TRANSPORT PROPERTIES OF STELLAR QUARK MATTER

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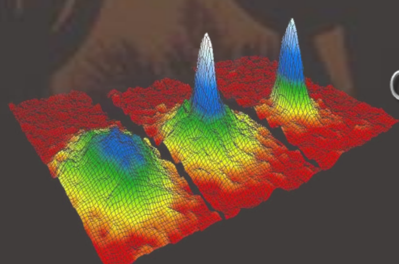
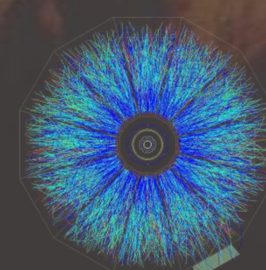


Quark-Gluon Plasma Meets Cold Atoms
September 25-27, 2008
GSI, Darmstadt, Germany



Introduction

- Compact stars as laboratory of extremes
- Stellar quark matter
- Physics properties of stars

Compact stars as a laboratory

- Mass:

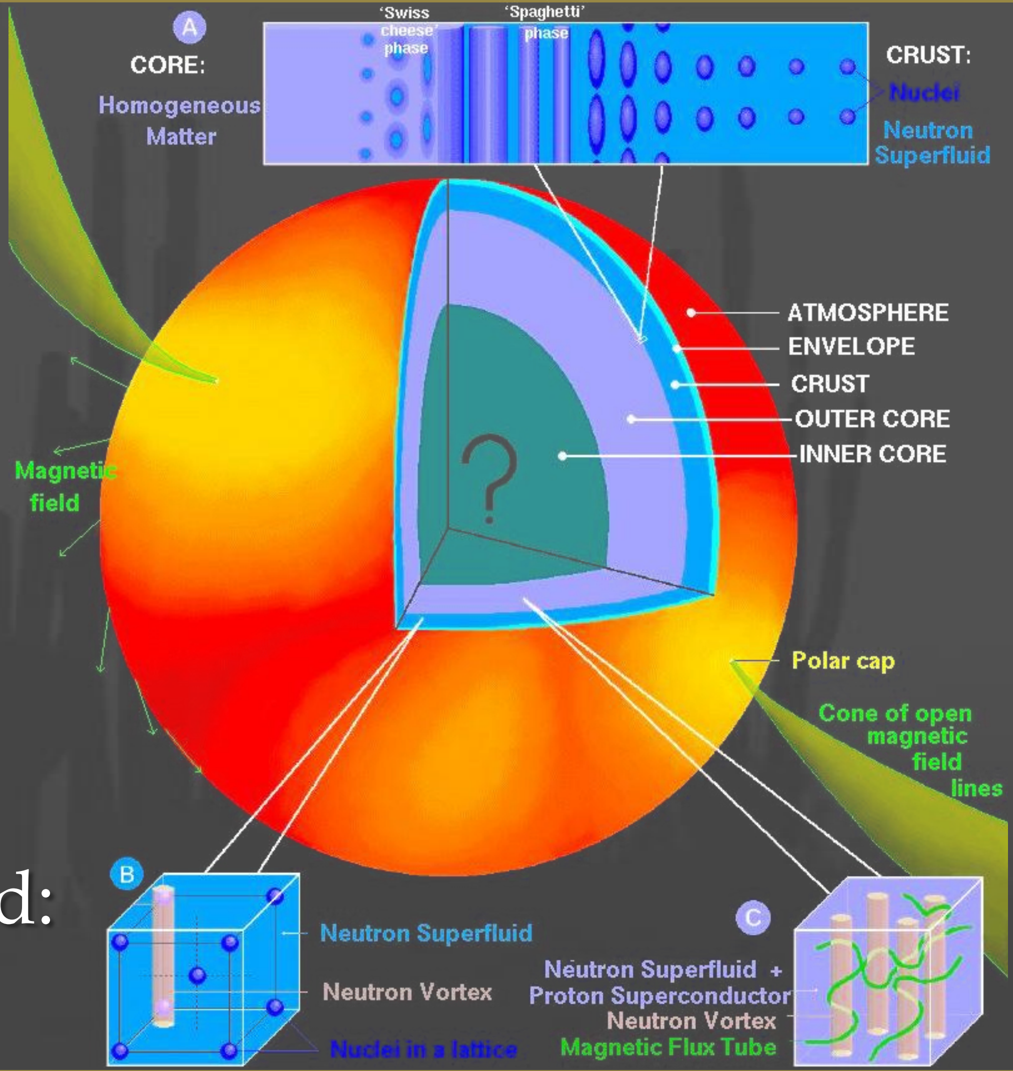
  $1.25M_\odot \lesssim M \lesssim 2M_\odot$
- Radius:

  $R \simeq 10$ km
- Period:

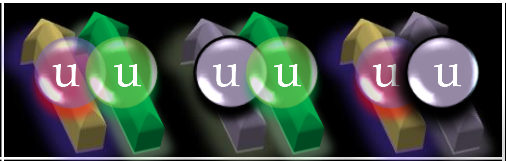
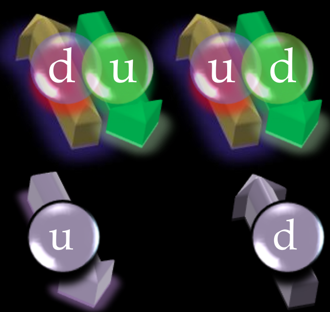
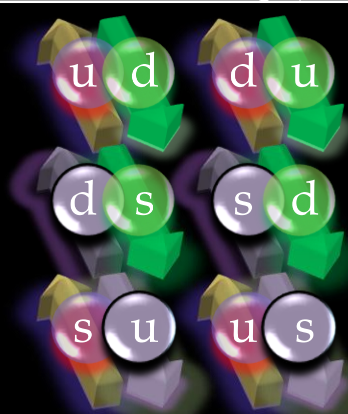
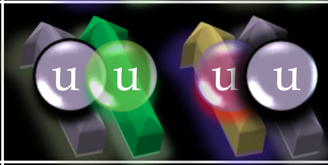
  $1.6\ \text{ms} \lesssim P \lesssim 12\ \text{s}$
- Core temperature:

