Magnetization of color-flavor-locked matter*

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*J.L. Noronha and I.A. Shovkovy, Phys. Rev. D 76, 105030 (2007), arXiv:0708.0307

Motivation

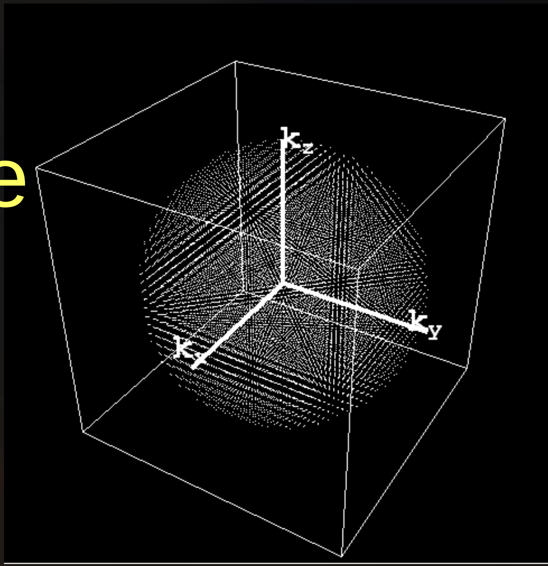
- Very dense (possibly, *deconfined*) baryonic matter exists inside neutron stars, $\rho \lesssim 10\rho_0$
- Neutron stars have rather large magnetic fields, i.e.,
  - Usual pulsars: $B_{\rm surf} \lesssim 10^{12}$ G
  - Magnetars: $B_{\rm surf} \lesssim 10^{15}$ G
- Upper limit for the field in the core: $B \lesssim 10^{18}$ G
- Note:

$$\sqrt{eB} \simeq 7.69 \times 10^{-8} \sqrt{B/1\mathrm{G}}\ \mathrm{MeV}$$

Neutron star

Color superconductivity

- At large density, quarks occupy all states within the Fermi sphere
- Attractive interaction and high degeneracy at Fermi sphere lead to *Cooper instability*



- Ground state is *color superconducting*, e.g., characterized by the following condensates

$$\left\langle \psi_{L,i}^{a,\alpha} \epsilon_{\alpha\beta} \psi_{L,j}^{b,\beta} \right\rangle = -\left\langle \psi_{R,i}^{a,\dot{\alpha}} \epsilon_{\dot{\alpha}\dot{\beta}} \psi_{R,j}^{b,\dot{\beta}} \right\rangle \sim \sum_{I=1}^{3} \varepsilon_{ijI} \epsilon^{abI} + \cdots$$

Symmetry of CFL ground state

- Global symmetries:
  - CFL

$$\underbrace{SU(3)_L}_{\mathbf{8}} \otimes \underbrace{SU(3)_R}_{\mathbf{8}} \otimes \underbrace{U(1)_B}_{\mathbf{1}} \otimes \underbrace{U(1)_A}_{\mathbf{1}} \Rightarrow \underbrace{SU(3)_{R+L+C}}_{\mathbf{8}}$$

  - CFL in a magnetic field (u versus d & s)

$$\underbrace{SU(2)_L}_{\mathbf{3}} \otimes \underbrace{SU(2)_R}_{\mathbf{3}} \otimes \underbrace{U(1)_B}_{\mathbf{1}} \otimes \underbrace{U(1)_A}_{\mathbf{1}} \otimes \underbrace{U(1)_A^-}_{\mathbf{1}} \Rightarrow \underbrace{SU(2)_{R+L+C}}_{\mathbf{3}}$$

- Gauge symmetry in both cases:

$$SU(3)_C \otimes U(1)_{\rm em} \Rightarrow \tilde{U}(1)_{\rm em}$$

In-medium electromagnetism

- The generator of $\tilde{U}(1)_{\rm em}$ is

$$\tilde{Q} = Q_f \otimes \mathbb{1}_c - \mathbb{1}_f \otimes Q_c$$

where

$$Q_c = -\lambda_8/\sqrt{3} = \mathrm{diag}(-1/3, -1/3, 2/3)$$

and

$$Q_f = \mathrm{diag}(-1/3, -1/3, 2/3)$$

- $\tilde{Q}$ charges of quarks are

s_b	s_g	s_r	d_b	d_g	d_r	u_b	u_g	u_r
0	0	-1	0	0	-1	+1	+1	0

Model

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + e\widetilde{Q}A\!\!\!/ + \mu\gamma_0)\psi + \sum_{\eta=1}^{3}\frac{G}{4}(\bar{\psi}P_\eta\psi_c)(\bar{\psi}_c\bar{P}_\eta\psi)$$

where $(P_\eta)^{ab}_{\alpha\beta} = i\gamma_5\,\epsilon^{ab\eta}\epsilon_{\alpha\beta\eta}$

