Dense baryon matter: progress and difficulties

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Outline

I. Introduction into color superconductivity

– Phase diagram of baryonic matter

– Color superconductivity

- 2SC (2 quark flavors)
- CFL (3 quark flavors)
- Spin-1 color superconductivity (1 quark flavor)

II. Cooper pairing under stress

– Neutrality vs. color superconductivity

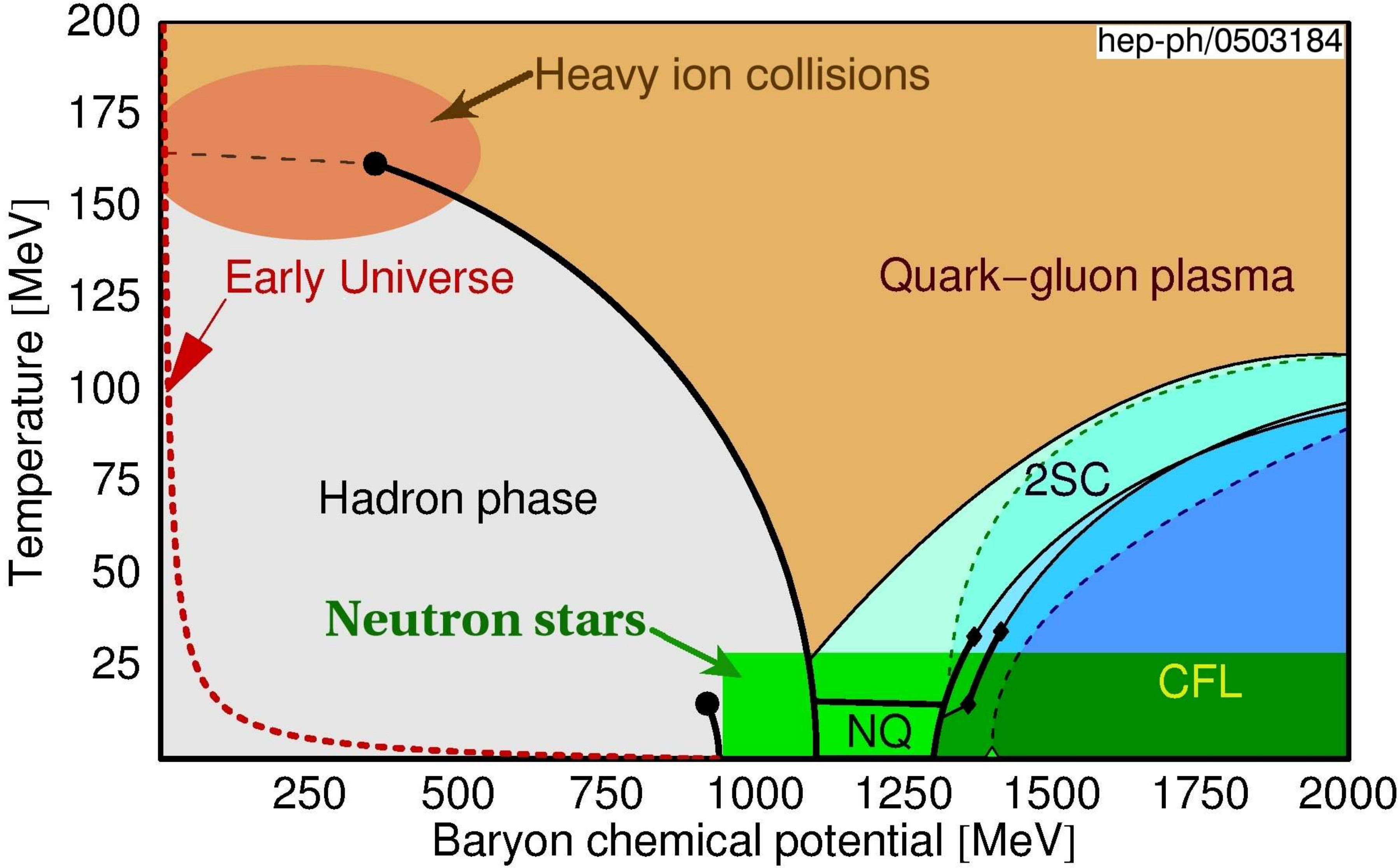
– Unconventional pairing in color superconductors

