The current crisis in the understanding

of

QCD phase diagram

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References

- I. Shovkovy and M. Huang, Phys. Lett. B **564** (2003) 205, `hep-ph/0302142`
- M. Huang and I. Shovkovy, Nucl. Phys. A **729** (2003) 835, `hep-ph/0307273`
- S. Rüster, I. Shovkovy, D. Rischke, Nucl. Phys. A **743** (2004) 127, `hep-ph/0405170`
- M. Huang and I. Shovkovy, Phys. Rev. D **70** (2004) 051501(R), `hep-ph/0407049`
- M. Huang and I. Shovkovy, `hep-ph/0408268`

QCD from first principles (Lattice calculations)

- Thermodynamical quantities $O(T, V, \mu)$ can be evaluated via Monte Carlo Simulatios,

$$\langle O(T, V, \mu) \rangle = \frac{1}{Z} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \; O \; \exp\left[-S_E(T, V, \mu)\right]$$

where

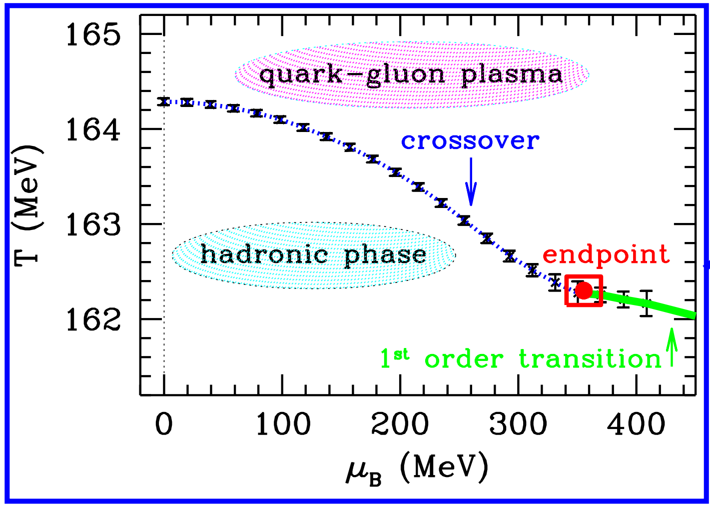
$$\begin{aligned} Z(T, V, \mu) &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \; \exp\left[-S_E(T, V, \mu)\right] \\ &= \int \mathcal{D}A \; \det M(\mu) \; \exp\left[-S_G(T, V, \mu)\right] \end{aligned}$$

- However, $\det M(\mu) \equiv |\det M(\mu)| e^{i\theta}$ is complex $\rightarrow$ probability interpretation of the integrand fails (sign problem)

- See, however, [Fodor&Katz, hep-lat/0106002], [Allton et al., hep-lat/0204010]

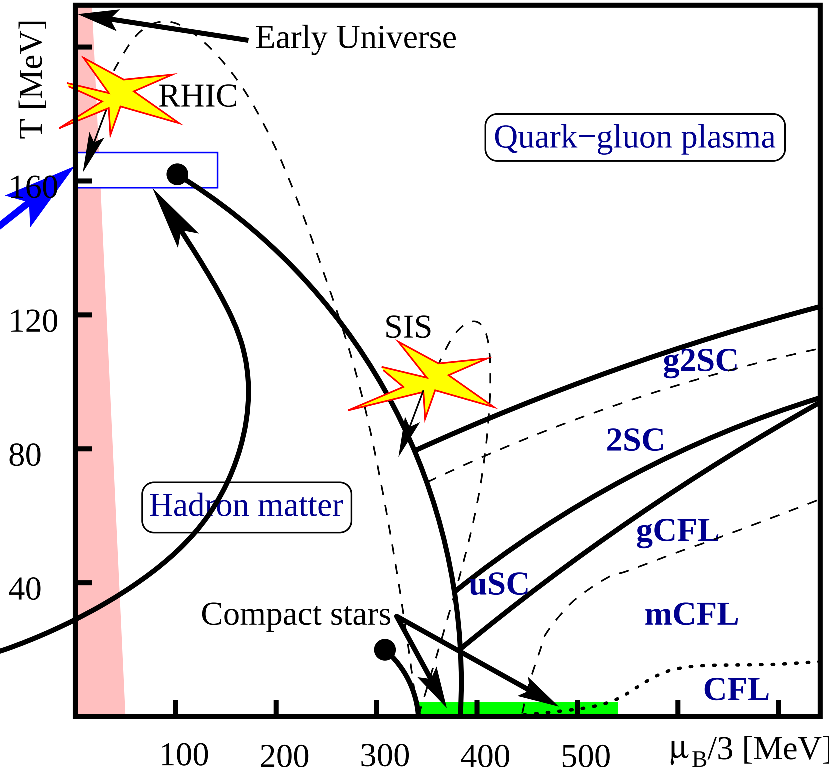
Phase diagram from lattice

This works only at small μ:



Fodor & Katz, hep−lat/0402006

(the other talks in this session)

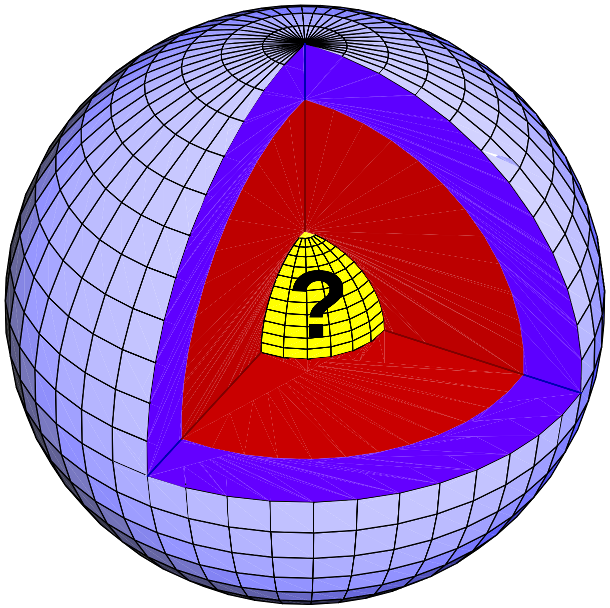


Dense quark matter ($T \lesssim \Lambda_{QCD} \lesssim \mu$)

Why are we interested in studying dense matter?

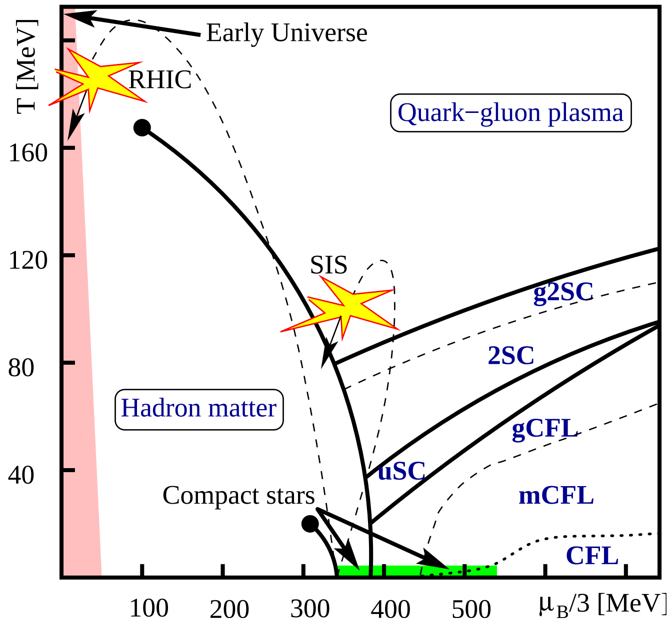
(i) Dense matter exists in the Universe

(densities in stars $\rho_c \gtrsim 5\rho_0$)



