

The current crisis in the understanding

of

QCD phase diagram

Igor A. Shovkovy



Frankfurt Institute for Advanced Studies
Johann W. Goethe-Universität
Frankfurt am Main, Germany

Collaborator(s)

- Mei Huang
- Stefan Rüster
- Dirk Rischke

References

- I. Shovkovy and M. Huang, Phys. Lett. B **564** (2003) 205, hep-ph/0302142
- M. Huang and I. Shovkovy, Nucl. Phys. A **729** (2003) 835, hep-ph/0307273
- S. Rüster, I. Shovkovy, D. Rischke, Nucl. Phys. A **743** (2004) 127, hep-ph/0405170
- M. Huang and I. Shovkovy, Phys. Rev. D **70** (2004) 051501(R), hep-ph/0407049
- M. Huang and I. Shovkovy, hep-ph/0408268

QCD from first principles (Lattice calculations)

- Thermodynamical quantities $O(T, V, \mu)$ can be evaluated via Monte Carlo Simulations,

$$\langle O(T, V, \mu) \rangle = \frac{1}{Z} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \ O \exp[-S_E(T, V, \mu)]$$

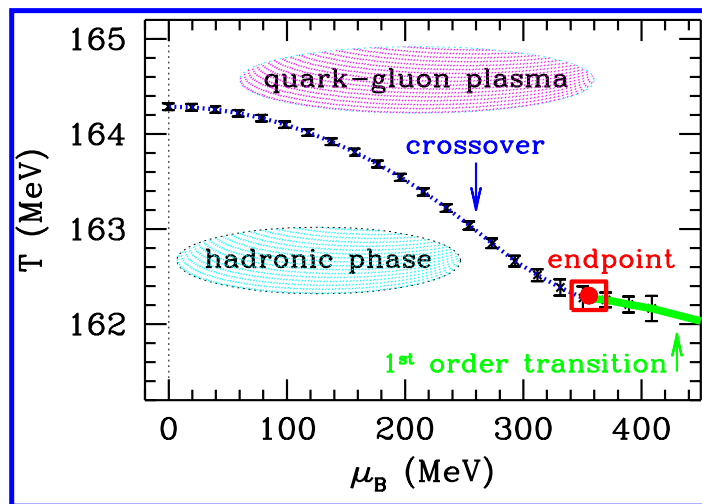
where

$$\begin{aligned} Z(T, V, \mu) &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \ \exp[-S_E(T, V, \mu)] \\ &= \int \mathcal{D}A \ \det M(\mu) \ \exp[-S_G(T, V, \mu)] \end{aligned}$$

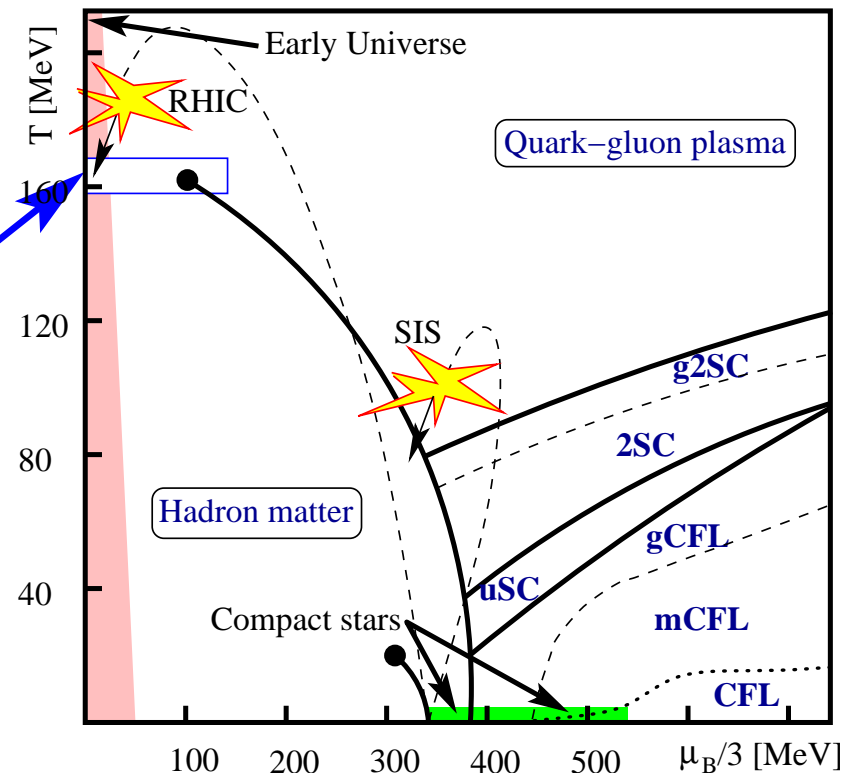
- However, $\det M(\mu) \equiv |\det M(\mu)| e^{i\theta}$ is complex \rightarrow probability interpretation of the integrand fails (sign problem)
- See, however, [Fodor&Katz, hep-lat/0106002], [Allton et al., hep-lat/0204010]

Phase diagram from lattice

This works only at small μ :



Fodor & Katz, hep-lat/0402006



Dense quark matter ($T \lesssim \Lambda_{QCD} \lesssim \mu$)

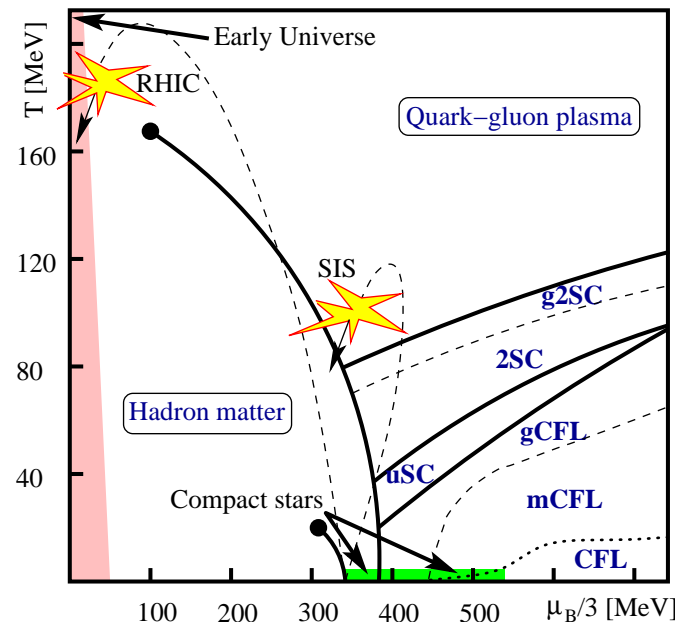
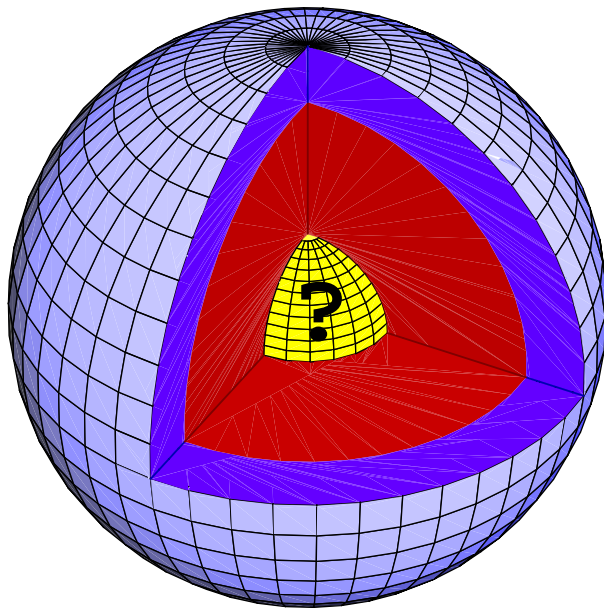
Why are we interested in studying dense matter?

