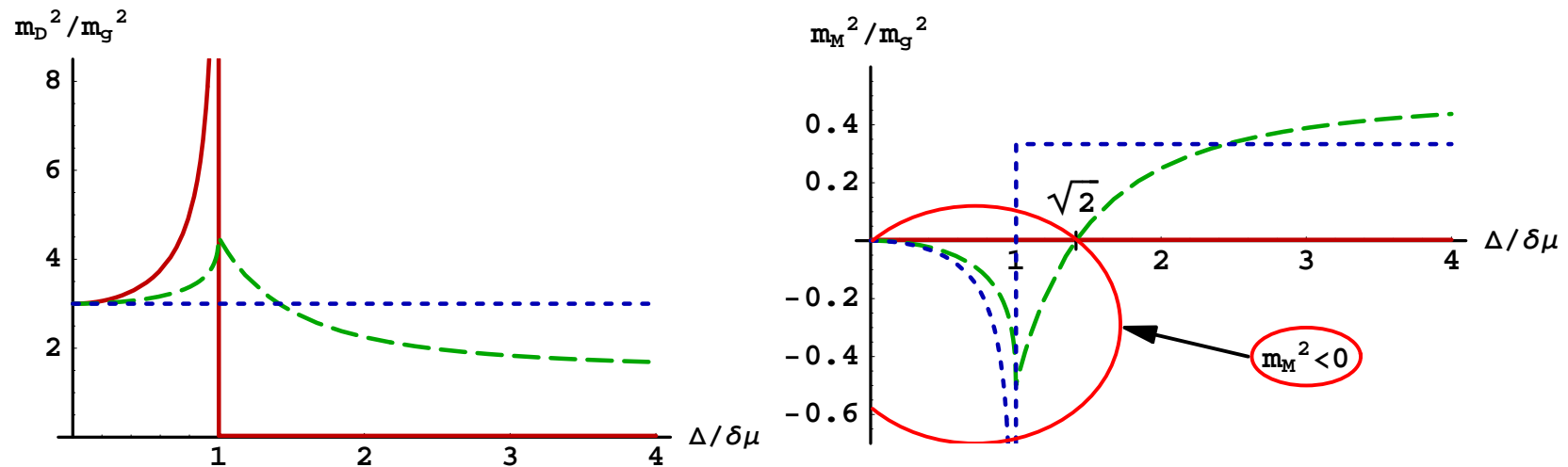
(Q.: Mixed phase? → A.: Unlikely if  $\sigma \gtrsim 20$   $\text{MeV/fm}^2$  [Shovkovy, Hanauske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054])

No Sarma instability if  $n_Q = 0$  is enforced *locally*!

# Chromomagnetic instability

Recent results for gluon screening masses

[Huang & Shovkovy, hep-ph/0407049]:



$A = 1, 2, 3$  — red solid line

$A = 4, 5, 6, 7$  — green long-dash line

$A = \tilde{8}$  — blue short-dash line

## Finite strange quark mass, $0 < m_s < \infty$

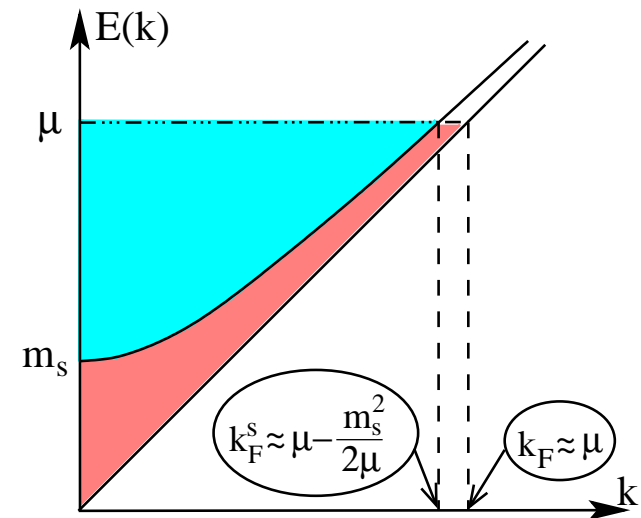
Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$

The ground state of strange quark matter may have:

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystalline pairing (nonzero momentum pairing, LOFF)

Recently, other possibilities were proposed as well ...