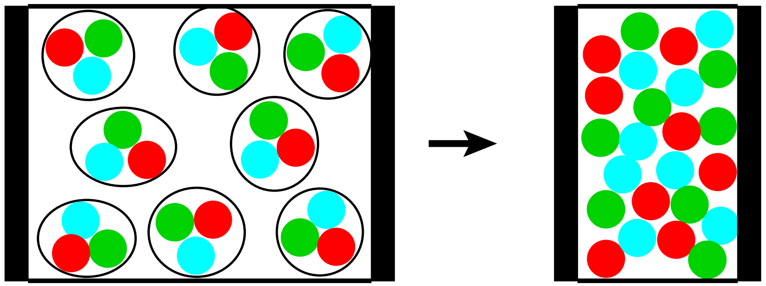
(ii) Fundamental properties of QCD

($\mu \gtrsim \Lambda_{QCD}$: no lattice results)



Dense matter might be deconfined

- "Squeezing" baryonic matter hard should produce quark matter:



- Conjecture: quark matter may exist in stars

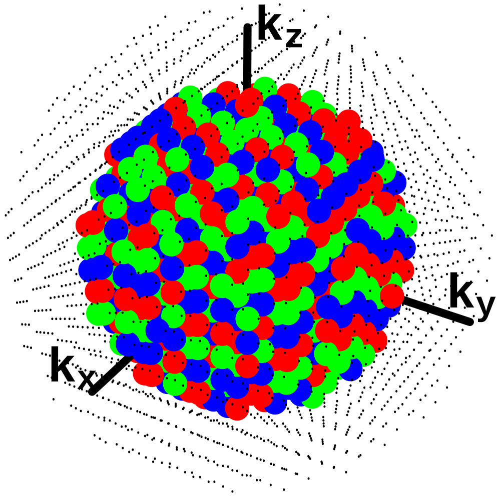
  [Ivanenko&Kurdgelaidze,'65], [Itoh'70], [Collins&Perry,'75]

- What is the ground state of quark matter?
- What is the effect of charge neutrality and β-equilibrium?

Ground state of dense quark matter

Educated guess:

(i) Quarks are fermions ($s = \frac{1}{2}$)
(ii) Interaction is weak ($\alpha_s \ll 1$)
} $\Rightarrow$ Fermi liquid (?)



(cf., electron gas in metals/alloys)

Further refinement:

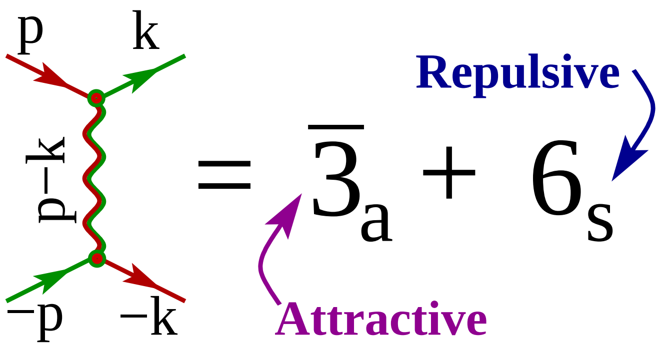
(i) Degenerate Fermi surface
(ii) Attractive interaction (?)
} $\Rightarrow$ Cooper instability

(cf., the Cooper instability in superconducting metals/alloys)

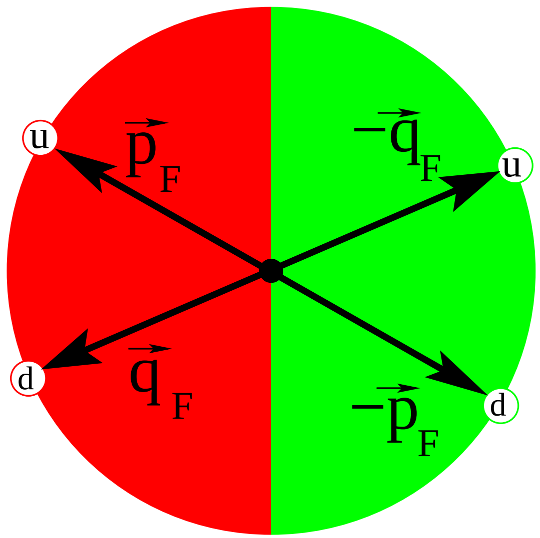
Color superconductivity in dense QCD

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84; Son,'99]

- $N_f = 2$: "up" and "down" quarks
- $N_c = 3$: "red", "green" and "blue"
- $p_F^{\mathrm{up}} = p_F^{\mathrm{down}} = \mu$
- Quark-quark interaction:



$$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$$



Cooper instability → color superconductivity

$$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle) \quad (\Leftarrow \text{Pauli principle})$$

2SC ground state properties

- Wave function of a Cooper-pair:
  Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|\mathrm{u,d}\rangle - |\mathrm{d,u}\rangle)_{1}$
- Diquark condensate (spin-0 gap $\sim$ 10 - 100 MeV):

$$\langle \left(\bar{\Psi}^C\right)^{\alpha}_{i} \gamma_5 \Psi^{\beta}_{j} \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

- Symmetry of ground state (2SC phase):
  - chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ — intact
  - baryon number $\mathrm{U}(1)_B \to \tilde{\mathrm{U}}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}} T_8$
  - approximate axial $\mathrm{U}(1)_A$ is broken $\to$ 1 pseudo-NG boson
  - Color gauge symmetry $\mathrm{SU}(3)_c \to \mathrm{SU}(2)_c$ by Anderson-Higgs mechanism $\to$ 5 massive gluons
  - Gauge symmetry $\mathrm{U}(1)_{\mathrm{em}} \to \tilde{\mathrm{U}}(1)_{\mathrm{em}}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$

Physical properties of 2SC phase

- Pressure/equation of state:

$$P \simeq \frac{\mu^4}{2\pi^2} - B + \boxed{\frac{\mu^2\Delta^2}{\pi^2}} \qquad \text{may be (un-)important}$$

- Transport/specific heat is dominated by
  - Unpaired "blue-up" and "blue-down" quarks
  - 1 pseudo-NG boson that results from breaking $U(1)_A$
  - 3 gluons of unbroken $\mathrm{SU}(2)_c$ (decoupled from blue quarks)
  - low energy photon of $\tilde{\mathrm{U}}(1)_{\mathrm{em}}$
- No superfluidity → no rotational vorticies
- No electromagnetic Meissner effect → no magnetic flux tubes
- Neutrino emissivity/cooling rate is large (direct URCA)

Color superconductivity, $N_f = 3$

- Wave function of a Cooper-pair:

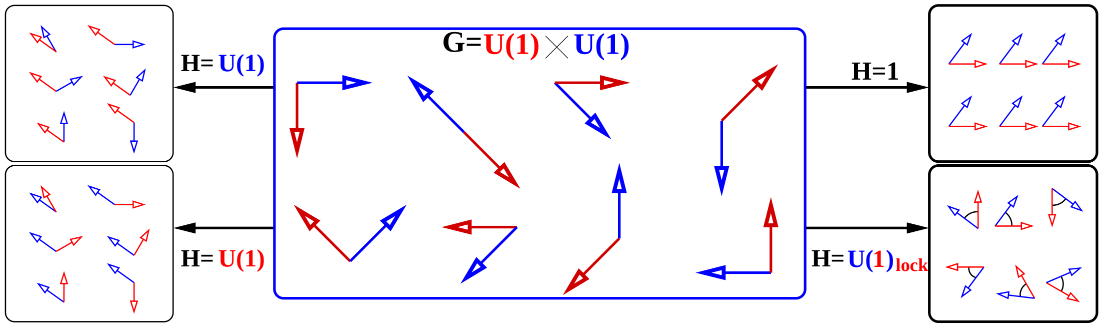
  Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|\mathrm{u,d}\rangle - |\mathrm{d,u}\rangle)_{\bar{3}}$

- Diquark condensate (spin-0 gap $\sim$ 10 - 100 MeV):

$$\langle \left(\bar{\Psi}_L^C\right)_i^\alpha \left(\Psi_L\right)_j^\beta \rangle = \langle \left(\bar{\Psi}_R^C\right)_i^\alpha \left(\Psi_R\right)_j^\beta \rangle \simeq \sum_I \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- Color-flavor locking: $SU(3)_L \times SU(3)_R \times SU(3)_c \to SU(3)_{L+R+c}$

  [Alford et al. hep-ph/9804403]



There are no $\langle q_L q_R \rangle$ condensates, but $SU(3)_L \times SU(3)_R$ chiral symmetry is broken down to $SU(3)_V$ through locking with color!

Color superconductivity, $N_f = 3$

- Symmetry of ground state (CFL phase):
  - chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$ $\rightarrow$ 8 (pseudo-)NG bosons, i.e., π^0, $\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$, η
  - baryon number $U(1)_B$ is broken $\rightarrow$ 1 NG boson (ϕ)
  - approximate axial $U(1)_A$ is broken $\rightarrow$ 1 pseudo-NG boson (η')
  - Color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism $\rightarrow$ 8 massive gluons
  - Gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
- Quasiparticle spectrum (9 quark quasiparticles):
  - octet under $SU(3)_{L+R+c}$ with gap Δ
  - singlet under $SU(3)_{L+R+c}$ with gap 2Δ

Physical properties of CFL phase

- Pressure/equation of state:

$$P \simeq \frac{3\mu^4}{4\pi^2} - B + \boxed{\frac{3\mu^2\Delta^2}{\pi^2}} \qquad \text{may be (un-)important}$$

- Transport/specific heat is dominated by [Shovkovy & Ellis, 2002]
  - 1 NG boson (ϕ) that results from breaking $U(1)_B$
  - low energy photon of $\tilde{\mathrm{U}}(1)_{\mathrm{em}}$
  - 1 pseudo-NG boson (η') that results from breaking $U(1)_A$
  - 8 ($\times$3 polarizations) light plasmons with mass $\sim \Delta$ (?)

    [Gusynin & Shovkovy, hep-ph/0108175]
- Superfluidity $\to$ rotational vorticies
- No electromagnetic Meissner effect $\to$ no magnetic flux tubes
- Neutrino emissivity/cooling rate is suppressed ($\sim e^{-\Delta/T}$)

Color superconductivity, $N_f = 1$

- Wave function of a Cooper-pair: $\left(|\bullet\bullet\rangle - |\bullet\bullet\rangle\right)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$
  - antisymmetric in color (attractive diquark $\bar{3}_c$ channel)
  - Pauli principle: symmetric in spin, i.e., spin-1 channel
- Diquark condensate (gap $\sim$ 10 - 100 keV):

$$\langle \left(\bar{\Psi}^C\right)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^\delta \sin\theta + \gamma_\perp^\delta(\vec{\mathbf{k}}) \cos\theta \right)$$

- Many possibilities, see Ph.D. thesis [A.Schmitt, nucl-th/0405076]:
  - Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
  - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta}$
  - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
  - A-phase: $\Delta_{c\delta} = \delta_{c3}\left(\delta_{\delta 1} + i\delta_{\delta 2}\right) \to$ largest pressure
- Meissner effect $\oplus$ type I superconductor $\to$ affect star properties

Importance of charge neutrality

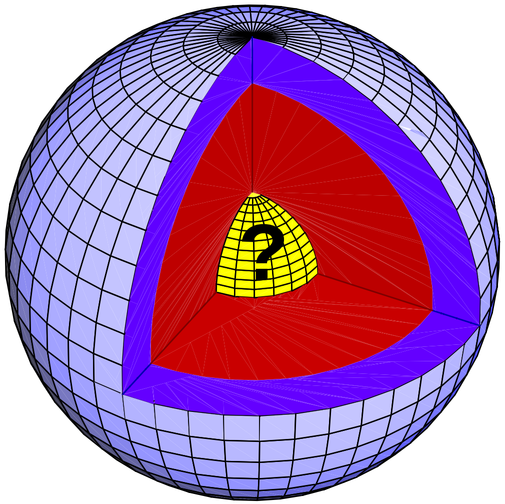
Matter in the bulk of a star is

(i) β-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:

$n_Q^{\text{el}} = 0, \qquad n_Q^{\text{color}} = 0$

Otherwise, a star is **not** stable!



- Coulomb energy (when $n_Q \neq 0$)

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3}\right)^2 \left(\frac{R}{1 \text{ km}}\right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \quad \Rightarrow \quad E_{\text{Coulomb}}^{\text{2SC}} \gg M_\odot c^2$

Neutrality vs. color superconductivity

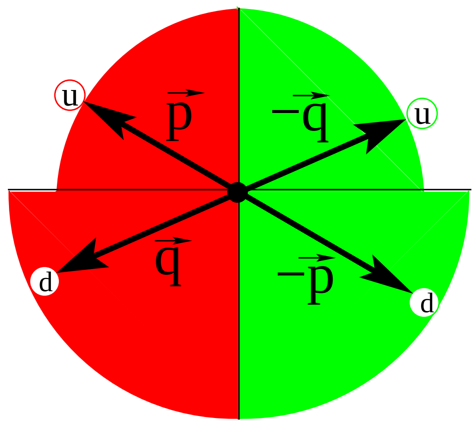
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in β-equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \quad \Rightarrow \quad \mu_d \approx 2^{1/3}\mu_u \;\Rightarrow\; \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{i.e.,} \qquad n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

The "best" Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Appearance of a gapless phase

Mismatch parameter μ_e is **not** a free model parameter,

$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \qquad \Rightarrow \qquad \mu_e = \mu_e(\mu, \Delta)$$

Three dynamical regimes determined by coupling strength η:

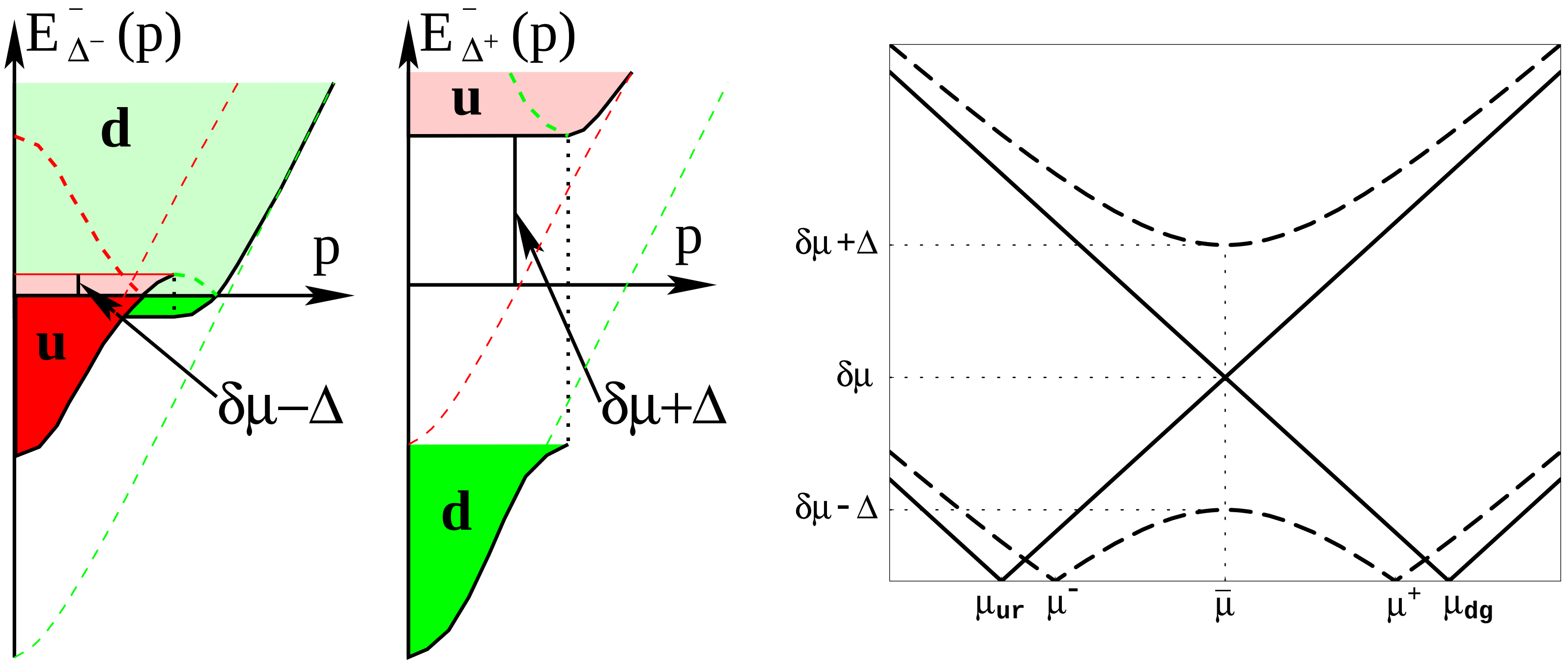
1. $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing: normal phase is the ground state

2. $\eta \gtrsim 0.8$ — strong coupling wins over the mismatch between the Fermi surfaces: 2SC is the ground state

3. $0.7 \lesssim \eta \lesssim 0.8$ — regime of intermediate coupling strength: the ground state is the gapless 2SC phase [hep-ph/0302142]

Quasiparticle spectrum in g2SC phase

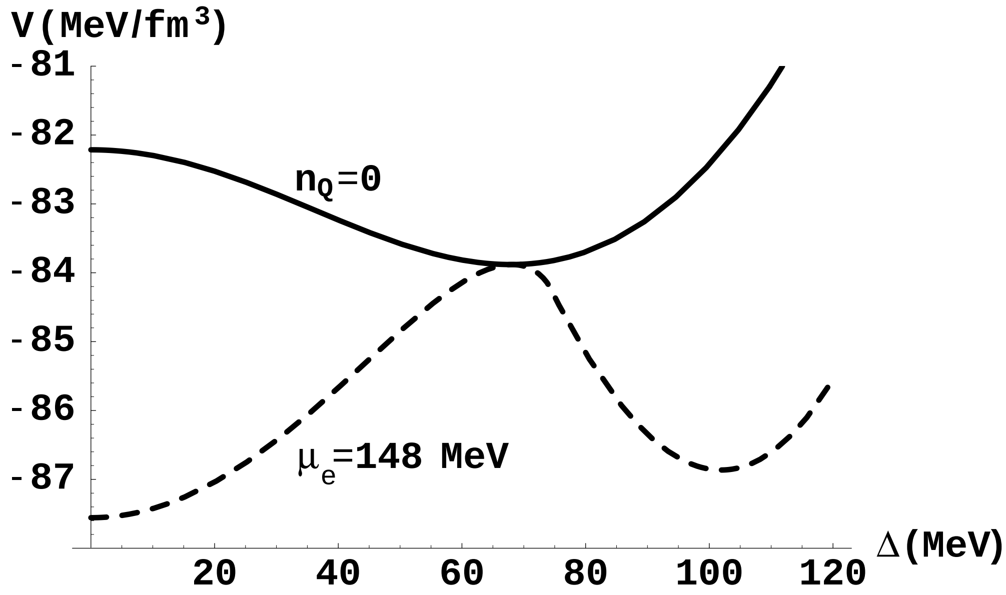
"Intermediate" coupling (gapless phase)



The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Is the g2SC phase stable?

Effective potential at $T = 0$ [I.S. & M.Huang, Phys. Lett. B564 (2003) 205]:

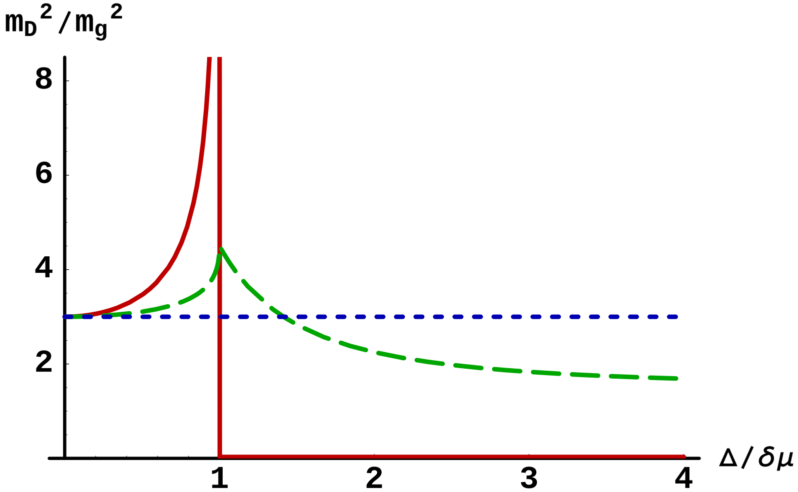


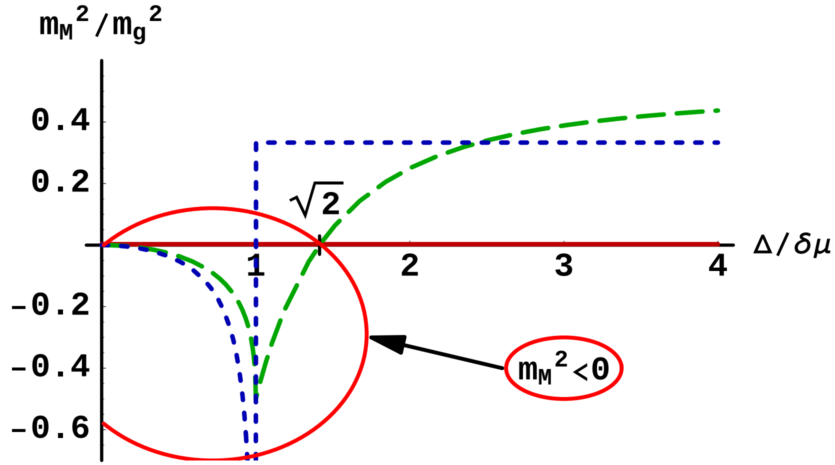
(Q.: Mixed phase? → A.: Unlikely if $\sigma \gtrsim 20$ MeV/fm^2 [Shovkovy, Hanauske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054])

No Sarma instability if $n_Q = 0$ is enforced *locally*!