  $10\ \text{keV} \lesssim T \lesssim 10\ \text{MeV}$
- Surface magnetic field:

  $10^8\ \text{G} \lesssim B \lesssim 10^{14}\ \text{G}$



Many possibilities

1 quark flavor (spin-1) *(e.g., only up)*		2 quark flavors *(e.g., up & down)*	3 quark flavors *(up, down & strange)*
CSL		2SC  + gapless, gluonic, crystalline, …	CFL  + gapless, crystalline, meson condensates, ...
Planar			
A/Polar			
Meissner effect: **Yes**		Meissner effect: **No**	Meissner effect: **No**
Superfluidity: **Yes**		Superfluidity: **No**	Superfluidity: **Yes**

- Which of the many phases may appear in stars?
- How to detect the presence of such phases?

Physical properties of interest

- Consequences of color superconductivity
  - Energy gap in the spectrum of quasiparticles

    (however, there are gapless modes in many phases)
  - Massless or light bosons at low energies

    (but such bosons do not appear in all phase)
  - Possible superfluidity

    (this is not the case in many phases)
  - Possible electromagnetic Meissner effect

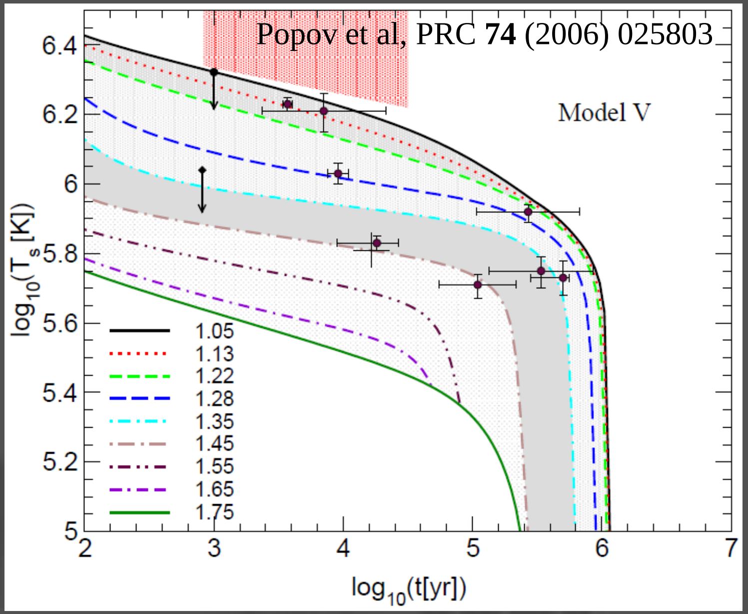
    (this is not the case in many phases)
- The bottom line: There are no universal footprints

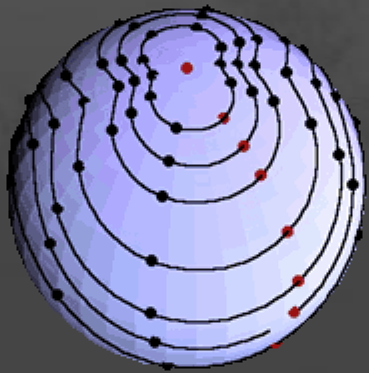
Possible avenues to explore…

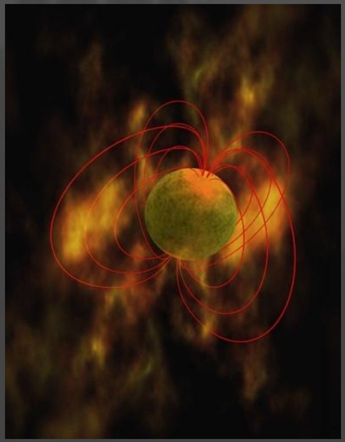
- Thermodynamics
  - Equation of state
- Magnetic properties
  - Induced/spontaneous magnetization
  - Magnetic field decay
- Transport
  - Heat/electric conductivities
  - Bulk/shear viscosities
- Emissivities/Opacities
  - Neutrino emission and mean free path
  - Rates of non-leptonic processes

Observational data as a tool

- Neutron star cooling
- Stellar glitches
- Gravitational waves & r-mode instability (figure by B. J. Owen)
- Magnetic properties
- Transient signals from protoneutron stars
- Mass, radius, velocity, etc.





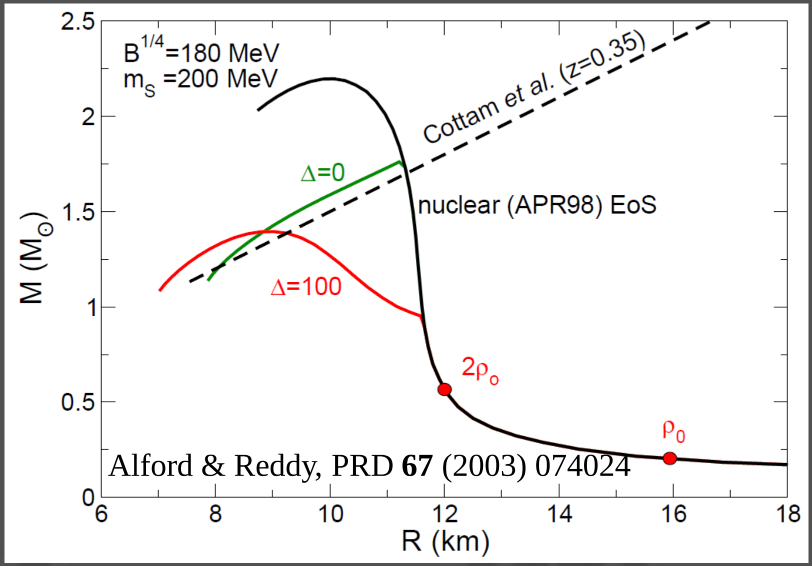


Thermodynamics

- Equation of state
- Specific heat
- Magnetization

Equation of state

- Pressure vs. energy density
  - Mass-radius relation
  - Maximum pulsar mass
  - Stellar compactness