Hubbard-Stratonovich transformation:

$$\frac{G}{4}(\bar{\psi}P_\eta\psi_c)(\bar{\psi}_c\bar{P}_\eta\psi) \to \frac{\phi_\eta}{2}(\bar{\psi}_c\bar{P}_\eta\psi) + \frac{\phi^*_\eta}{2}(\bar{\psi}P_\eta\psi_c) - \frac{|\phi_\eta|^2}{G}$$

In general, $\phi_\eta = \Delta_\eta + \widetilde{\phi}_\eta$

Mean field approximation

Pairing and gaps

Because of the $SU(2)$ flavor symmetry (d ⇔ s)

$$\underbrace{\Delta_1}_{ud} = \underbrace{\Delta_2}_{us} = \Delta \quad \text{and} \quad \underbrace{\Delta_3}_{sd} = \phi$$

Note that $\mathbf{m_s=0}$ is used in this study

Some small effects in weak magnetic fields are neglected

Fukushima & Warringa, Phys. Rev. Lett. **100**, 032007 (2008)

Charge neutrality is **not** enforced

This becomes an issue only in ultra-strong magnetic fields

Fukushima & Warringa, Phys. Rev. Lett. **100**, 032007 (2008)

Gibbs free energy

$$\mathcal{G} = \frac{B^2}{8\pi} - \frac{HB}{4\pi} + \mathcal{F} - \mathcal{F}_{\rm vac}$$

where

$$\mathcal{F} = \frac{2\Delta^2}{G} + \frac{\phi^2}{G} - \Gamma(T,\mu,\Delta,\phi,B)$$

and

$$\Gamma(T,\mu,\Delta,\phi,B) = \frac{1}{2}\ln\det\mathcal{S}^{-1}$$

$$\Gamma = \underbrace{3\,\Gamma_0(\phi) + \Gamma_0(\Delta_1) + \Gamma_0(\Delta_2)}_{\text{3+1+1 neutral quasi-quarks}} + \underbrace{4\,\Gamma_B(\Delta)}_{\text{2+2 charged quasi-quarks}}$$

where $\Delta_{1/2} = \frac{1}{2}\left(\sqrt{\phi^2 + 8\Delta^2} \pm \phi\right)$

