Color Superconductivity

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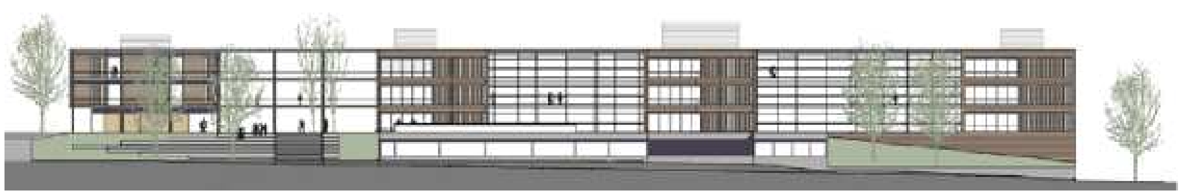


Outline

I. Introduction into color superconductivity

– Dense matter in nature

– Color superconductivity

- $N_f = 2$ color superconductivity (2SC)
- $N_f = 3$ color-flavor locking (CFL)
- Spin-1 color superconductivity ($N_f = 1$)

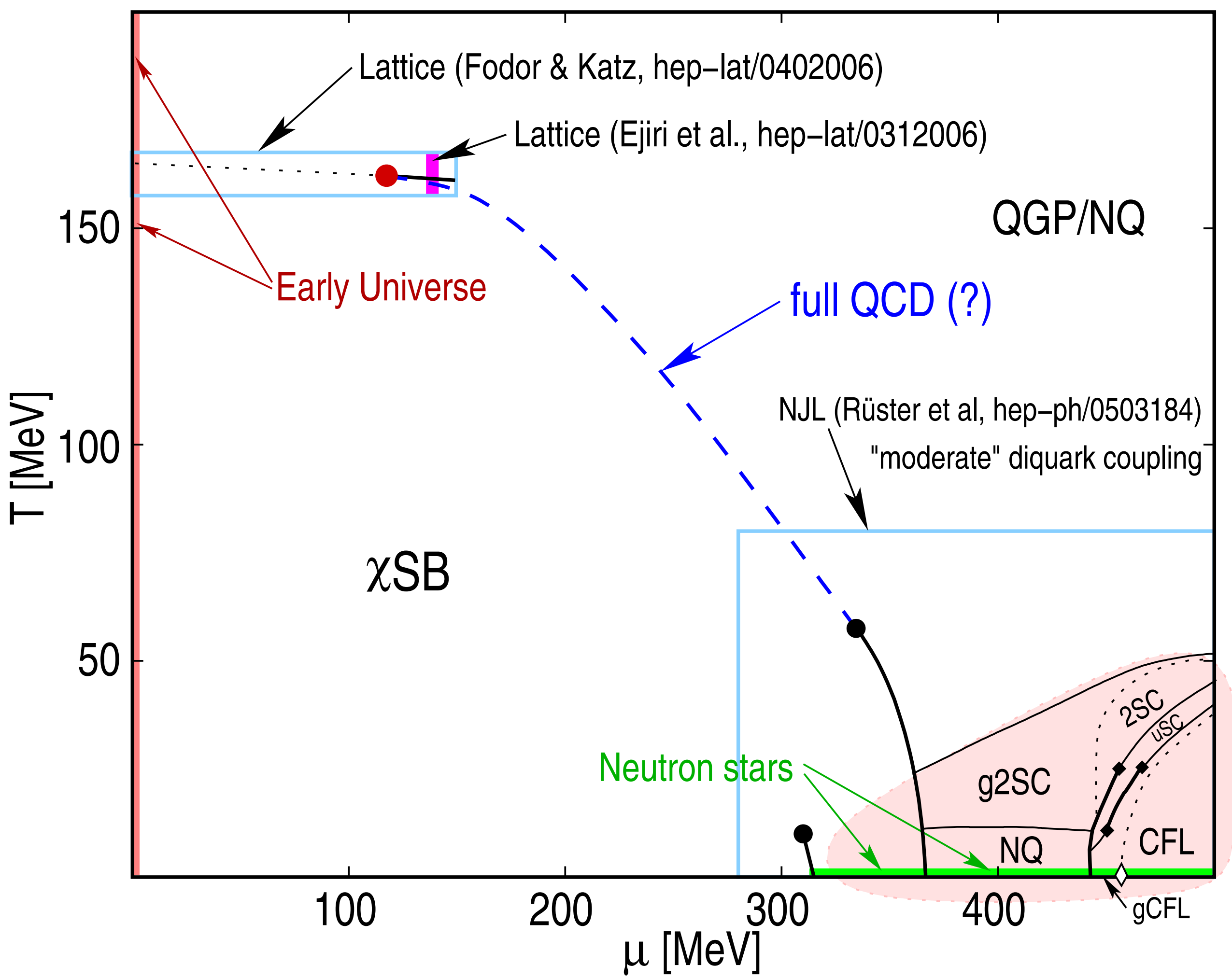
II. Color superconductivity in "stellar" matter