[Fraga et al, PRD **63** (2001) 121702]
[Lugones & Horvath, PRD **66** (2002) 074017]
[Alford & Reddy, PRD **67** (2003) 074024]
[Baldo et al, PLB **562** (2003) 163]
[I.S. et al, PRD **67** (2003) 103004]
[Banik & Bandyopadhyay, PRD **67** (2003) 123003]
[Buballa et al, PLB **595** (2004) 36]
[Rüster & Rischke, PRD **69** (2004) 045011]
[Alford et al, Astrophys. J. **629** (2005) 969]
[Blaschke et al, PRC **75** (2007) 065804]
…

$$n_{\text{eff}}^{\text{CFL}} = 3$$

$$n_{\text{eff}}^{\text{2SC}} = 1$$

$$P \simeq \frac{N_c N_f \mu^4}{12\pi^2} - B + n_{\text{eff}} \frac{\mu^2 \Delta^2}{\pi^2} + \mathcal{O}\left(\Delta^4\right)$$

$$\mathcal{E} \simeq \frac{N_c N_f \mu^4}{4\pi^2} + B + n_{\text{eff}} \frac{\mu^2 \Delta^2}{\pi^2} \left(1 + \frac{2\mu}{\Delta}\frac{\partial \Delta}{\partial \mu}\right) + \mathcal{O}\left(\Delta^4\right)$$

Specific heat

- Affecting cooling rate

  [I.S. & Ellis, PRC **66** (2002) 015802]

  [Casalbuoni et al, PLB **575** (2003) 181]

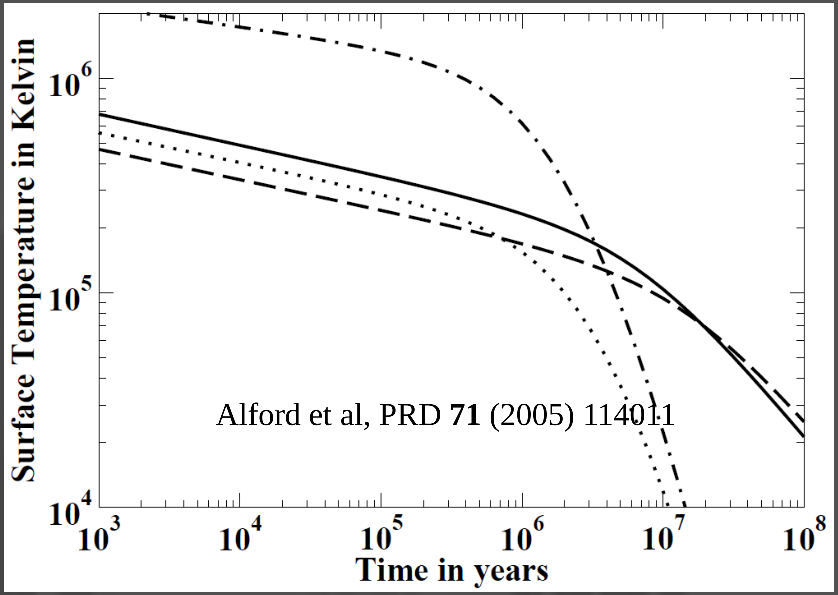
  [Ipp et al, PRD **69** (2004) 011901]

  [Alford et al, PRD **71** (2005) 114011]

  [Anglani et al, PRD **74** (2006) 074005]

  [Schmitt et al, PRD **73** (2006) 034012]

  …



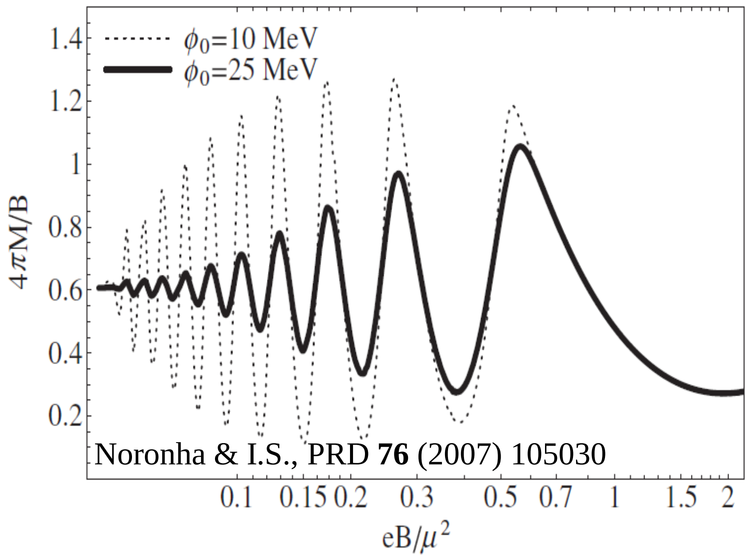
  - CFL (φ-boson): $c_V = \frac{2\pi^2}{15v^3}T^3$
  - 2SC and spin-1 phases (unpaired quarks):

$$c_V = T \sum \mu_f^2 \left[\frac{1}{3} + \frac{2}{3}K(\varphi_f)\right]$$

  - C_V is particularly sensitive to quadratic gapless modes (e.g., as in gCFL phase): $c_V \propto \mu^2\sqrt{\Delta}\sqrt{T}$

Magnetization

- Neutron stars have large magnetic fields, i.e.,
  - Normal pulsars: $B_{surf} \lesssim 10^{12}$ G
  - Magnetars: $B_{surf} \lesssim 10^{15}$ G
- Upper limit for the magnetic field in the core: $B_{core} \lesssim 10^{18}$ G



- Large magnetization effects, instabilities, magnetic domains [Noronha & I.S., PRD **76** (2007) 105030]
- Spontaneous magnetization [Ferrer & Incera, PRD **76** (2007) 114012]
- Domain walls, ferromagnetism [Son & Stephanov, PRD 77 (2008) 014021]