## Gapless $N_f = 3$ quark matter

- Distorted color-flavor pairing:

$$\Delta_{ij}^{\alpha\beta} \simeq \Delta_1 \epsilon_{1ij} \varepsilon^{1\alpha\beta} + \Delta_2 \epsilon_{2ij} \varepsilon^{2\alpha\beta} + \Delta_3 \epsilon_{3ij} \varepsilon^{3\alpha\beta} + \dots$$

- Control (mismatch) parameter:

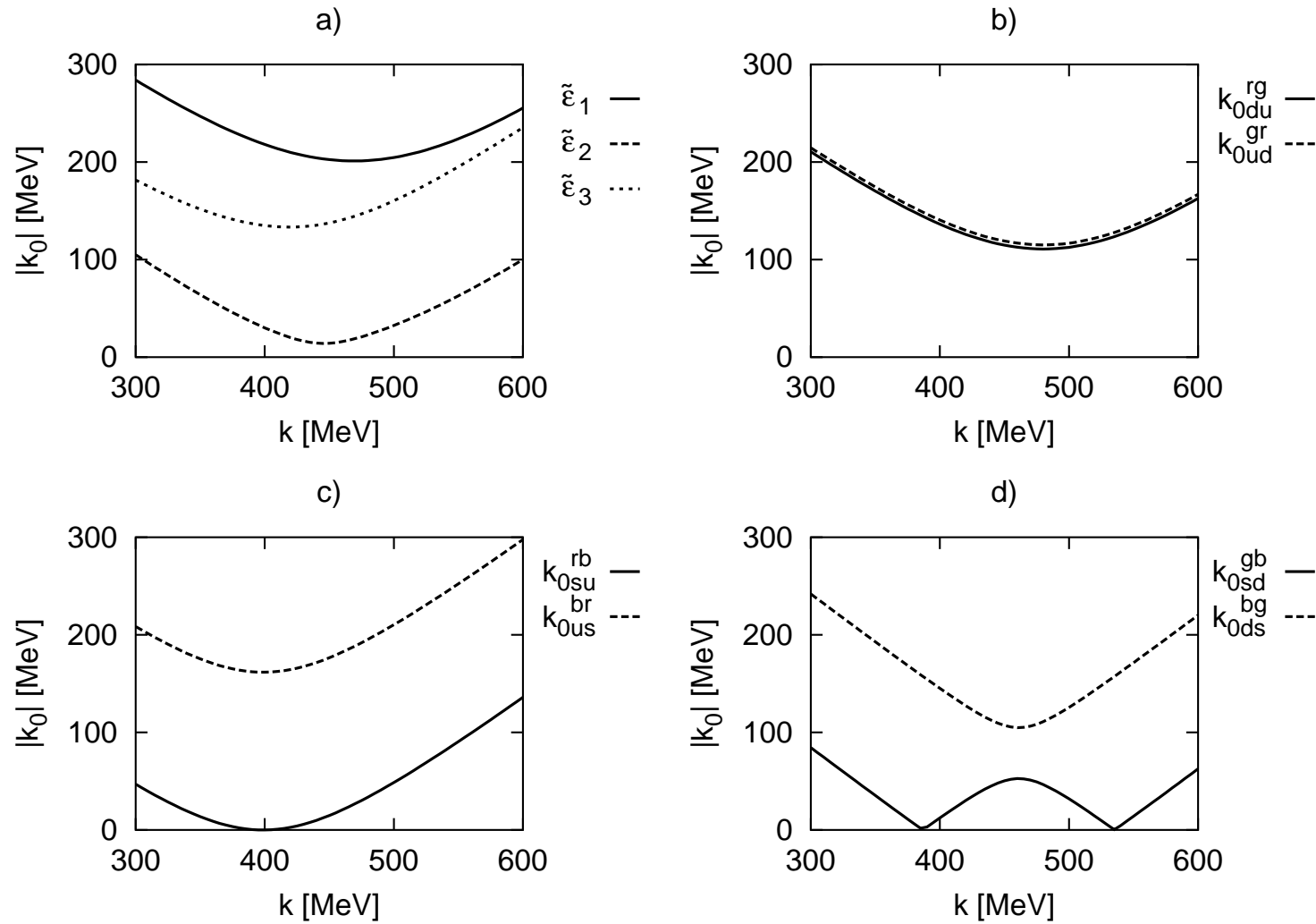
$$\delta\mu \equiv \frac{\mu_{bd} - \mu_{gs}^{\text{eff}}}{2} \approx -\frac{\mu_8}{2} + \frac{m_s^2}{4\mu} \approx \boxed{\frac{m_s^2}{2\mu}}$$

where  $\mu_{gs}^{\text{eff}} \simeq \mu_{gs} - \frac{m_s^2}{2\mu}$  and  $\mu_8 \simeq -\frac{m_s^2}{2\mu}$  (blue color is special)

