

The Many Phases

of Color-Superconducting Quark Matter

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Outline

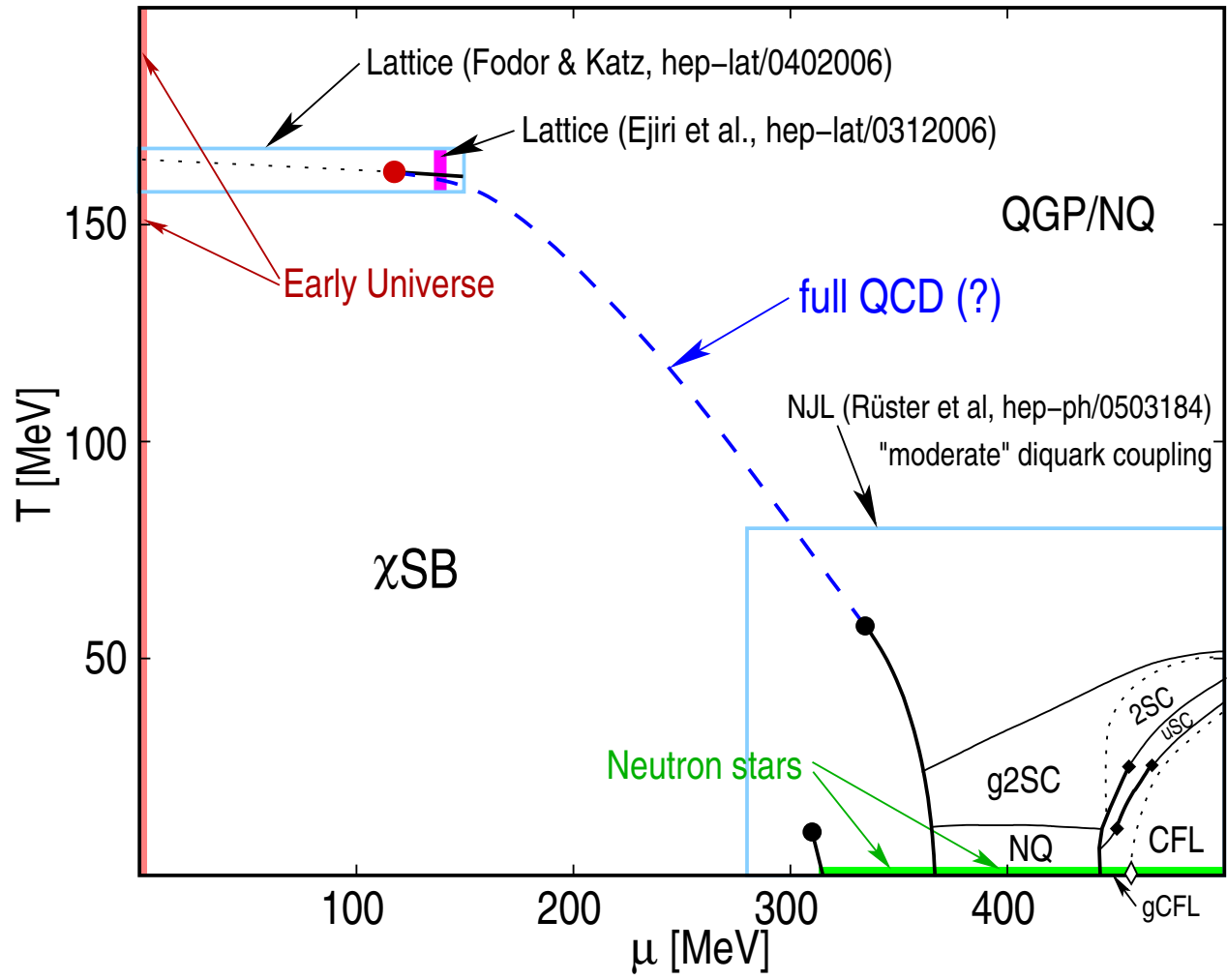
I. Introduction into color superconductivity

- General properties of dense matter
- Cooper instability and the ground state
- $N_f = 2$ color superconductivity
- Color-flavor locked phase ($N_f = 3$)
- Spin-1 color superconductivity ($N_f = 1$)

II. Color superconductivity in neutral matter

- Neutrality vs. color superconductivity
- Gapless phases of color superconductivity
- Other possible phases
- Current status