Transport properties

- Thermal conductivity
- Electrical conductivity
- Bulk viscosity
- Shear viscosity
- Neutrino emission
- Neutrino mean free path

Thermal conductivity

- CFL [I.S. & Ellis, PRC **66** (2002) 015802]:
  - κ is dominated by the Nambu-Goldstone boson φ:

$$\kappa = \frac{2\pi^2 T^3}{45 v^2} l_\varphi$$

where $l_\varphi \propto \mu^8/T^9$ is the large angle scattering mean free path

- 2SC (and almost all spin-1 phases):
  - κ is dominated by electrons and unpaired quarks

$$\kappa_e \propto \frac{\mu_e^2}{\alpha} \quad \text{and} \quad \kappa_q \propto \frac{\mu_q^2}{\alpha}$$

where only the in-medium electromagnetism (and no gluon exchange) is taken into account [Heiselberg & Pethick, PRD **48** (1993) 2916]

- Quark matter cores should be almost perfectly isothermal

Electrical conductivity

- CFL [I.S. & Ellis, PRC **66** (2002) 015802]:
  - σ_e is dominated by thermal electron-positron pairs:

$$\sigma_e \sim \frac{2T^{3/2}}{\alpha\sqrt{m_e}L_e} \quad \text{where} \quad L_e \simeq \ln\frac{T}{m_D \max(\alpha, v_e)}$$

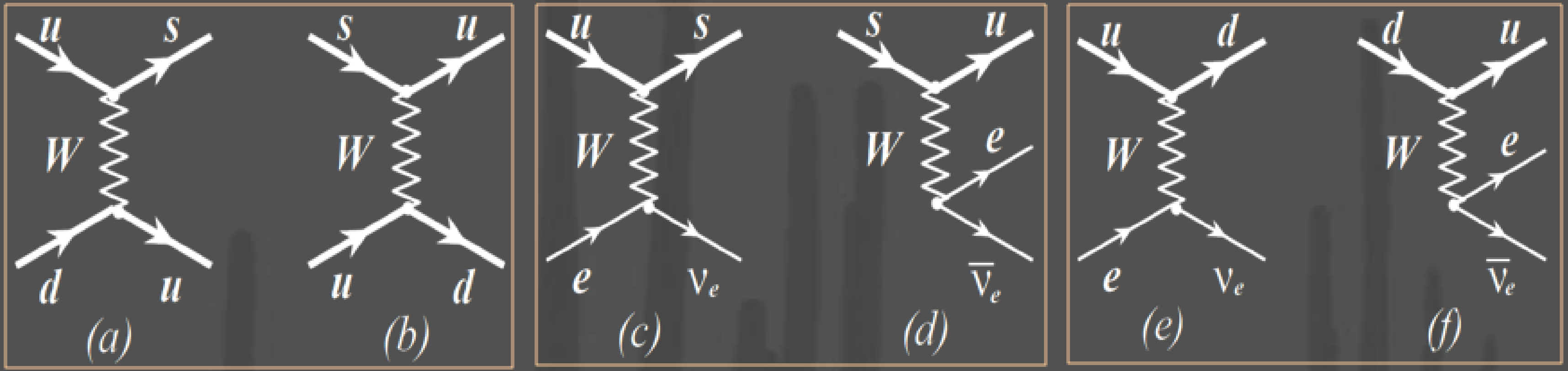
is the Coulomb logarithm and m_D is the electron Debye mass

- 2SC (and almost all spin-1 phases):
  - σ_e is dominated by electrons and unpaired quarks

$$\sigma_e \propto \frac{\mu_e^{8/3}}{\alpha^{2/3}T^{5/3}}$$

where again only the in-medium electromagnetism contributes [Heiselberg & Pethick, PRD 48 (1993) 2916]

Bulk viscosity in quark matter I



$\Gamma_a - \Gamma_b = -\lambda_1 \delta\mu_1$

$\Gamma_c - \Gamma_d = -\lambda_2 \delta\mu_2$

$\Gamma_e - \Gamma_f = -\lambda_3 (\delta\mu_2 - \delta\mu_1)$

$$\lambda_1 \simeq \frac{64}{5\pi^3} G_F^2 \cos^2\theta_C \sin^2\theta_C \mu_d^5 T^2$$

$$\lambda_2 \simeq \frac{17}{40\pi} G_F^2 \sin^2\theta_C \mu_s m_s^2 T^4$$

$$\lambda_3 \simeq \frac{17}{15\pi^2} G_F^2 \cos^2\theta_C \alpha_s \mu_d \mu_u \mu_e T^4$$