Chromomagnetic instability in g2SC phase

Results for gluon screening masses
[Huang & Shovkovy, hep-ph/0407049]:





$A = 1, 2, 3$ — red solid line

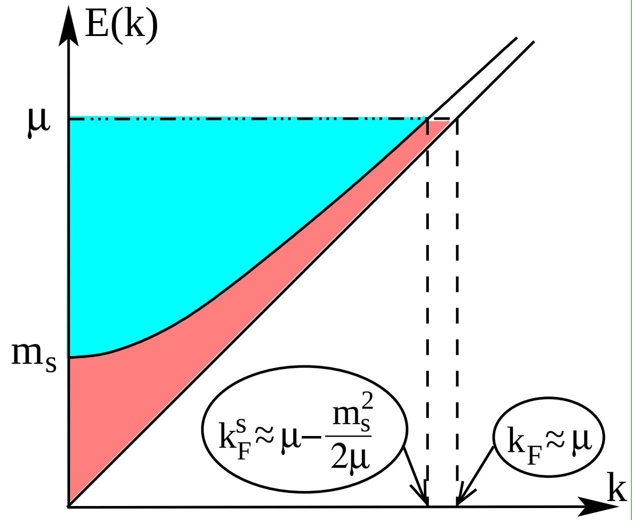
$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

Finite strange quark mass, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$



The ground state of strange quark matter may have:

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystaline pairing (nonzero momentum pairing, LOFF)

Recently, other possibilities were proposed as well . . .

Gapless $N_f = 3$ quark matter

- Distorted color-flavor pairing:

$$\Delta_{ij}^{\alpha\beta} \simeq \Delta_1 \; \epsilon_{1ij}\varepsilon^{1\alpha\beta} + \Delta_2 \; \epsilon_{2ij}\varepsilon^{2\alpha\beta} + \Delta_3 \; \epsilon_{3ij}\varepsilon^{3\alpha\beta} + \ldots$$

- Control (mismatch) parameter:

$$\delta\mu \equiv \frac{\mu_{bd} - \mu_{gs}^{\text{eff}}}{2} \approx -\frac{\mu_8}{2} + \frac{m_s^2}{4\mu} \approx \boxed{\frac{m_s^2}{2\mu}}$$

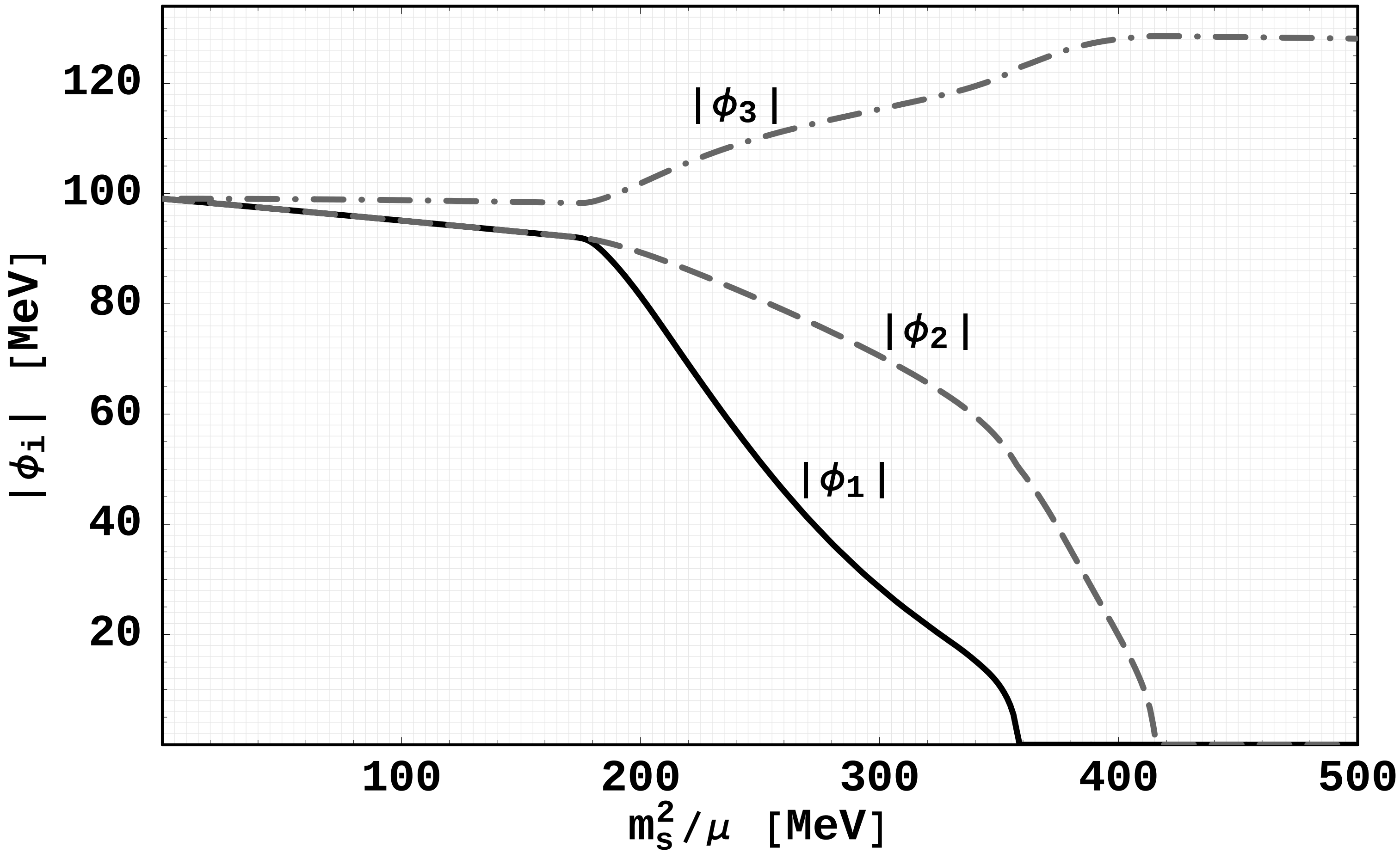
where $\mu_{gs}^{\text{eff}} \simeq \mu_{gs} - \frac{m_s^2}{2\mu}$ and $\mu_8 \simeq -\frac{m_s^2}{2\mu}$ (blue color is special)