(i) Dense matter exists in the Universe

(densities in stars $\rho_c \gtrsim 5\rho_0$)

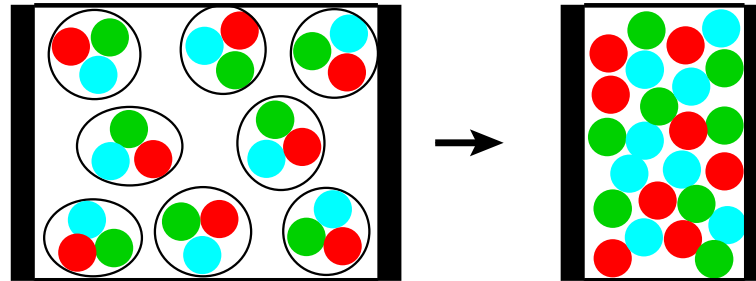
(ii) Fundamental properties of QCD

($\mu \gtrsim \Lambda_{QCD}$: no lattice results)



Dense matter might be deconfined

- “Squeezing” baryonic matter hard should produce quark matter:

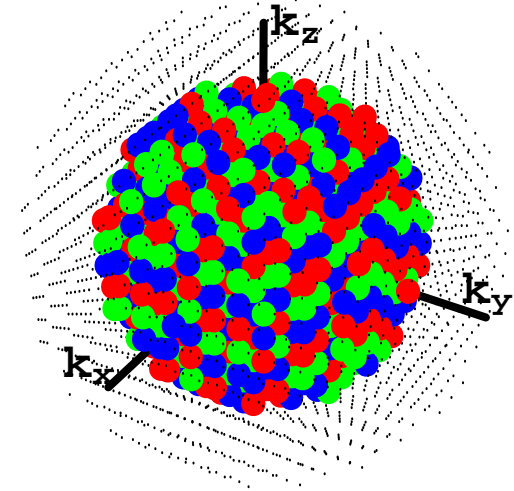


- **Conjecture:** quark matter may exist in stars
[Ivanenko&Kurdgelaidze,'65], [Itoh'70], [Collins&Perry,'75]
- What is the ground state of quark matter?
- What is the effect of charge neutrality and β -equilibrium?

Ground state of dense quark matter

Educated guess:

- (i) Quarks are fermions ($s = \frac{1}{2}$) }
(ii) Interaction is weak ($\alpha_s \ll 1$) } \Rightarrow Fermi liquid (?)
(cf., electron gas in metals/alloys)



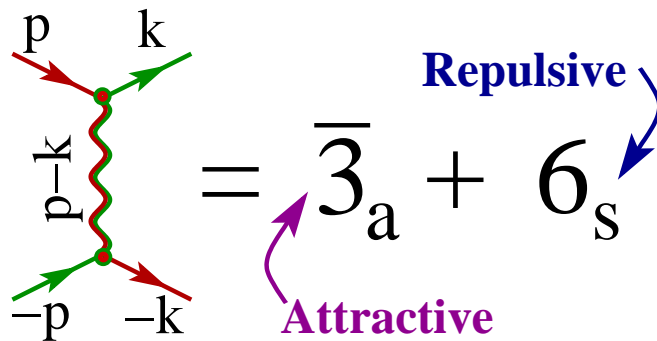
Further refinement:

- (i) Degenerate Fermi surface }
(ii) Attractive interaction (?) } \Rightarrow Cooper instability
(cf., the Cooper instability in superconducting metals/alloys)

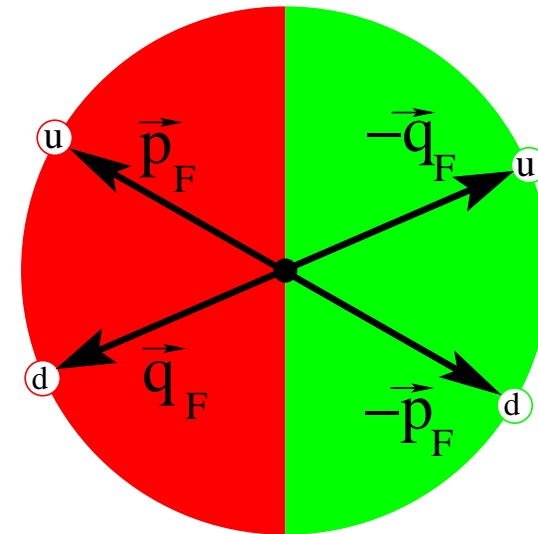
Color superconductivity in dense QCD

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84; Son,'99]

- $N_f = 2$: “up” and “down” quarks
- $N_c = 3$: “red”, “green” and “blue”
- $p_F^{\text{up}} = p_F^{\text{down}} = \mu$
- Quark-quark interaction:



$$\langle \mathbf{u}_p \mathbf{d}_{-p} \rangle = - \langle \mathbf{u}_q \mathbf{d}_{-q} \rangle \neq 0$$



Cooper instability \rightarrow color superconductivity

$$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle) \quad (\Leftarrow \text{Pauli principle})$$

2SC ground state properties

- Wave function of a Cooper-pair:

Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_1$

- Diquark condensate (spin-0 gap $\sim 10 - 100$ MeV):

$$\langle (\bar{\Psi}^C)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

- Symmetry of ground state (2SC phase):

- chiral $SU(2)_L \times SU(2)_R$ — intact

- baryon number $U(1)_B \rightarrow \tilde{U}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}}T_8$

- approximate axial $U(1)_A$ is broken \rightarrow 1 pseudo-NG boson

- Color gauge symmetry $SU(3)_c \rightarrow SU(2)_c$ by Anderson-Higgs mechanism \rightarrow 5 massive gluons

- Gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}}T_8$

Physical properties of 2SC phase

- Pressure/equation of state:

$$P \simeq \frac{\mu^4}{2\pi^2} - B + \frac{\mu^2 \Delta^2}{\pi^2} \quad \text{may be (un-)important}$$

- Transport/specific heat is dominated by
 - Unpaired “blue-up” and “blue-down” quarks
 - 1 pseudo-NG boson that results from breaking $U(1)_A$
 - 3 gluons of unbroken $SU(2)_c$ (decoupled from blue quarks)
 - low energy photon of $\tilde{U}(1)_{em}$
- No superfluidity \rightarrow no rotational vortices
- No electromagnetic Meissner effect \rightarrow no magnetic flux tubes
- Neutrino emissivity/cooling rate is large (direct URCA)

Color superconductivity, $N_f = 3$

- Wave function of a Cooper-pair:

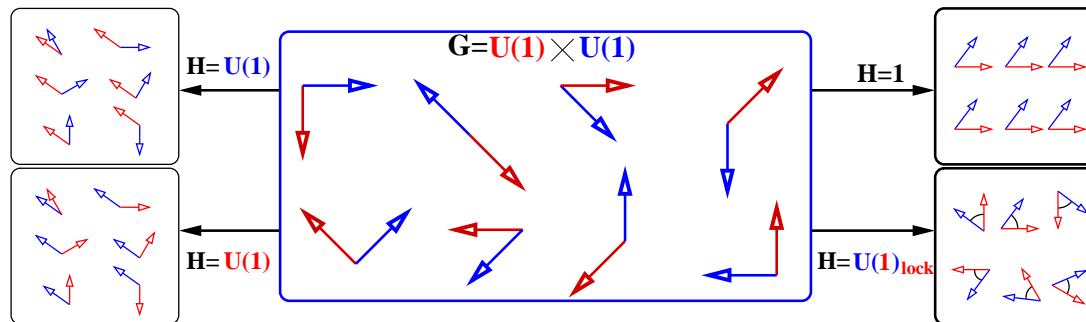
Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_{\bar{3}}$

