Graphene: Symmetry breaking in the carbon Flatland*

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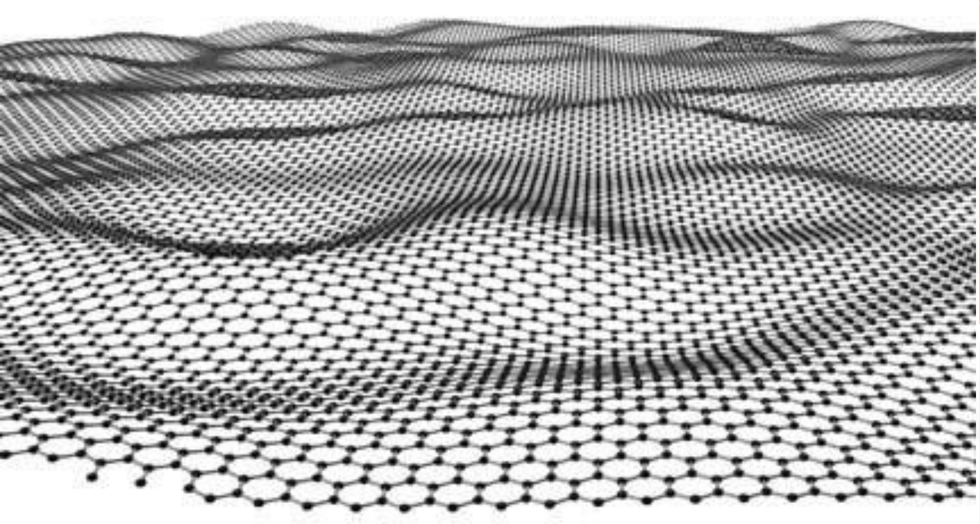
*E. Gorbar, V. Gusynin, V. Miransky, I. Shovkovy, arXiv:0806.0846, Phys. Rev. B 78 (2008), 085437

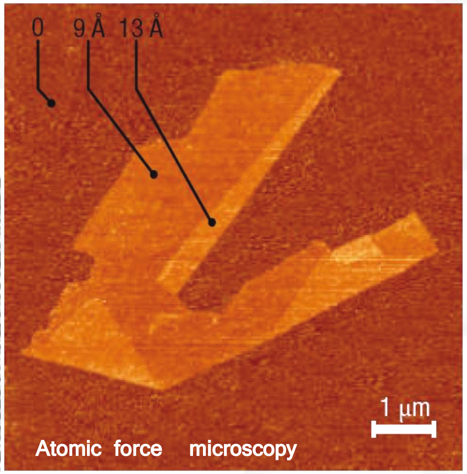
What is graphene?

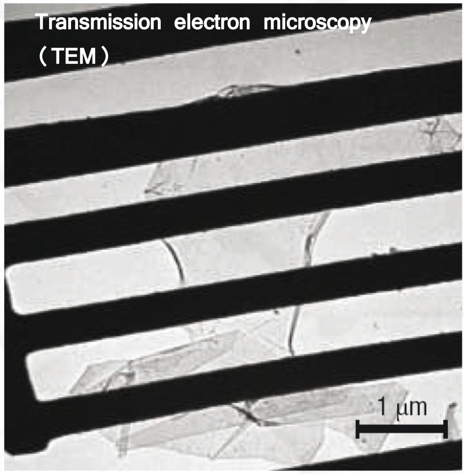
- It is a single atomic layer of graphite, see