- Gapless CFL phase with $\Delta_1 < \Delta_2 < \Delta_3$:

$$T = 0: \qquad \delta\mu \equiv \frac{m_s^2}{2\mu} > \Delta_0$$

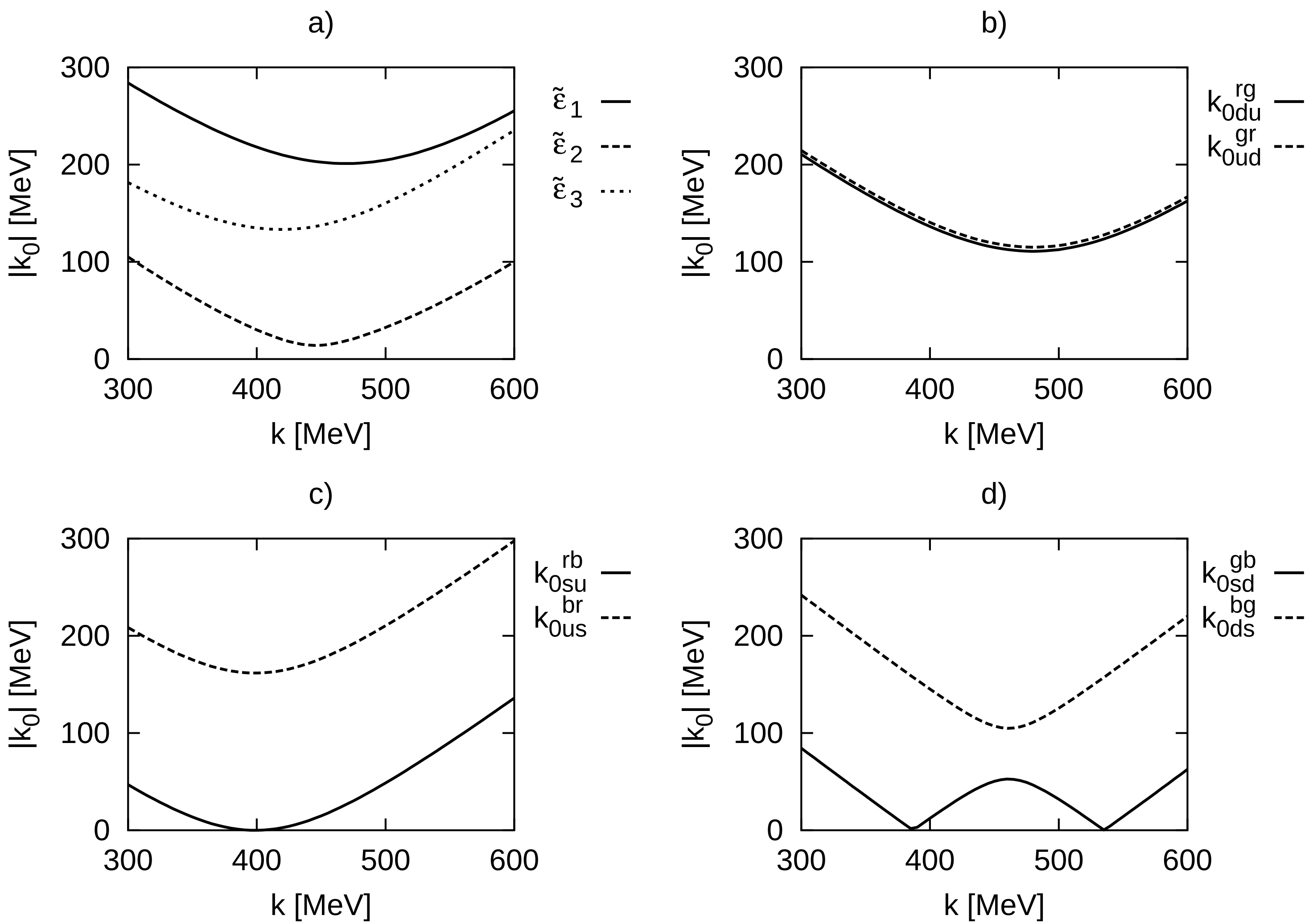
[Alford et al. hep-ph/0311286]

Gap parameters



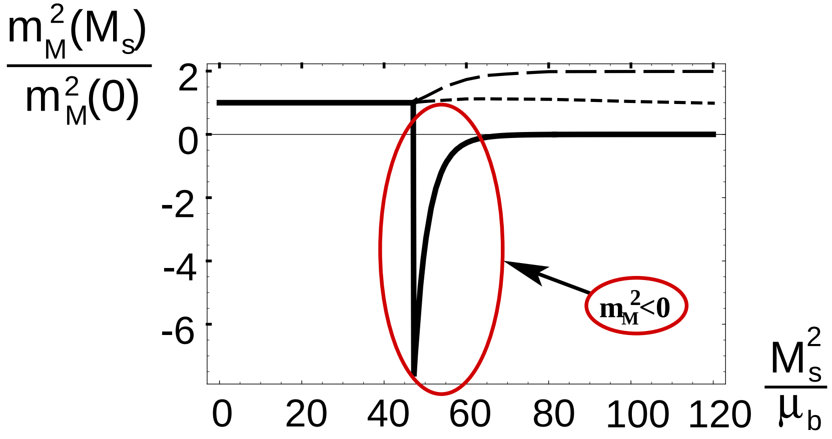
$\mu = 500$ MeV

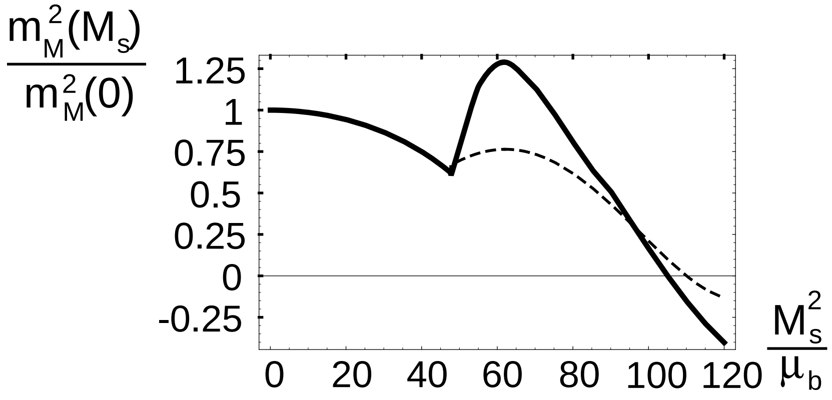
Quasiparticle spectrum in gCFL phase



Chromomagnetic instability in gCFL phase

Recent results for Meissner screening masses
[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:





$A = 1, 2$ — solid line

$A = 3$ — short-dashed line

$A = 8$ — long-dashed line

$A = 4, 5$ — dashed line

$A = 6, 7$ — solid line

Nonzero temperature

- There can exist many phases at $T \neq 0$

  [Iida et al, hep-ph/0312363], [Rüster et al, hep-ph/0405170], [Fukushima al, hep-ph/0408322]

- Zoo of phases:

$$
\begin{array}{rllll}
\text{CFL:} & \Delta_1 \neq 0, & \Delta_2 \neq 0, & \Delta_3 \neq 0, & (\mu_e \approx 0) \\
\text{mCFL:} & \Delta_1 \neq 0, & \Delta_2 \neq 0, & \Delta_3 \neq 0, & (\mu_e \neq 0) \\
\text{uSC:} & \Delta_1 = 0, & \Delta_2 \neq 0, & \Delta_3 \neq 0, & \\
\text{dSC:} & \Delta_1 \neq 0, & \Delta_2 = 0, & \Delta_3 \neq 0, & \\
\text{2SC:} & \Delta_1 = 0, & \Delta_2 = 0, & \Delta_3 \neq 0, & \\
\text{NQM:} & \Delta_1 = 0, & \Delta_2 = 0, & \Delta_3 = 0, &
\end{array}
$$