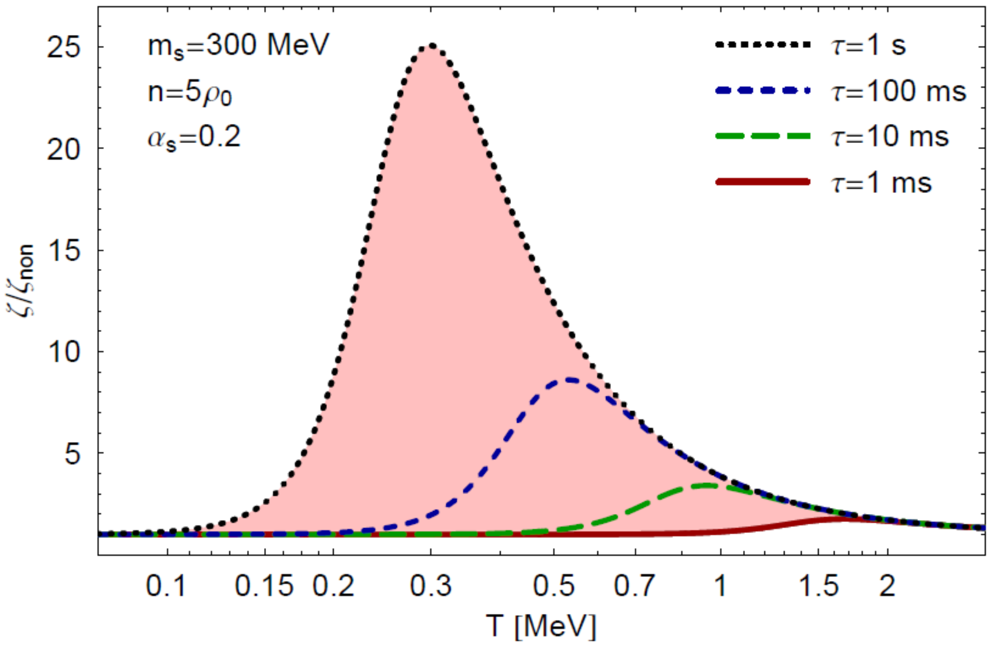
$$\lambda_1 \gg \lambda_2, \lambda_3$$

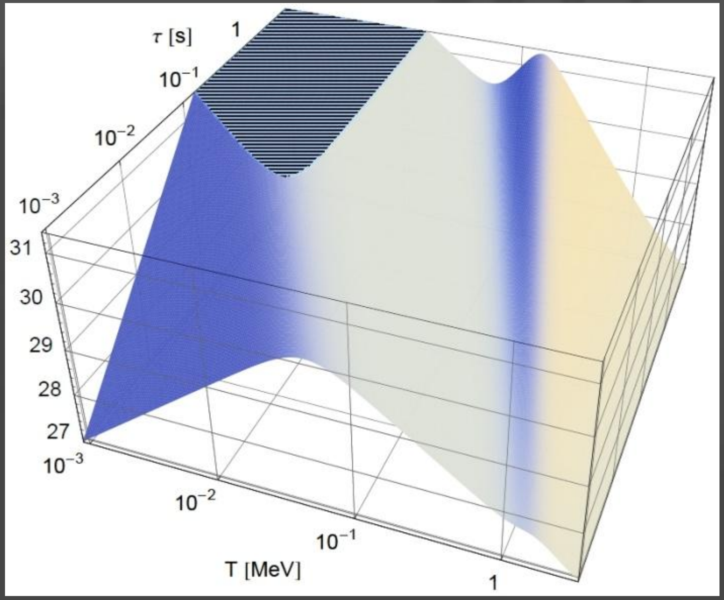
Bulk viscosity in quark matter II

- Non-leptonic contribution: $$\zeta_{\text{non}} \simeq \frac{\lambda_1 C_1^2}{\omega^2 + (\lambda_1 A_1/n)^2}$$

where $A_1 \propto n^{1/3}$ and $C_1 \propto m_s^{\,2}/n^{1/3}$

- Contributions of the semi-leptonic processes are large when $\omega \lesssim \omega_0$ where $\omega_0 = [\lambda_1(\lambda_2+\lambda_3)]^{1/2}/n^{2/3}$





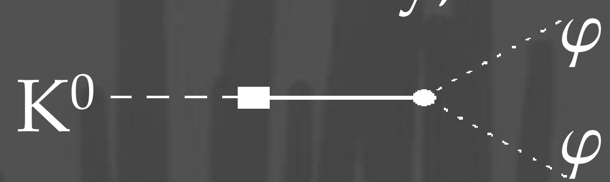
[Sad, I.S., & Rischke, PRD **75** (2007) 125004]

Bulk viscosity (CFL)

- The quasiparticle contributions are suppressed, [Madsen, PRL **85** (2000) 10]

$$\zeta^{\text{CFL}} \simeq \zeta^{\text{normal}} \exp\left(-\frac{2\Delta}{T}\right)$$

- At low temperature, the bulk viscosity is dominated by the kaon decay, $K^0 \to \varphi + \varphi$ [Alford et al, PRC **75** (2007) 055209]



- The effect of kaon condensate [Alford et al, arXiv:0806.0285]

$$\zeta_{\text{CFL}-K^0} \simeq \frac{80\, G_{ds}^2 f_\pi^2 f_H^2}{\pi} \frac{\delta m^2\, T^7}{\omega^2 \mu_q^4}$$

- The contribution due to $\varphi \to \varphi + \varphi$ [Manuel & Llanes-Estrada, JCAP **0708** (2007) 001] :

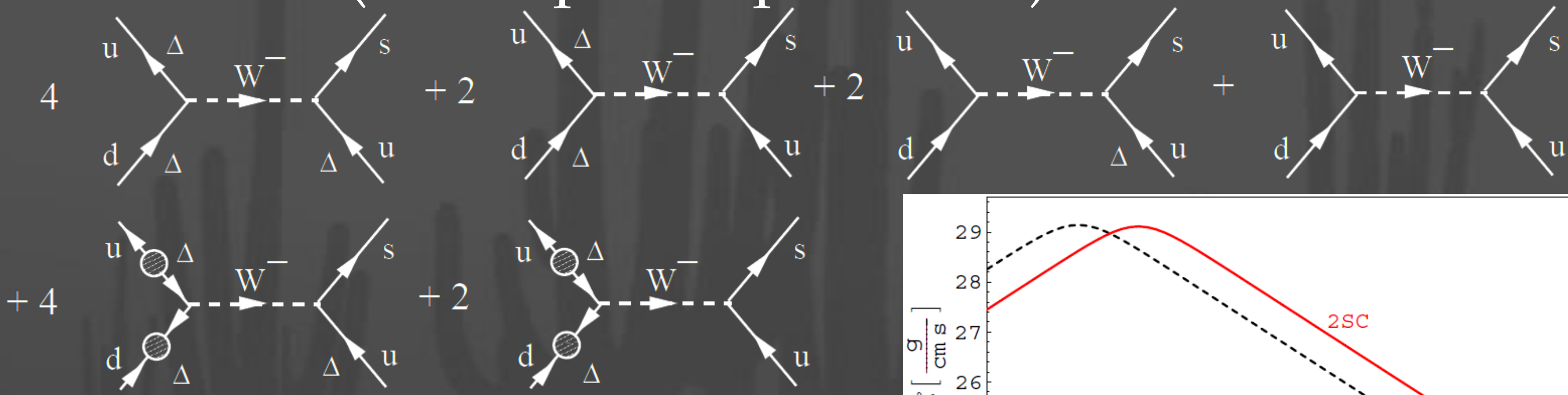
$$\zeta_{\text{CFL}}^{\varphi} = 0.011 \frac{M_s^4}{T}$$