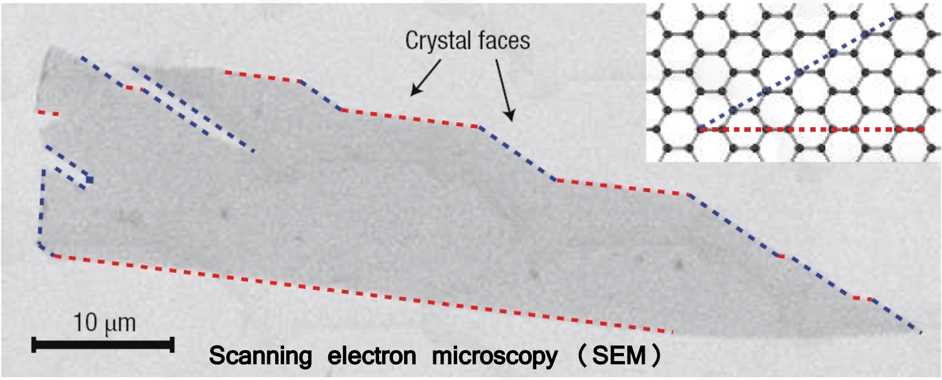
[Novoselov et al., Science 306, **666** (2004)]



2D crystal with hexagonal lattice of carbon atoms

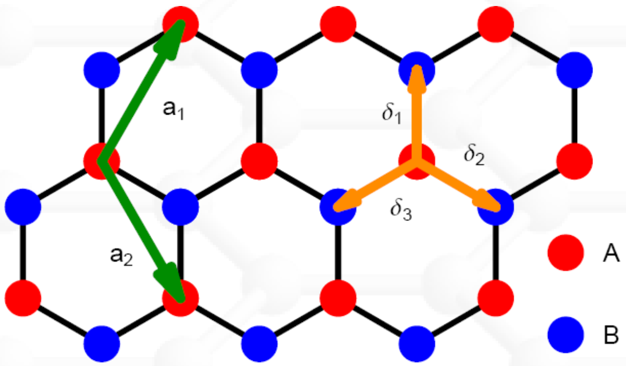






Lattice in coordinate/reciprocal space

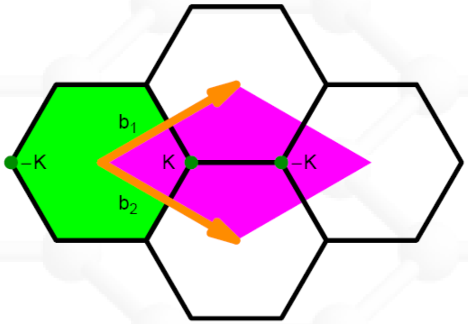
- Two carbon atoms per primitive cell
- Translation vectors



$$\mathbf{a}_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad \mathbf{a}_2 = a\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

where a is the lattice constant

- Reciprocal lattice vectors



$$\mathbf{b}_1 = 2\pi/a(1, 1/\sqrt{3}), \; \mathbf{b}_2 = 2\pi/a(1, -1/\sqrt{3})$$

Tight binding model

- There are strong covalent sigma-bonds between nearest neighbors
- Hamiltonian

$$H = -t \sum_{\mathbf{n},\boldsymbol{\delta}_i,\sigma} \left[a^{\dagger}_{\mathbf{n},\sigma} \exp\left(\frac{ie}{\hbar c} \boldsymbol{\delta}_i \mathbf{A} \right) b_{\mathbf{n}+\boldsymbol{\delta},\sigma} + \mathrm{c.c.} \right]$$

where $a_{\mathbf{n},\sigma}$ and $b_{\mathbf{n}+\delta,\sigma}$ are the annihilation operators of electrons with spin $\sigma = \uparrow, \downarrow$