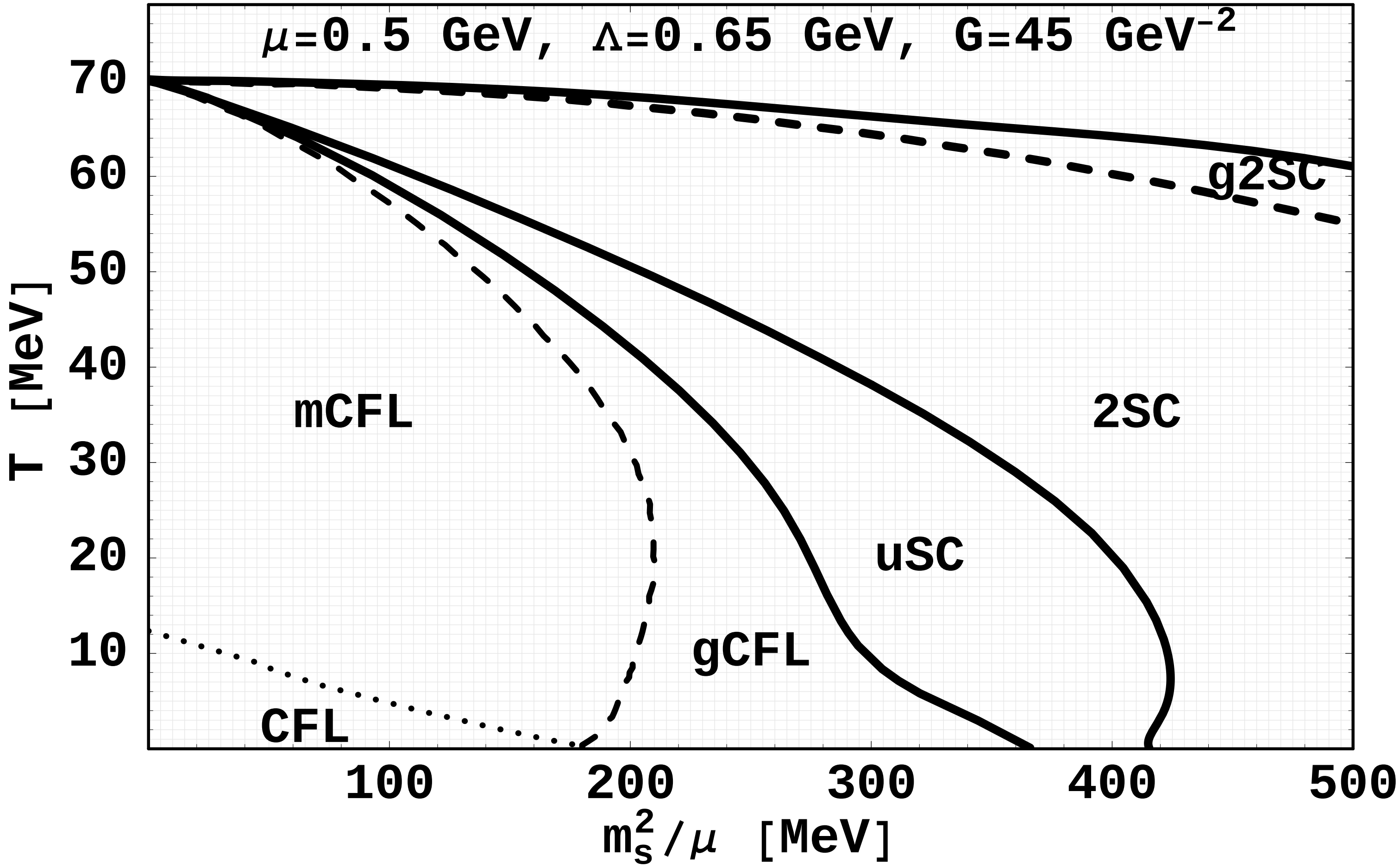
plus g2SC and gCFL as special cases of 2SC and mCFL.

Overview of phases with strangeness

- Color-flavor locked (CFL) phase
  - "Enforced pairing": $n_u = n_d = n_s$ $(T \simeq 0)$

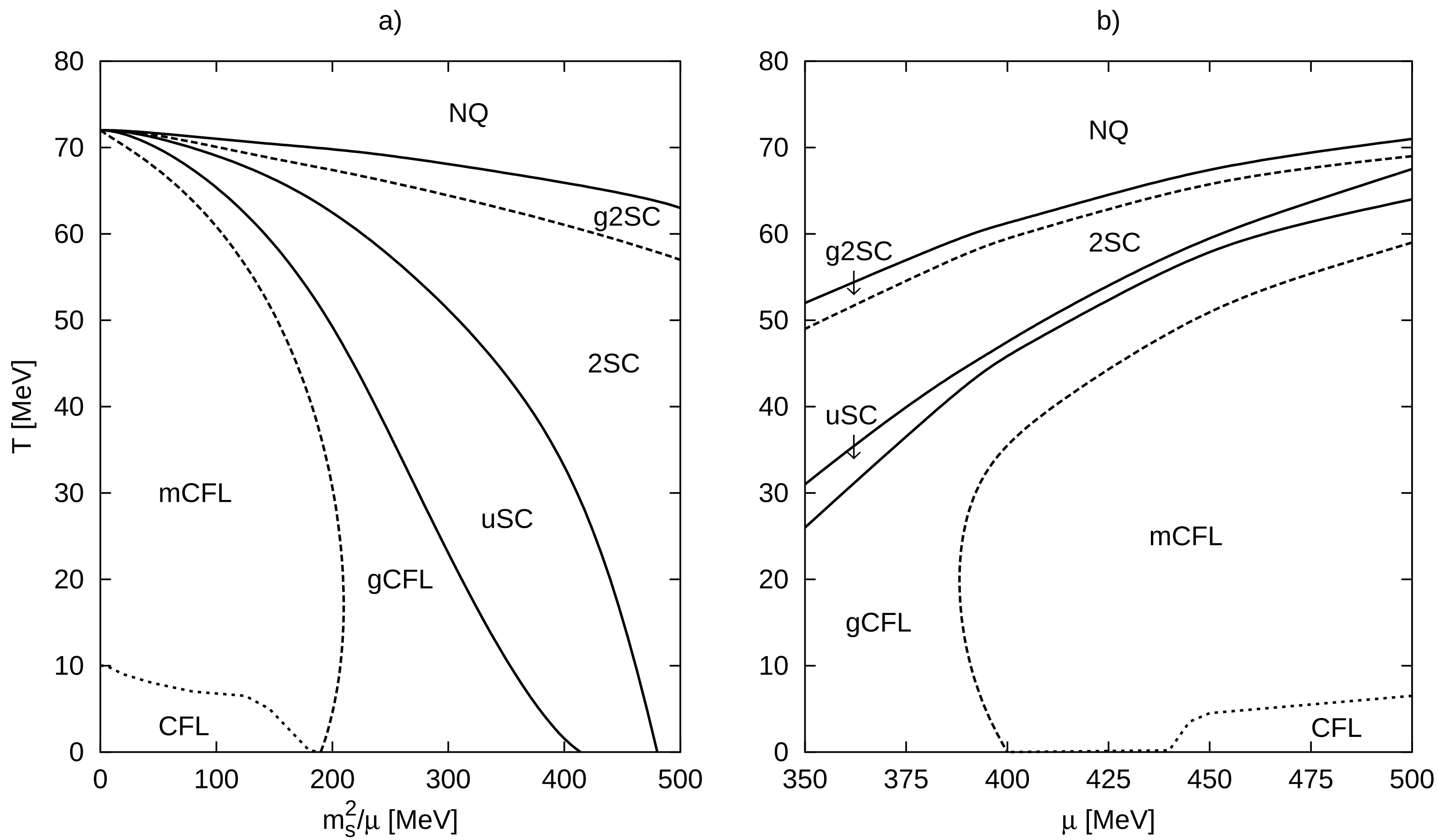
    [Rajagopal, Wilczek, 2001]
  - Natural insulator, $n_{\text{el}} \simeq 0$
  - Little specific heat and low neutrino emissivity
- Metallic CFL phase $(n_{\text{el}} \neq 0)$
  - $T = 0$: gapless CFL phase (no "enforced pairing")
  - $T \neq 0$: thermal effects $\to n_{\text{el}} \neq 0$
  - Large specific heat and high neutrino emissivity
- uSC phase: only *ud*- & *us*-pairing (no *ds*-pairing)
- dSC phase: only *du*- & *ds*-pairing (no *us*-pairing)