$T - \mu$ phase diagram of QCD

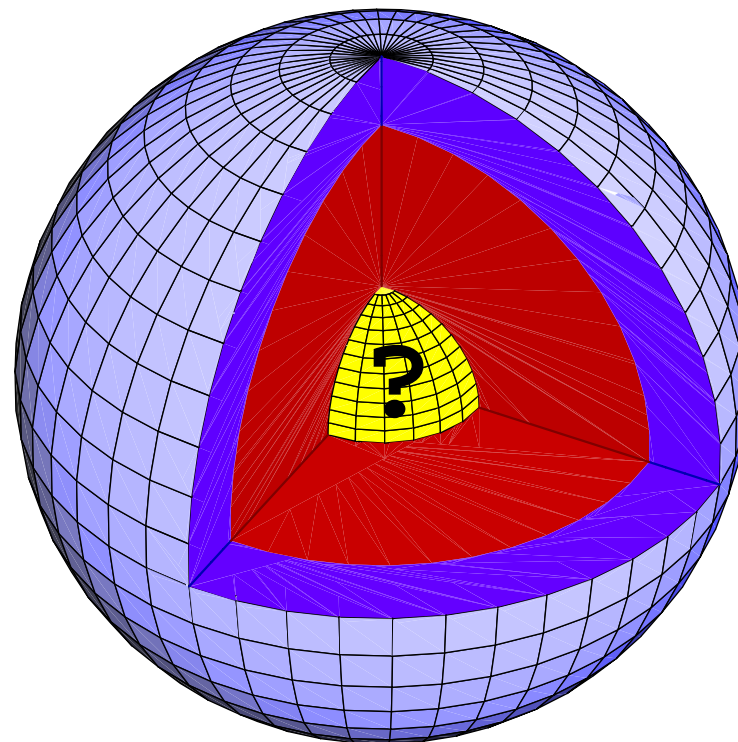


Dense baryonic matter in Nature

Dense baryonic matter exists in the Universe

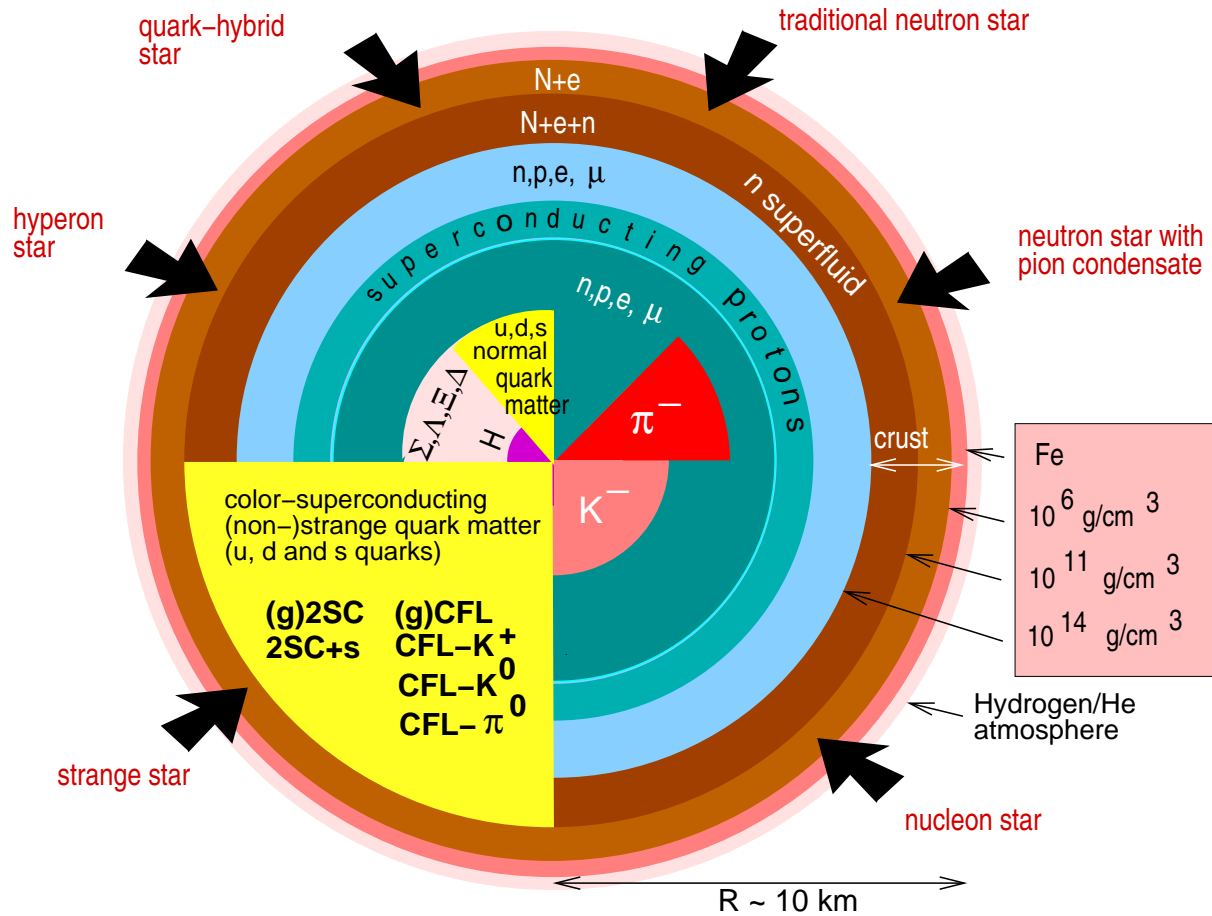
Compact (neutron) stars

- Radius:
 $R \simeq 10 \text{ km}$
- Mass:
 $1.25M_{\odot} \lesssim M \lesssim 2M_{\odot}$
- Core temperature:
 $10 \text{ keV} \lesssim T \lesssim 10 \text{ MeV}$
- Surface magnetic field:
 $10^8 \text{ G} \lesssim B \lesssim 10^{14} \text{ G}$



What is the state of matter at the highest stellar densities, $\rho_c \gtrsim 5\rho_0$?

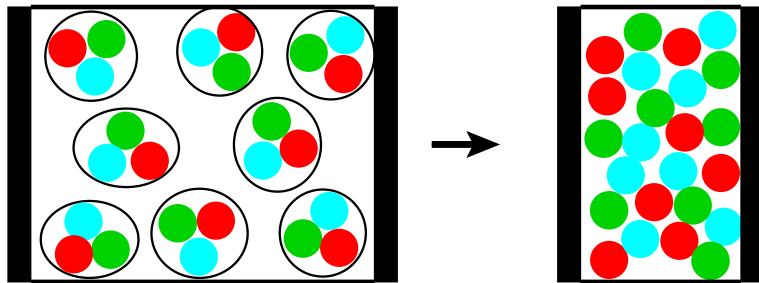
Phases of matter inside compact stars



[figure from F. Weber, astro-ph/0407155 (modified)]

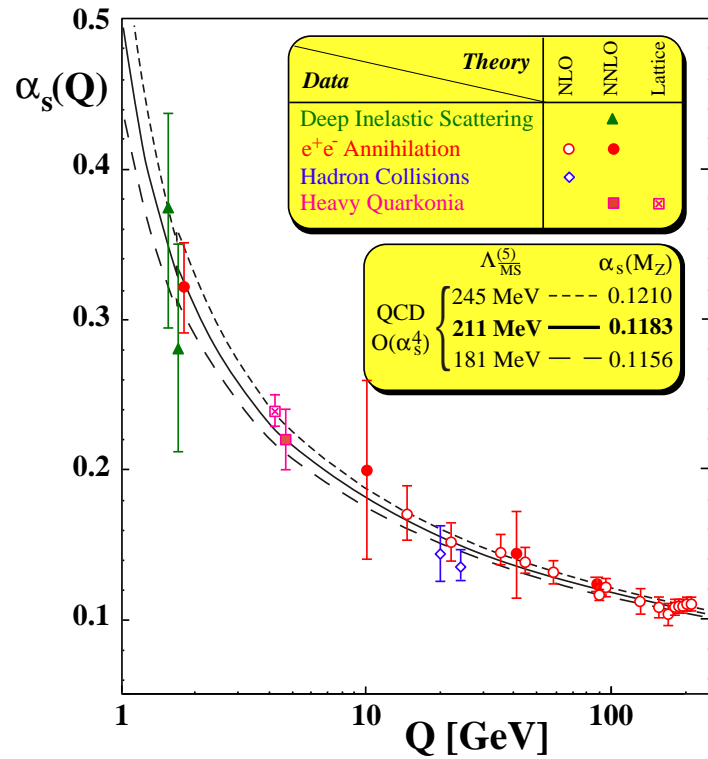
Very dense baryonic matter

- “Squeezing” baryonic matter hard should produce quark matter:



- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
 $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,'73]
- Very dense quark matter is **weakly** interacting [Collins&Perry,'75]

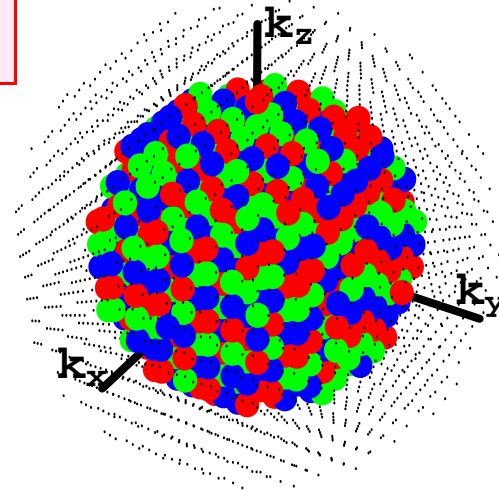
☹️ Note: realistic densities in stars are not sufficiently large:
 $\rho \lesssim 10\rho_0$, where $\rho_0 \approx 0.15 \text{ fm}^{-3} \Rightarrow \mu \lesssim 500 \text{ MeV}$