– Neutrality vs. color superconductivity

– Unconventional pairing in color superconductors

– Summary of the current status

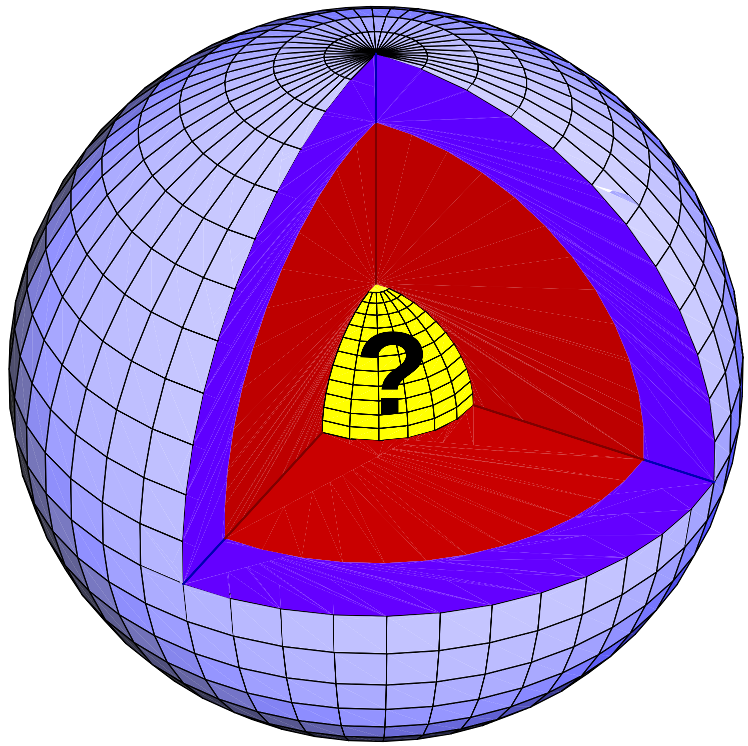
$T-\mu$ phase diagram of QCD



Dense baryonic matter in Nature

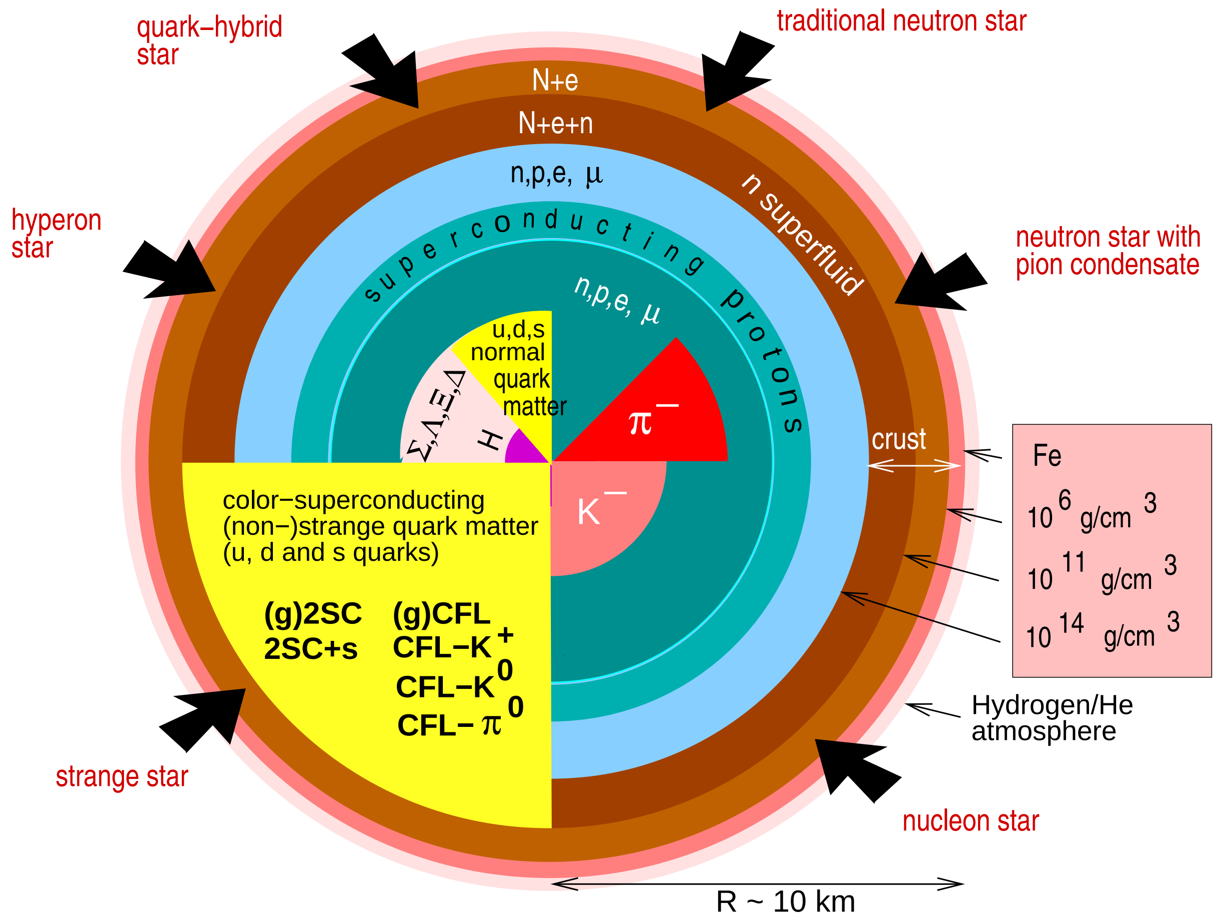
Compact (neutron) stars

- Radius:
  $R \simeq 10$ km
- Mass:
  $1.25 M_\odot \lesssim M \lesssim 2 M_\odot$
- Core temperature:
  10 keV $\lesssim T \lesssim 10$ MeV
- Surface magnetic field:
  10^8 G $\lesssim B \lesssim 10^{14}$ G



What is the state of matter at the highest stellar densities, $\rho_c \gtrsim 5\rho_0$?