– Gapless, crystalline and other phases

– Current status

– Summary

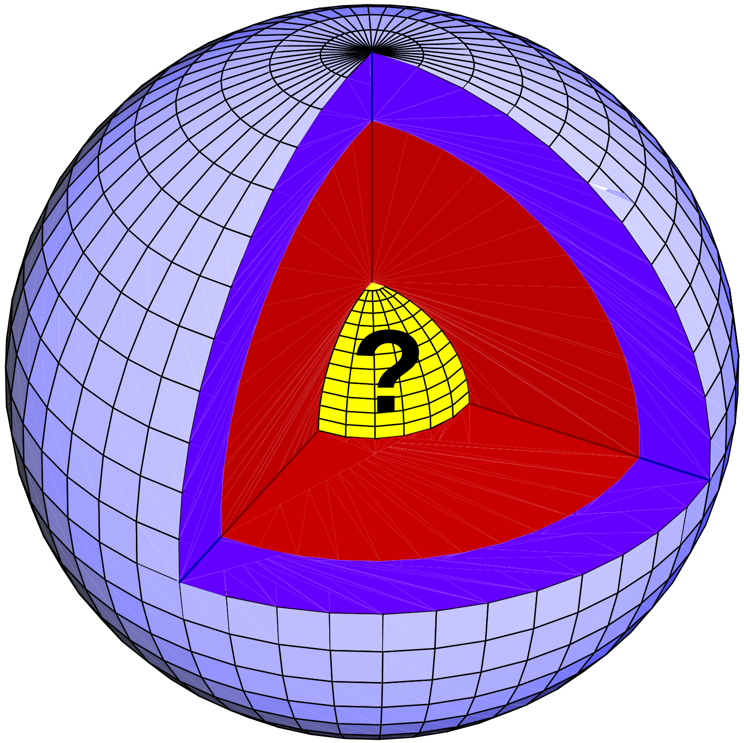
$T - \mu$ phase diagram of QCD



Dense baryonic matter in Nature

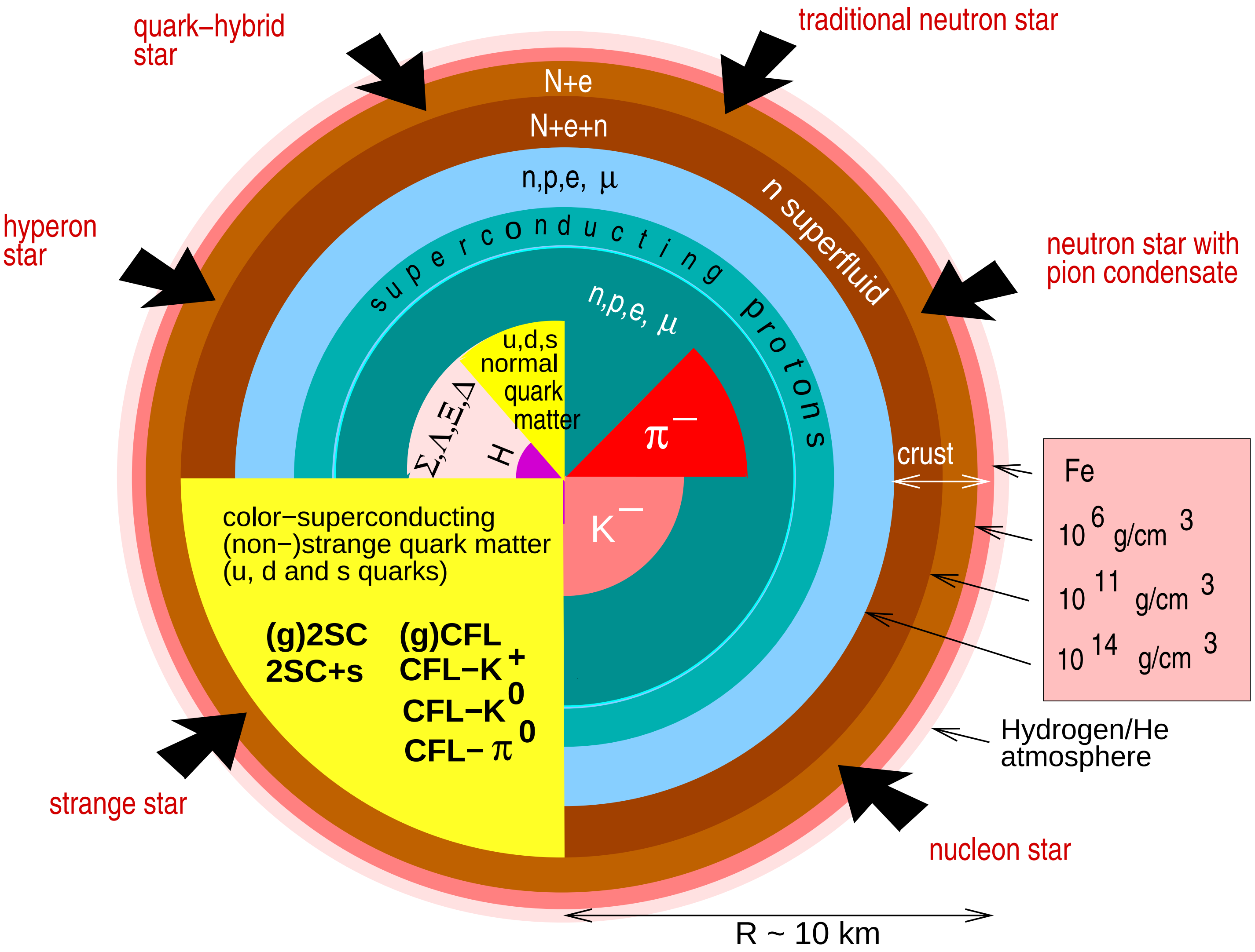
Compact (neutron) stars

- Radius:
  $R \simeq 10$ km
- Mass:
  $1.25 M_\odot \lesssim M \lesssim 2 M_\odot$
- Core temperature:
  10 keV $\lesssim T \lesssim 10$ MeV
- Surface magnetic field:
  10^8 G $\lesssim B \lesssim 10^{14}$ G
- Rotational period:
  1.6 ms $\lesssim P \lesssim 12$ s



Central densities in stars should be rather high: $\rho_c \gtrsim 5\rho_0$

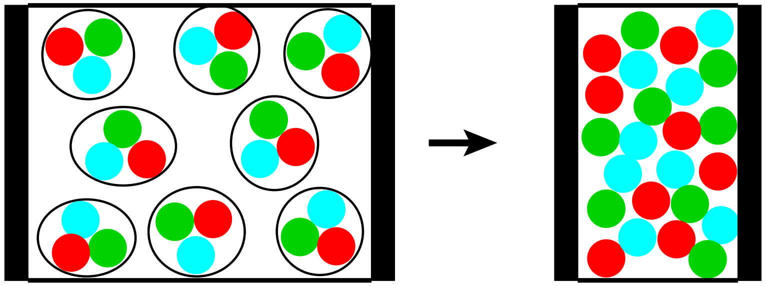
Possible phases of matter inside stars

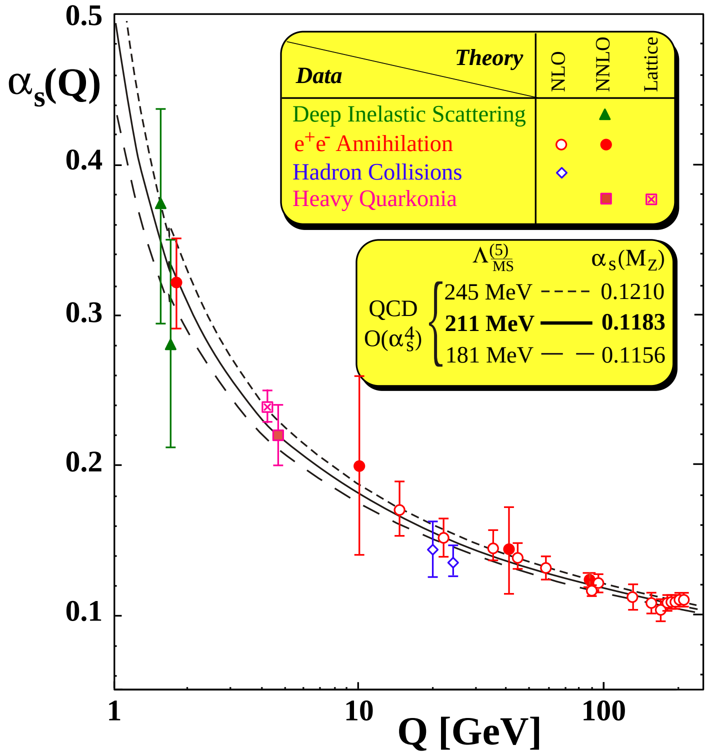


[figure from F. Weber, astro-ph/0407155 (modified)]

Very dense baryonic matter

Baryons at high density → quark matter





- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
  $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,'73]

⇒ **Weakly** interacting regime

[Collins&Perry,'75]