Bulk viscosity (2SC)

- 2SC [Sad, I.S. & Rischke, PRD 75 (2007) 065016]:

$$\zeta_{2SC} = \frac{\lambda_3 C_3^2}{\omega^2 + (\lambda_3 A_3/n)^2} \quad \text{where} \quad \lambda_3 = (1/3)\lambda_{unpaired}$$

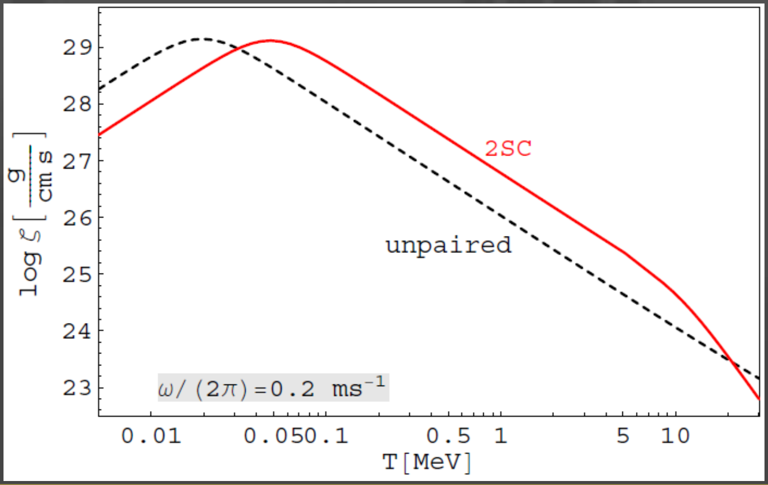
- 2SC+s (non-leptonic processes):



Note: $\lambda_1 = (1/9)\lambda_{unpaired}$

[Alford & Schmitt, J.Phys.G**34** (2007) 67]

[Madsen, PRL **85** (2000) 10]



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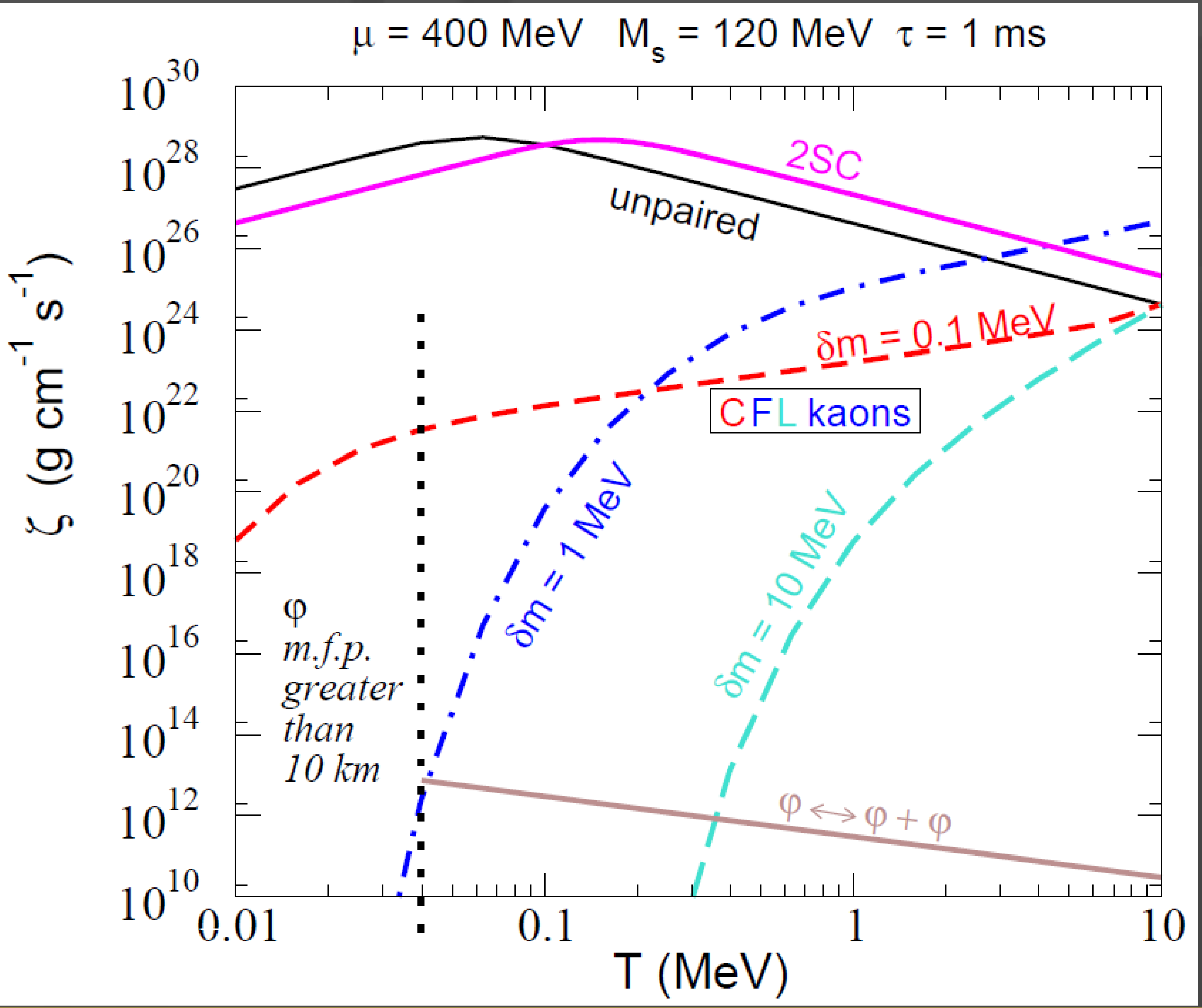
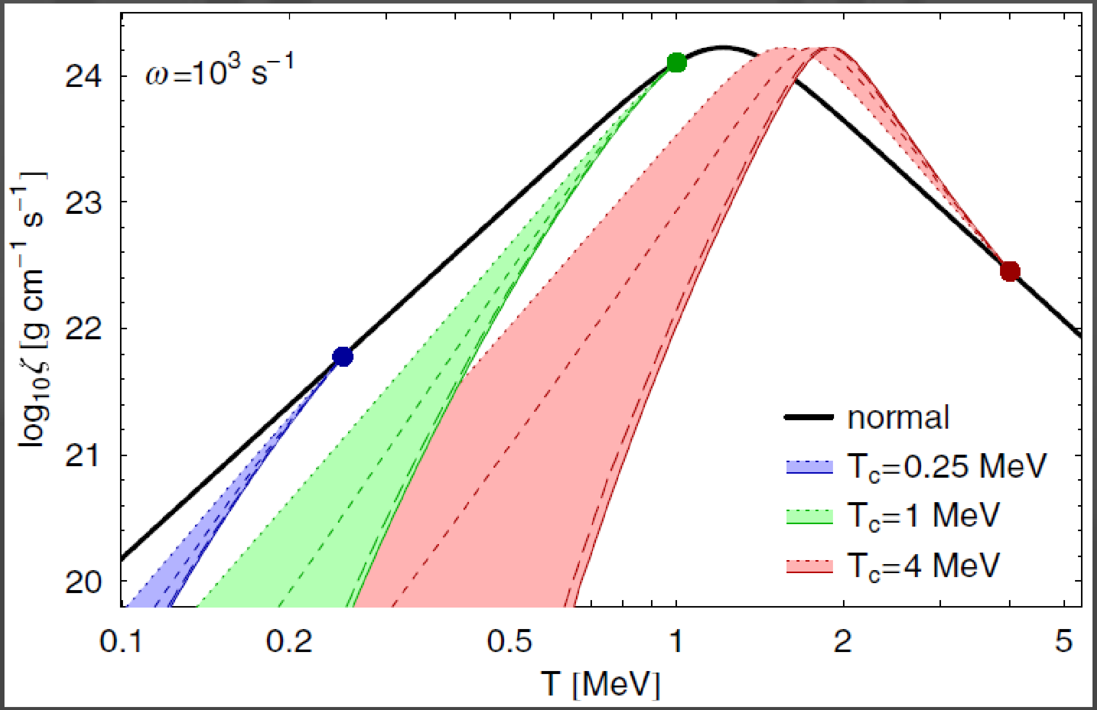
Bulk viscosity: unpaired, CFL, 2SC