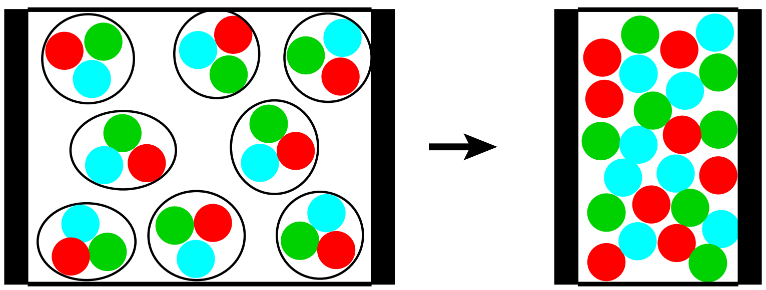
Possible phases of matter inside stars

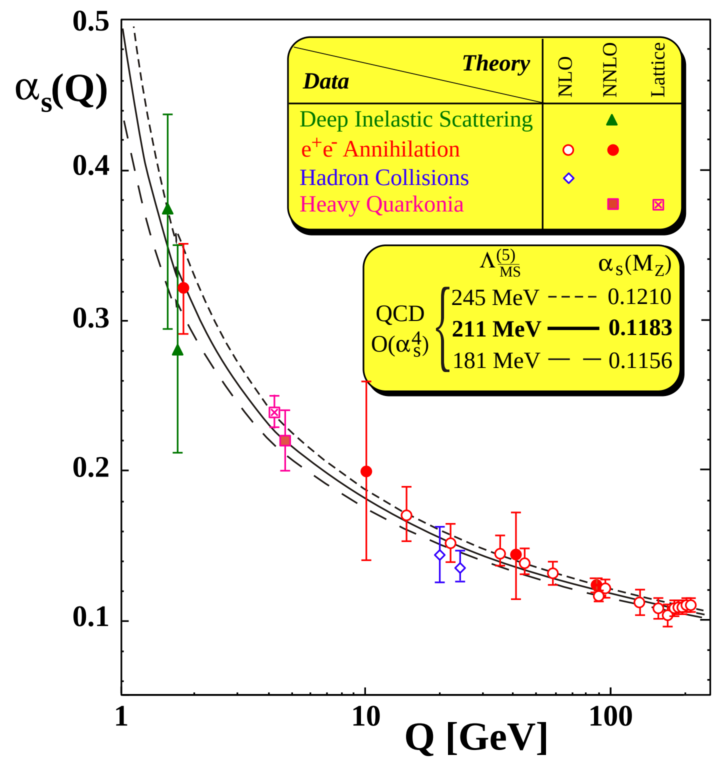


[figure from F. Weber, astro-ph/0407155 (modified)]

Very dense baryonic matter

Baryons at high density → quark matter





- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
  $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,'73]

⇒ **Weakly** interacting regime
[Collins&Perry,'75]

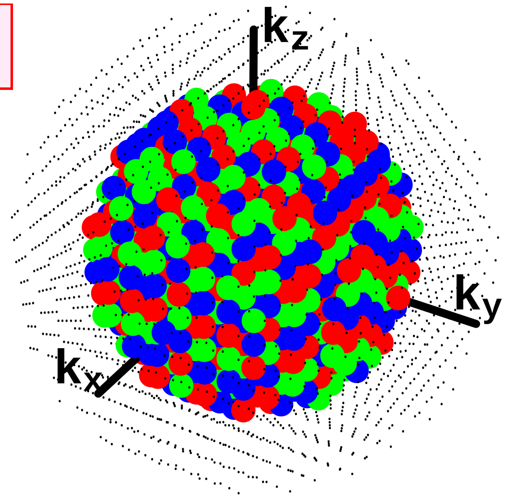
☹ Note: realistic densities in stars may not be sufficiently large:

$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3} \quad \Rightarrow \quad \mu \lesssim 500 \text{ MeV}$$

Ground state of dense quark matter

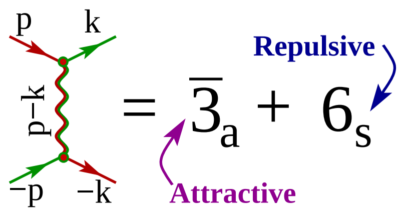
Noninteracting quarks:

(i) Deconfined quarks ($\mu \gg \Lambda_{QCD}$)
(ii) Pauli principle ($s = \frac{1}{2}$)
} $\Rightarrow$



Interacting quarks:

(i) Effective models ($\mu \gtrsim \Lambda_{QCD}$)
(ii) One-gluon exchange ($\mu \gg \Lambda_{QCD}$)



} $\Rightarrow$ Cooper instability

$\Downarrow$

Color superconductivity

$$\langle \left(\bar{\Psi}^C\right)_i^{\alpha} \gamma_5 \Psi_j^{\beta} \rangle \neq 0$$

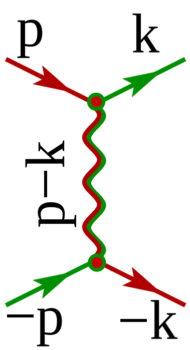
$N_f = 2$ color superconductivity (2SC)

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84]

- $N_f = 2$, i.e., "up" and "down" quarks
- $N_c = 3$, i.e., "red", "green" and "blue"