Note: realistic densities in stars may not be sufficiently large:

$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3} \quad \Rightarrow \quad \mu \lesssim 500 \text{ MeV}$$

Two complimentary approaches

(i) QCD (from first principles):

$$\mathcal{L}_{\rm QCD} = \bar{\psi}^{\alpha}_{f} \left(i\gamma^{\mu}\partial_{\mu} + \gamma^{0}\mu_{f} + gT^{a}_{\alpha\beta}\gamma^{\mu}A^{a}_{\mu} - m_{f} \right) \psi^{\beta}_{f} - \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu}$$

– predictions are reliable when $\mu \gg \Lambda_{QCD}$

(ii) Phenomenological (e.g., NJL-type) models fitted to reproduce basic properties of vacuum QCD and/or nuclear matter, e.g.,

$$\mathcal{L}_{\rm NJL} = \bar{\psi}^{\alpha}_{f} \left(i\gamma^{\mu}\partial_{\mu} + \gamma^{0}\mu_{f} - m_{f} \right) \psi^{\beta}_{f} + \frac{g^{2}}{2}(\bar{\psi}\gamma^{\mu}T^{a}\psi)(\bar{\psi}\gamma_{\mu}T^{a}\psi)$$

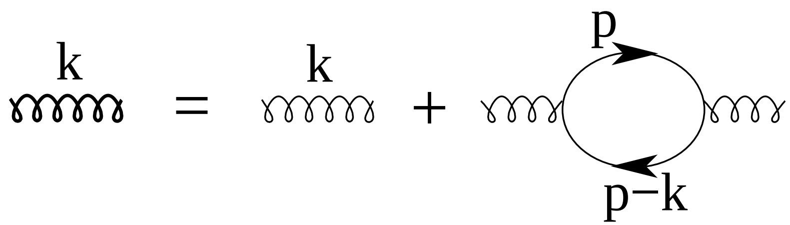
– may work only when $\rho \lesssim \rho_0$

Note: densities of interest: $3\rho_0 \lesssim \rho \lesssim 10\rho_0$

QCD: one-gluon exchange (with screening)

Gluon propagator in dense medium:

$$\mathcal{D}^{-1}_{\mu\nu}(k) = D^{-1}_{0,\mu\nu}(k) - \Pi_{\mu\nu}(k),$$ i.e.,



Electrical Debye screening and magnetic dynamical screening [Son,hep-ph/9812287]:

$$i\mathcal{D}_{\mu\nu}(k_4,|\vec{k}|) \simeq -\frac{O^{(el)}_{\mu\nu}}{k_4^2+|\vec{k}|^2+2M_D^2} - \frac{|\vec{k}|O^{(mag)}_{\mu\nu}}{|\vec{k}|^3+\pi M_D^2|k_4|/2},$$

where $M_D^2 = \alpha_s N_f \mu^2/\pi$ is the Debye mass

Magnetic interaction is long-ranged in (near-)static limit, $k_4 \lesssim |\vec{k}| \ll \mu$

QCD: gap equation & solution

Gap equation:

$$\Delta_p^{\pm} = \frac{4\pi\alpha_s}{3}\int \frac{d^4k}{(2\pi)^4}\left(\frac{\Delta_k^- \,\mathrm{Tr}\left(\gamma_\mu\Lambda_k^+\gamma_\nu\Lambda_p^\pm\right)}{k_0^2-(|\mathbf{k}|-\mu)^2-|\Delta_k^-|^2}+\frac{\Delta_k^+ \,\mathrm{Tr}\left(\gamma_\mu\Lambda_k^-\gamma_\nu\Lambda_p^\pm\right)}{k_0^2-(|\mathbf{k}|+\mu)^2-|\Delta_k^+|^2}\right)\mathcal{D}_{k-p}^{\mu\nu}$$

Approximate solution for $\Delta_0 = |\Delta_k^-|_{k\simeq\mu}$

$$\Delta_0 \simeq \lambda\,\mu \qquad \underbrace{\exp\left(-\frac{3\pi^{3/2}}{2^{3/2}\sqrt{\alpha_s}}\right)}_{\text{long-range magnetic gluons}}$$

where

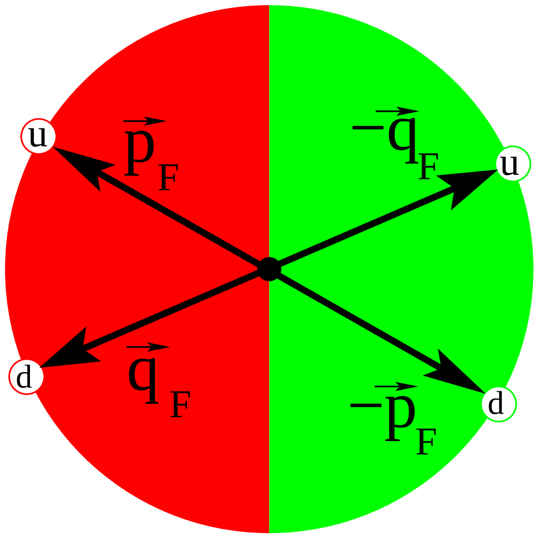
$$\lambda = \underbrace{\frac{2(4\pi)^{3/2}}{\alpha_s^{5/2}}}_{\text{electric gluons}} \times \underbrace{\exp\left(-\frac{4+\pi^2}{8}\right)}_{\text{quark self-energy corrections}} \times(\text{higher order corrections})$$

$N_f = 2$ color superconductivity (2SC)

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84]

- $N_f = 2$: "up" and "down"
- $N_c = 3$: "red", "green" and "blue"

$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$



Diquark condensate:

$$\phi_3 = \left\langle \left(\bar{\Psi}^C\right)_i^{\alpha} \varepsilon^{ij} \varepsilon_{\alpha\beta 3} C\gamma^5 \Psi_j^{\beta} \right\rangle \neq 0$$

(Pauli principle)

Note:

- Same colors and same flavors do not pair (spin-0 channel)
- Only two colors of quarks participate in Cooper pairing

Symmetries of 2SC state

- Diquark condensate:

$$\langle \left(\bar{\Psi}^C\right)_i^{\alpha} \gamma_5 \Psi_j^{\beta} \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

[Alford et al, hep-ph/9711395], [Rapp et al, hep-ph/9711396]

✓ baryon number $\mathrm{U}(1)_B \to \tilde{\mathrm{U}}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}} T_8$
(quark matter is not superfluid)

✓ gauge symmetry $\mathrm{U}(1)_{\mathrm{em}} \to \tilde{\mathrm{U}}(1)_{\mathrm{em}}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$
(there is no Meissner effect)

– chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ — intact

– approximate axial $\mathrm{U}(1)_A$ is broken → 1 pseudo-NG boson

– color gauge symmetry $\mathrm{SU}(3)_c \to \mathrm{SU}(2)_c$ (Higgs mechanism)

$N_f = 3$ color superconductivity

Diquark condensate:

$$\left\langle \left(\bar{\psi}^C\right)_i^a \varepsilon^{ijk} \varepsilon_{abc} C\gamma^5 \psi_j^b \right\rangle \sim \delta_k^c$$

or, in terms of chiral fields,

$$\left\langle \psi_{L,i}^{a,\alpha} \varepsilon^{ijk} \varepsilon_{abc} \varepsilon_{\alpha\beta} \psi_{L,j}^{b,\beta} \right\rangle = -\left\langle \psi_{R,i}^{a,\dot{\alpha}} \varepsilon^{ijk} \varepsilon_{abc} \varepsilon_{\dot{\alpha}\dot{\beta}} \psi_{R,j}^{b,\dot{\beta}} \right\rangle \sim \delta_k^c$$

Color-flavor locking:

[Alford, Rajagopal, & Wilczek, hep-ph/9804403]

$\langle \mathrm{LL} \rangle$	is invariant under	$g_{\mathrm{L,flavor}} \otimes g_{\mathrm{color}}$	if	$g_{\mathrm{color}} \equiv g^{-1}_{\mathrm{L,flavor}}$
$\langle \mathrm{RR} \rangle$	is invariant under	$g_{\mathrm{R,flavor}} \otimes g_{\mathrm{color}}$	if	$g_{\mathrm{color}} \equiv g^{-1}_{\mathrm{R,flavor}}$

i.e., residual global symmetry is $SU(3)_{\mathrm{L+R}}$

As in vacuum, there appear 8 corresponding Nambu-Goldstone bosons:

$$\pi^0,\ \pi^\pm,\ K^0,\ \bar{K}^0,\ K^\pm,\ \eta$$

Symmetries of CFL ground state

✓ chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
$\rightarrow$ 8 (pseudo-)NG bosons, i.e., π^0, $\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$, η
(almost like in vacuum QCD)

✓ baryon number $U(1)_B$ is broken $\rightarrow$ 1 NG boson (ϕ)
(quark matter is superfluid)

✓ gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
(there is no Meissner effect)

- approximate axial $U(1)_A$ is broken $\rightarrow$ 1 pseudo-NG boson (η')
- color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism
$\rightarrow$ 8 massive gluons

$N_f = 1$ color superconductivity

- Cooper pair: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$
- Diquark condensate:

$$\langle \left(\bar{\Psi}^C\right)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^\delta \sin\theta + \gamma_\perp^\delta (\vec{\mathbf{k}}) \cos\theta \right)$$

[Iwasaki & Iwado, 1995], [Schäfer, hep-ph/0006034], [Alford et al, hep-ph/0210106]

- Many possibilities, e.g., see [Schmitt, nucl-th/0412033]:
  - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta}$ → largest pressure (?)
  - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
  - Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
  - A-phase: $\Delta_{c\delta} = \delta_{c3}\left(\delta_{\delta 1} + i\delta_{\delta 2}\right)$ → unusual neutrino emission
- Many similarities with superfluidity in ^{3}He ...

Color superconductivity inside stars

Matter in the bulk of a star must be

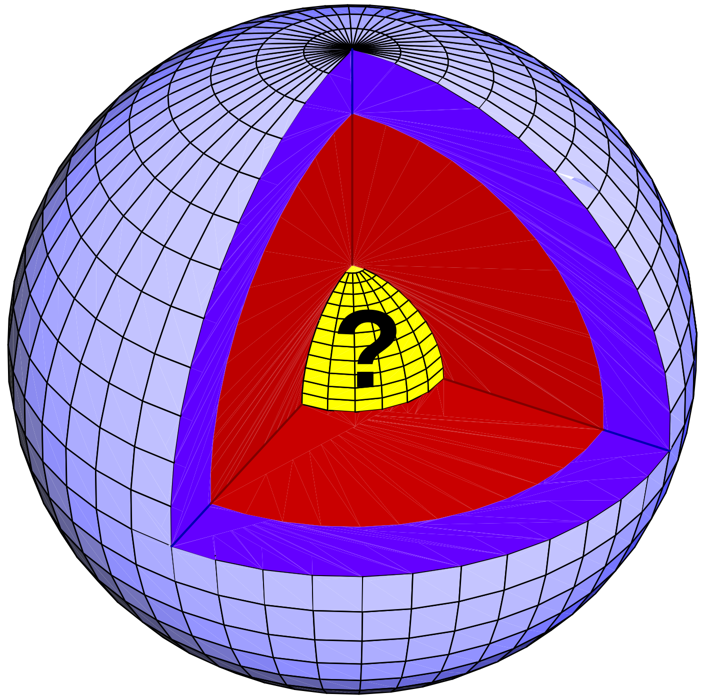
(i) in β-equilibrium: $\mu_d = \mu_u + \mu_e$,
i.e., the weak processes

$$u + e^- \rightarrow d + \nu$$

$$\& \quad d \rightarrow u + e^- + \bar{\nu}$$

have equal rates;

(ii) electrically and color neutral:
$n_Q^{\text{el}} = 0, \qquad n_Q^{\text{color}} = 0$



If $n_Q \neq 0$, the Coulomb energy is

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3}\right)^2 \left(\frac{R}{1\ \text{km}}\right)^5$$

e.g., if $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \quad \Rightarrow \quad E_{\text{Coulomb}}^{\text{2SC}} \gg M_\odot c^2$

Unconventional Cooper pairing, $N_f = 2$

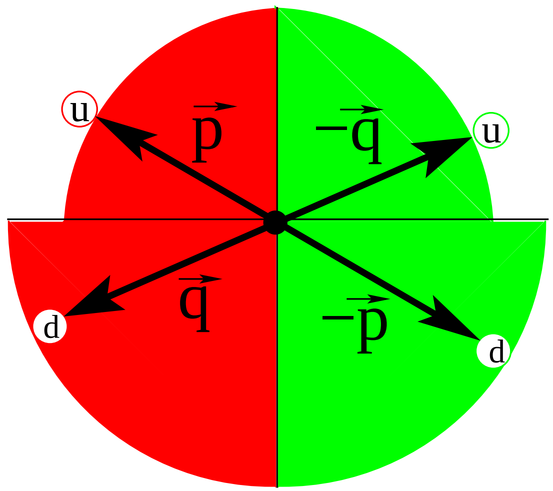
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter appears when $n_d \approx 2n_u$
- Electrons, required in β equilibrium, **cannot** help:

$$n_d \approx 2n_u \quad \text{where} \quad n_d = \frac{\mu_d^3}{\pi^2}, \quad n_u = \frac{\mu_u^3}{\pi^2}$$

$$\text{i.e.,} \quad \mu_d \approx 2^{1/3}\mu_u \;\Rightarrow\; \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u \;\Rightarrow\; n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

Therefore, Cooper pairing is unavoidably distorted by the "mismatch" parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



What happens then?