Gap equations and magnetization

$$\Delta = \frac{G}{4}\left(\frac{\partial\Gamma}{\partial\Delta}\right)$$

$$\phi = \frac{G}{2}\left(\frac{\partial\Gamma}{\partial\phi}\right)$$

$$B = H + 4\pi M$$

$$M = (\partial\Gamma/\partial B)|_{\text{extremum}}$$

Note: thermodynamic stability requires

$(\partial H/\partial B)_\mu > 0$ and $\vec{H}\cdot\vec{B} > 0$

Model parameters and regularization

$\mu = 500$ MeV

$h_\Lambda = \exp(-\varepsilon^2/\Lambda^2)$ with $\Lambda = 1$ GeV

e.g.,

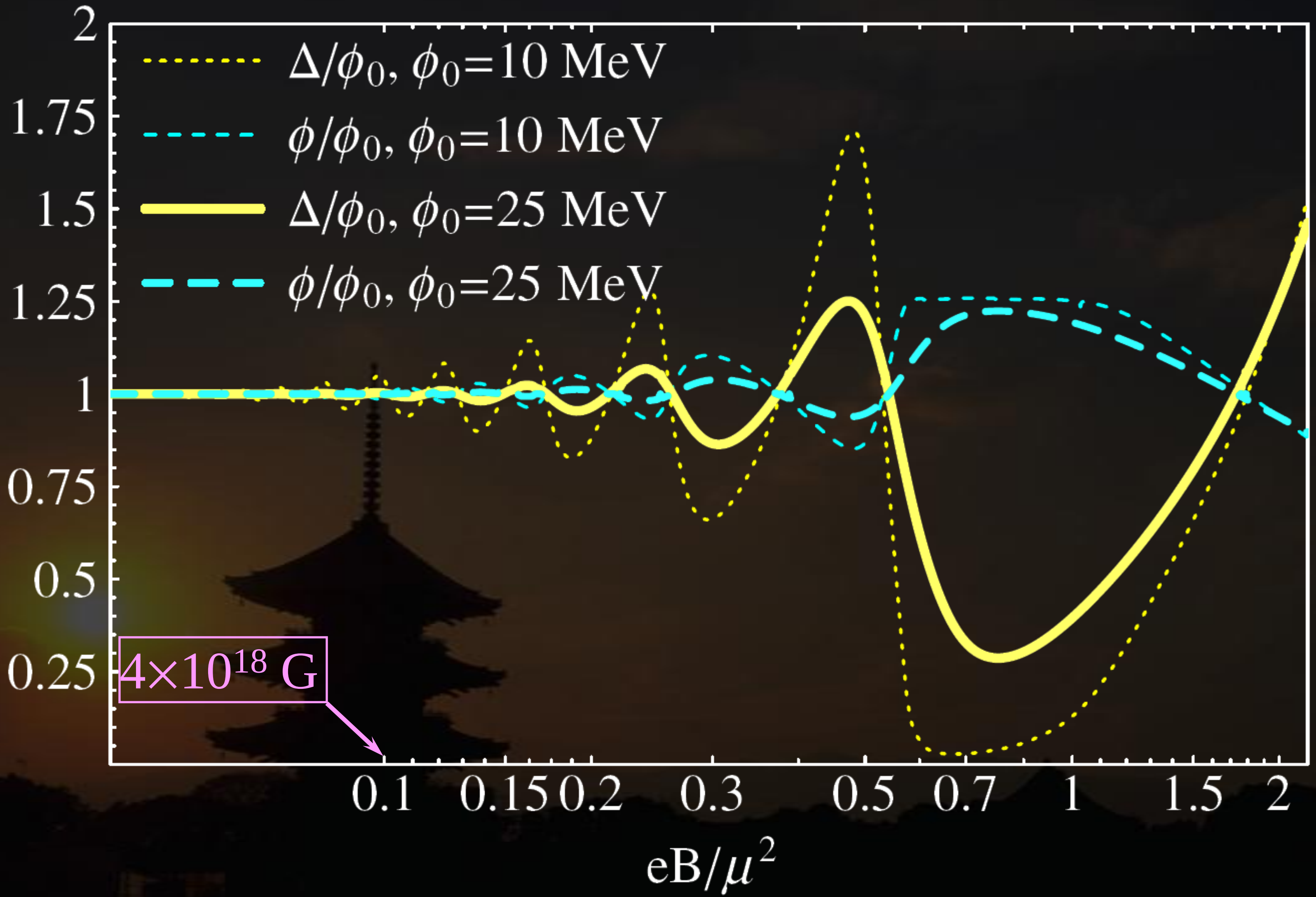
$$\Gamma_0(\phi) = \int \frac{d^3\vec{p}}{(2\pi)^3} h_\Lambda \left[E_0^+(\phi) + E_0^-(\phi)\right]$$