$\langle u_p\, d_{-p} \rangle = -\langle u_q\, d_{-q} \rangle \neq 0$

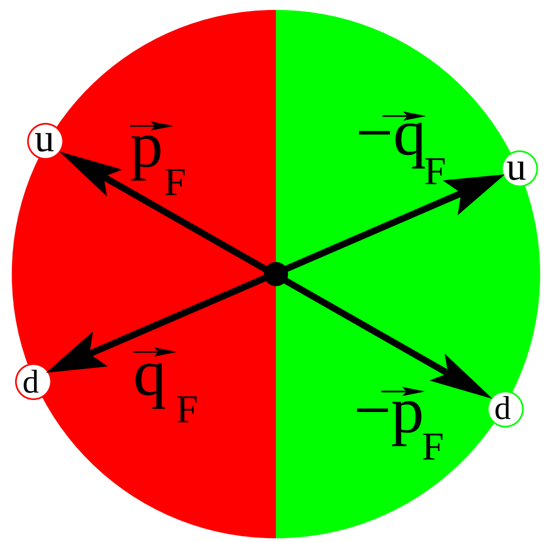
Gluon-exchange interaction:



$= \bar{3}_a + 6_s$

Repulsive (6_s)

Attractive ($\bar{3}_a$)



Cooper pairs:

$$\left(|\bullet\bullet\rangle - |\bullet\bullet\rangle\right)_{\bar{3}} \otimes \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)_0 \otimes \left(|u,d\rangle - |d,u\rangle\right)$$

Symmetries of 2SC state

- Diquark condensate:

$$\langle \left(\bar{\Psi}^C\right)_i^{\alpha} \gamma_5 \Psi_j^{\beta} \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

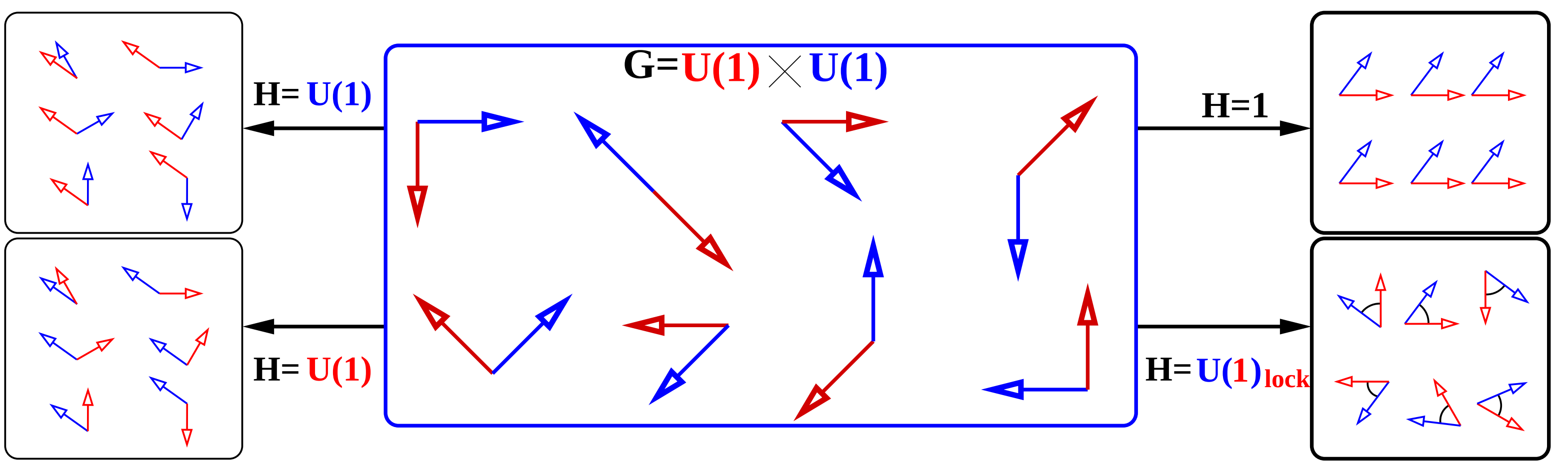
  - chiral $SU(2)_L \times SU(2)_R$ — intact
  - baryon number $U(1)_B \to \tilde{U}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}} T_8$ (quark matter is not superfluid)
  - gauge symmetry $U(1)_{em} \to \tilde{U}(1)_{em}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$ (there is no Meissner effect)
  - approximate axial $U(1)_A$ is broken $\to$ 1 pseudo-NG boson
  - color gauge symmetry $SU(3)_c \to SU(2)_c$ (Higgs mechanism)

$N_f = 3$ color superconductivity

- Cooper pair: $(|\color{red}{\bullet}\color{green}{\bullet}\rangle - |\color{green}{\bullet}\color{red}{\bullet}\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|\mathrm{u,d}\rangle - |\mathrm{d,u}\rangle)_{\bar{3}}$
- Diquark condensate:

$$\langle \left(\bar{\Psi}^C_L\right)^\alpha_i \left(\Psi_L\right)^\beta_j \rangle = \langle \left(\bar{\Psi}^C_R\right)^\alpha_i \left(\Psi_R\right)^\beta_j \rangle \simeq \sum_I \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_c \to \mathrm{SU}(3)_{L+R+c}$ via "color-flavor locking", without $\langle \bar{q}_L q_R \rangle$ condensates! [Alford et al. hep-ph/9804403]



Symmetries of CFL ground state

- chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
  $\rightarrow$ 8 (pseudo-)NG bosons, i.e., π^0, $\pi^{\pm}$, $K^{\pm}$, K^0, $\bar{K}^0$, η
  (almost like in vacuum QCD)
- baryon number $U(1)_B$ is broken $\rightarrow$ 1 NG boson (ϕ)
  (quark matter is superfluid)
- approximate axial $U(1)_A$ is broken $\rightarrow$ 1 pseudo-NG boson (η')
- color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism $\rightarrow$ 8 massive gluons
- gauge symmetry $U(1)_{\mathrm{em}} \rightarrow \tilde{U}(1)_{\mathrm{em}}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
  (there is no Meissner effect)

