Towards phase diagram of neutral dense matter

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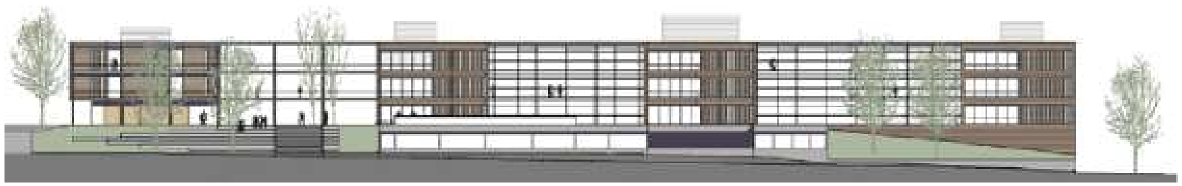

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Outline

- Introduction and motivation
- The use of NJL type effective models at finite density

- Results for the phase diagram(s) of dense QCD
  - Self-consistent treatment of quark masses
  - Effect of neutrino trapping
  - Achievements, uncertainties and failures
- Current "state of the art"

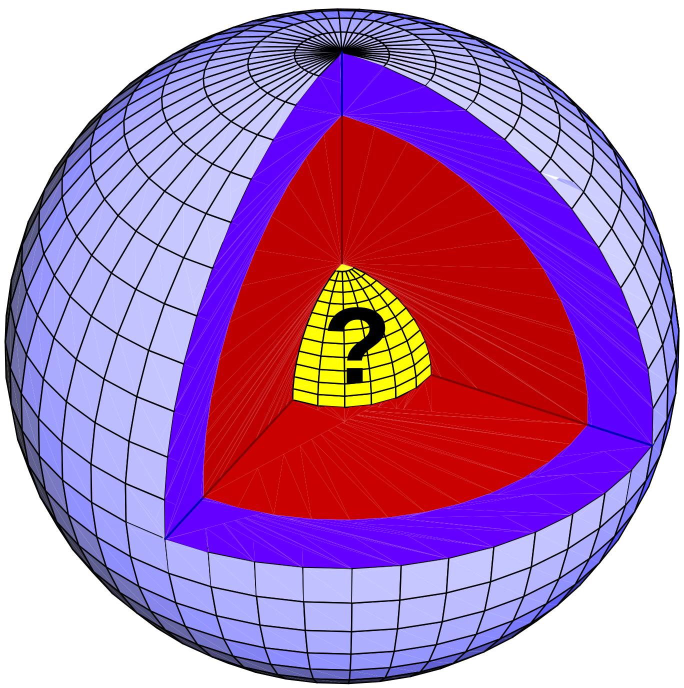
- Conclusions
- Outlook

Dense baryonic matter in Nature

Very dense baryonic matter does exist in the Universe

Compact (neutron) stars

- Radius:
  $R \simeq 10$ km
- Mass:
  $1.25 M_\odot \lesssim M \lesssim 2 M_\odot$
- Core temperature:
  10 keV $\lesssim T \lesssim 10$ MeV
- Surface magnetic field:
  10^8 G $\lesssim T \lesssim 10^{14}$ G



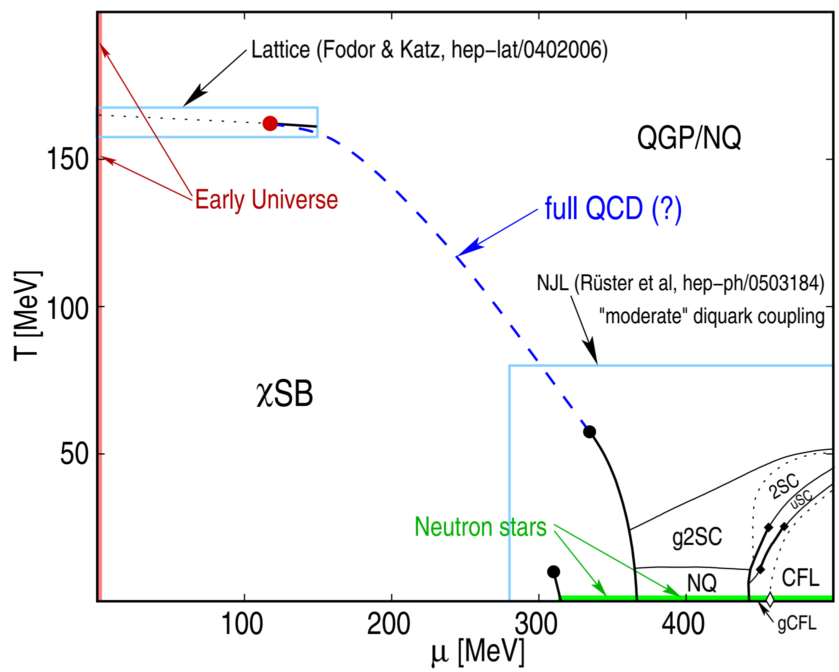
Central densities in stars should be rather high: $\rho_c \gtrsim 5\rho_0$

QCD at large baryon density

One would like to know the fundamental properties of QCD at

$$T \lesssim \Lambda_{QCD} \lesssim \mu$$

- No reliable lattice results at finite density, $\mu \gtrsim \Lambda_{QCD}$
- Effective models of QCD are not reliable
- Effects of charge neutrality and β-equilibrium are not under control
- Difficulties in determining stable ground states

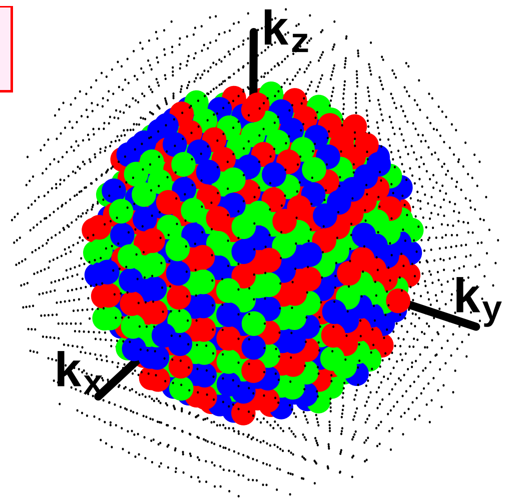


So, how hard could it be?