Gapless 2SC phase

Competition: $\delta\mu$ vs. Δ_0 (where Δ_0 is the gap at $\delta\mu = 0$)

The "winner" is determined by the diquark coupling strength
[Shovkovy&Huang, Phys. Lett. **B 564** (2003) 205]

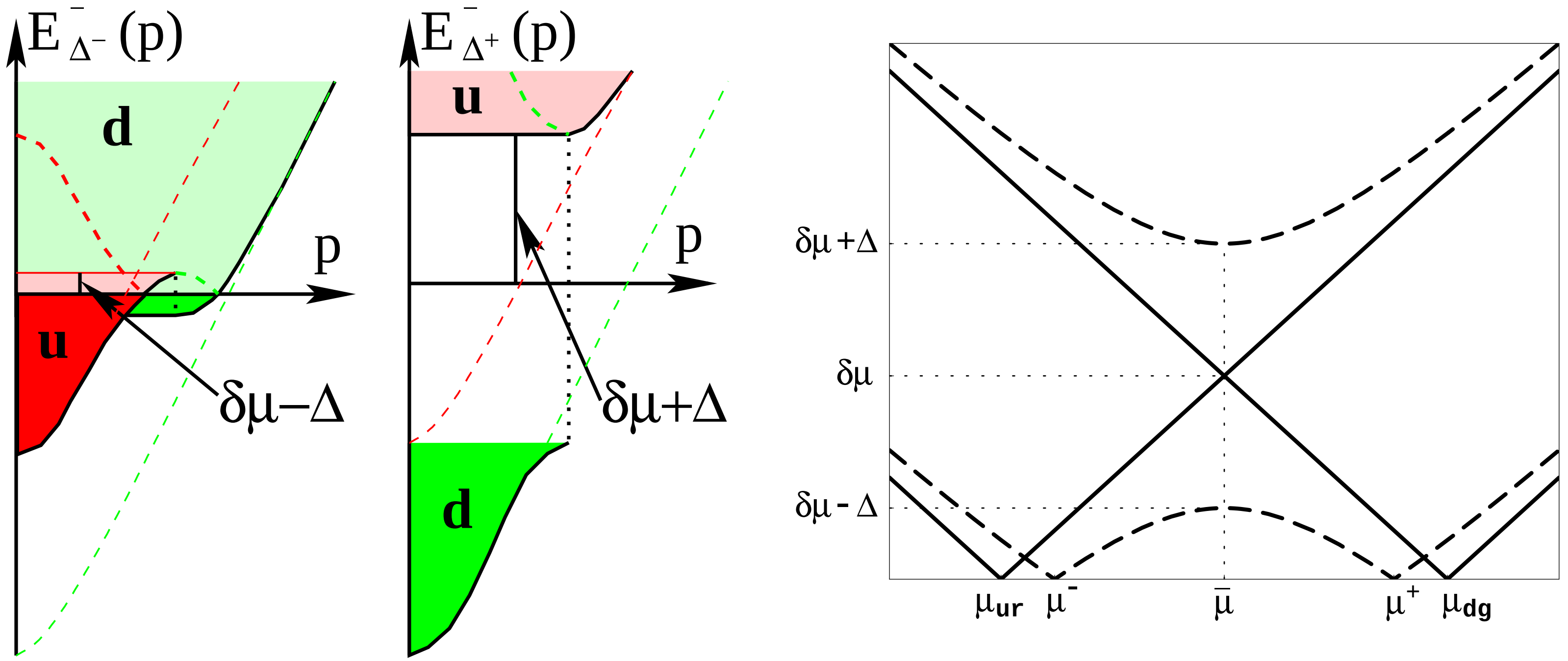
1. $\delta\mu \gtrsim \Delta_0$ — the mismatch does not allow Cooper pairing:
   normal phase is the ground state

2. $\delta\mu \lesssim \frac{1}{2}\Delta_0$ — coupling is strong enough to win over the mismatch:
   2SC is the ground state

3. $\frac{1}{2}\Delta_0 \lesssim \delta\mu \lesssim \Delta_0$ — regime of intermediate coupling strength:
   the ground state is the gapless 2SC phase

Quasiparticle spectrum in g2SC phase

"Intermediate" coupling

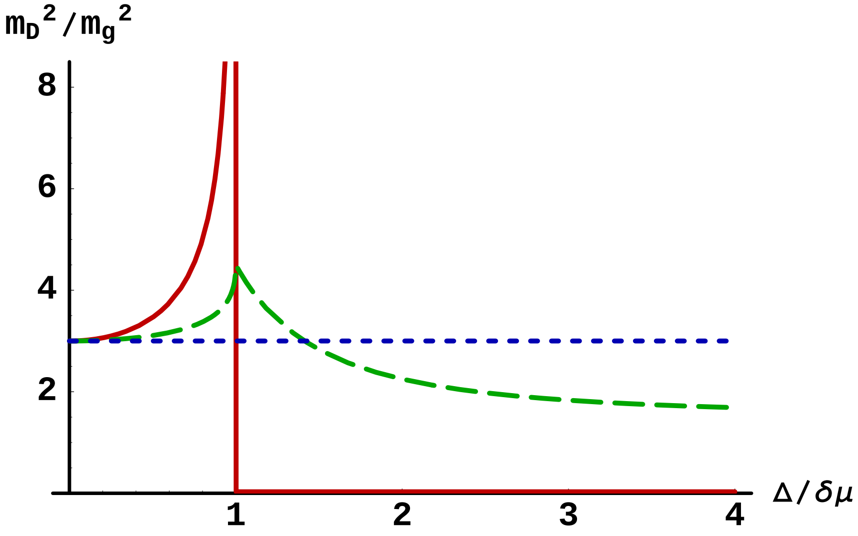


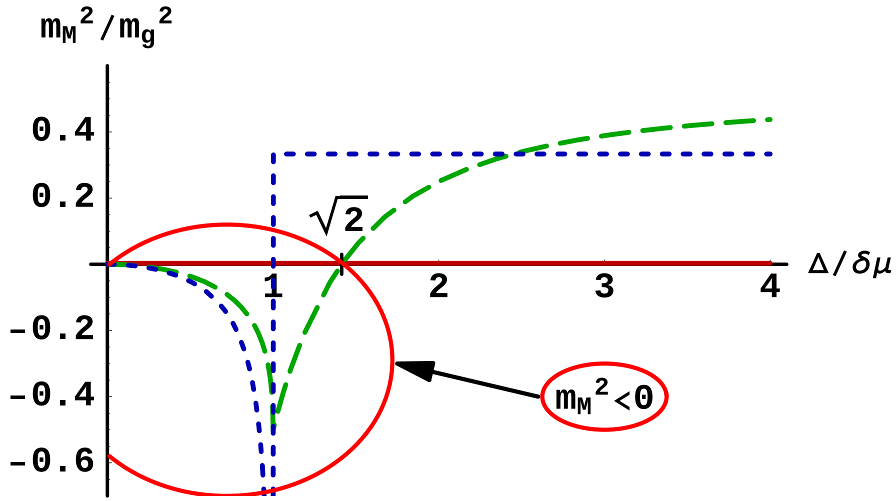
The energy gaps in the quasiparticle spectra are