[Alford & Schmitt, arXiv:0709.4251]

Bulk viscosity (spin-1 SCS)

- All, but CSL phase, have unpaired quarks
- Then, bulk viscosity is qualitatively the same as in the normal phase [Sad, I.S. & Rischke, PRD 75 (2007) 065016]
- The result in the gapped version of CSL phase



Shear viscosity

- CFL:
  - Dominated by massless φ-bosons
    [Manuel et al, JHEP **09** (2005) 076] :

$$\eta_{CFL} \simeq 1.3 \times 10^{-4} \frac{\mu_e^8}{T^5}$$

- 2SC (and almost all spin-1 phases):
  - Dominated by unpaired quasiparticles:

$$\eta_{2SC} \propto \frac{\mu_e^{14/3}}{\alpha^{2/3} T^{5/3}}$$

where again only the in-medium electromagnetism contributes [Heiselberg & Pethick, PRD **48** (1993) 2916]

Neutrino emission (CFL)

- $T \ll \Delta$: dominated by massless φ-bosons

$$\epsilon \sim \frac{G_F^2}{f_\pi^2 \mu^4} T^{15} \simeq 10^{-11} T_9^{15} \mu_{100}^{-6} \text{ erg cm}^{-3} \text{ s}^{-1}$$

[Jaikumar, Prakash & Schäfer, Phys. Rev. D **66** (2002) 063003]

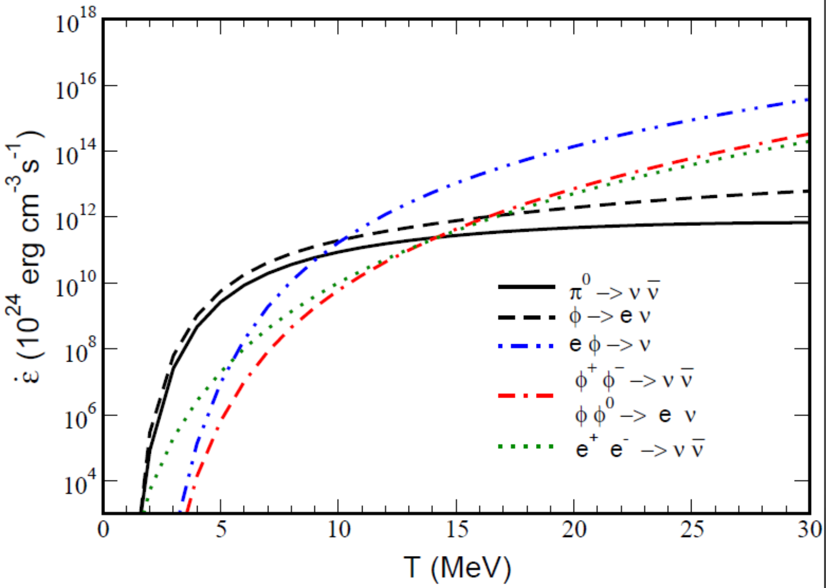
- c.f., typical values at not very low temperatures

$T = 5$ MeV:

$\epsilon \simeq 5 \times 10^{33} \text{ erg cm}^{-3} \text{ s}^{-1}$

$T = 10$ MeV:

$\epsilon \simeq 2 \times 10^{35} \text{ erg cm}^{-3} \text{ s}^{-1}$



[Reddy, Sadzikowski & Tachibana, Nucl. Phys. A **714** (2003) 337]

Neutrino emission (2SC & spin-1)