Ground state of dense quark matter

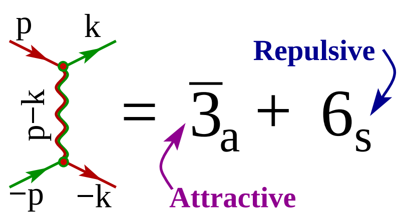
Noninteracting quarks:

(i) Deconfined quarks ($\mu \gg \Lambda_{QCD}$)
(ii) Pauli principle ($s = \frac{1}{2}$) $\Rightarrow$



Interacting quarks:

(i) Effective models ($\mu \gtrsim \Lambda_{QCD}$)
(ii) One-gluon exchange ($\mu \gg \Lambda_{QCD}$)



$\Rightarrow$ Cooper instability

$\Downarrow$

Color superconductivity

$$\langle \left(\bar{\Psi}^C\right)_i^{\alpha} \gamma_5 \Psi_j^{\beta} \rangle \neq 0$$

Unconventional Cooper pairing

- Wave function of a spin-0 Cooper pair:
  $(|{\color{red}\bullet}{\color{green}\bullet}\rangle - |{\color{green}\bullet}{\color{red}\bullet}\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes \boxed{(|\mathrm{u}_{\mathbf{p}}, \mathrm{d}_{-\mathbf{p}}\rangle - |\mathrm{d}_{\mathbf{p}}, \mathrm{u}_{-\mathbf{p}}\rangle)_{1,\bar{3}}}$
- In β-equilibrium, quarks have non-equal Fermi momenta:

$$p_F^{(u)} \neq p_F^{(d)} \neq p_F^{(s)}$$

Charge neutrality ($N_f = 2$)

$$\mu_d = \mu_u + \mu_e$$

$$\frac{2}{3} n_u - \frac{1}{3} n_d - n_e = 0$$

$$\delta\mu \equiv \frac{p_F^{(d)} - p_F^{(u)}}{2} = \boxed{\frac{\mu_e}{2}}$$

Color neutrality ($N_f = 3$)

$$m_s \gg m_u, m_d$$

CFL: strange $\Leftrightarrow$ blue

$$\delta\mu \equiv \frac{p_F^{(bd)} - p_F^{(gs)}}{2} \approx \boxed{\frac{m_s^2}{2\mu}}$$

[Alford,Kouvaris&Rajagopal,hep-ph/0311286]

How does the mismatch $\delta\mu \neq 0$ affect Cooper pairing?

Gapless 2SC phase

Competition:

$\delta\mu$ vs. Δ_0 $\big($here Δ_0 is the gap at $\delta\mu = 0\big)$

The "winner" is determined by the diquark coupling strength
[Shovkovy&Huang, hep-ph/0302142]

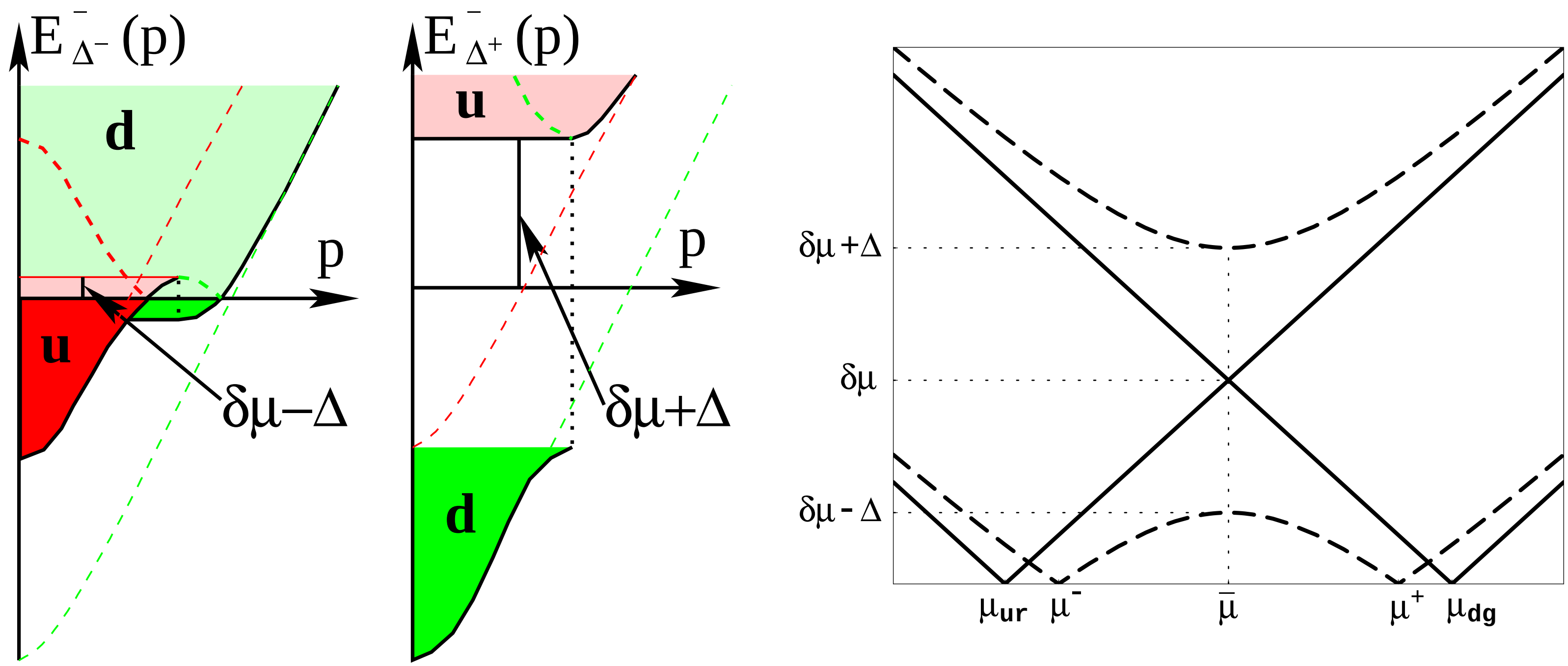
1. $\delta\mu \gtrsim \Delta_0$ — the mismatch does not allow Cooper pairing: normal phase is the ground state