$N_f = 1$ color superconductivity

- Cooper pair: $\left(|\bullet\bullet\rangle - |\bullet\bullet\rangle\right)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$
- Diquark condensate:

$$\langle \left(\bar{\Psi}^C\right)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^\delta \sin\theta + \gamma_\perp^\delta(\vec{\mathbf{k}}) \cos\theta\right)$$

- Many possibilities, e.g., see [A.Schmitt, nucl-th/0412033]:
  - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta}$ $\rightarrow$ largest pressure (?)
  - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
  - Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
  - A-phase: $\Delta_{c\delta} = \delta_{c3}\left(\delta_{\delta 1} + i\delta_{\delta 2}\right)$ $\rightarrow$ unusual neutrino emission

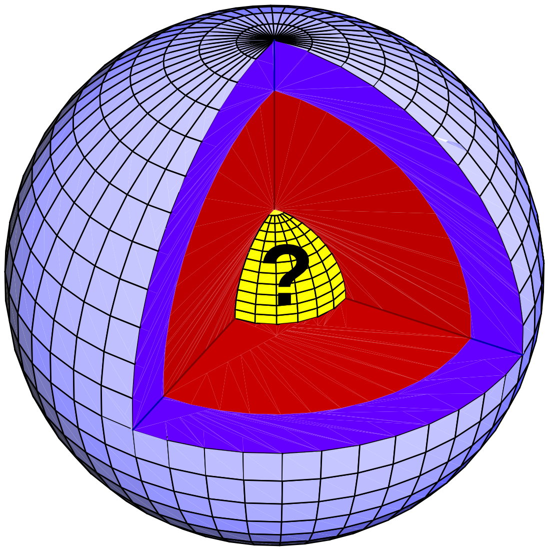
Color superconductivity in stellar matter

Matter in the bulk of a star is

(i) β-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:

$n_Q^{\text{el}} = 0, \qquad n_Q^{\text{color}} = 0$



If $n_Q \neq 0$, the Coulomb energy is

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3}\right)^2 \left(\frac{R}{1\ \text{km}}\right)^5$$

e.g., if $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \quad \Rightarrow \quad E_{\text{Coulomb}}^{\text{2SC}} \gg M_\odot c^2$

Unconventional Cooper pairing, $N_f = 2$

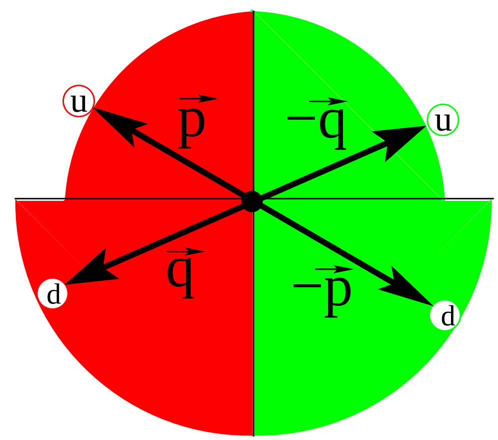
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter appears when $n_d \approx 2n_u$
- Electrons, required in β equilibrium, **cannot** help:

$$n_d \approx 2n_u \quad \Rightarrow \quad \mu_d \approx 2^{1/3}\mu_u \;\Rightarrow\; \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{i.e.,} \qquad n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

Cooper pairing is distorted by "mismatch" parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Gapless 2SC phase

Competition: $\delta\mu$ vs. Δ_0 (where Δ_0 is the gap at $\delta\mu = 0$)

The "winner" is determined by the diquark coupling strength
[Shovkovy&Huang, Phys. Lett. **B 564** (2003) 205]

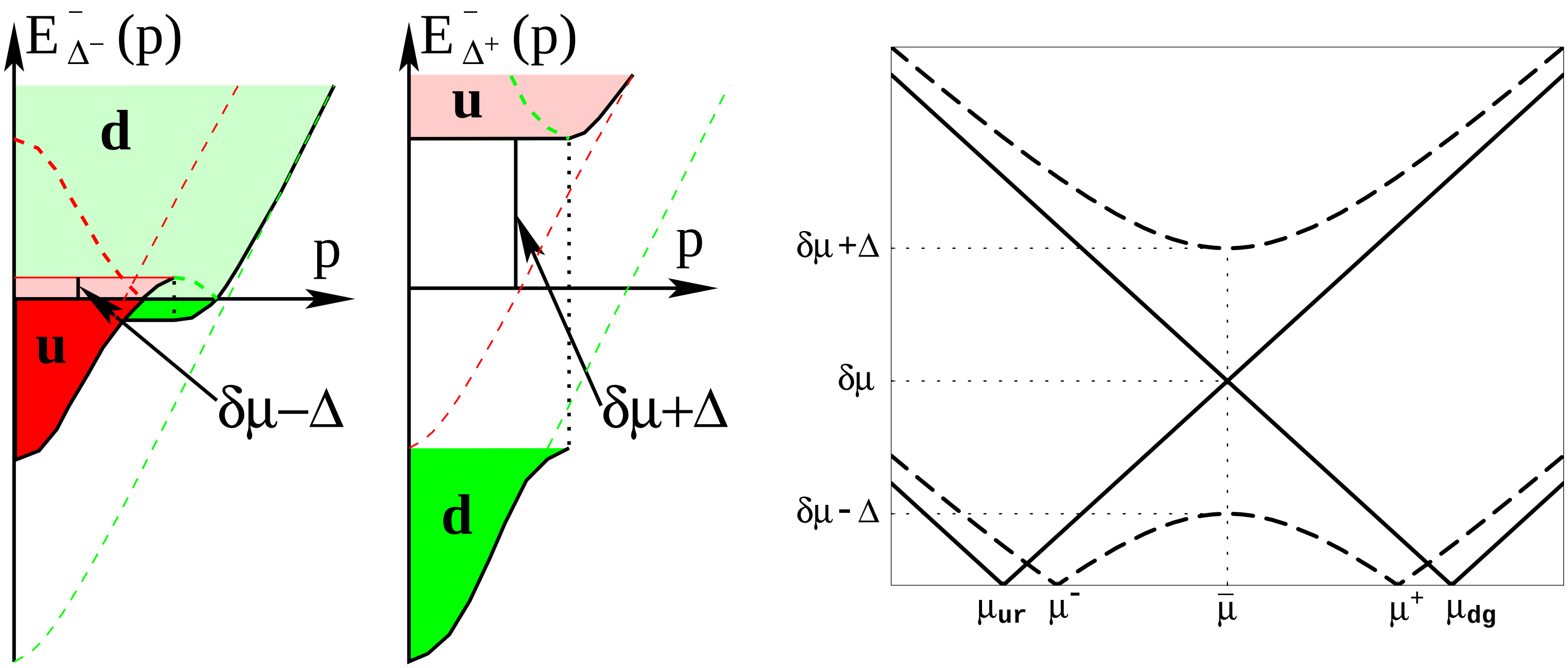
1. $\delta\mu \gtrsim \Delta_0$ — the mismatch does not allow Cooper pairing: normal phase is the ground state