- Gapless CFL phase with  $\Delta_1 < \Delta_2 < \Delta_3$ :

$$T = 0 : \quad \delta\mu \equiv \frac{m_s^2}{2\mu} > \Delta_0 \quad [\text{Alford et al. hep-ph/0311286}]$$

# Quasiparticle spectrum in gCFL phase





## Nonzero temperature

- There can exist many phases at  $T \neq 0$

[Iida et al, hep-ph/0312363], [Rüster et al, hep-ph/0405170], [Fukushima et al, hep-ph/0408322]

- Zoo of phases:

CFL:  $\Delta_1 \neq 0, \quad \Delta_2 \neq 0, \quad \Delta_3 \neq 0, \quad (\mu_e \approx 0)$

mCFL:  $\Delta_1 \neq 0, \quad \Delta_2 \neq 0, \quad \Delta_3 \neq 0, \quad (\mu_e \neq 0)$

uSC:  $\Delta_1 = 0, \quad \Delta_2 \neq 0, \quad \Delta_3 \neq 0,$

dSC:  $\Delta_1 \neq 0, \quad \Delta_2 = 0, \quad \Delta_3 \neq 0,$

2SC:  $\Delta_1 = 0, \quad \Delta_2 = 0, \quad \Delta_3 \neq 0,$

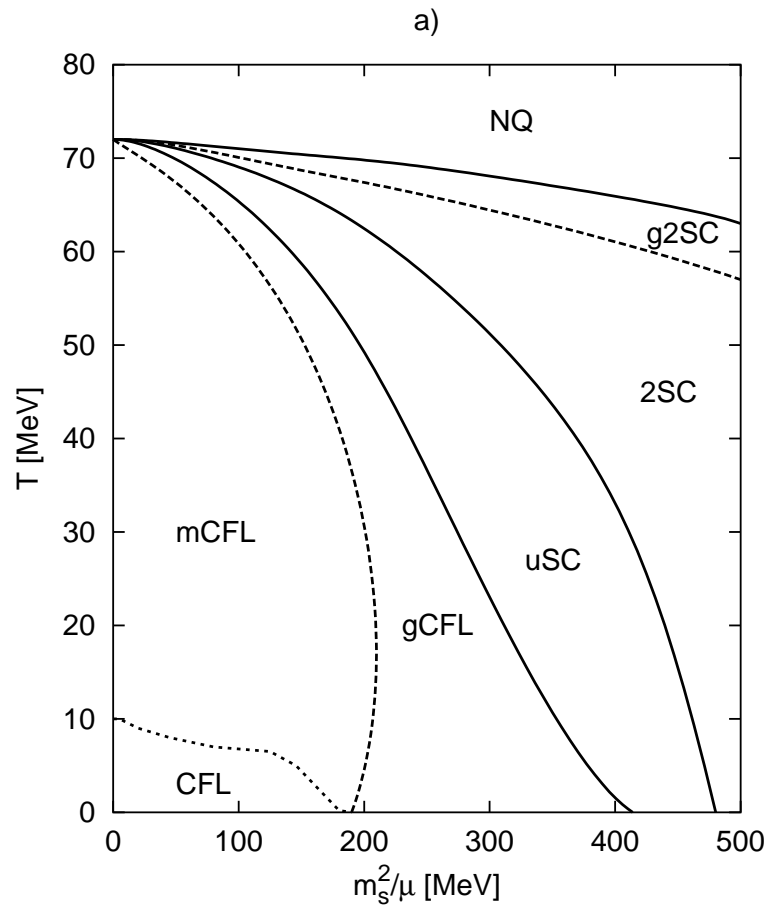
NQM:  $\Delta_1 = 0, \quad \Delta_2 = 0, \quad \Delta_3 = 0,$

plus g2SC and gCFL as special cases of 2SC and mCFL.