Coupling constants

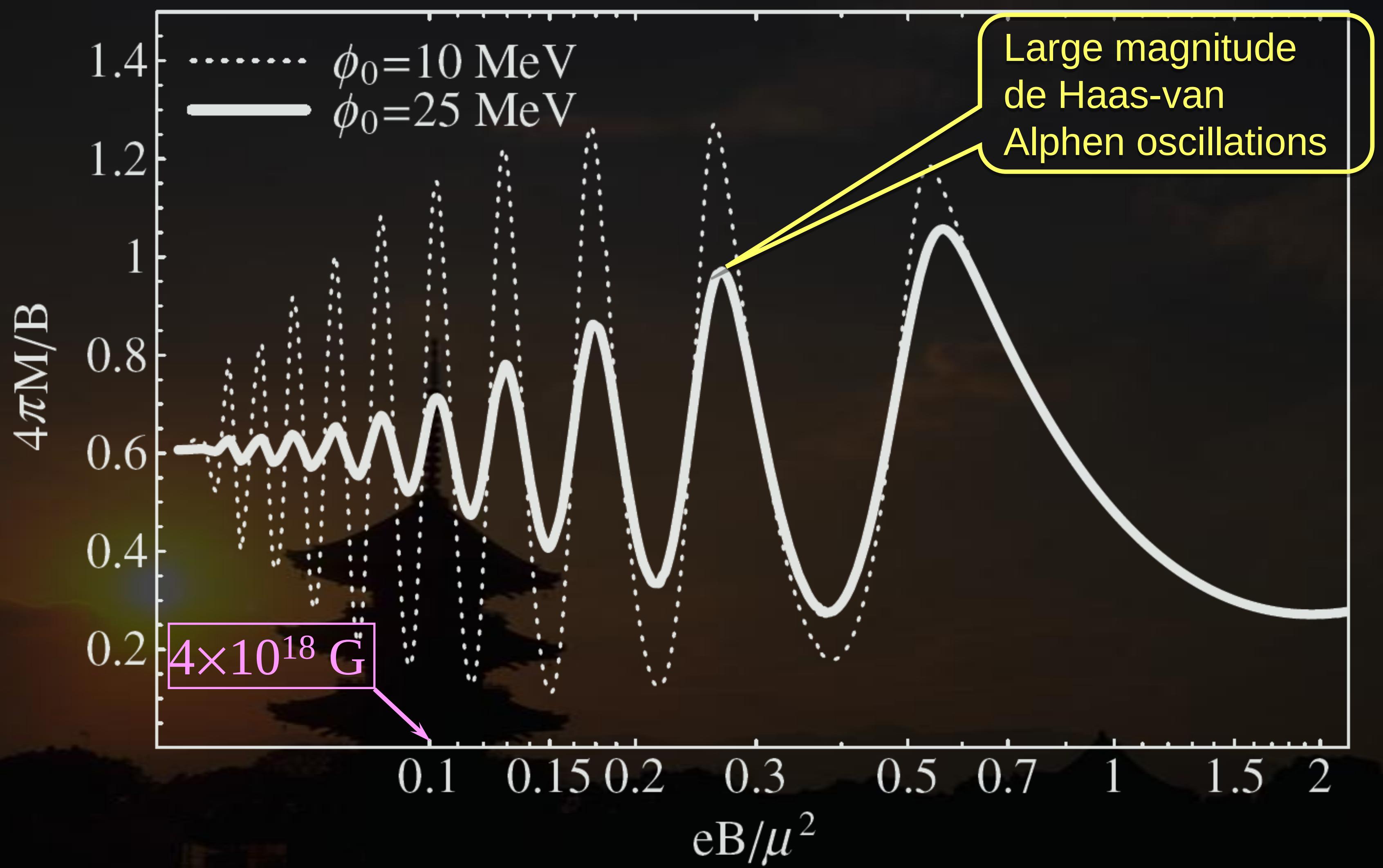
Set I: $\phi_0 = 10$ MeV, $G = 4.32$ GeV^{-2}