2. $\delta\mu \lesssim \frac{1}{2}\Delta_0$ — coupling is strong enough to win over the mismatch: 2SC is the ground state

3. $\frac{1}{2}\Delta_0 \lesssim \delta\mu \lesssim \Delta_0$ — regime of intermediate coupling strength: the ground state is the gapless 2SC phase

Quasiparticle spectrum in g2SC phase

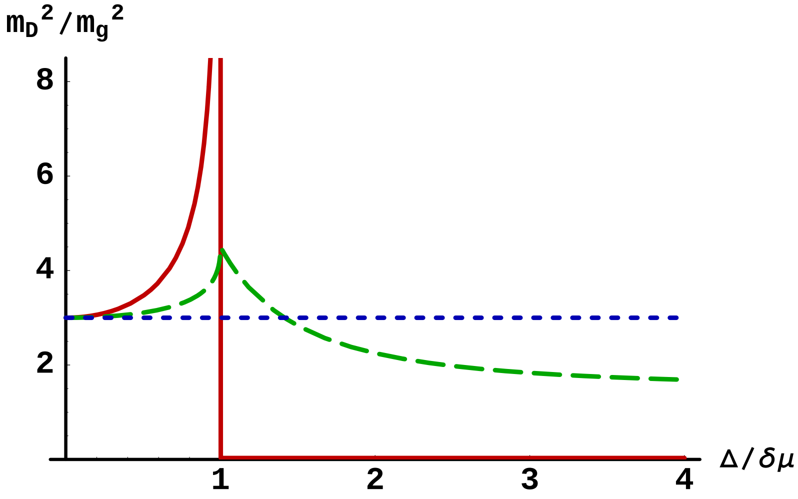
"Intermediate" coupling

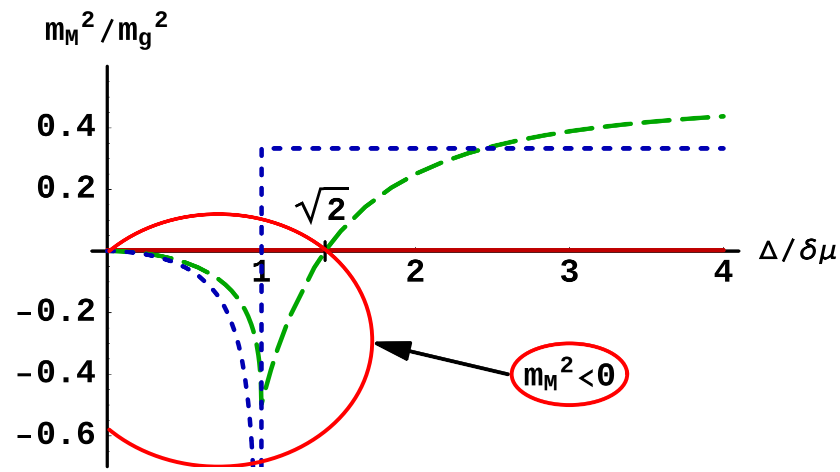


The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Chromomagnetic instability

Recent results for gluon screening masses
[Huang & Shovkovy, hep-ph/0407049]:





$A = 1, 2, 3$ — red solid line

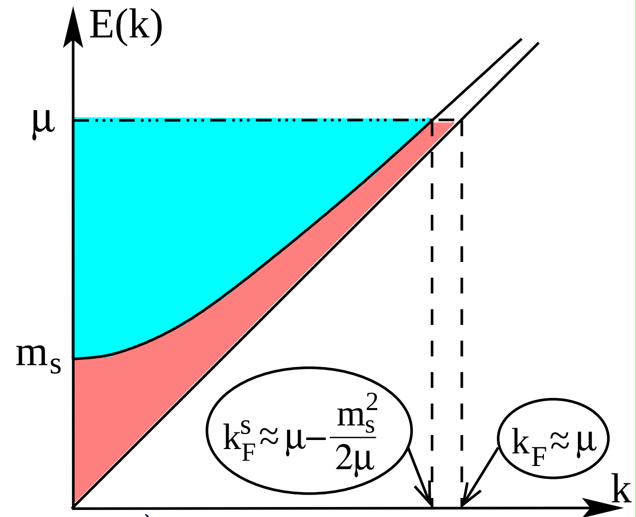
$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

$N_f = 2+1$ color superconductivity, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$



The ground state of strange quark matter:

(?) only condensates of same flavor (spin-1 channel)

(?) only superconductivity of up and down quarks (2SC or g2SC)

(?) crystaline pairing (nonzero momentum pairing, LOFF)

(?) . . .