- The nearest neighbor vectors are

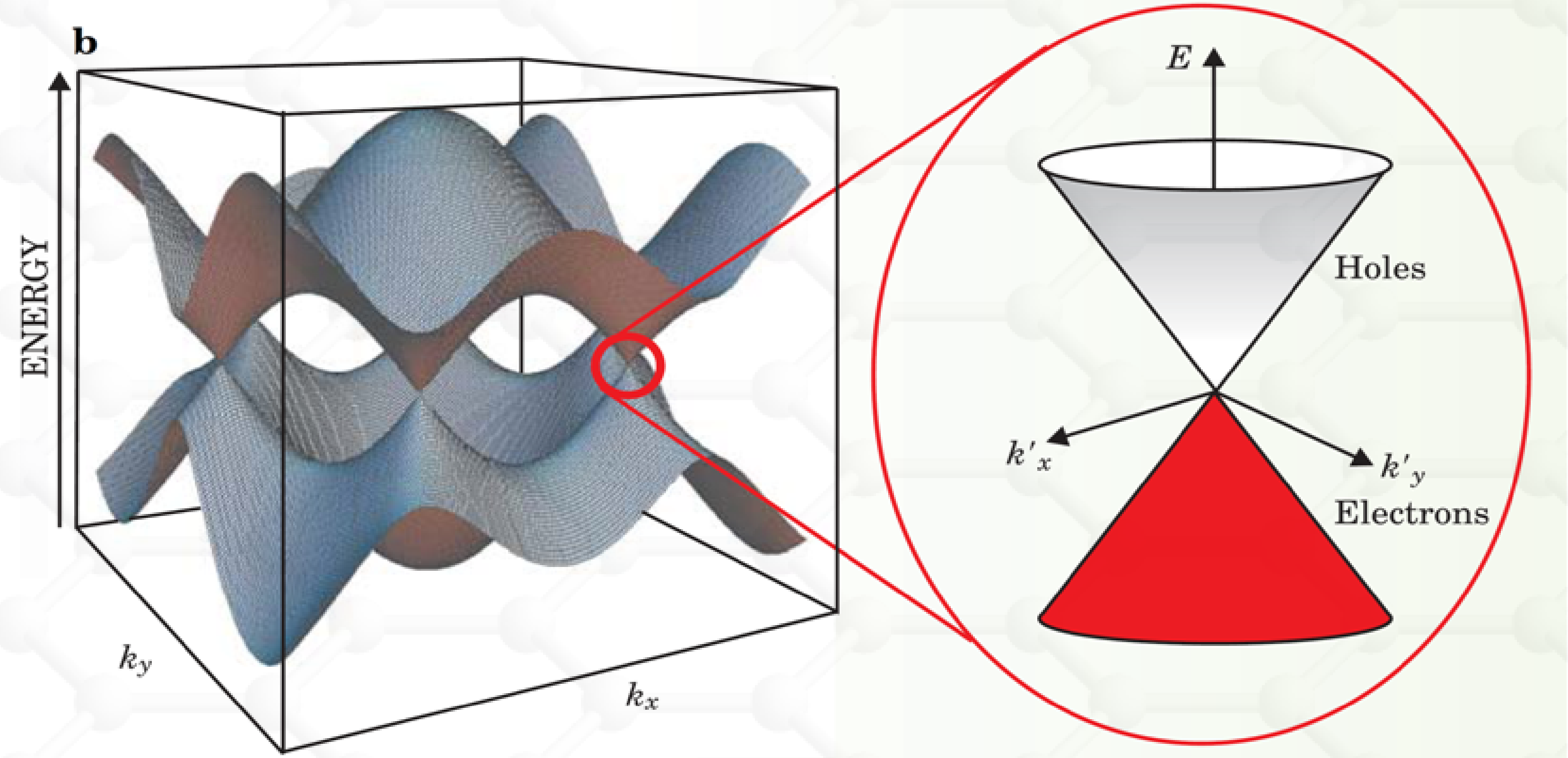
$$\boldsymbol{\delta}_1 = (\mathbf{a}_1 - \mathbf{a}_2)/3\,, \quad \boldsymbol{\delta}_2 = \mathbf{a}_1/3 + 2\mathbf{a}_2/3\,,$$
$$\boldsymbol{\delta}_3 = -\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2 = -2\mathbf{a}_1/3 - \mathbf{a}_2/3$$

Low energy Dirac fermions

$$\mathcal{L} = \sum_{\sigma=\pm 1} \bar{\Psi}_\sigma(t,\mathbf{r})[i\gamma^0(\hbar\partial_t - i\mu_\sigma) + i\hbar v_F\gamma^1 D_x + i\hbar v_F\gamma^2 D_y]\Psi_\sigma(t,\mathbf{r})$$