- Diquark condensate (spin-0 gap $\sim 10 - 100$ MeV):

$$\langle (\bar{\Psi}_L^C)_i^\alpha (\Psi_L)_j^\beta \rangle = \langle (\bar{\Psi}_R^C)_i^\alpha (\Psi_R)_j^\beta \rangle \simeq \sum \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- Color-flavor locking: $SU(3)_L \times SU(3)_R \times SU(3)_c \rightarrow SU(3)_{L+R+c}$

[Alford et al. hep-ph/9804403]



There are no $\langle q_L q_R \rangle$ condensates, but $SU(3)_L \times SU(3)_R$ chiral symmetry is broken down to $SU(3)_V$ through locking with color!

Color superconductivity, $N_f = 3$

- Symmetry of ground state (CFL phase):
 - chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
→ 8 (pseudo-)NG bosons, i.e., $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$
 - baryon number $U(1)_B$ is broken → 1 NG boson (ϕ)
 - approximate axial $U(1)_A$ is broken → 1 pseudo-NG boson (η')
 - Color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism → 8 massive gluons
 - Gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$
- Quasiparticle spectrum (9 quark quasiparticles):
 - octet under $SU(3)_{L+R+c}$ with gap Δ
 - singlet under $SU(3)_{L+R+c}$ with gap 2Δ

Physical properties of CFL phase

- Pressure/equation of state:

$$P \simeq \frac{3\mu^4}{4\pi^2} - B + \frac{3\mu^2 \Delta^2}{\pi^2} \quad \text{may be (un-)important}$$

- Transport/specific heat is dominated by [Shovkovy & Ellis, 2002]
 - 1 NG boson (ϕ) that results from breaking $U(1)_B$
 - low energy photon of $\tilde{U}(1)_{\text{em}}$
 - 1 pseudo-NG boson (η') that results from breaking $U(1)_A$
 - 8 ($\times 3$ polarizations) light plasmons with mass $\sim \Delta$ (?)

[Gusynin & Shovkovy, hep-ph/0108175]

- Superfluidity \rightarrow rotational vortices
- No electromagnetic Meissner effect \rightarrow no magnetic flux tubes
- Neutrino emissivity/cooling rate is suppressed ($\sim e^{-\Delta/T}$)

Color superconductivity, $N_f = 1$

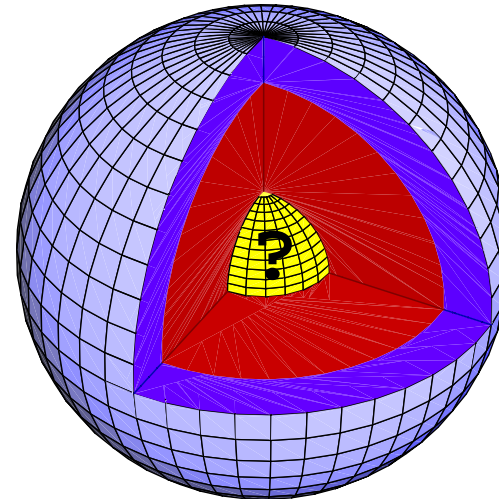
- Wave function of a Cooper-pair: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$
 - antisymmetric in color (attractive diquark $\bar{3}_c$ channel)
 - Pauli principle: symmetric in spin, i.e., spin-1 channel
- Diquark condensate (gap $\sim 10 - 100$ keV):

$$\langle (\bar{\Psi}^C)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^\delta \sin \theta + \gamma_\perp^\delta(\vec{\mathbf{k}}) \cos \theta \right)$$
- Many possibilities, see Ph.D. thesis [A.Schmitt, nucl-th/0405076]:
 - Polar phase: $\Delta_{c\delta} = \delta_{c3} \delta_{\delta 3}$
 - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta}$
 - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3} \delta_{\delta 3}$
 - A-phase: $\Delta_{c\delta} = \delta_{c3} (\delta_{\delta 1} + i\delta_{\delta 2}) \rightarrow$ largest pressure
- Meissner effect \oplus type I superconductor \rightarrow affect star properties

Importance of charge neutrality

Matter in the bulk of a star is

- (i) β -equilibrated: $\mu_d = \mu_u + \mu_e$
- (ii) electrically and color neutral:
 $n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$



Otherwise, a star is **not** stable!

- Coulomb energy (when $n_Q \neq 0$)

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_{\odot} c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left(\frac{R}{1 \text{ km}} \right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2\text{SC}} \gg M_{\odot} c^2$

Neutrality vs. color superconductivity

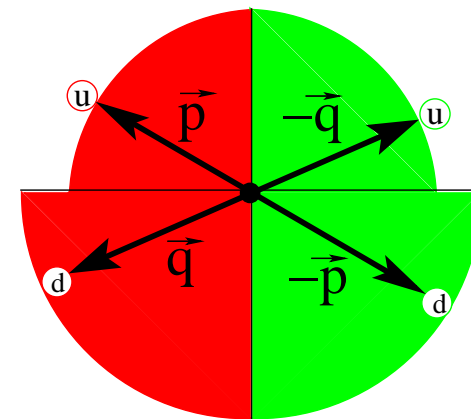
- The “best” 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in β -equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u$$

i.e., $n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$

The “best” Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Appearance of a gapless phase

Mismatch parameter μ_e is **not** a free model parameter,

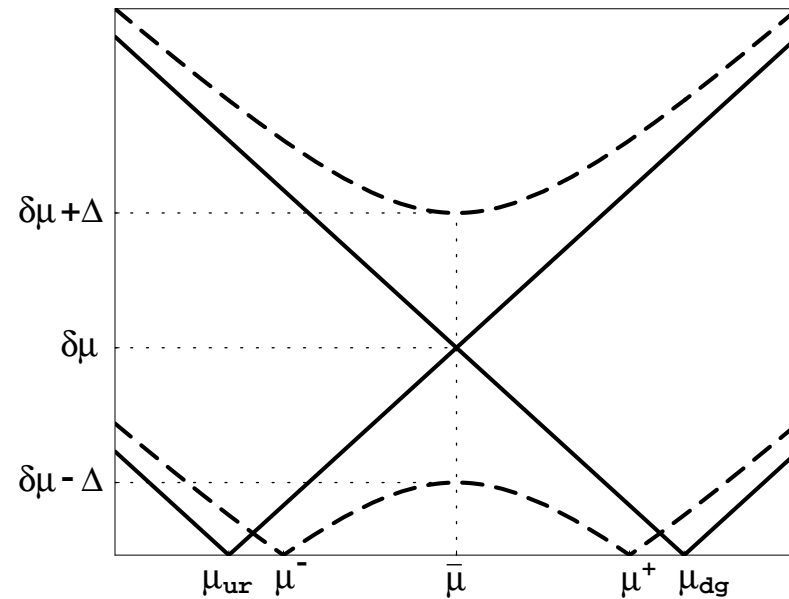
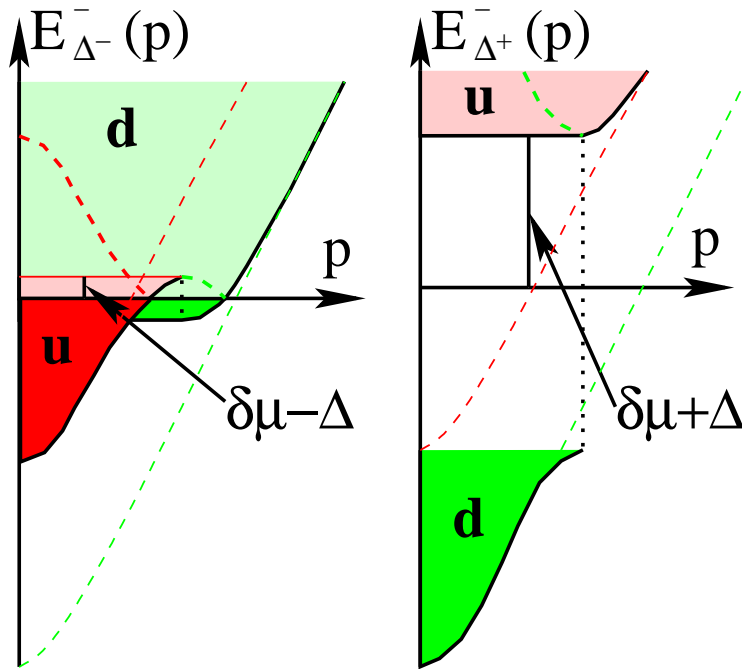
$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \quad \Rightarrow \quad \mu_e = \mu_e(\mu, \Delta)$$