Set II: $\phi_0 = 25$ MeV, $G = 5.15$ GeV^{-2}

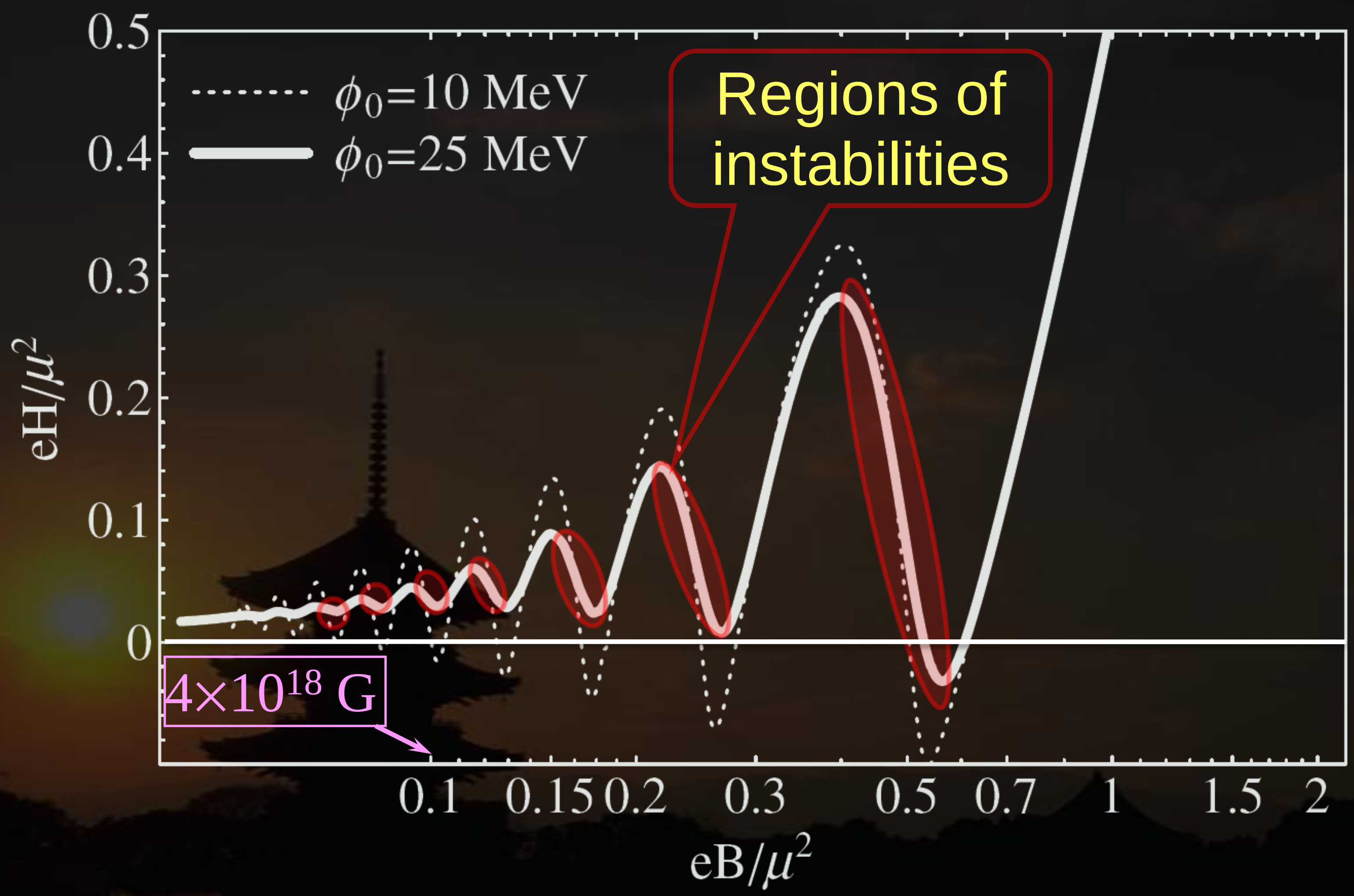
Results for the gaps



Magnetization $M = (\partial\Gamma/\partial B)|_{\text{extremum}}$



H vs. B



0.5
0.4
0.3
0.2
0.1
0
eH/μ^2
ϕ_0=10 MeV
ϕ_0=25 MeV
Regions of instabilities
4×10^18 G
0.1
0.15
0.2
0.3
0.5
0.7
1
1.5
2
eB/μ^2