P. R. Wallace, Phys Rev **71,** 622 (1947)

G.W. Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)



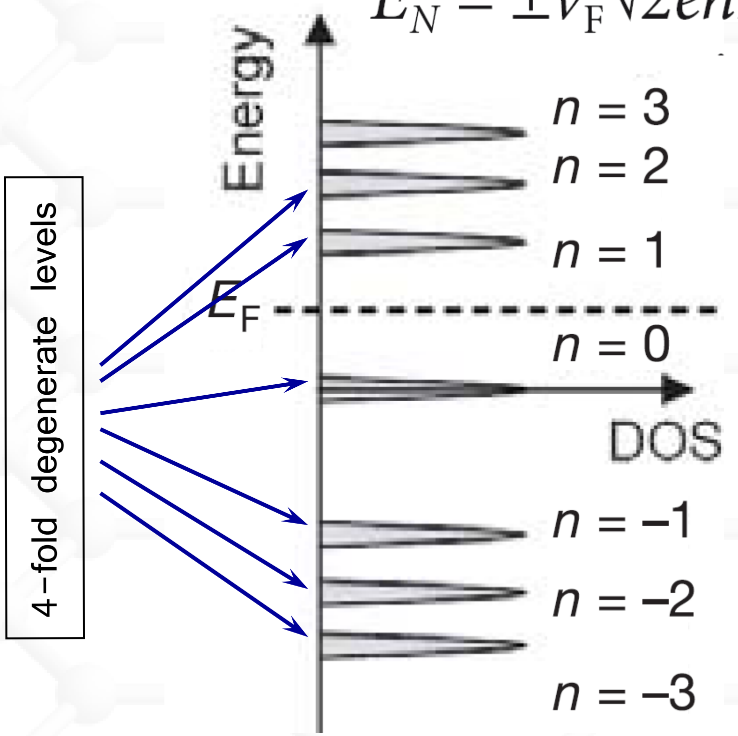
Quantum Hall effect in graphene

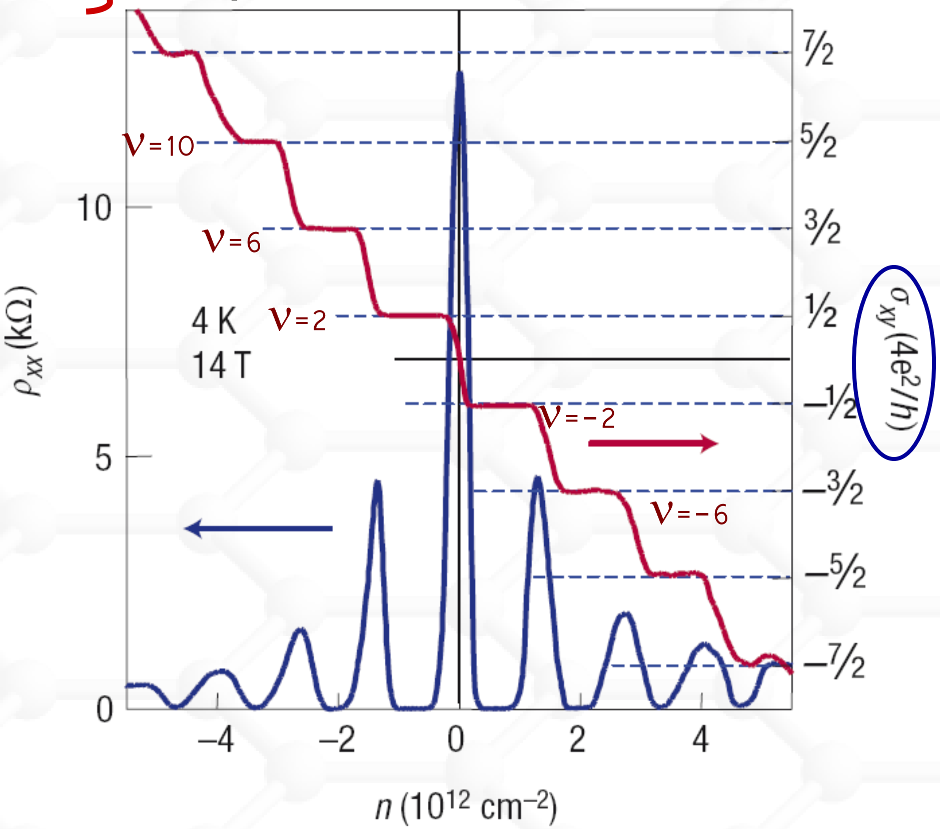
[1] Zheng & Ando, PRB **65**, 245420 (2002)
[2] Gusynin & Sharapov, PRL **95**, 146801 (2005)
[3] Peres, Guinea, & Castro Neto, PRB **73**, 125411 (2006)
[4] Novoselov et al., Nature **438**, 197 (2005)
[5] Zhang et al., Nature **438**, 201 (2005)