Three dynamical regimes determined by coupling strength η :

1. $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing:
normal phase is the ground state
2. $\eta \gtrsim 0.8$ — strong coupling wins over the mismatch between the Fermi surfaces: 2SC is the ground state
3. $0.7 \lesssim \eta \lesssim 0.8$ — regime of intermediate coupling strength:
the ground state is the gapless 2SC phase [hep-ph/0302142]

Quasiparticle spectrum in g2SC phase

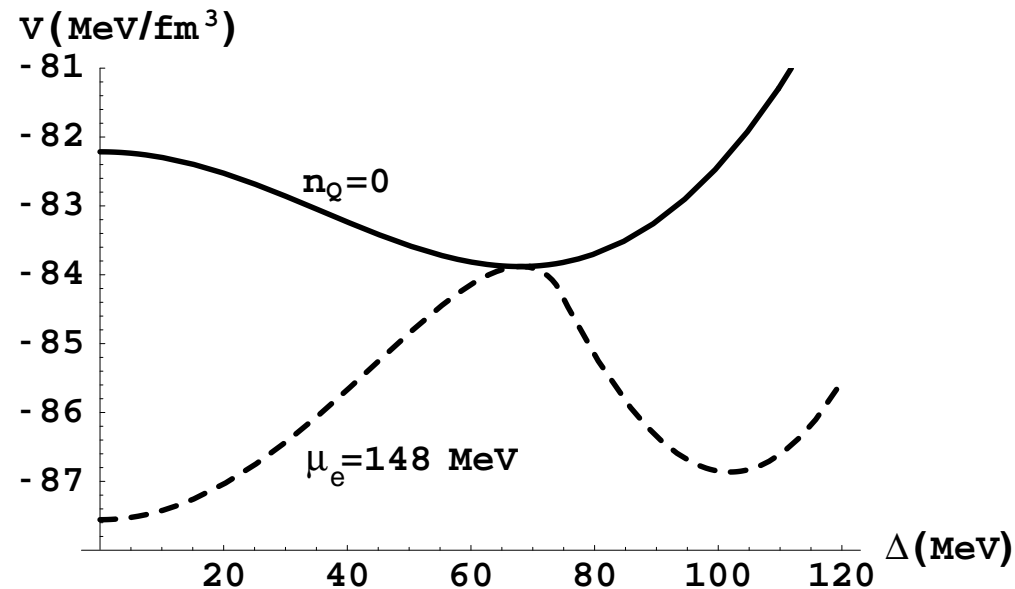
“Intermediate” coupling (gapless phase)



The energy gaps in the quasiparticle spectra are 0 & $\Delta + \delta\mu$

Is the g2SC phase stable?

Effective potential at $T = 0$ [I.S. & M.Huang, Phys. Lett. B564 (2003) 205]:

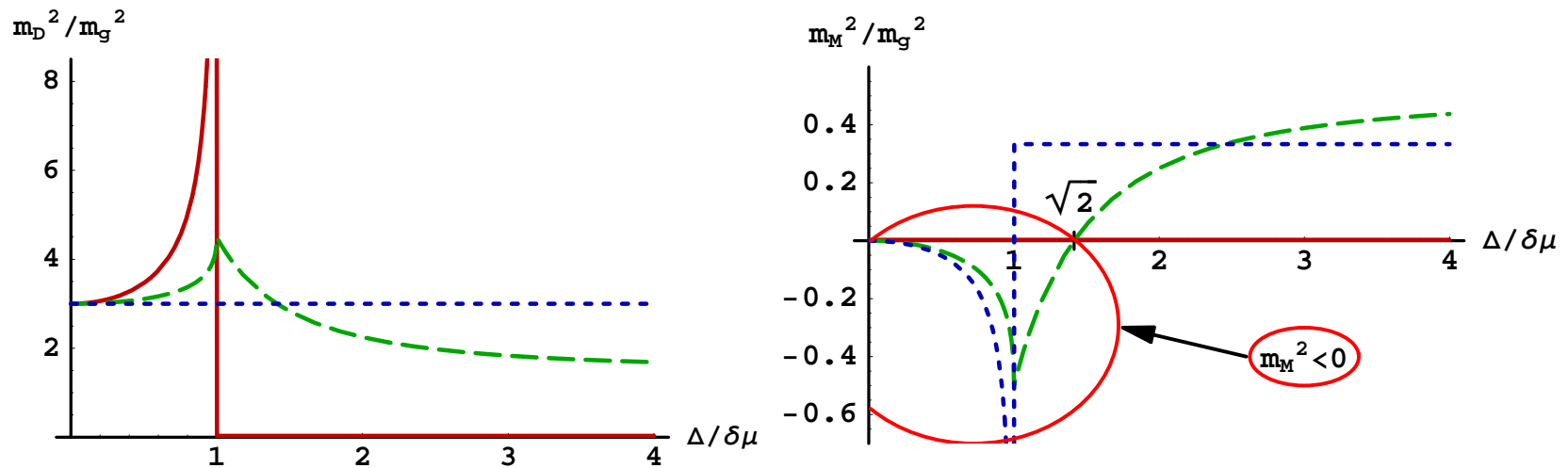


(Q.: Mixed phase? → A.: Unlikely if $\sigma \gtrsim 20 \text{ MeV}/\text{fm}^2$ [Shovkovy, Hanauske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054])

No Sarma instability if $n_Q = 0$ is enforced *locally*!