2. $\delta\mu \lesssim \frac{1}{2}\Delta_0$ — coupling is strong enough to win over the mismatch: 2SC is the ground state

3. $\frac{1}{2}\Delta_0 \lesssim \delta\mu \lesssim \Delta_0$ — regime of intermediate coupling strength: the ground state is the gapless 2SC phase

Quasiparticle spectrum in g2SC phase

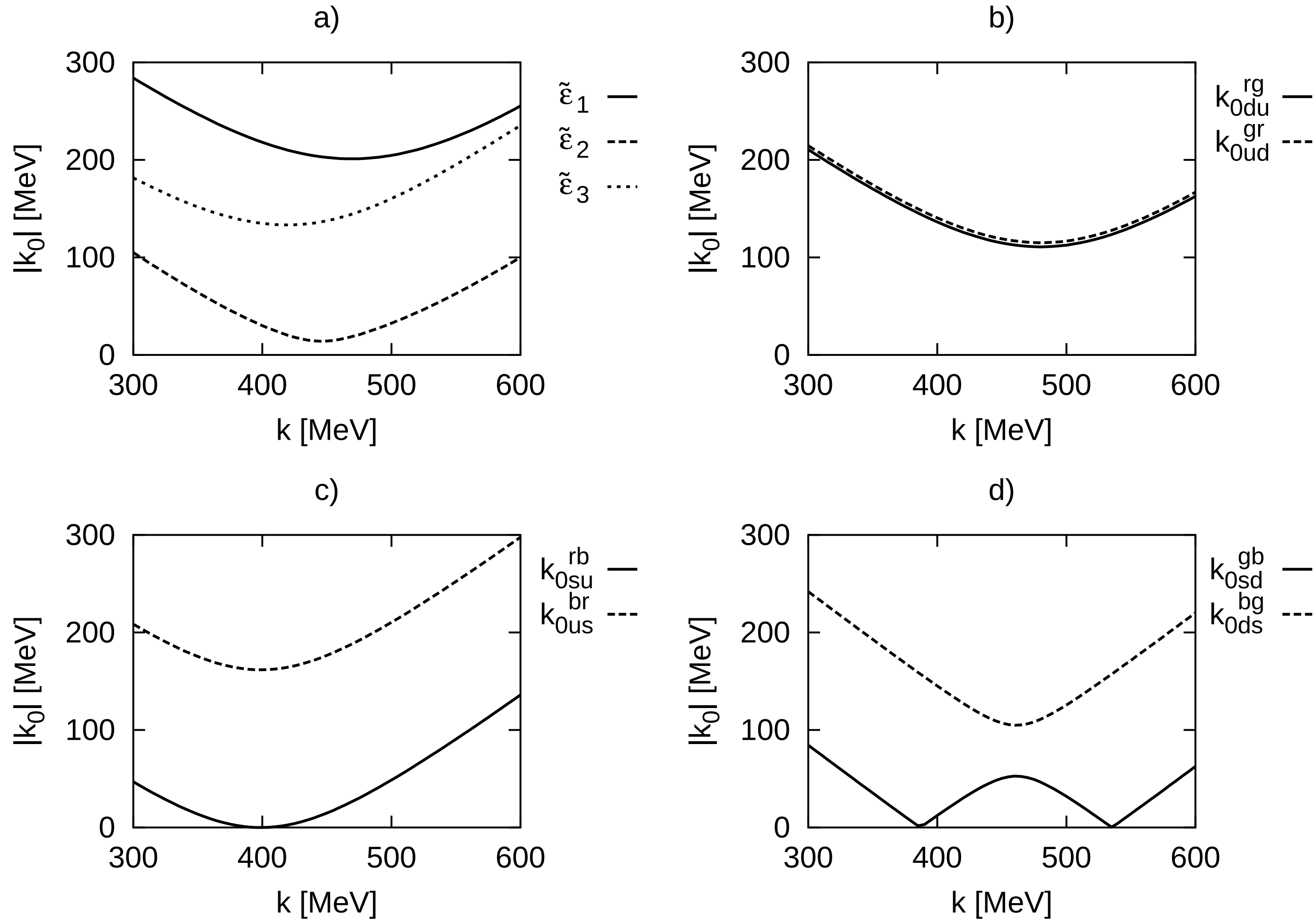
[Huang&Shovkovy hep-ph/0307273]



The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Quasiparticle spectrum in gCFL phase

[Alford,Kouvaris&Rajagopal, hep-ph/0311286]; [Rüster et al, hep-ph/0405170]



Phase diagram of neutral dense matter

NJL model: [Rehberg,Klevansky&Hüfner, Phys.Rev.C **53**, 410 (1996)]

$$\begin{aligned}
\mathcal{L} &= \bar\psi\,(i\partial\!\!\!/ - \hat m\,)\psi + G_S \sum_{a=0}^{8}\left[\left(\bar\psi\lambda_a\psi\right)^2 + \left(\bar\psi i\gamma_5\lambda_a\psi\right)^2\right] \\
&+ G_D \sum_{\gamma,c}\left[\bar\psi^a_\alpha i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta\right]\left[(\bar\psi_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma\right] \\
&- K\left\{\det_f\left[\bar\psi\,(1+\gamma_5)\,\psi\right] + \det_f\left[\bar\psi\,(1-\gamma_5)\,\psi\right]\right\}
\end{aligned}$$