$$\sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{h}\left(n + \frac{1}{2}\right)$$

Experiment

$$E_N = \pm v_F \sqrt{2e\hbar BN}$$





Quantum Hall Effect at large *B*

There are new plateaus at

$\nu=0, \ \nu=\mp1, \ \nu=\mp4$

i.e., the degeneracy of some Landau levels is lifted

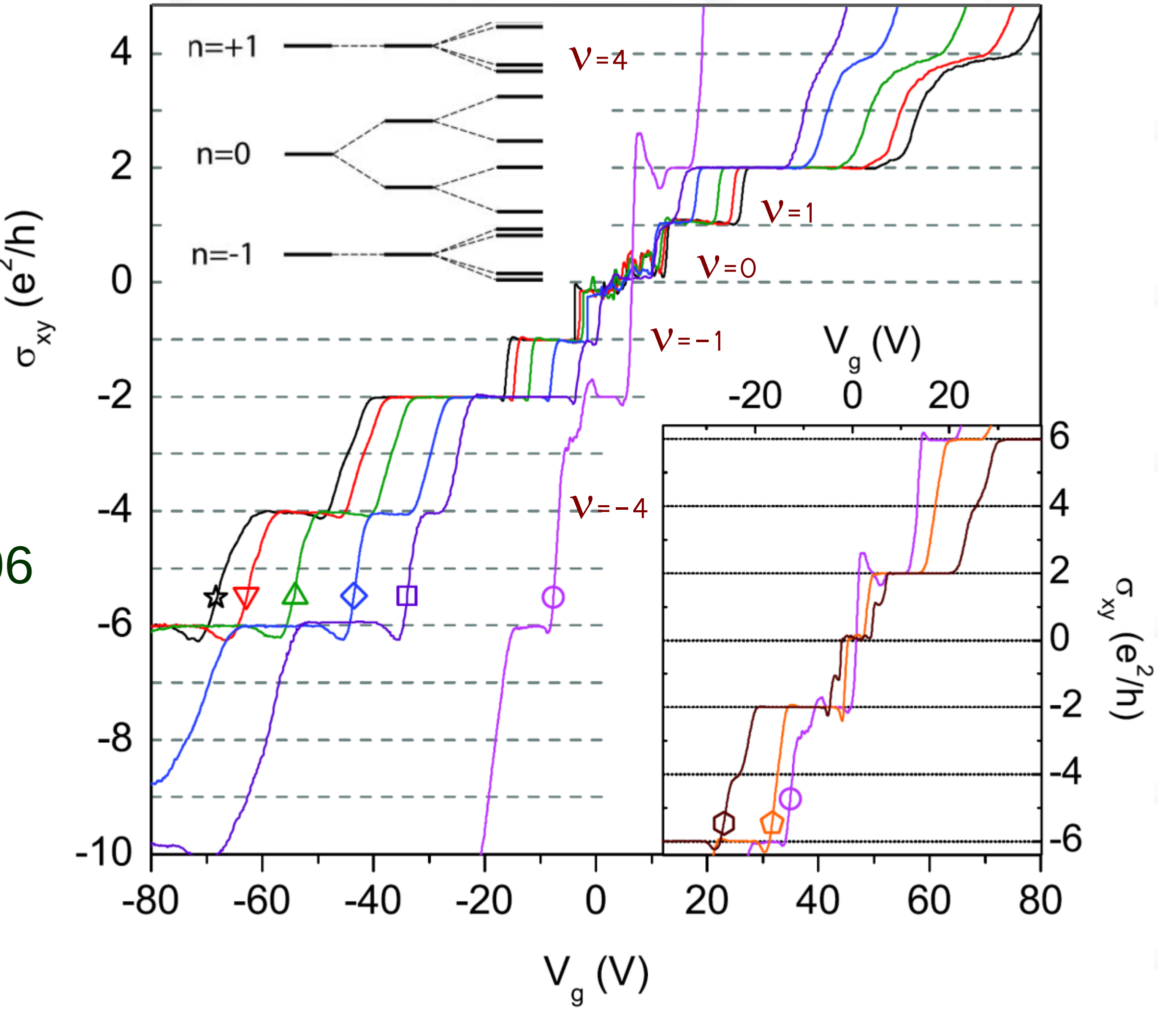
See also

Abanin et al., PRL **98**, 196806 (2007)

Jiang et al., PRL **99**, 106802 (2007)

Checkelsky et al., PRL 100, 206801 (2008)

Zhang et al., PRL **96,** 136806 (2006)



Magnetic catalysis (MC) scenario

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Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

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(Received 11 May 1994)

It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

$$E_n = \sqrt{2n|eB|} \Rightarrow E_n = \sqrt{2n|eB| + \Delta_0^2}$$

where $\Delta_0 \sim \sqrt{|eB|}$ $\Longrightarrow$ $\nu=0$

First proposed for graphene in

D.V. Khveshchenko, PRL **87**, 206401 (2001); ibid. **87**, 246802 (2001)
E.V. Gorbar, V.P. Gusynin, V.A. Miransky, and I.A. Shovkovy, PRB **66**, 045108 (2002).

Quantum Hall Ferromagnetism (QHF)

Arovas, Karlhelde, & Lilliehook, PRB **59**, 13147 (1999)

Ezawa & Hasebe, PRB **65**, 075311 (2002)