Chromomagnetic instability in g2SC phase

Results for gluon screening masses
 [Huang & Shovkovy, hep-ph/0407049]:



- $A = 1, 2, 3$ — red solid line
- $A = 4, 5, 6, 7$ — green long-dash line
- $A = \tilde{8}$ — blue short-dash line

Finite strange quark mass, $0 < m_s < \infty$

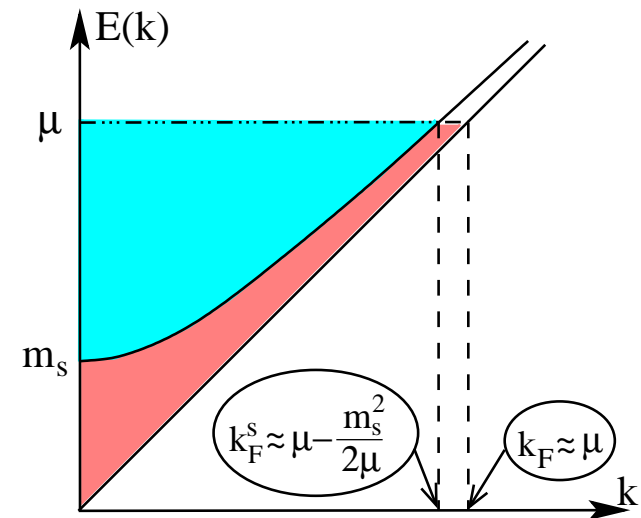
Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$

The ground state of strange quark matter may have:

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystalline pairing (nonzero momentum pairing, LOFF)

Recently, other possibilities were proposed as well ...



Gapless $N_f = 3$ quark matter

- Distorted color-flavor pairing:

$$\Delta_{ij}^{\alpha\beta} \simeq \Delta_1 \epsilon_{1ij} \epsilon^{1\alpha\beta} + \Delta_2 \epsilon_{2ij} \epsilon^{2\alpha\beta} + \Delta_3 \epsilon_{3ij} \epsilon^{3\alpha\beta} + \dots$$

- Control (mismatch) parameter:

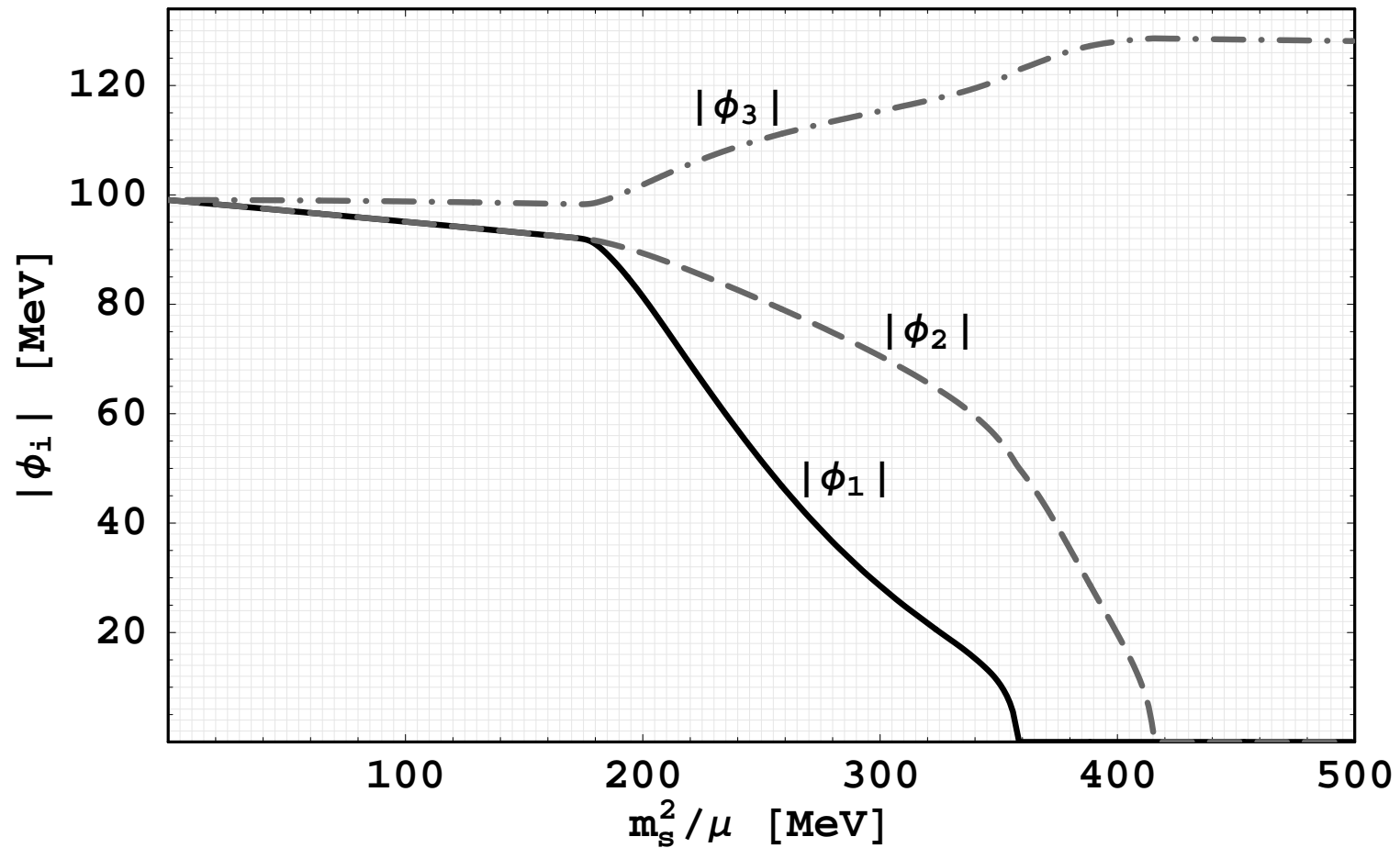
$$\delta\mu \equiv \frac{\mu_{bd} - \mu_{gs}^{\text{eff}}}{2} \approx -\frac{\mu_8}{2} + \frac{m_s^2}{4\mu} \approx \boxed{\frac{m_s^2}{2\mu}}$$

where $\mu_{gs}^{\text{eff}} \simeq \mu_{gs} - \frac{m_s^2}{2\mu}$ and $\mu_8 \simeq -\frac{m_s^2}{2\mu}$ (blue color is special)

- Gapless CFL phase with $\Delta_1 < \Delta_2 < \Delta_3$:

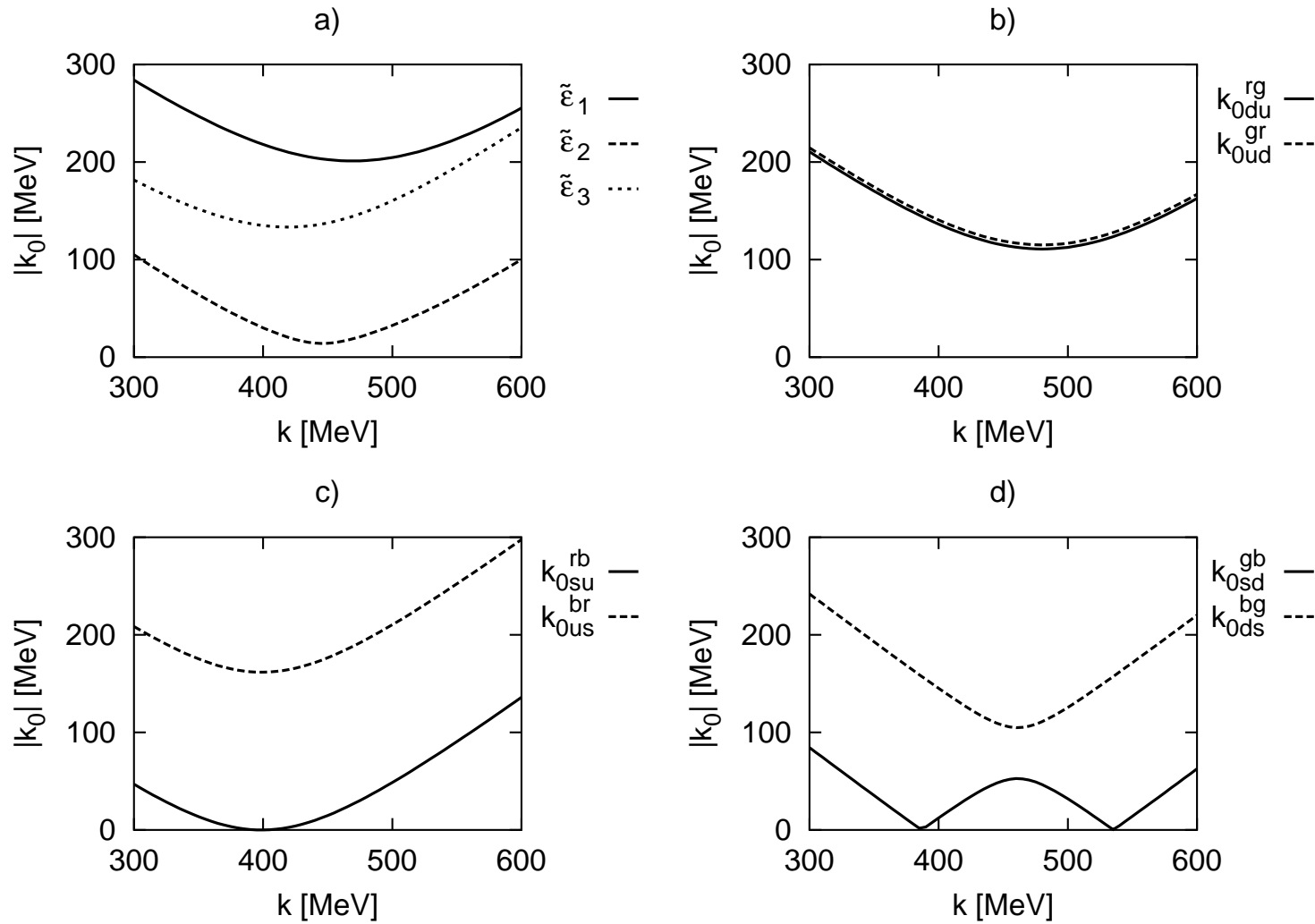
$$T = 0 : \quad \delta\mu \equiv \frac{m_s^2}{2\mu} > \Delta_0 \quad [\text{Alford et al. hep-ph/0311286}]$$

Gap parameters



$\mu = 500$ MeV

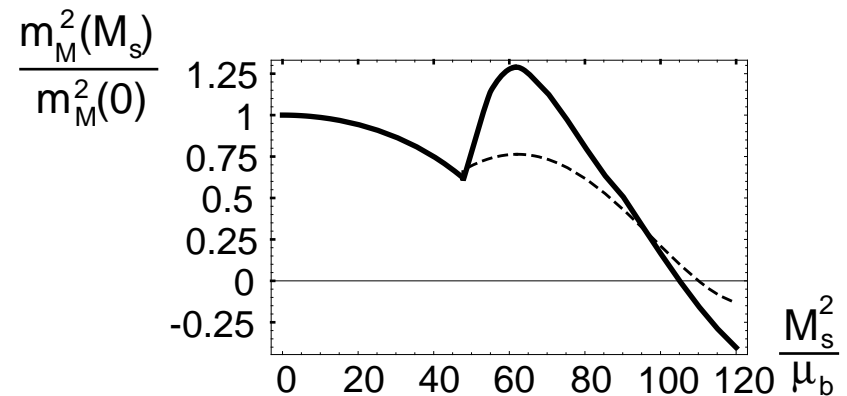
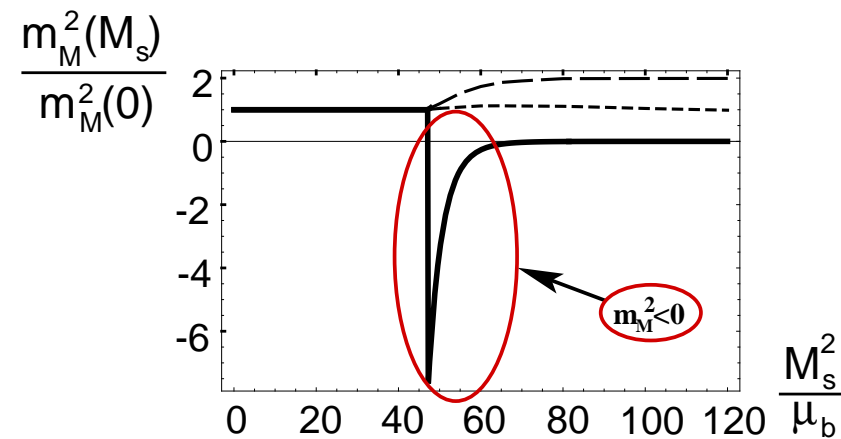
Quasiparticle spectrum in gCFL phase



Chromomagnetic instability in gCFL phase

Recent results for Meissner screening masses

[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:



$A = 1, 2$ — solid line
 $A = 3$ — short-dashed line
 $A = 8$ — long-dashed line

$A = 4, 5$ — dashed line
 $A = 6, 7$ — solid line

Nonzero temperature

- There can exist many phases at $T \neq 0$

[Iida et al, hep-ph/0312363], [Rüster et al, hep-ph/0405170], [Fukushima et al, hep-ph/0408322]

- Zoo of phases:

$$\text{CFL:} \quad \Delta_1 \neq 0, \quad \Delta_2 \neq 0, \quad \Delta_3 \neq 0, \quad (\mu_e \approx 0)$$

$$\text{mCFL:} \quad \Delta_1 \neq 0, \quad \Delta_2 \neq 0, \quad \Delta_3 \neq 0, \quad (\mu_e \neq 0)$$

$$\text{uSC:} \quad \Delta_1 = 0, \quad \Delta_2 \neq 0, \quad \Delta_3 \neq 0,$$

$$\text{dSC:} \quad \Delta_1 \neq 0, \quad \Delta_2 = 0, \quad \Delta_3 \neq 0,$$