0 & $\Delta + \delta\mu$

Chromomagnetic instability

Recent results for gluon screening masses
[Huang & Shovkovy, hep-ph/0407049]:





$A = 1, 2, 3$ — red solid line

$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

Origin of instability

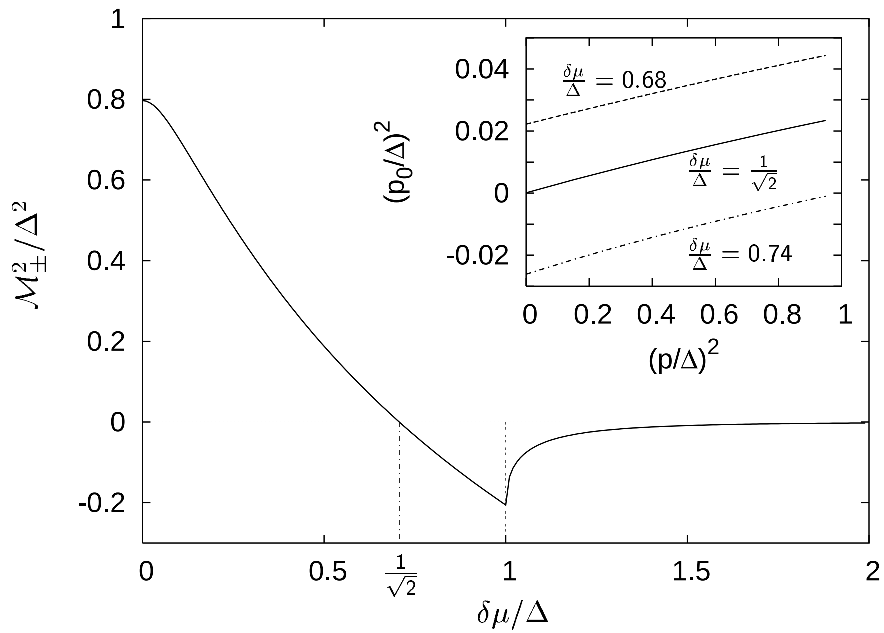
Collective modes with quantum numbers of gluons [Gorbar et al, hep-ph/0602221]

- 4-7th: $p_0^2 = m^2 + v^2 p^2$

  $m^2 > 0$ for $\Delta > \sqrt{2}\delta\mu$

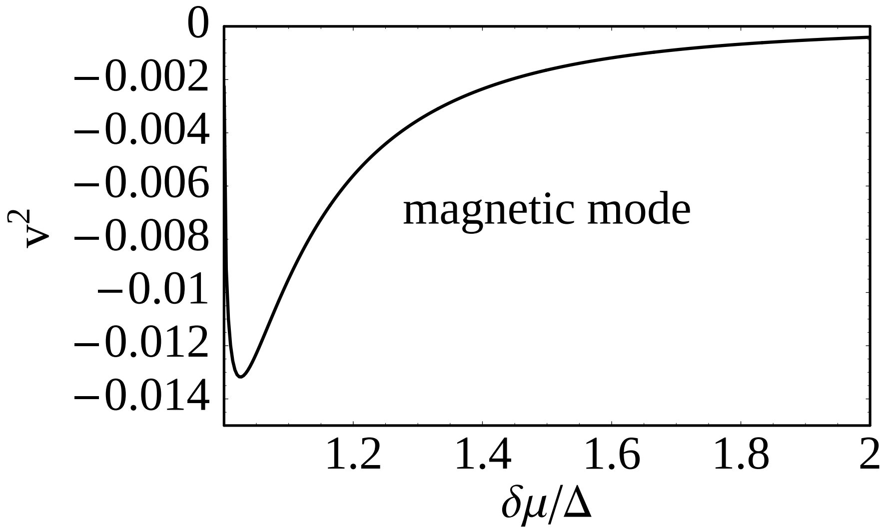
  $m^2 < 0$ for $\Delta < \sqrt{2}\delta\mu$

- 8th: $p_0^2 = v^2 p^2$ with $v^2 < 0$
  appearing only for $\Delta < \delta\mu$



Two types of tachyons → two type of ground states

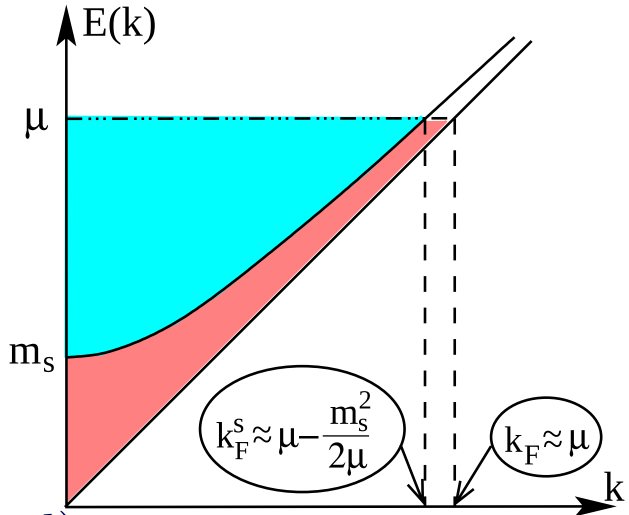
- 4-7th: $\langle A^{(4)} \rangle = \text{const}$
  [Gorbar et al, hep-ph/0507303]
- 8th: $\langle A^{(8)}(x) \rangle \neq \text{const}$
  i.e., more than 1-wave LOFF



$N_f = 2+1$ color superconductivity, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \boxed{\frac{m_s^2}{2\mu}}$$



Then, the ground state is defined by:

(?) only condensates of same flavor (spin-1 channel)

(?) only superconductivity of up and down quarks (2SC or g2SC)