Ground state of dense quark matter

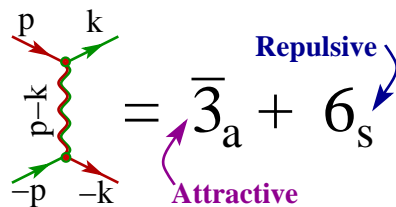
Noninteracting quarks:

- (i) Deconfined quarks ($\mu \gg \Lambda_{QCD}$)
 - (ii) Pauli principle ($s = \frac{1}{2}$)
- } \Rightarrow



Interacting quarks:

- (i) Effective models ($\mu \gtrsim \Lambda_{QCD}$)
 - (ii) One-gluon exchange ($\mu \gg \Lambda_{QCD}$)
- } \Rightarrow Cooper instability



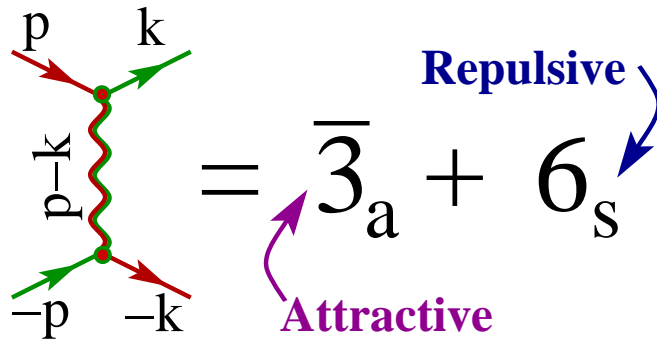
Color superconductivity

$$\langle (\bar{\Psi}^C)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \neq 0$$

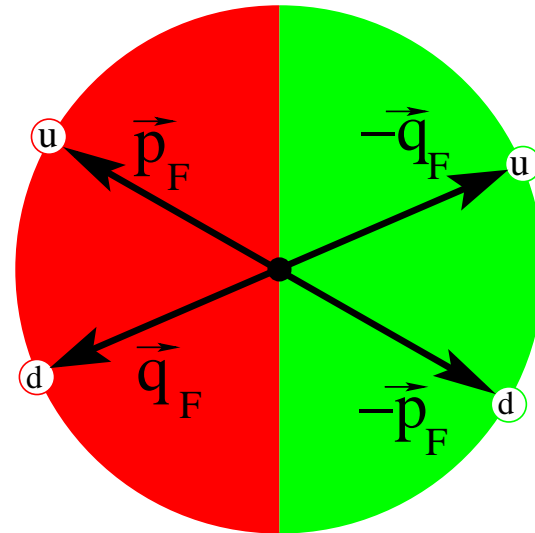
$N_f = 2$ color superconductivity

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84]

- Assumption: $p_F^{\text{up}} \approx p_F^{\text{down}} \approx \mu$
- $N_c = 3$: “red”, “green” and “blue”
- Quark-quark interaction in QCD:



$$\langle \mathbf{u}_p \mathbf{d}_{-p} \rangle = - \langle \mathbf{u}_q \mathbf{d}_{-q} \rangle \neq 0$$

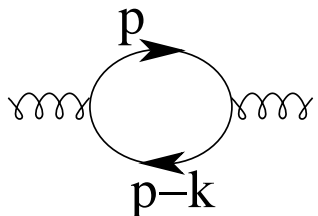


Cooper instability \rightarrow color superconductivity

$$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle) \quad (\Leftarrow \text{Pauli principle})$$

QCD: screening of one-gluon interaction

Hard-dense-loop approximation:

$$\mathcal{D}_{\mu\nu}^{-1}(k) = D_{0,\mu\nu}^{-1}(k) - \Pi_{\mu\nu}(k), \quad \text{i.e.,} \quad \text{wavy line} \stackrel{k}{=} \text{wavy line} + \text{wavy line} \circlearrowleft \text{wavy line}$$


Electrical Debye screening and **magnetic dynamical screening**

[Son,hep-ph/9812287]:

$$i\mathcal{D}_{\mu\nu}(k_4, |\vec{k}|) \simeq -\frac{O_{\mu\nu}^{(el)}}{k_4^2 + |\vec{k}|^2 + 2M_D^2} - \frac{|\vec{k}| O_{\mu\nu}^{(mag)}}{|\vec{k}|^3 + \pi M_D^2 |k_4|/2},$$

where $M_D^2 = \alpha_s N_f \mu / \pi$ is the Debye mass

Region of dominant interaction: $\Delta \ll k_4 \lesssim |\vec{k}| \ll \mu$

😊 Note: QCD analysis is reliable only when $\Delta \gg \Lambda_{QCD}$

Schwinger-Dyson (gap) equation

[hep-ph/9906478;hep-ph/9906512;nucl-th/9907041;hep-ph/9909574;hep-ph/9910225]

$$\text{---}\overrightarrow{p}\text{---} = \text{---}\overrightarrow{p}\text{---} + \text{---}\overrightarrow{p-k}\text{---} \text{ (with a wavy loop labeled } k \text{ attached to the line)}$$

This reduces to

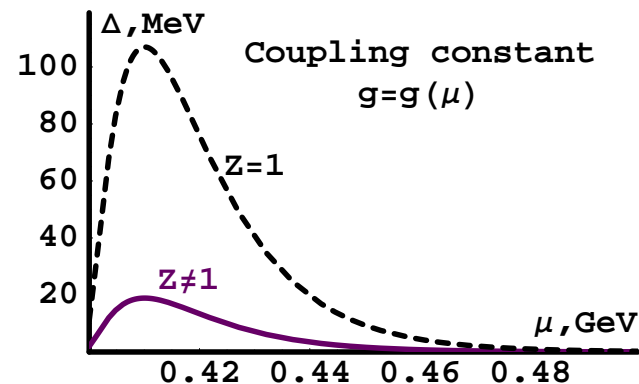
$$\Delta(p_4) \simeq \frac{g^2}{18\pi^2} \int \frac{dq_4 \Delta(q_4)}{\sqrt{q_4^2 + \Delta^2}} \ln \frac{\Lambda}{|q_4 - p_4|}$$

where

$$\Lambda = 2(4\pi)^4 \mu g^{-5} \exp\left(-\frac{4+\pi^2}{8}\right)$$

Solution for the gap:

$$\Delta(0) \simeq \Lambda \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$