Parameters:

$$\begin{aligned}
m_{u,d} &= 5.5\ \text{MeV} \\
m_s &= 140.7\ \text{MeV} \\
G_S\Lambda^2 &= 1.835 \\
K\Lambda^5 &= 12.36 \\
\Lambda &= 602.3\ \text{MeV}
\end{aligned}
\quad\Rightarrow\quad
\begin{aligned}
m_\pi &= 135.0\ \text{MeV} \\
m_K &= 497.7\ \text{MeV} \\
m_{\eta'} &= 957.8\ \text{MeV} \\
f_\pi &= 92.4\ \text{MeV} \\
m_\eta &= 514.8\ \text{MeV}
\end{aligned}$$

General approach

Quark chemical potentials:

$$\mu_{ab}^{\alpha\beta} = \left(\mu\delta^{\alpha\beta} + \mu_Q Q_f^{\alpha\beta}\right)\delta_{ab} + \left[\mu_3 \left(T_3\right)_{ab} + \mu_8 \left(T_8\right)_{ab}\right]\delta^{\alpha\beta}$$

Dynamically generated quark masses:

$$\hat{M} = \mathrm{diag}_f(M_u, M_d, M_s), \quad \text{with} \quad M_\alpha = m_\alpha - 4G_S\sigma_\alpha + 2K\sigma_\beta\sigma_\gamma$$

Allowed condensates:

$$\Delta_c \sim \epsilon^{\alpha\beta c}\epsilon_{abc}\langle(\bar{\psi}_C)^a_\alpha i\gamma_5\psi^b_\beta\rangle \qquad \text{(no sum over color “}c\text{”)}$$

$$\sigma_\alpha \sim \langle\bar{\psi}^a_\alpha\psi^a_\alpha\rangle \qquad \text{(no sum over flavor “}\alpha\text{”)}$$

Gap equations and neutrality constraints

Pressure:

$$p = p_L - \frac{1}{4G_D}\sum_{c=1}^{3}|\Delta_c|^2 - 2G_S\sum_{\alpha=1}^{3}\sigma_\alpha^2 + 4K\sigma_u\sigma_d\sigma_s + \frac{1}{2}\ln\det\frac{S^{-1}}{T}$$

Coupled set of 9 equations:

$$\frac{\partial p}{\partial \sigma_\alpha} = 0 \qquad \frac{\partial p}{\partial \Delta_c} = 0$$

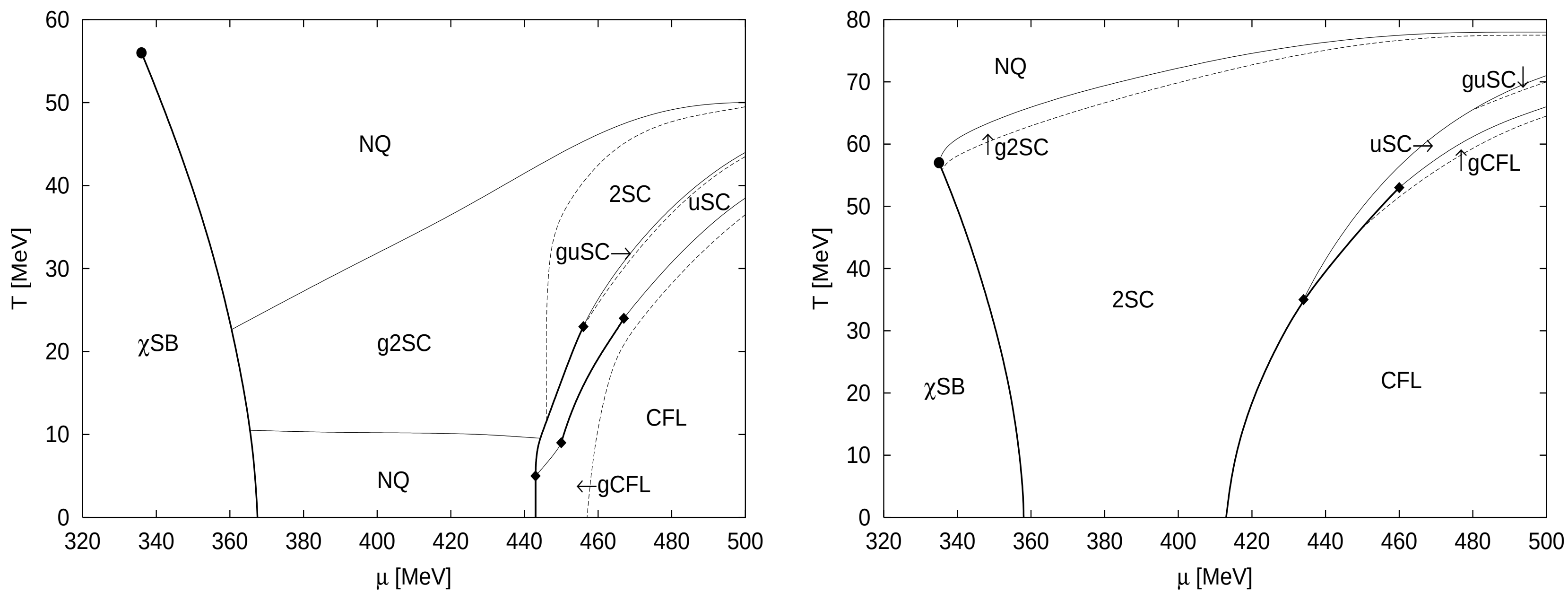
$$n_Q \equiv \frac{\partial p}{\partial \mu_Q} = 0 \qquad n_3 \equiv \frac{\partial p}{\partial \mu_3} = 0 \qquad n_8 \equiv \frac{\partial p}{\partial \mu_8} = 0$$