- Dominated by the direct Urca processes involving unpaired quarks [Iwamoto, PRL **44** (1980) 1637]

$$\epsilon_\nu = \frac{457}{630}\alpha_s G_F^2 T^6 \mu_e \mu_u \mu_d \left[\frac{1}{3} + \frac{2}{3} G(\varphi_u, \varphi_d)\right]$$

[Schmitt, I.S., Wang, Phys. Rev. D **73** (2006) 034012]
[Jaikumar et al, Phys. Rev. C **73** (2006) 042801]

- Cooper pair breaking/recombination processes [Jaikumar & Prakash, Phys. Lett. B **516** (2001) 345]

$$\epsilon_q^{\nu\bar{\nu}} \cong 1.4 \times 10^{20} N_\nu T_9^7 F a_q \left(\frac{n_B}{n_0}\right)^{2/3} \text{erg cm}^{-3}\,\text{s}^{-1}$$

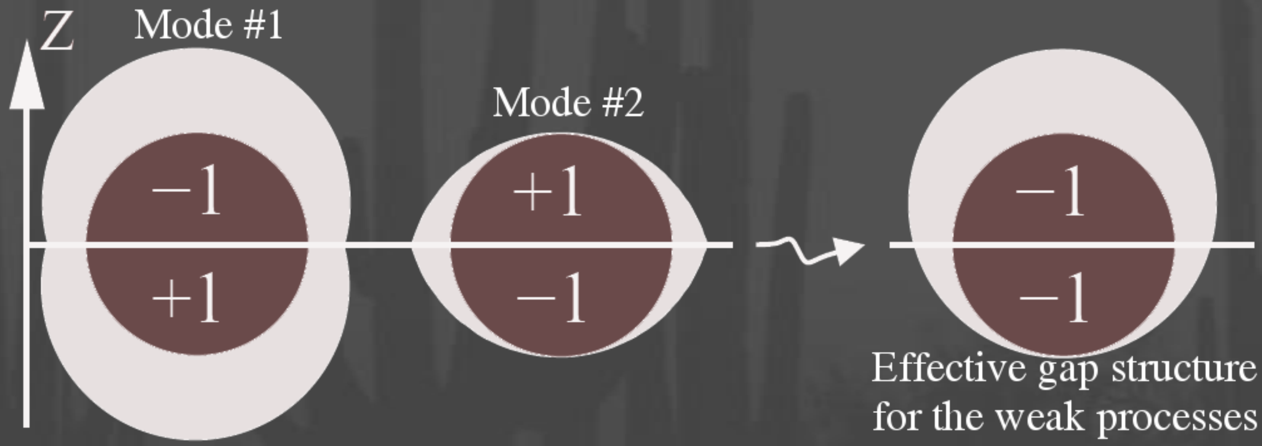
where $F \propto \exp(-2\Delta/T)$ for $T \ll \Delta$
and $F \propto (\Delta/T_c)^2$ for $T \to T_c$

Neutrino mini-rocket

- The emission in A-phase (spin-1 phase) is spatially asymmetric

  [Schmitt, I.S., Wang, Phys. Rev. Lett. **94** (2005) 211101]

  Erratum: [Schmitt, I.S., Wang, Phys. Rev. Lett. **95** (2005) 159902]



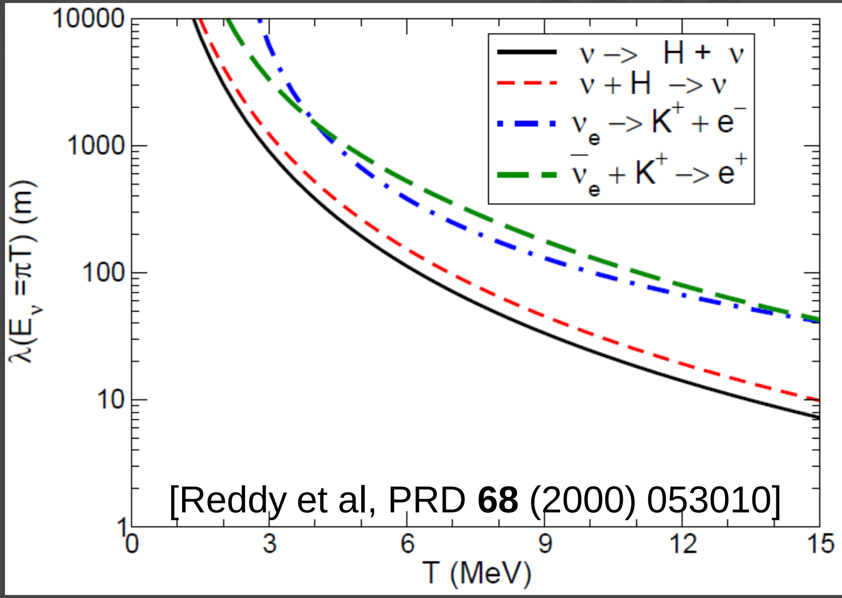
- However, the kick to the pulsar is very small

$$\delta v_{\max} \sim 10^{-4}\ \mathrm{km/s}$$

Neutrino mean free path

- CFL is special because neutrino mean free path may be large even at moderately high *T*
- Thus, it may affect the early evolution of a newly born neutron star
- The mean free path is dominated by Nambu-Goldstone bosons

[Carter & Reddy, PRD **62** (2000) 103002]
[Reddy et al, PRD **68** (2000) 053010]
[Jaikumar et al, PRD **66** (2002) 063003]
[Kundu & Reddy, PRC **70** (2004) 055803]



Outlook

- There is no unique footprint of color superconductivity
- However, many transport properties are strongly affected by color superconductivity
- Transport and thermodynamics may point in favor of quark stars
- Notably, CFL phase has most distinguishable properties

In lieu of summary

If quark stars exist in stars, we'll get a chance to understand the theory of strong interactions in one of its most unusual realizations

If dense quark matter and color superconductivity do not exist in stars, we advance our theoretical techniques and apply them to other degenerate fermionic systems

Arizona State University

Thank you