$$\text{2SC:} \quad \Delta_1 = 0, \quad \Delta_2 = 0, \quad \Delta_3 \neq 0,$$

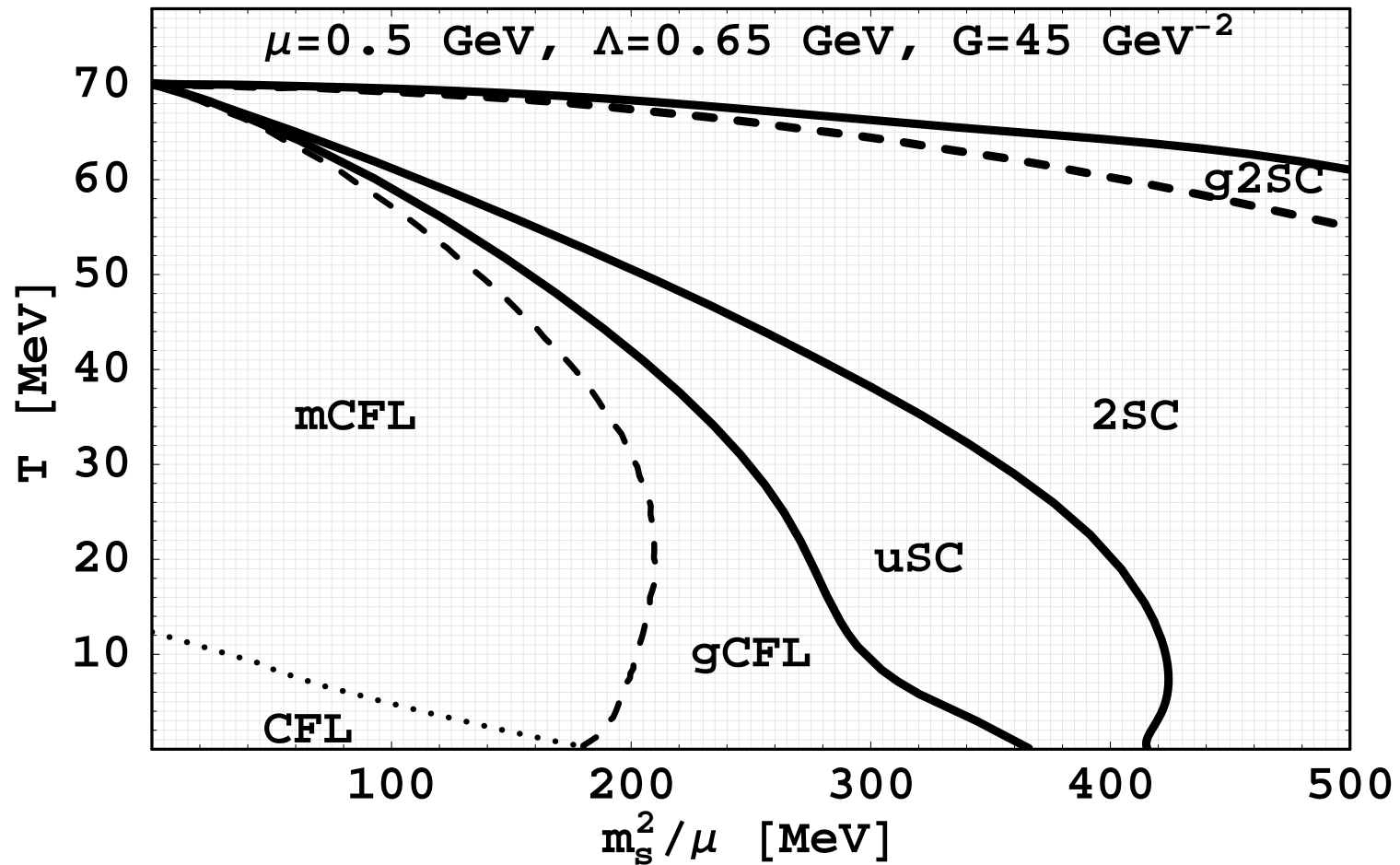
$$\text{NQM:} \quad \Delta_1 = 0, \quad \Delta_2 = 0, \quad \Delta_3 = 0,$$

plus g2SC and gCFL as special cases of 2SC and mCFL.

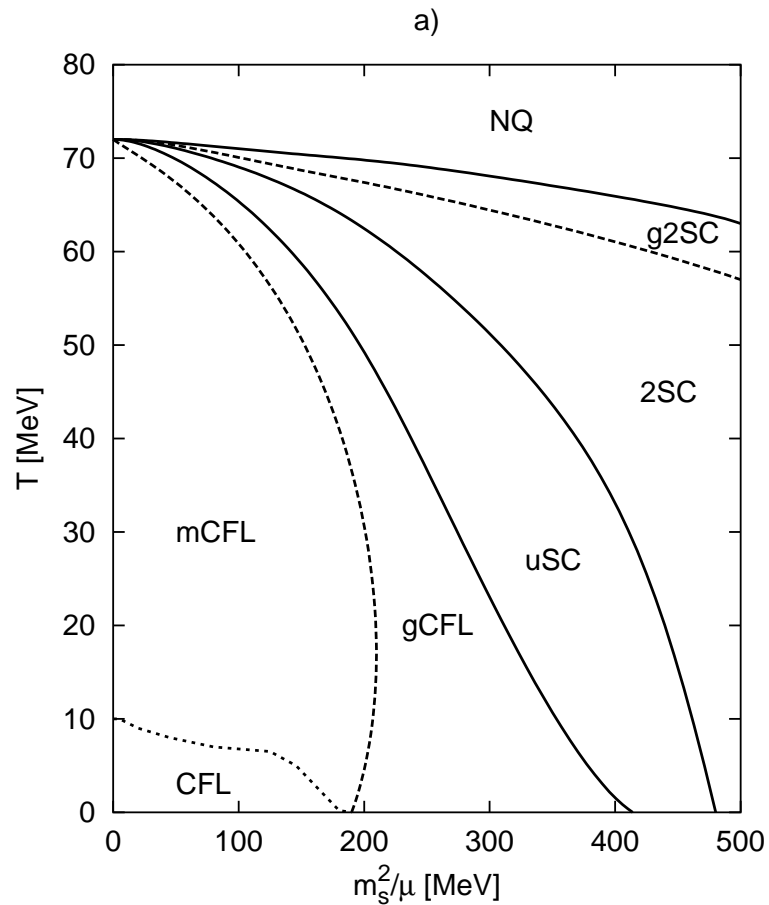
Overview of phases with strangeness

- Color-flavor locked (CFL) phase
 - “Enforced pairing”: $n_u = n_d = n_s$ ($T \simeq 0$)
[Rajagopal, Wilczek, 2001]
 - Natural insulator, $n_{el} \simeq 0$
 - Little specific heat and low neutrino emissivity
- Metallic CFL phase ($n_{el} \neq 0$)
 - $T = 0$: gapless CFL phase (no “enforced pairing”)
 - $T \neq 0$: thermal effects $\rightarrow n_{el} \neq 0$
 - Large specific heat and high neutrino emissivity
- uSC phase: only ud - & us -pairing (no ds -pairing)
- dSC phase: only du - & ds -pairing (no us -pairing)