Note: charge neutrality is enforced locally (no mixed phases allowed)

Phase diagram, $\mu_{\nu_e} = 0$

(without neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, hep-ph/0503184]



$G_D = \frac{3}{4} G_S$ (intermediate coupling) $\qquad$ $G_D = G_S$ (strong coupling)

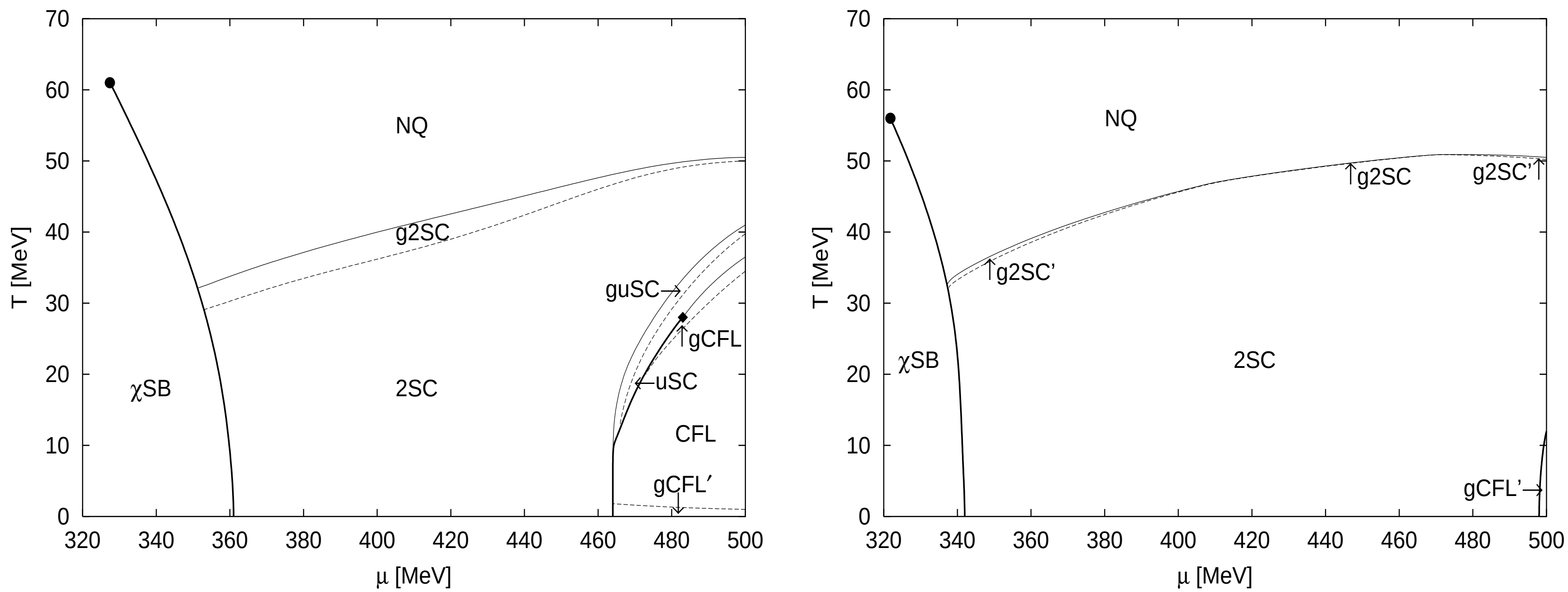
Note: Gapless phases play little role at strong coupling, $G_D = G_S$

See also [Blaschke, Fredriksson, Grigorian, Sandin & Öztaš, hep-ph/0503194]

Phase diagram, $\mu_{\nu_e} \neq 0$

(with neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, work in progress]