(?) gapless CFL phase ($\oplus$ yet unknown stabilization mechanism)
[Alford, Kouvaris & Rajagopal, hep-ph/0311286]

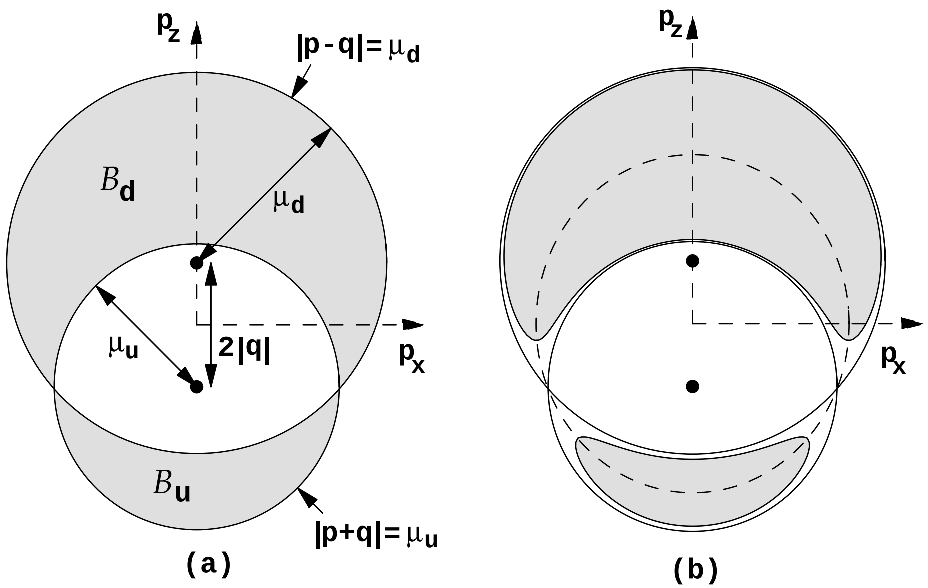
(?) crystalline pairing (nonzero momentum pairing, LOFF)
[Alford, Bowers & Rajagopal, hep-ph/0008208]

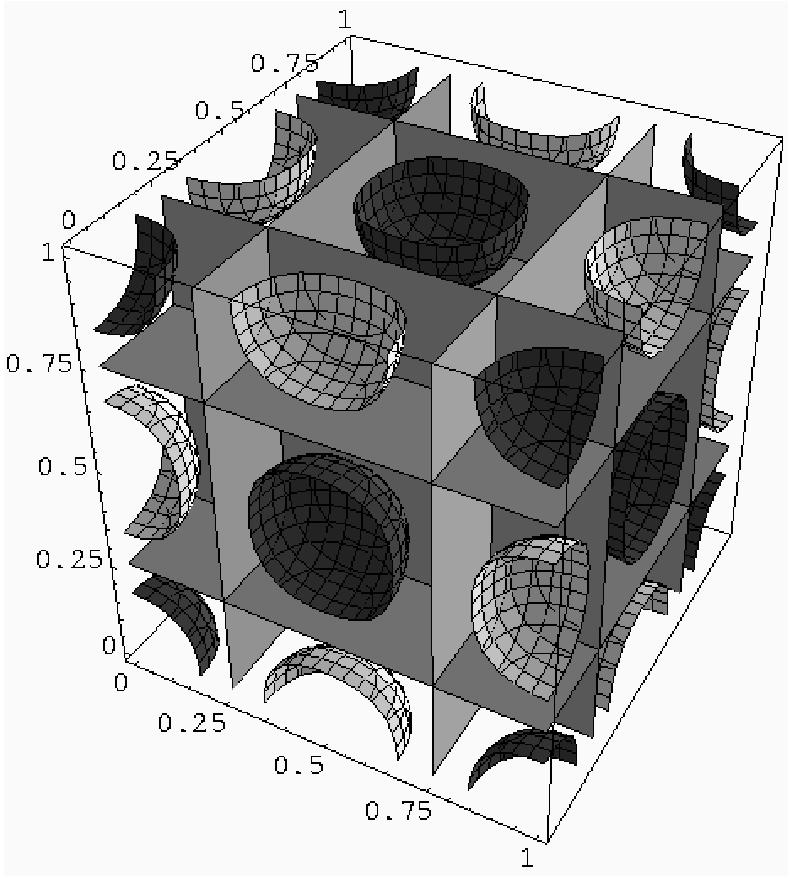
(?) P-wave kaon condensates
[Schafer, hep-ph/0508190]

Crystalline phase (LOFF)

[Alford, Bowers & Rajagopal, hep-ph/0008208]

Cooper pairs with nonzero momenta:



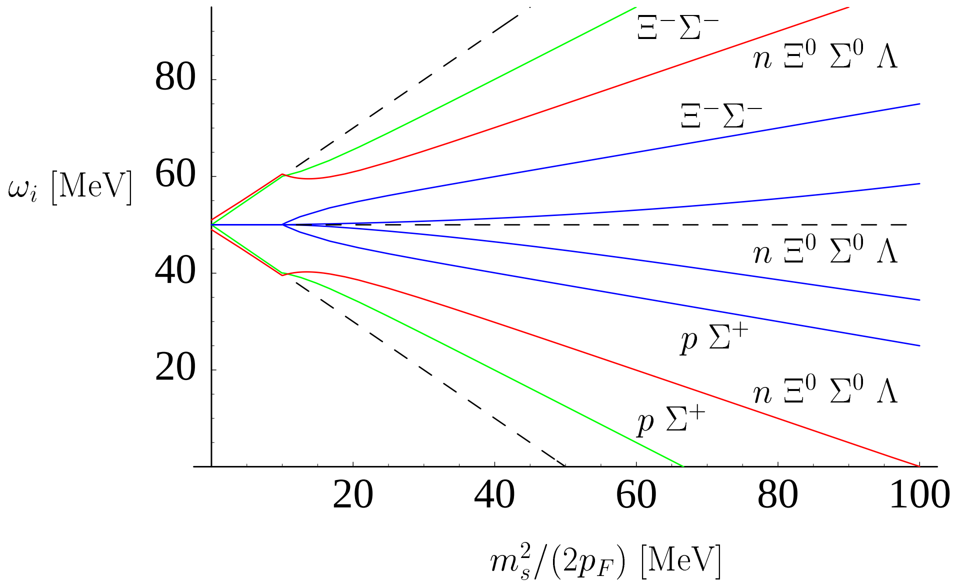


[Bowers, hep-ph/0305301]

P-wave kaon condensation

Baryon spectrum in CFL phase

[Kryjevski & Schafer, hep-ph/0407329]