Instanton induced interaction

Local interaction [Alford et al., '98; Rapp et al., '98]

$$\mathcal{L}_{\text{int}}^{(\text{eff})} \simeq -\frac{G}{8N_c^2(N_c - 1)} \left[\left(\psi^T C \tau_2 \lambda_A^a \gamma_5 \psi \right) \left(\bar{\psi} \tau_2 \lambda_A^a \gamma_5 C \bar{\psi}^T \right) + \dots \right]$$

where τ_2 and λ_A^a are antisymmetric Pauli and Gell-Mann matrices

Diquark condensate:

$$\Delta^3 \sim \varepsilon^{3\alpha\beta} \langle \psi_\alpha^i C (\tau_2)_{ij} \gamma_5 \psi_\beta^j \rangle$$

Choice of parameters:

$$\text{e.g., } m_q^{(\text{vacuum})} = 400 \text{ MeV}$$

$$\text{(or } N_{\text{inst}}/V \simeq 1.44 \text{ fm}^{-4}) \quad \Rightarrow$$

$$G \simeq 490 \text{ GeV}^{-2}$$

$$\text{Cut-off: } \Lambda \lesssim 1 \text{ GeV}$$

Result for the gap:

$$\Delta \simeq 100 \text{ MeV}$$

Symmetries of 2SC ground state

- Diquark condensate:

$$\langle (\bar{\Psi}^C)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

When $\Delta \neq 0$,

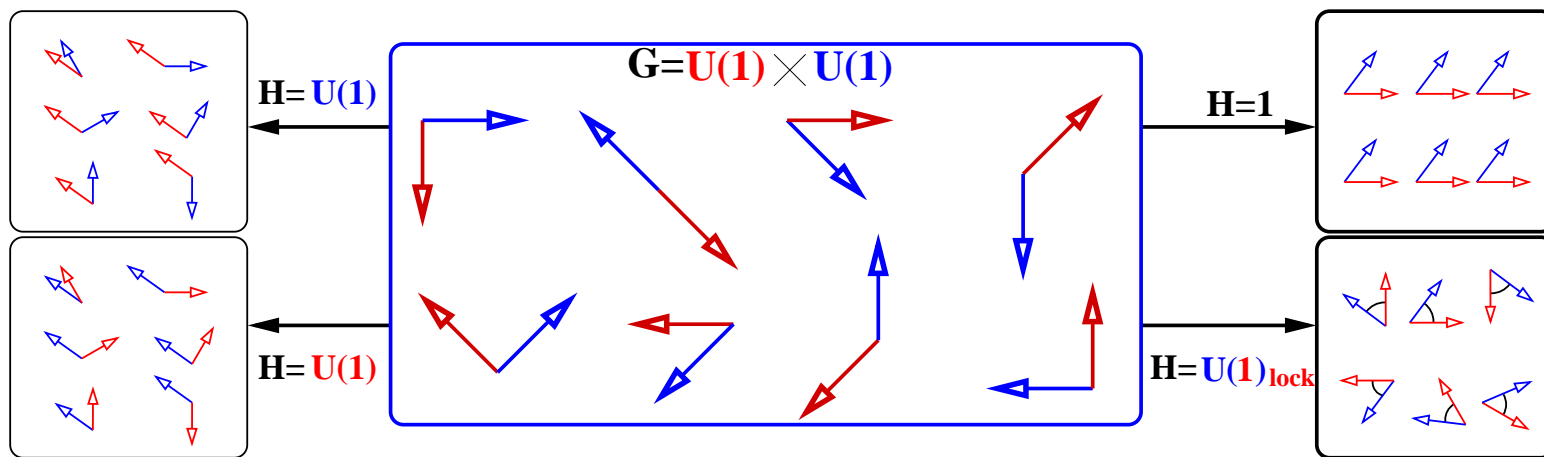
- chiral $SU(2)_L \times SU(2)_R$ — intact
- baryon number $U(1)_B \rightarrow \tilde{U}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}} T_8$
- gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$
- approximate axial $U(1)_A$ is broken \rightarrow 1 pseudo-NG boson
- color gauge symmetry $SU(3)_c \rightarrow SU(2)_c$ by Anderson-Higgs mechanism \rightarrow 5 massive gluons

$N_f = 3$ color superconductivity

- Cooper pair: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_{\bar{3}}$
- Diquark condensate:

$$\langle (\bar{\Psi}_L^C)_i^\alpha (\Psi_L)_j^\beta \rangle = \langle (\bar{\Psi}_R^C)_i^\alpha (\Psi_R)_j^\beta \rangle \simeq \sum_I \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- $SU(3)_L \times SU(3)_R \times SU(3)_c \rightarrow SU(3)_{L+R+c}$ via “color-flavor locking”, without $\langle \bar{q}_L q_R \rangle$ condensates! [Alford et al. hep-ph/9804403]



Symmetries of CFL ground state

- chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
 \rightarrow 8 (pseudo-)NG bosons, i.e., $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$
(like in vacuum QCD)
- baryon number $U(1)_B$ is broken \rightarrow 1 NG boson (ϕ)
(quark matter is superfluid)
- approximate axial $U(1)_A$ is broken \rightarrow 1 pseudo-NG boson (η')
- color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism \rightarrow 8 massive gluons
- gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$
(there is no Meissner effect)

$N_f = 1$ color superconductivity

- Cooper pair: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$

- Diquark condensate:

$$\langle (\bar{\Psi}^C)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^\delta \sin \theta + \gamma_\perp^\delta(\vec{\mathbf{k}}) \cos \theta \right)$$

- Many possibilities, e.g., see [A.Schmitt, nucl-th/0412033]:

- Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta} \rightarrow$ largest pressure (?)

- Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$

- Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$

- A-phase: $\Delta_{c\delta} = \delta_{c3} (\delta_{\delta 1} + i\delta_{\delta 2})$

Color superconductivity in neutral matter

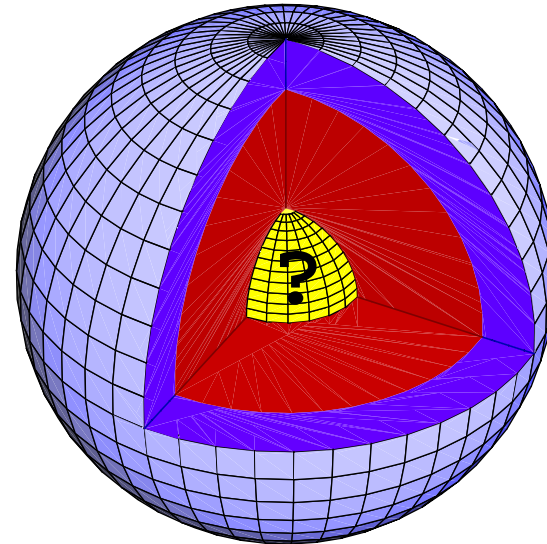
Matter in the bulk of a star is

- (i) β -equilibrated: $\mu_d = \mu_u + \mu_e$
- (ii) electrically and color neutral:
 $n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$

If $n_Q \neq 0$, the Coulomb energy is

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_{\odot} c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left(\frac{R}{1 \text{ km}} \right)^5$$

$$\text{e.g., if } 10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{\text{2SC}} \gg M_{\odot} c^2$$