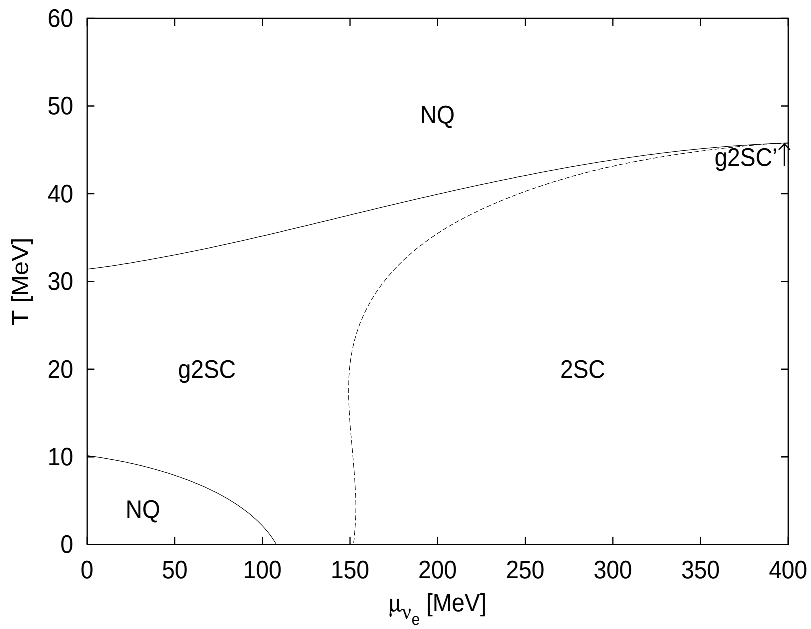


$\mu_{\nu_e} = 200$ MeV $\mu_{\nu_e} = 400$ MeV

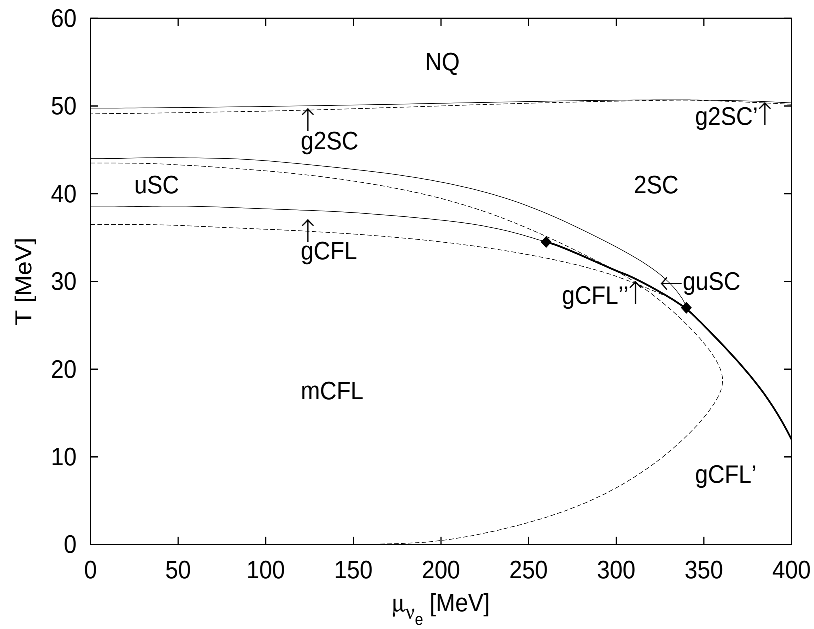
Note: gapless phases play little role already at $G_D = \frac{3}{4} G_S$

$T-\mu_{\nu_e}$ phase diagram

(preliminary results)



$\mu = 400$ MeV (outer core)

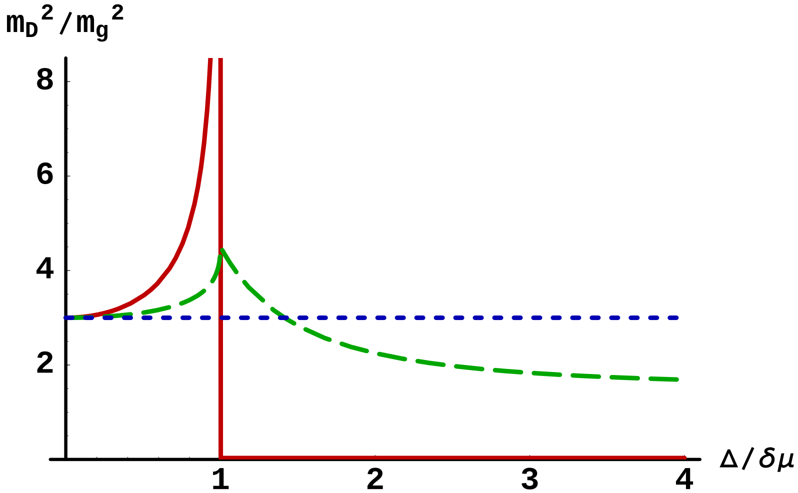


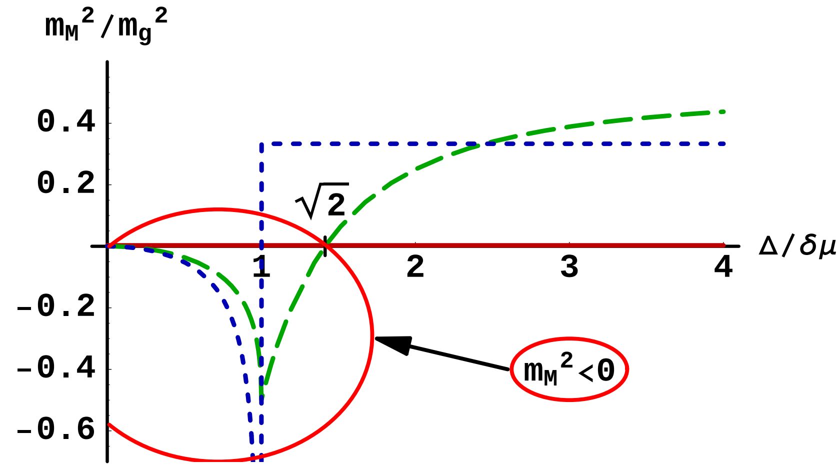
$\mu = 500$ MeV (inner core)

[Rüster, Werth, Buballa, Shovkovy & Rischke, work in progress]

The problem of instabilities

Chromomagnetic instability in the g2SC phase
[Huang & Shovkovy, hep-ph/0407049; hep-ph/0408268]:





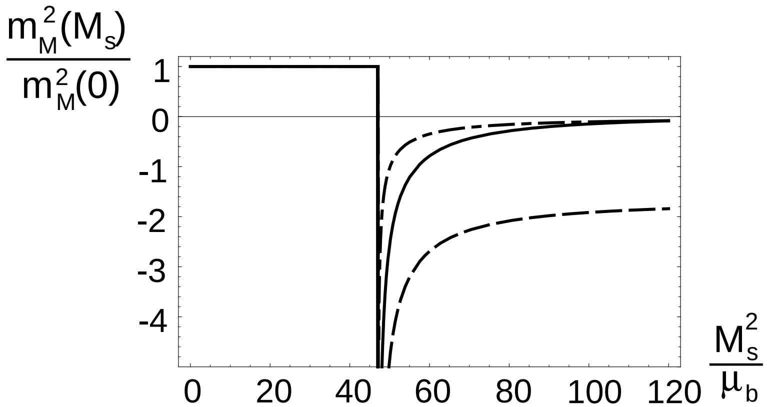
$A = 1, 2, 3$ — red solid line

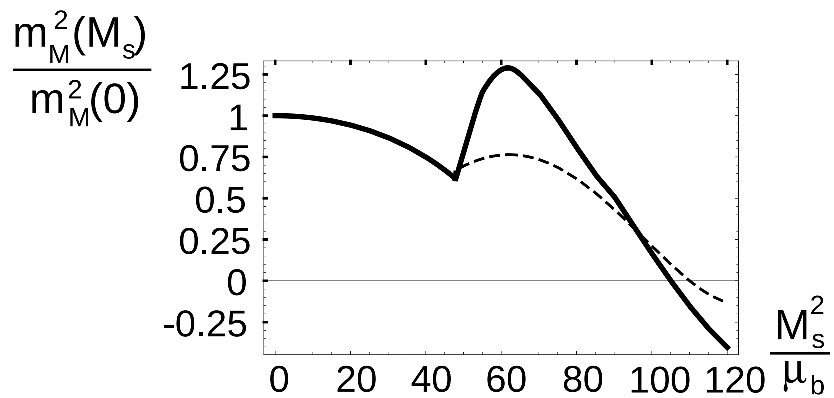
$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

Chromomagnetic instability in gCFL phase

Similar results for Meissner screening masses
[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:





$A = 1, 2$ — solid line

$A = 3$ — short-dashed line

$A = 8$ — long-dashed line

$A = 4, 5$ — dashed line

$A = 6, 7$ — solid line

See also [Wu & Yip, `cond-mat/0303185`], [Alford & Wang, `hep-ph/0501078`]

State of the art

(i) g2SC $\rightarrow$ mixed phase
[Reddy & Rupak, nucl-th/0405054]

Problems:
(i) it cannot work for the gCFL phase
(ii) screening effects may increase surface tension
(iii) conclusions change if electromagnetism is stronger

(ii) g2SC $\rightarrow$ crystalline (LOFF) phase [Alford, et al. hep-ph/0008208],
[Giannakis & Ren, hep-th/0504053]

Problems:
(i) it works only in a narrow window of parameters
$0.7 \lesssim \delta\mu/\Delta_0 \lesssim 0.75$ (cf., $\frac{1}{2} \lesssim \delta\mu/\Delta_0 \lesssim 1$ in g2SC)
(ii) neutrality condition has not yet been imposed

Conclusions

- Neutrality and β-equilibrium strongly affect the properties of dense quark matter
- Phase diagram of neutral dense matter has a very rich structure
- Some features of the QCD phase diagram at $\mu \gtrsim \Lambda_{QCD}$ start to develop
- There is a fundamental problem in current understanding of gapless phases and their instabilities
- Several promising alternatives to gapless phases do exist (e.g., LOFF and mixed phases)

Outlook

- One needs to clarify the precise nature of instabilities
- The "price" of imposing neutrality in the LOFF phase should be studied in detail
- The possibility of mixed phases should be subjected to close scrutiny (e.g., along the lines of [Maruyama, et al. nucl-th/0503027])
- The possibility of spontaneously induced currents in gapless phases should be studied (e.g., along the lines of [Huang, hep-ph/0504235])
- One should look into other possible ways of stabilizing phases with unconventional Cooper pairing (e.g., gauge field condensates, or meson condensates, etc.)

Collaborator(s)