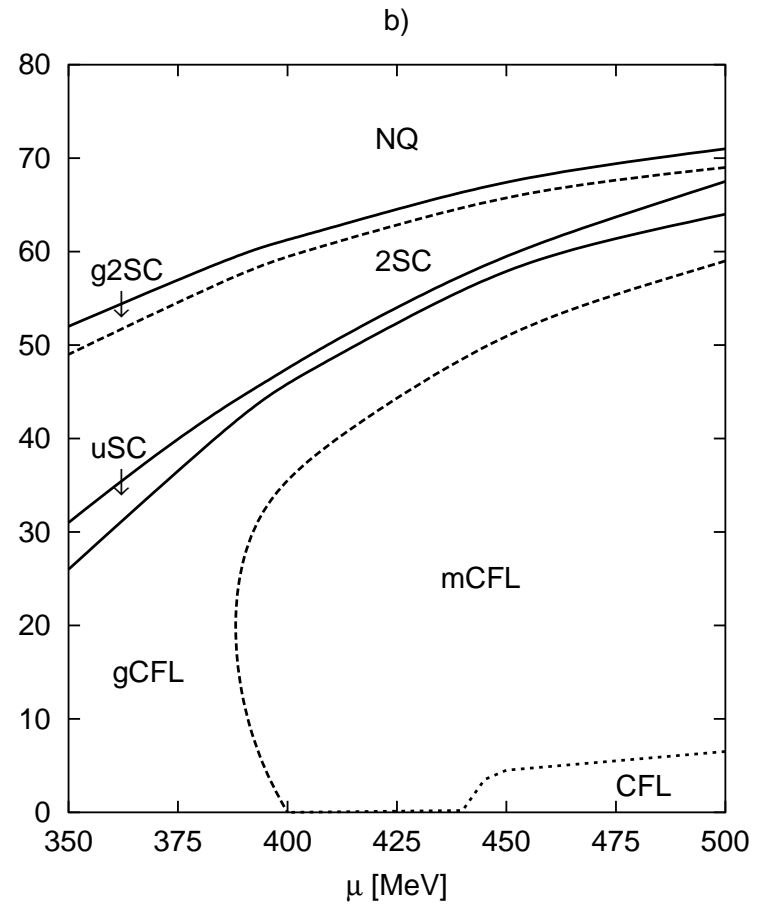
Phase diagram in $T-m_s^2/\mu$ plane



Phase diagram in $T-\mu$ plane



a) $\mu = 500$ MeV



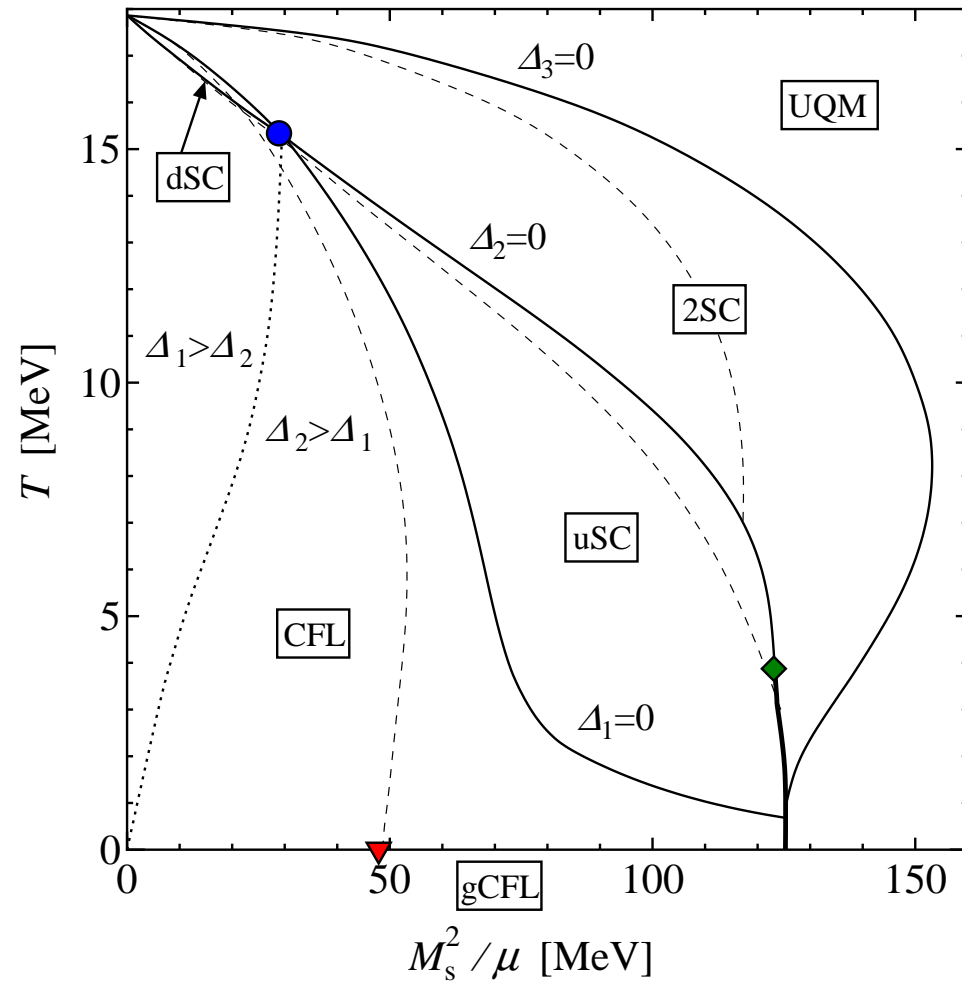
b) $m_s = 250$ MeV

Yet another version of the phase diagram ...

This is taken from

[K. Fukushima, C. Kouvaris,
K. Rajagopal, hep-ph/0408322]

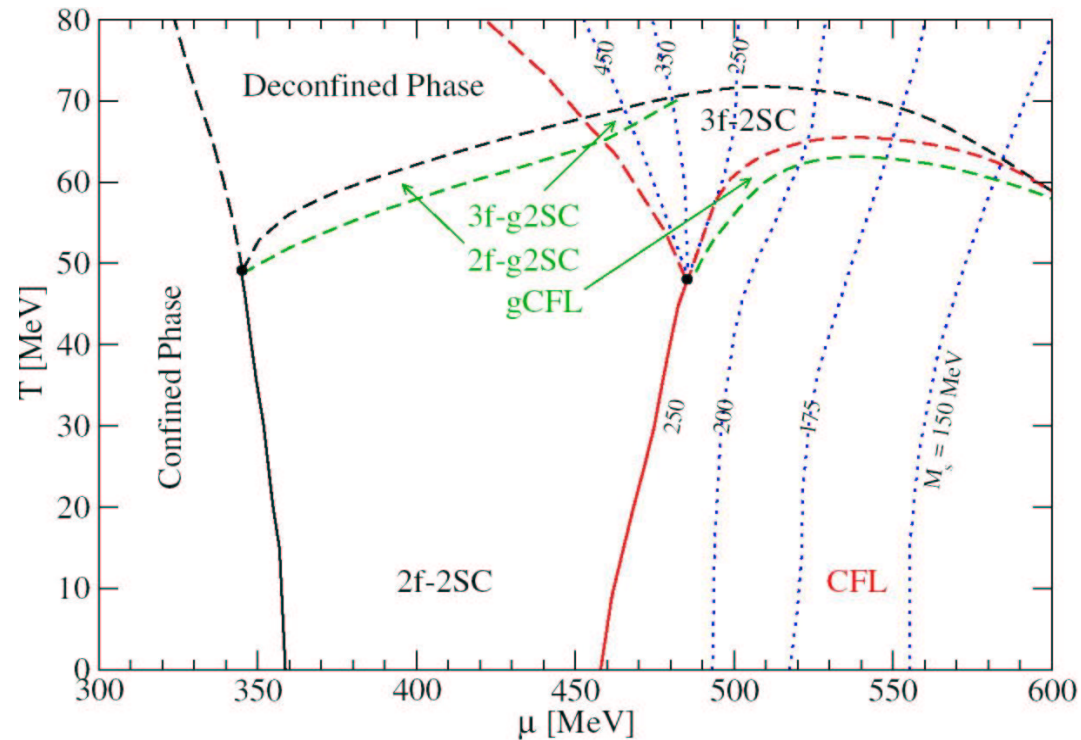
Here the quark chemical
potential is kept fixed,
 $\mu = 500$ MeV



Phase diagram in $T-\mu$ plane

[D. Blaschke, S. Fredriksson, H. Grigorian, F. Sandin, A. Öztas] taken from the talk by H. Grigorian at VI and RTN Collaboration Meeting, GSI, 22 October, 2004:

Phase diagram



In this study, m_s is determined self-consistently

Current status

- Sufficiently cold and dense matter is a color superconductor
- Neutrality and β -equilibrium may strongly affect the properties of dense matter
- There can exist many different CSC phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K⁰, CFL+ η)
- Some features of $T - \mu$ phase diagram start to develop
- Because of the chromomagnetic instability, there is an apparent crisis in the understanding of the phase diagram at large μ