Unconventional Cooper pairing, $N_f = 2$

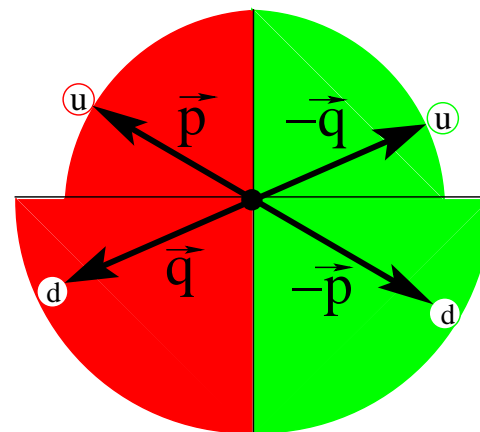
- The “best” 2SC phase appears when $n_d \approx n_u$
- Neutral matter appears when $n_d \approx 2n_u$
- Electrons, required by β equilibrium, **cannot** help:

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u$$

$$\text{i.e.,} \quad n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$$

Cooper pairing is distorted by the following “mismatch” parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Gapless 2SC phase

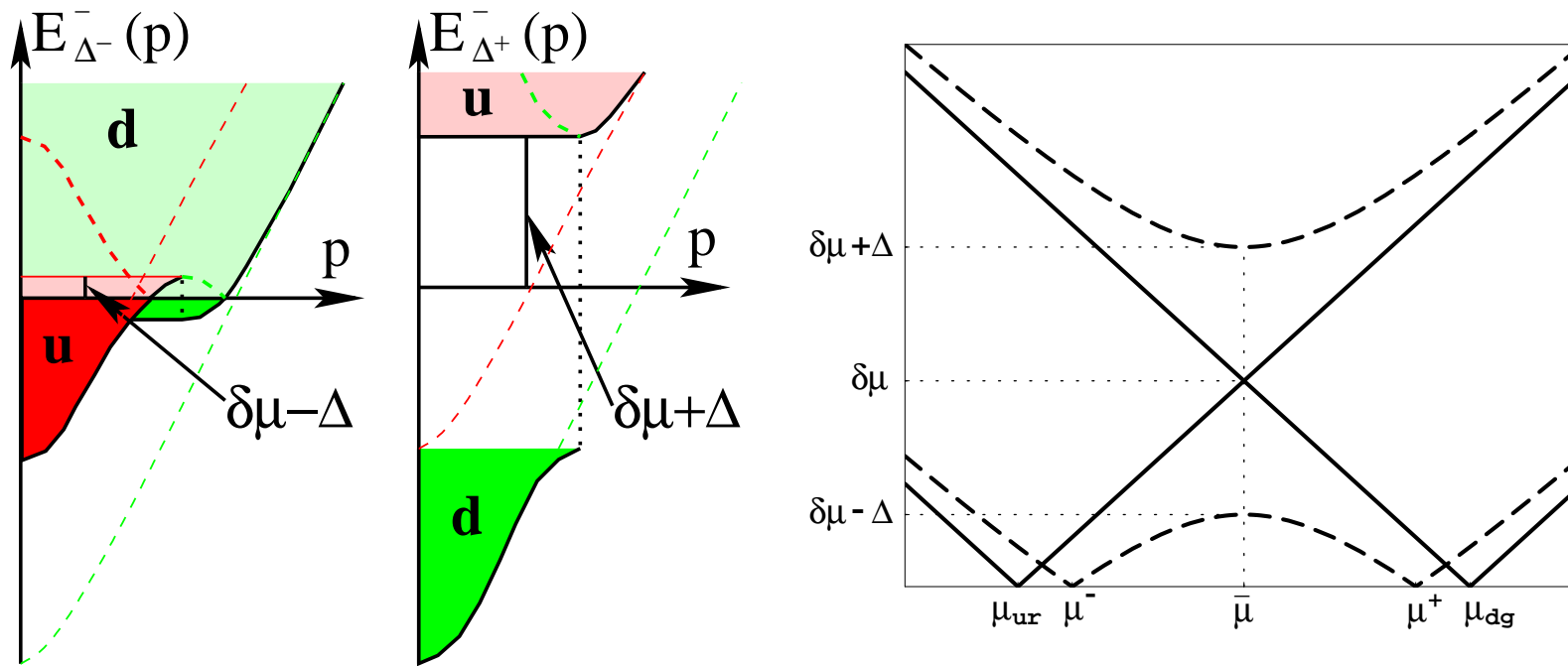
Competition: $\delta\mu$ vs. Δ_0 (where Δ_0 is the gap at $\delta\mu = 0$)

The “winner” is determined by the diquark coupling strength
[Shovkovy&Huang, hep-ph/0302142]

1. $\delta\mu \gtrsim \Delta_0$ — the mismatch does not allow Cooper pairing:
normal phase is the ground state
2. $\delta\mu \lesssim \frac{1}{2}\Delta_0$ — coupling is strong enough to win over the
mismatch: 2SC is the ground state
3. $\frac{1}{2}\Delta_0 \lesssim \delta\mu \lesssim \Delta_0$ — regime of intermediate coupling strength:
the ground state is the gapless 2SC phase

Quasiparticle spectrum in g2SC phase

“Intermediate” coupling

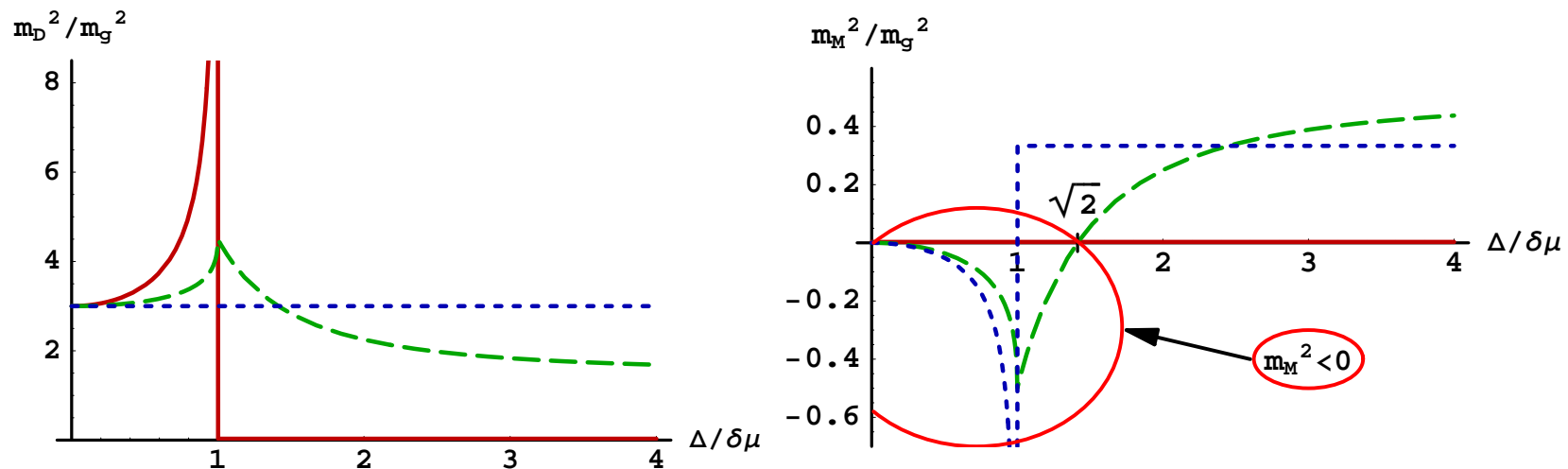


The energy gaps in the quasiparticle spectra are 0 & $\Delta + \delta\mu$

Chromomagnetic instability

Recent results for gluon screening masses

[Huang & Shovkovy, hep-ph/0407049]:



- $A = 1, 2, 3$ — red solid line
- $A = 4, 5, 6, 7$ — green long-dash line
- $A = \tilde{8}$ — blue short-dash line

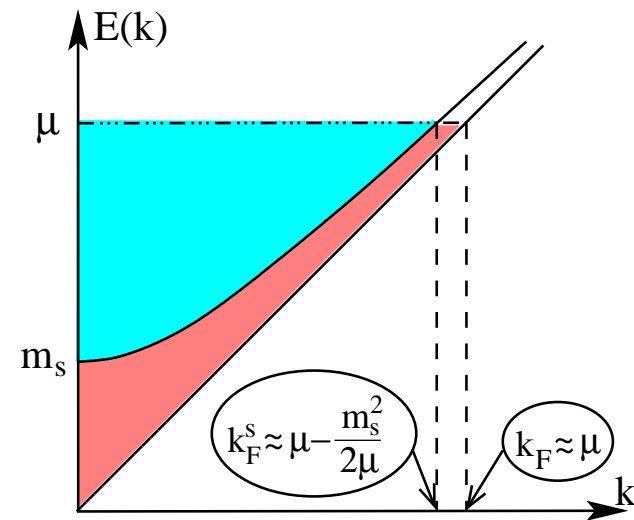
$N_f = 2 + 1$ color superconductivity, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$

The ground state of strange quark matter may have:

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystalline pairing (nonzero momentum pairing, LOFF)
- ...



Gapless CFL phase

[Alford, Kouvaris & Rajagopal, hep-ph/0311286]

- Distorted color-flavor pairing:

$$\Delta_{ij}^{\alpha\beta} \simeq \Delta_1 \epsilon_{1ij} \epsilon^{1\alpha\beta} + \Delta_2 \epsilon_{2ij} \epsilon^{2\alpha\beta} + \Delta_3 \epsilon_{3ij} \epsilon^{3\alpha\beta} + \dots$$

- Control (mismatch) parameter:

$$\delta\mu \equiv \frac{\mu_{bd} - \mu_{gs}^{\text{eff}}}{2} \approx -\frac{\mu_8}{2} + \frac{m_s^2}{4\mu} \approx \frac{m_s^2}{2\mu}$$

where $\mu_{gs}^{\text{eff}} \simeq \mu_{gs} - \frac{m_s^2}{2\mu}$ and $\mu_8 \simeq -\frac{m_s^2}{2\mu}$ (blue color is special)

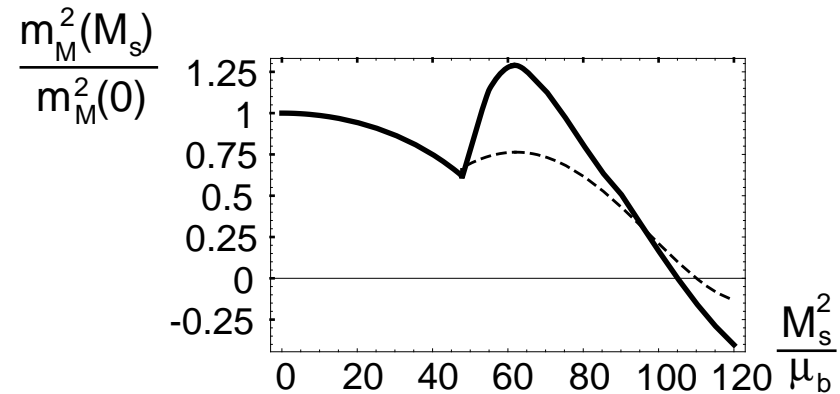
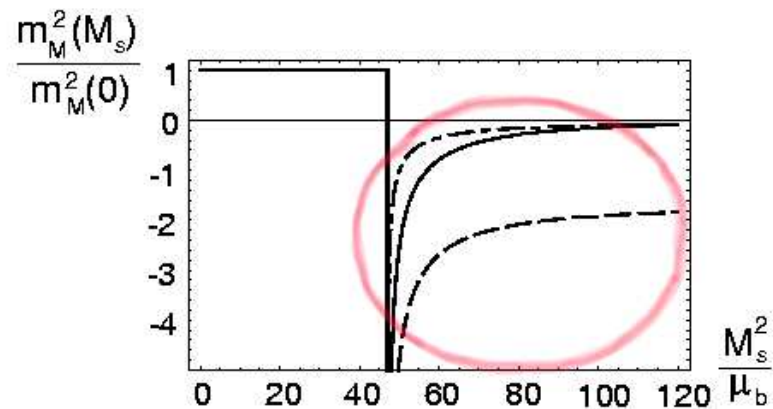
- At $T = 0$, the gapless CFL phase occurs when

$$\delta\mu \equiv \frac{m_s^2}{2\mu} > \Delta_0$$

Chromomagnetic instability in gCFL phase

Recent results for Meissner screening masses

[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:



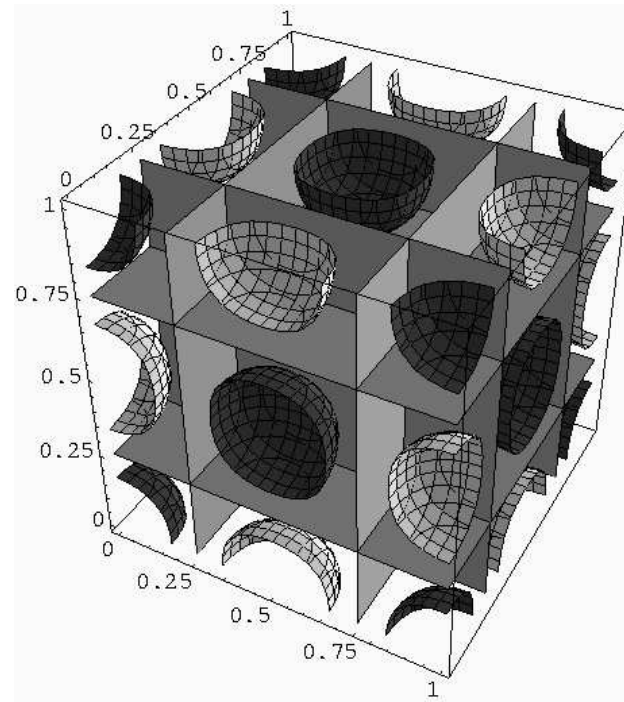
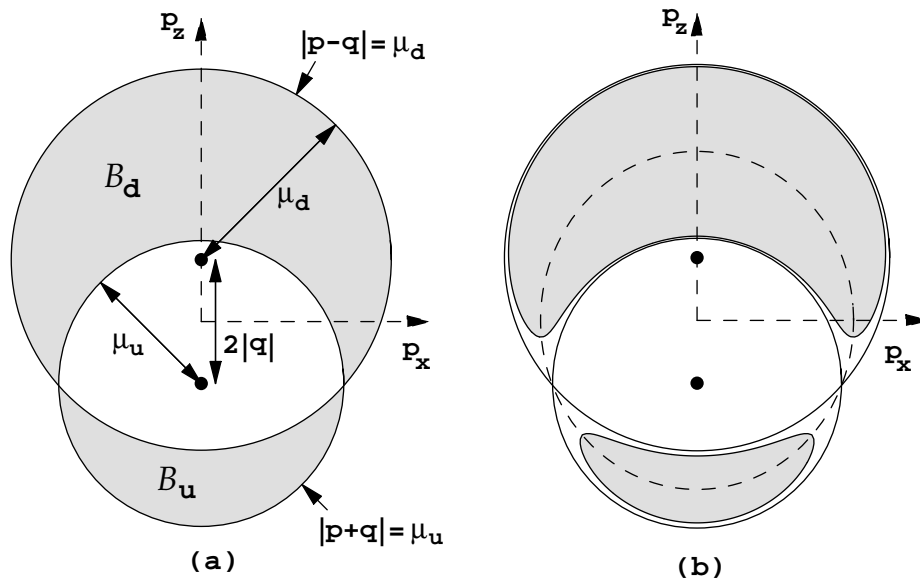
$A = 1, 2$ — solid line
 $A = 3$ — short-dashed line
 $A = 8$ — long-dashed line

$A = 4, 5$ — dashed line
 $A = 6, 7$ — solid line

Alternatives. I

Crystalline color superconductivity (LOFF phase)
[Alford, Bowers & Rajagopal, hep-ph/0008208]

Cooper pairs with nonzero momenta:

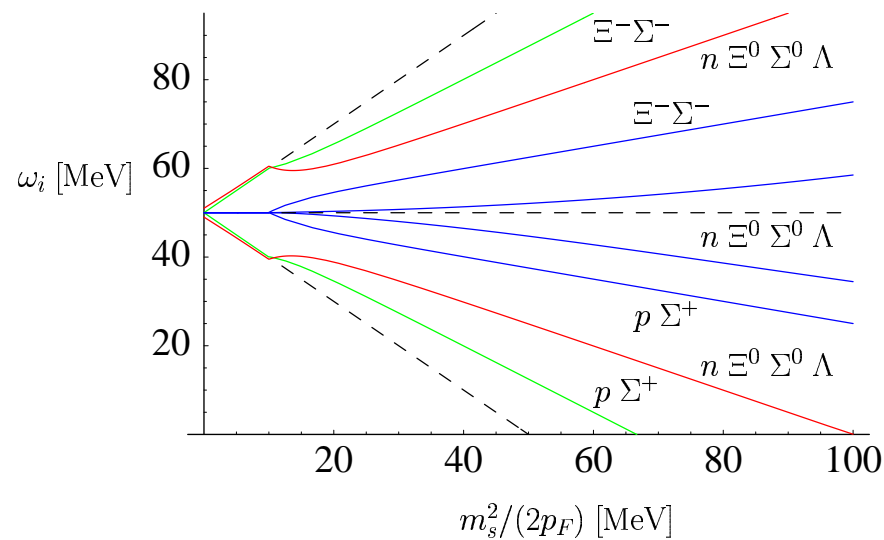


[Bowers, hep-ph/0305301]

Alternatives. II

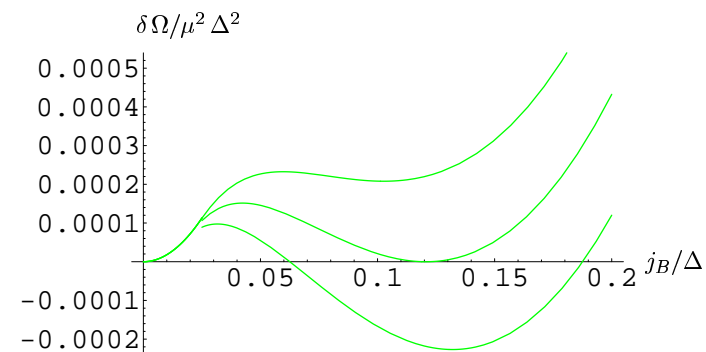
Baryon spectrum in the CFL phase

[Kryjevski & Schafer, hep-ph/0407329]



Solution (?)

P-wave meson condensation



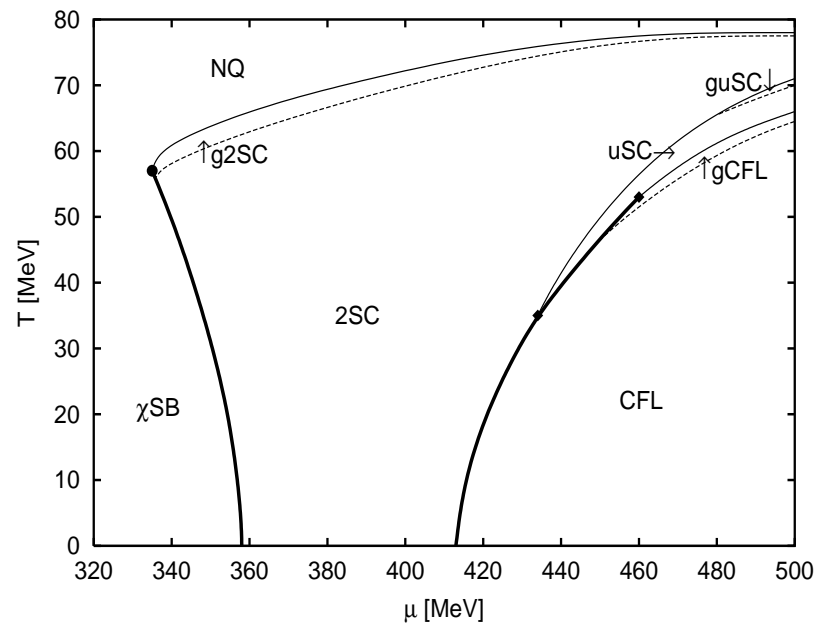
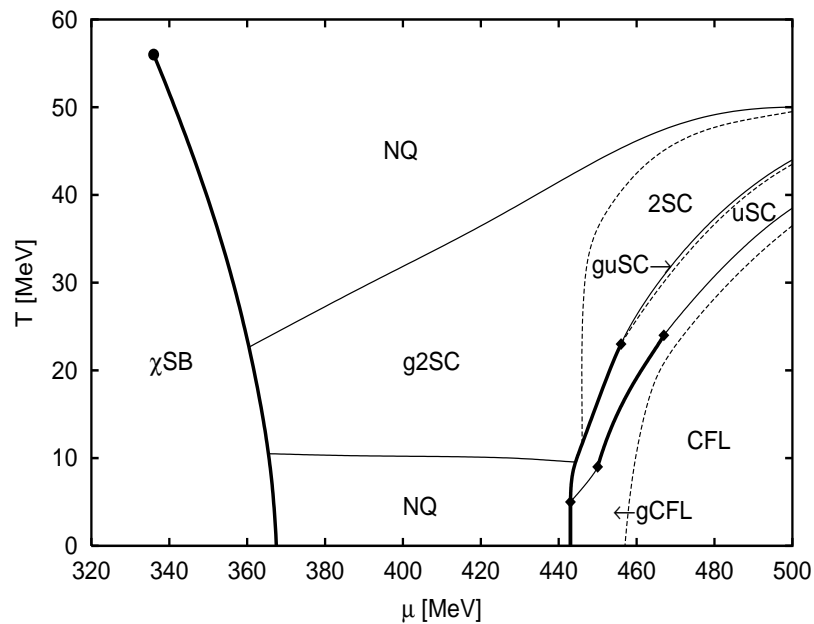
[Kryjevski, hep-ph/0508180],