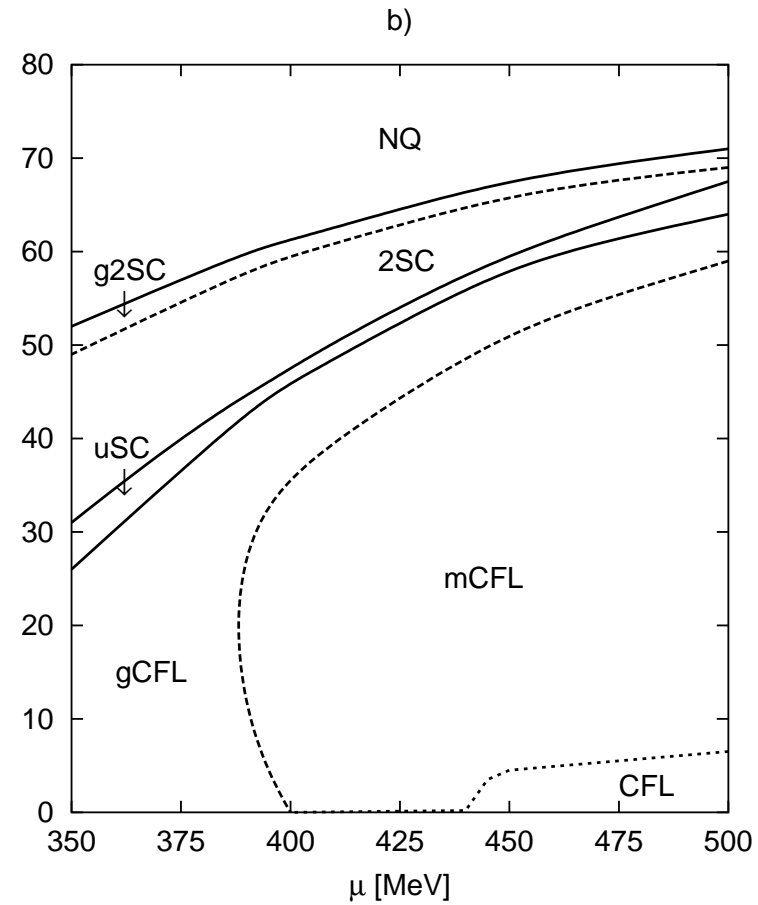
## Overview of phases with strangeness

- Color-flavor locked (CFL) phase
  - “Enforced pairing”:  $n_u = n_d = n_s$  ( $T \simeq 0$ )  
[Rajagopal, Wilczek, 2001]
  - Natural insulator,  $n_{\text{el}} \simeq 0$
  - Little specific heat and low neutrino emissivity
- Metallic CFL phase ( $n_{\text{el}} \neq 0$ )
  - $T = 0$ : gapless CFL phase (no “enforced pairing”)
  - $T \neq 0$ : thermal effects  $\rightarrow n_{\text{el}} \neq 0$
  - Large specific heat and high neutrino emissivity
- uSC phase: only  $ud$ - &  $us$ -pairing (no  $ds$ -pairing)
- dSC phase: only  $du$ - &  $ds$ -pairing (no  $us$ -pairing)

# Phase diagram



a)  $\mu = 500$  MeV



b)  $m_s = 250$  MeV

## Current status

- Sufficiently cold and dense matter is a color superconductor
- Neutrality and  $\beta$ -equilibrium may strongly affect the properties of dense matter
- There can exist many different CSC phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K<sup>0</sup>, CFL+ $\eta$ )
- Some features of  $T - \mu$  phase diagram start to develop
- A search for signature-type observables of color superconductivity inside stars is under way

## Outlook

- A systematic study of competition between different phases in dense QCD should be completed
- Physical properties (transport, in particular) of QCD phases should be addressed in detail
- The status of gapless phases should be resolved (addressing, e.g., the chromomagnetic instability, spontaneously induced currents)
- The most promising observable(s), (dis-)proving the presence of CSC inside stars, should be proposed
- A rigorous approach to treat QCD at nonzero densities should be developed