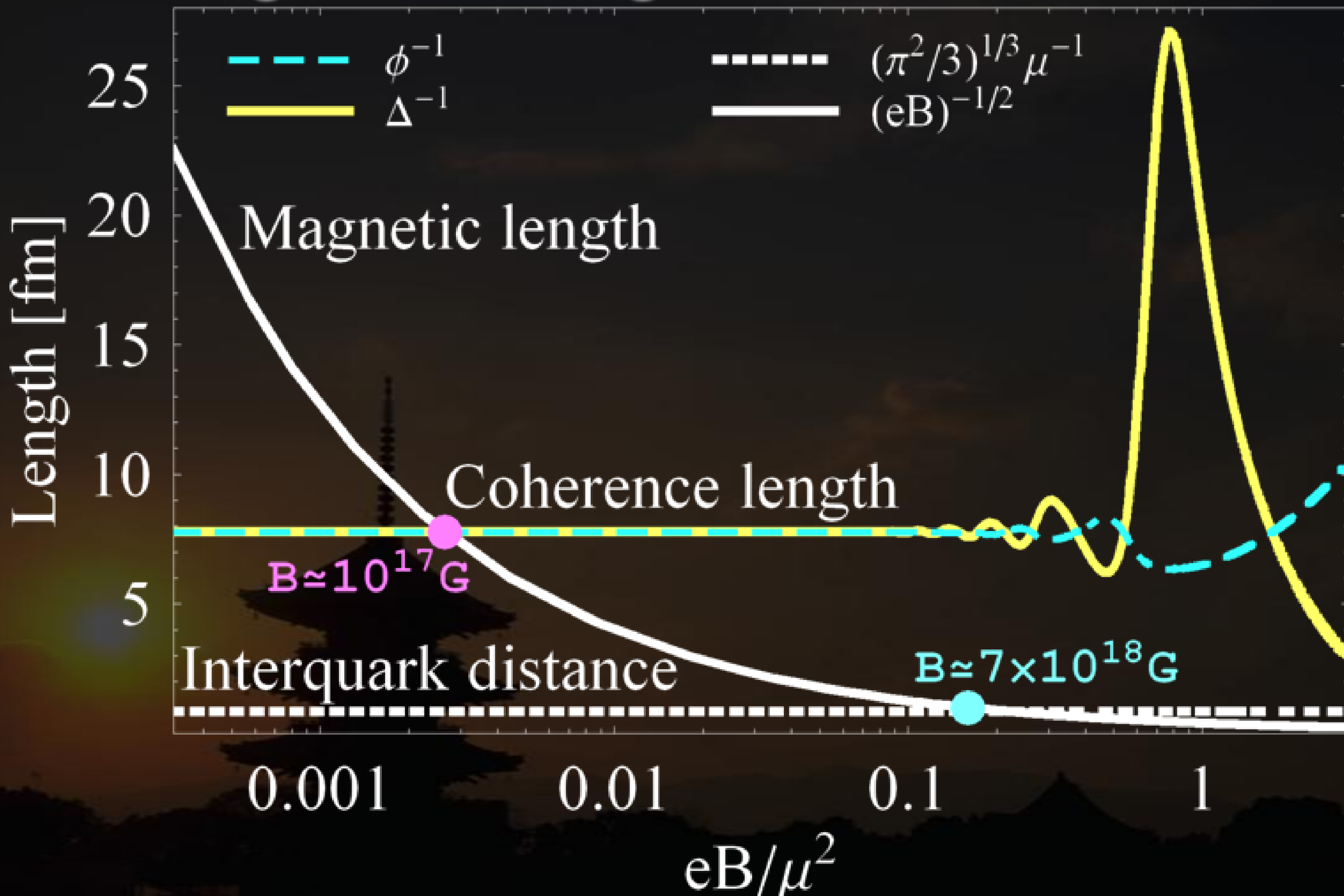
Outcome of instabilities

- *Mixed phases* with domains of non-equal magnetization
- *First order phase transitions* with jumps in the value of the $\boldsymbol{B}$ field and magnetization
- Other types inhomogeneities?

Implications for magnetars…

- Re-arrangement of the magnetic domains with different magnetization could be a source of sudden energy release
- This could be the source of energy bursts in soft gamma ray repeaters (SGR’s)
- This could also lead to random bursts of neutrinos from re-heated regions in old stars

Magnetic length vs. other scales



Open questions

- What is the source of the magnetic field in *CFL* quark matter? (Is it *curCFL* phase?)
- What are the effects of the magnetic field on the Nambu-Goldstone bosons?
- What is the effect of the magnetic field on the mass-radius relation of a star?
- Are there any plasma instabilities in the CFL phase and what is their role?

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Summary

- Magnetic properties of quark matter are potentially of phenomenological interest
- Magnetization reveals strong oscillations in quark (and not in hadronic) matter
- Further studies are needed…