[Schafer, hep-ph/0508190]

\Rightarrow No instabilities

Phase diagrams of neutral dense QCD

[Rüster, Werth, Buballa, Shovkovy & Rischke, hep-ph/0503184]



$G_D = \frac{3}{4}G_S$ (intermediate coupling)

$G_D = G_S$ (strong coupling)

Note: Gapless phases play little role at strong coupling, $G_D = G_S$

See also [Blaschke, et.al., hep-ph/0503194]

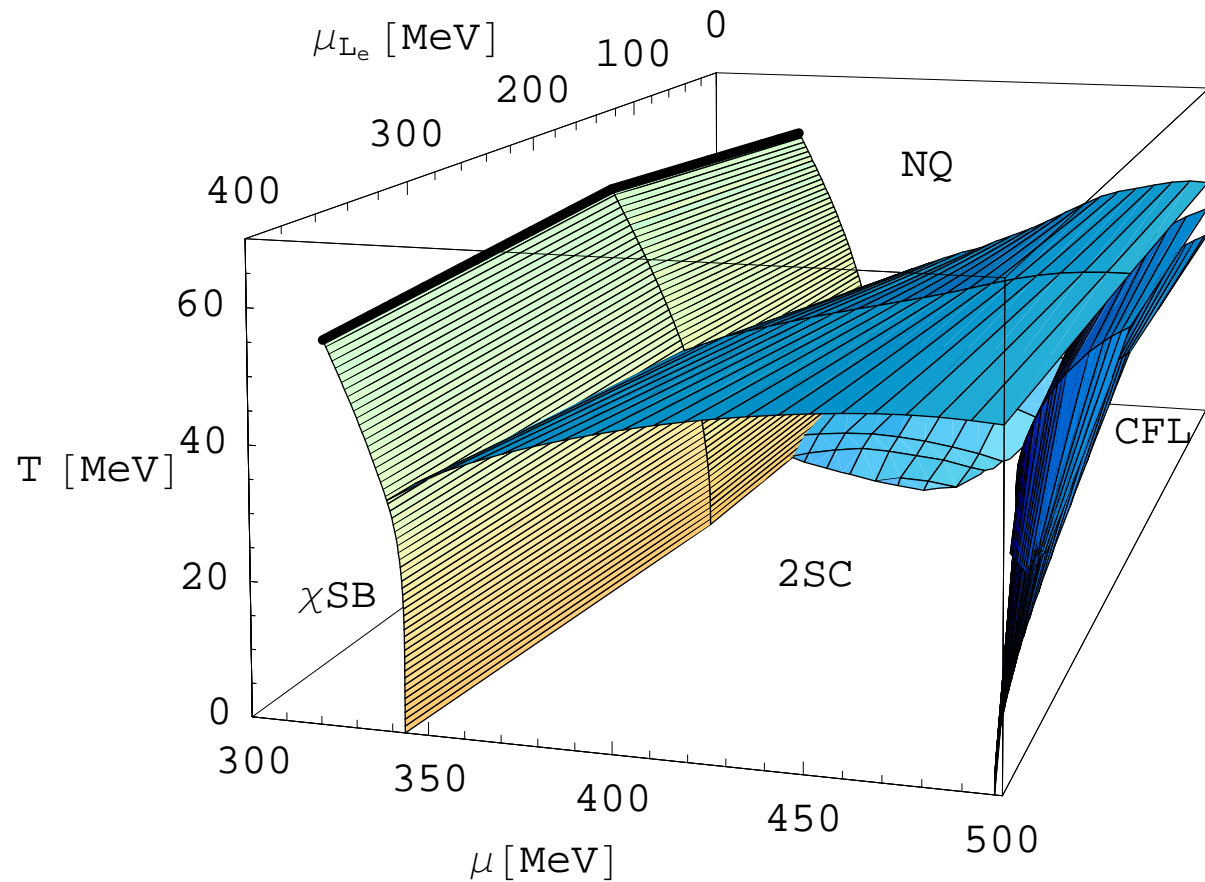
3D phase diagram

Neutrino trapping

$$\mu_{L_e} > 0$$

inside hot matter

$$T \lesssim 40 \text{ MeV}$$



[Rüster, et.al., hep-ph/0509073]

Current status

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative
 - In this limit, QCD can be studied from first principles
- Some features of $T - \mu$ phase diagram start to develop
 - In particular, sufficiently dense matter is a color superconductor
- Neutrality and β -equilibrium strongly affect the properties of CSC matter
 - There can exist many different phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K⁰, CFL+ η)
- Current problems: (i) instabilities of gapless phases, (ii) inhomogeneous ground states, (iii) search for observables, etc.

Some reviews on color superconductivity

- ▶ K. Rajagopal and F. Wilczek, “The condensed matter physics of QCD” [hep-ph/0011333](#)
- ▶ M. Alford, “Color superconducting quark matter”
Ann. Rev. Nucl. Part. Sci. **51**, 131 (2001) [hep-ph/0102047](#)
- ▶ T. Schäfer, “Quark matter” [hep-ph/0304281](#)
- ▶ D. H. Rischke, “The quark-gluon plasma in equilibrium”
Prog. Part. Nucl. Phys. **52**, 197 (2004) [nucl-th/0305030](#)
- ▶ M. Buballa, “NJL model analysis of quark matter at large density”
Phys. Rept. **407**, 205 (2005) [hep-ph/0402234](#)
- ▶ I. A. Shovkovy, “Two lectures on color superconductivity”
[nucl-th/0410091](#)