- Stefan Rüster
- Verena Werth
- Dirk Rischke
- Michael Buballa

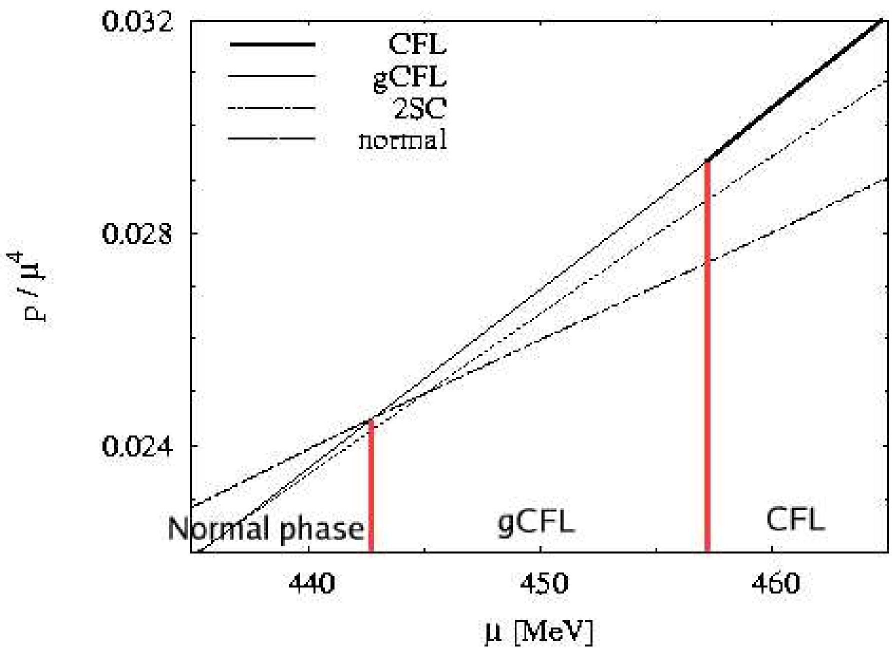
References

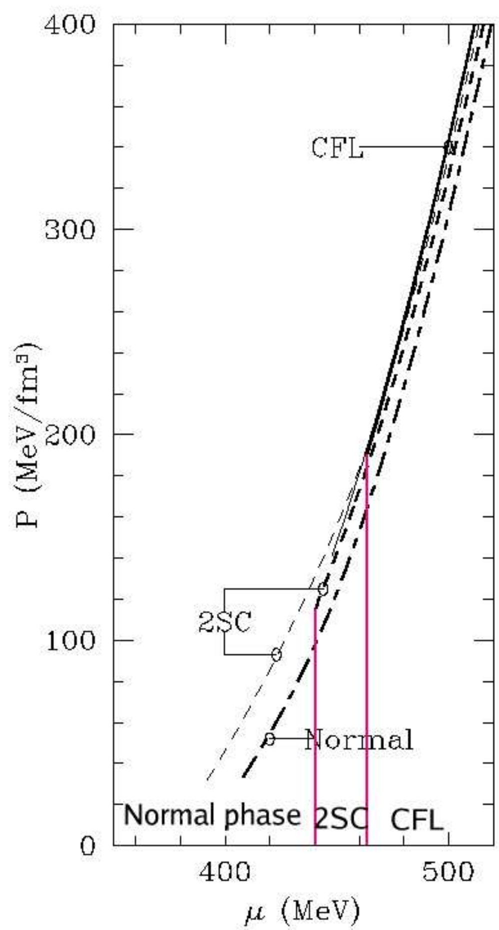
- S. B. Rüster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, `hep-ph/0503184`
- I. A. Shovkovy, S. B. Rüster, D. H. Rischke, `nucl-th/0411040`
- S. B. Rüster, I. A. Shovkovy, D. H. Rischke, Nucl. Phys. A **743** (2004) 127, `hep-ph/0405170`

Extras: I. Comparison with hep-ph/0205201

Comparison with [Steiner et al., hep-ph/0205201]⇒

[Rüster et al., hep-ph/0503184] ⇓





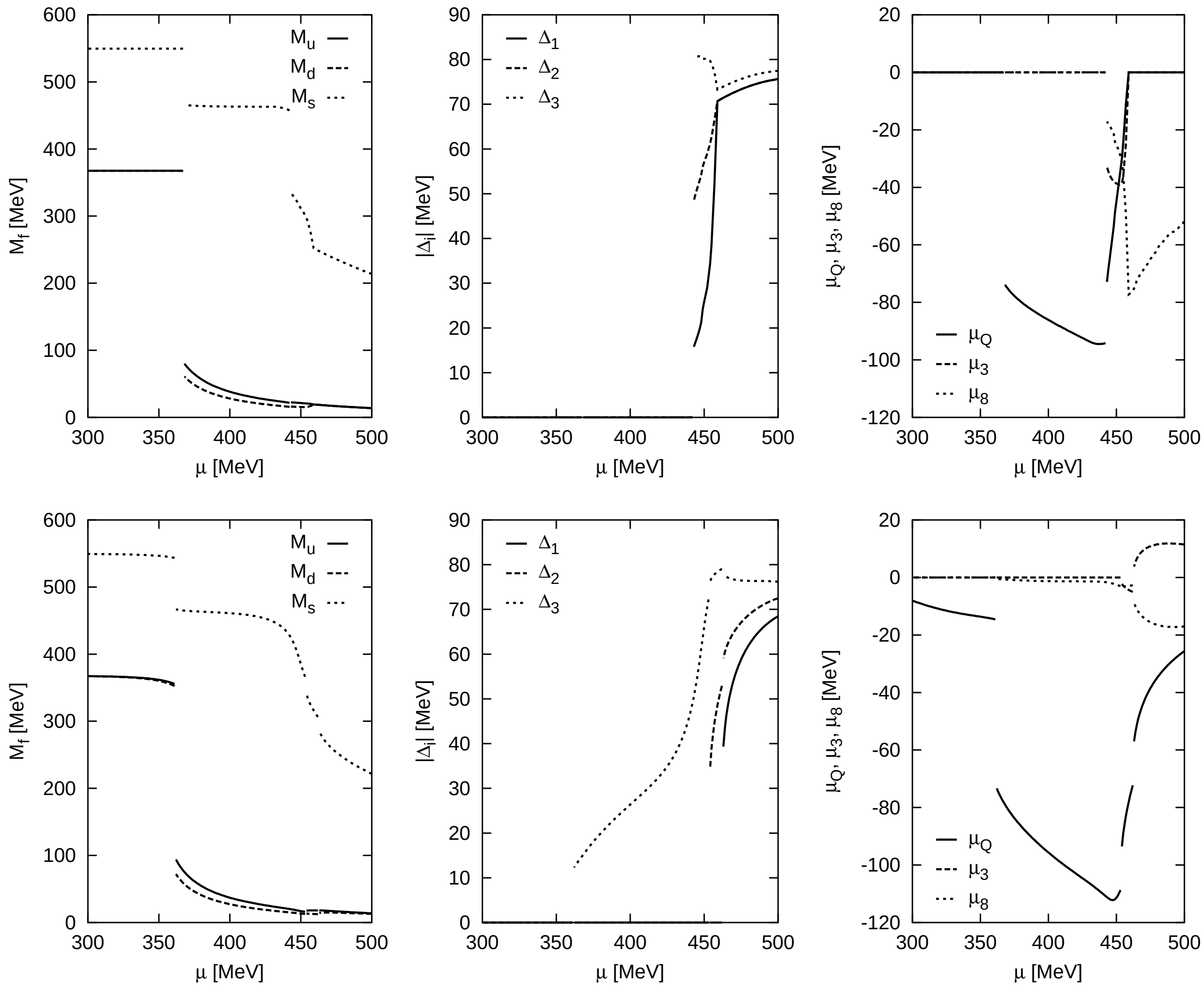
Within the same model [Rehberg,Klevansky&Hüfner, Phys.Rev.C **53**, 410 (1996)]

Extras: II. Solutions for M_f, $|\Delta_i|$, $|\mu_q|$

$G_D = \frac{3}{4} G_S$

$T = 0$ MeV $\Rightarrow$

$T = 20$ MeV $\Rightarrow$



Extras: II. Solutions for M_f, $|\Delta_i|$, $|\mu_q|$

$G_D = G_S$

$T = 0$ MeV $\Rightarrow$

$T = 40$ MeV $\Rightarrow$