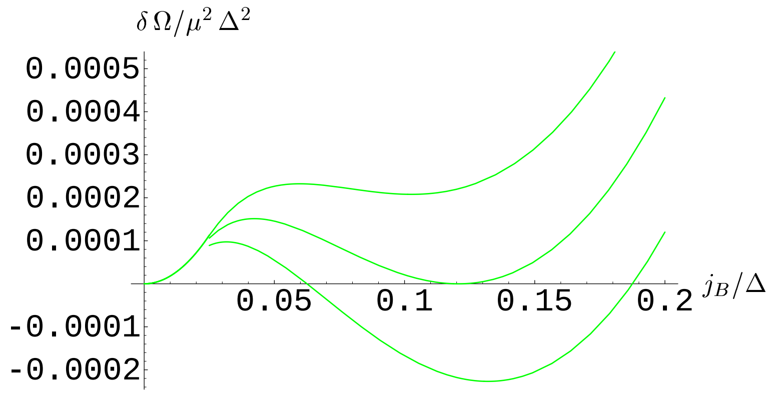


See also

[Son & Stephanov, cond-mat/0507586] $\Rightarrow$

Within the framework of effective theory:

P-wave meson condensation



[Kryjevski, hep-ph/0508180],

[Schafer, hep-ph/0508190]

$\Downarrow$

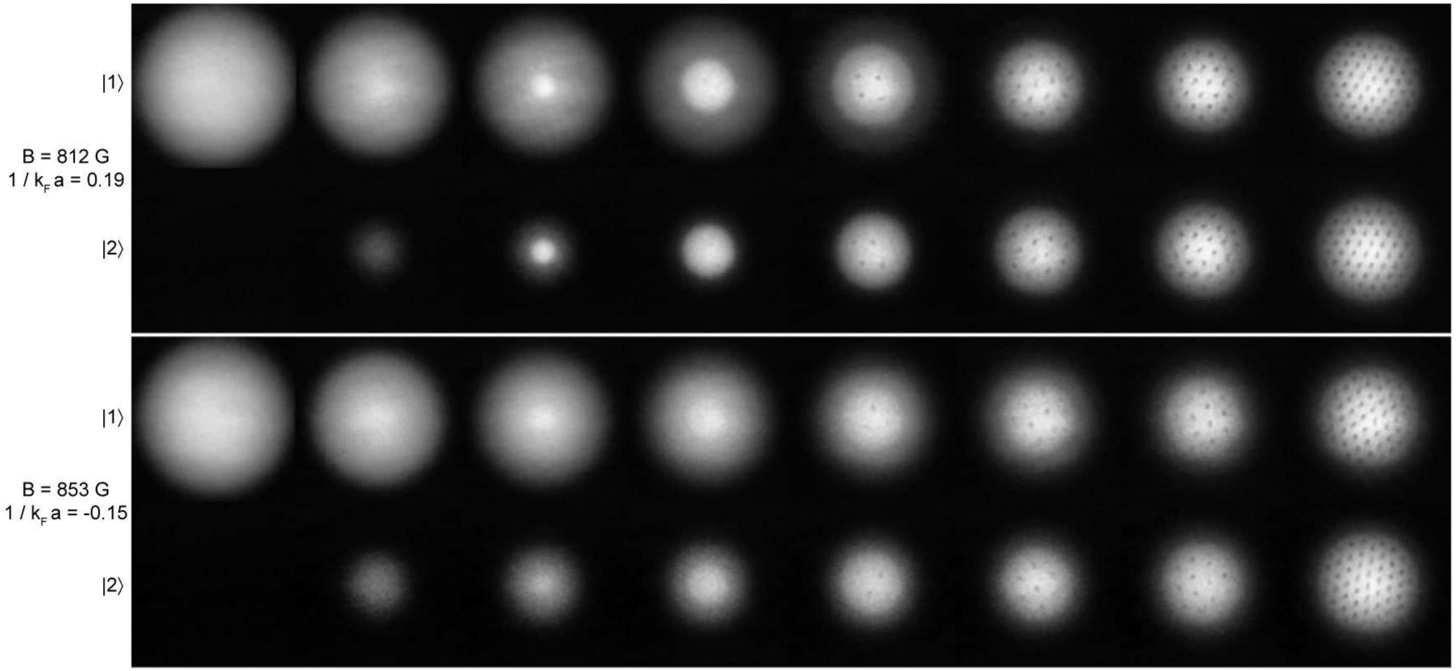
No instabilities

Down to earth . . .

Dense quark matter may be "modeled" in a tabletop experiment

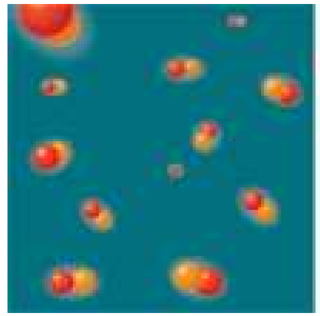
by studying trapped cold gases of fermionic atoms
(e.g., ^{6}Li or ^{40}K)

First experimental results:

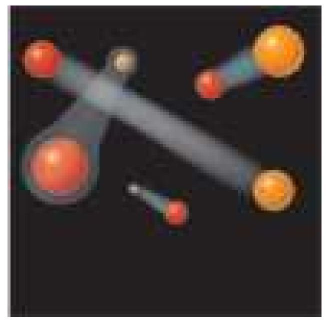


⋆

BEC limit



BCS limit



Zwierlein *et. al.*,, cond-mat/0511197⋆
Partridge *et. al.*,, cond-mat/0511752

Summary

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative
- In this limit, QCD can be studied from first principles
- Under conditions in stars, Cooper pairing is unconventional
- There may exist many different phases in the QCD phase diagram as well as in stars
- Physics of stars and physics of matter around us might be closer related than one might naively expect ...

Many problems remain:

(i) instabilities of gapless phases

(ii) inhomogeneous ground states

(iii) search for observables, etc.

Some reviews on color superconductivity

- K. Rajagopal and F. Wilczek, "The condensed matter physics of QCD" `hep-ph/0011333`
- M. Alford, "Color superconducting quark matter" Ann. Rev. Nucl. Part. Sci. **51**, 131 (2001) `hep-ph/0102047`
- T. Schäfer, "Quark matter" `hep-ph/0304281`
- D. H. Rischke, "The quark-gluon plasma in equilibrium" Prog. Part. Nucl. Phys. **52**, 197 (2004) `nucl-th/0305030`
- M. Buballa, "NJL model analysis of quark matter at large density" Phys. Rept. **407**, 205 (2005) `hep-ph/0402234`
- I. A. Shovkovy, "Two lectures on color superconductivity" Found. Phys. **35**, 1309 (2005) `nucl-th/0410091`