Gapless CFL phase

[Alford, Kouvaris & Rajagopal, hep-ph/0311286]

- Distorted color-flavor pairing:

$$\Delta_{ij}^{\alpha\beta} \simeq \underbrace{\Delta_1\ \epsilon_{1ij}\varepsilon^{1\alpha\beta}}_{\langle d_b s_g\rangle \& \langle d_g s_b\rangle} + \underbrace{\Delta_2\ \epsilon_{2ij}\varepsilon^{2\alpha\beta}}_{\langle u_b s_r\rangle \& \langle u_r s_b\rangle} + \underbrace{\Delta_3\ \epsilon_{3ij}\varepsilon^{3\alpha\beta}}_{\langle u_g d_r\rangle \& \langle u_r d_g\rangle} + \ldots$$

- Mismatch parameter:

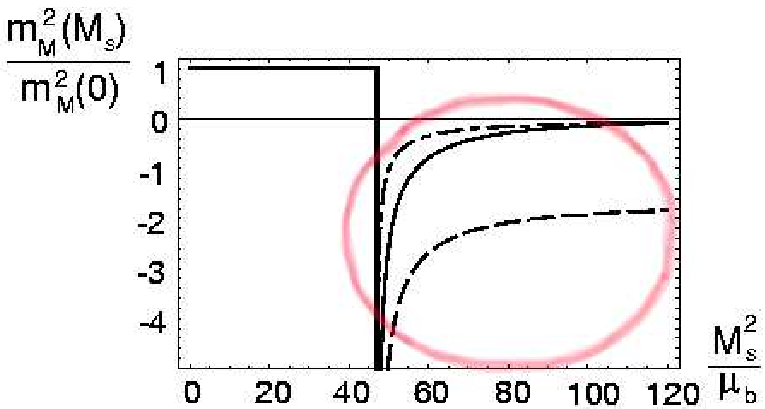
$$\delta\mu \approx \frac{m_s^2}{2\mu}$$

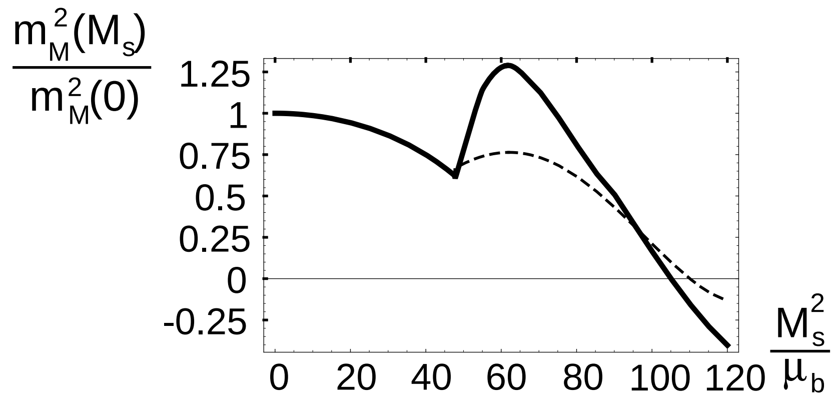
- At $T=0$, the gapless CFL phase occurs when

$$\delta\mu \equiv \frac{m_s^2}{2\mu} > \Delta_0$$

Chromomagnetic instability in gCFL phase

Recent results for Meissner screening masses
[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:





$A = 1, 2$ — solid line

$A = 3$ — short-dashed line

$A = 8$ — long-dashed line

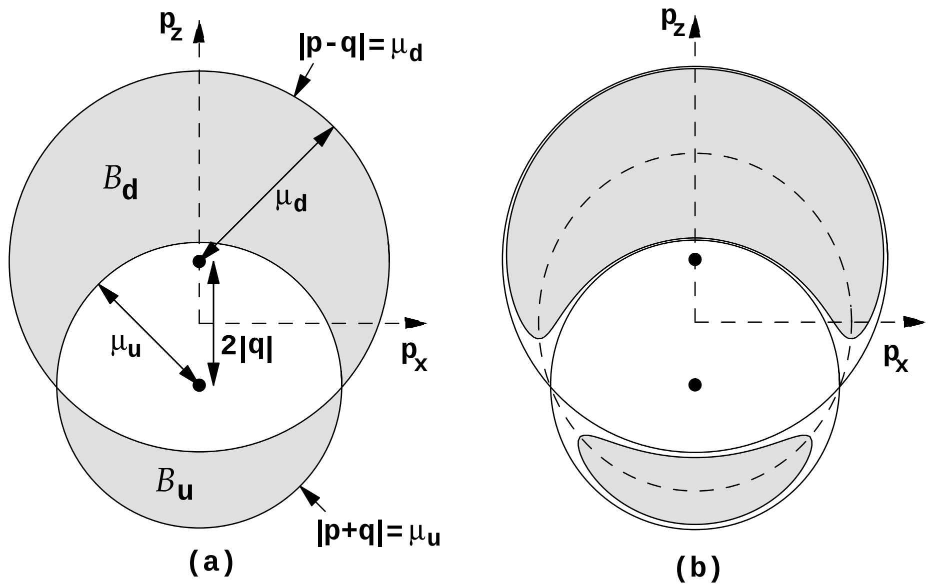
$A = 4, 5$ — dashed line

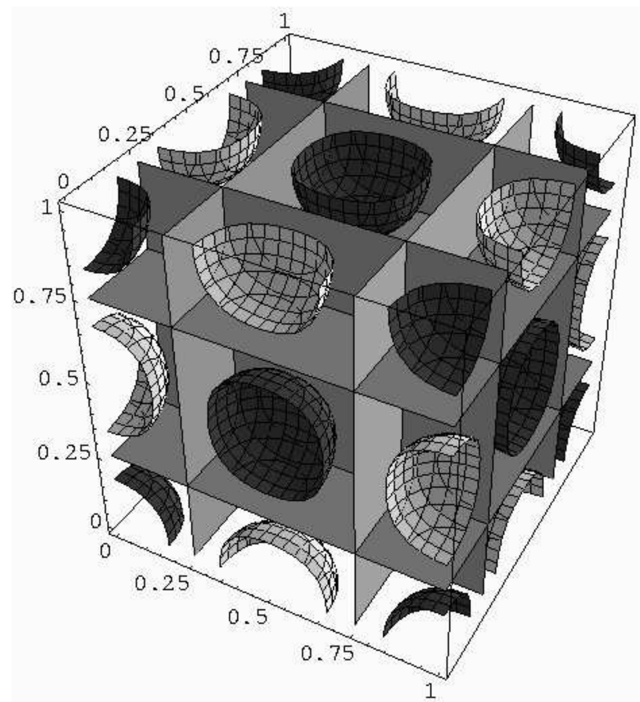
$A = 6, 7$ — solid line

Alternatives. I

Crystalline color superconductivity (LOFF phase)
[Alford, Bowers & Rajagopal, hep-ph/0008208]

Cooper pairs with nonzero momenta:



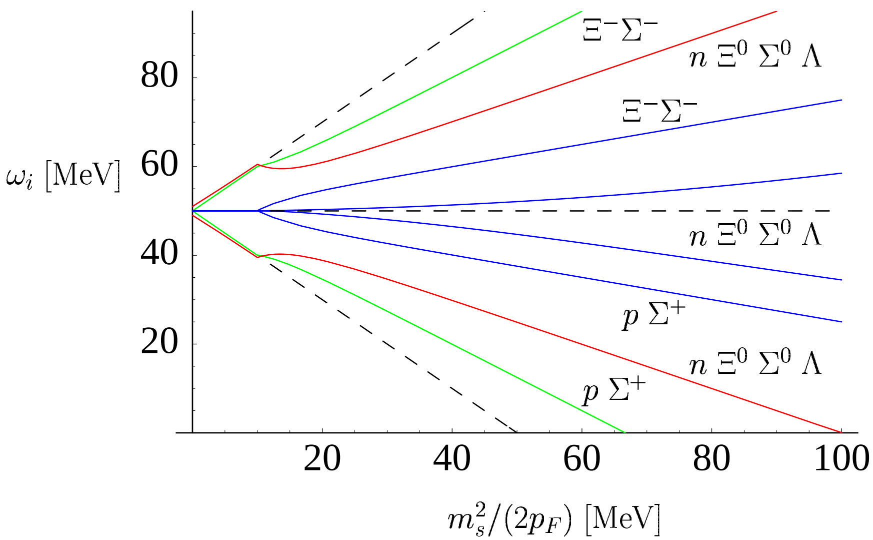


[Bowers, hep-ph/0305301]

Alternatives. II

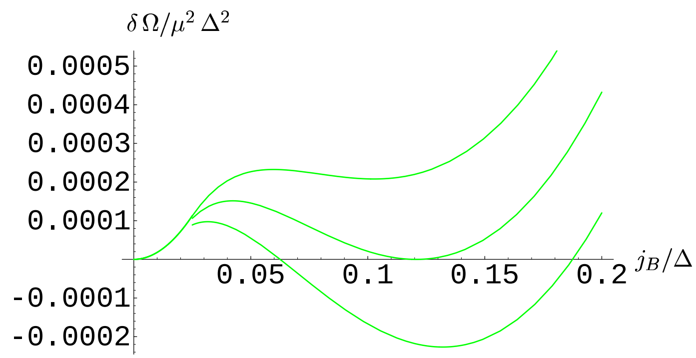
Baryon spectrum in the CFL phase

[Kryjevski & Schafer, hep-ph/0407329]



Solution (?)

P-wave meson condensation



[Kryjevski, hep-ph/0508180],
[Schafer, hep-ph/0508190]

⇒ No instabilities

Current status

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative

– In this limit, QCD can be studied from first principles

- Some features of $T - \mu$ phase diagram start to develop

– In particular, sufficiently dense matter is a color superconductor

- Neutrality and β-equilibrium strongly affect the properties of CSC matter

– There can exist many different phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K^0, CFL+η)

- Current problems: (i) instabilities of gapless phases, (ii) inhomogeneous ground states, (iii) search for observables, etc.

Some reviews on color superconductivity

- K. Rajagopal and F. Wilczek, "The condensed matter physics of QCD" hep-ph/0011333
- M. Alford, "Color superconducting quark matter" Ann. Rev. Nucl. Part. Sci. **51**, 131 (2001) hep-ph/0102047
- T. Schäfer, "Quark matter" hep-ph/0304281
- D. H. Rischke, "The quark-gluon plasma in equilibrium" Prog. Part. Nucl. Phys. **52**, 197 (2004) nucl-th/0305030
- M. Buballa, "NJL model analysis of quark matter at large density" Phys. Rept. **407**, 205 (2005) hep-ph/0402234
- I. A. Shovkovy, "Two lectures on color superconductivity" Found. Phys. **35** (2005), in press nucl-th/0410091