Phase diagram in T–m_s^2/μ plane



The quark chemical potential is kept fixed, $\mu = 500$ MeV

Phase diagram in T–μ plane



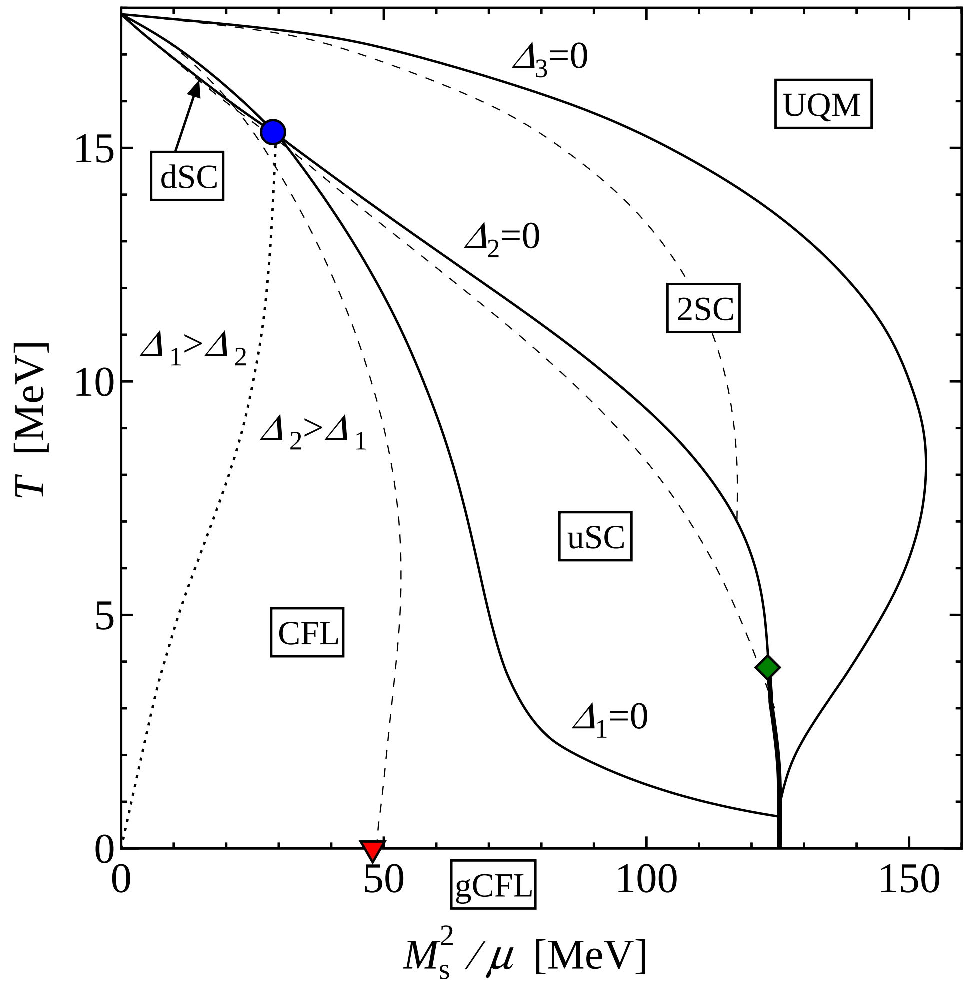
a) $\mu = 500$ MeV   b) $m_s = 250$ MeV

Yet another version of the phase diagram ...

This is taken from

[K. Fukushima, C. Kouvaris, K. Rajagopal, hep-ph/0408322]

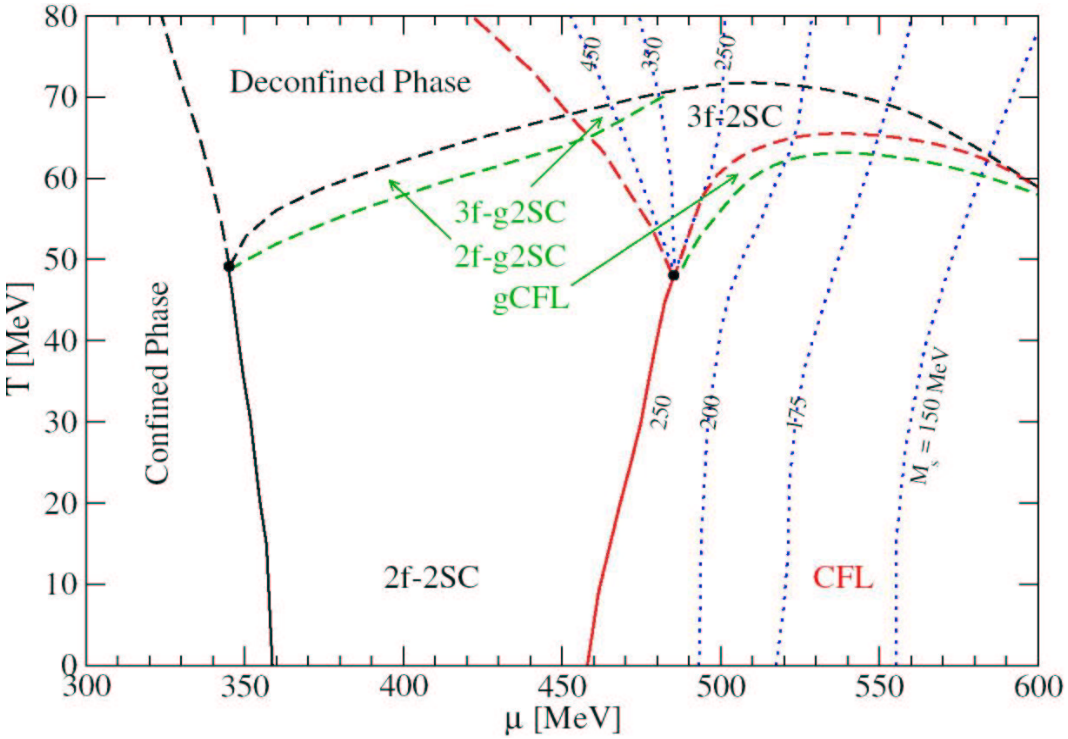
Here the quark chemical potential is kept fixed, $\mu = 500$ MeV



Phase diagram in T–μ plane

[D. Blaschke, S. Fredriksson, H. Grigorian, F. Sandin, A. Öztas] taken from the talk by H. Grigorian at VI and RTN Collaboration Meeting, GSI, 22 October, 2004:

Phase diagram



In this study, m_s is deteremined self-consistently

Current status

- Sufficiently cold and dense matter is a color superconductor
- Neutrality and β-equilibrium may strongly affect the properties of dense matter
- There can exist many different CSC phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K^0, CFL+η)
- Some features of $T - \mu$ phase diagram start to develop
- Because of the chromomagnetic instability, there is an apparent crisis in the understanding of the phase diagram at large μ