- Spin/valley degeneracy of the half-filled Landau level is lifted by the exchange (repulsive Coulomb) interaction
- This is similar to the **Hund's Rule(s)** in atomic physics
- In the lowest energy state, the coordinate part of the wave function is *antisymmetric* (with the electrons being as far apart as possible)

  i.e., it is *symmetric* in the spin/valley indices
- This is nothing else but ferromagnetism

General Approach

Model Hamiltonian

[Gorbar, Gusynin, Miransky, Shovkovy, **arXiv:0806.0846**, PRB **78** (2008) 085437]

$$H = H_0 + H_C + \int d^2\mathbf{r}\,[\underbrace{\mu_B B \Psi^\dagger \sigma^3 \Psi}_{\textit{Zeeman term}} - \mu_0 \Psi^\dagger \Psi]$$

where

$$H_0 = v_F \int d^2\mathbf{r}\,\overline{\Psi}\left(\gamma^1 \pi_x + \gamma^2 \pi_y\right)\Psi,$$

is the Dirac Hamiltonian, and

$$H_C = \frac{1}{2}\int d^2\mathbf{r}\, d^2\mathbf{r}'\, \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) U_C(\mathbf{r}-\mathbf{r}')\Psi^\dagger(\mathbf{r}')\Psi(\mathbf{r}')$$

is the Coulomb interaction term.

Note that $\Psi_{\underbrace{s}_{\textit{Spin index}}}^T = (\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, \psi_{K'As})$

$v_F \approx 10^6$ m/s

Symmetry

- The Hamiltonian $H = H_0 + H_C$ possesses "flavor" *U(4)* symmetry
- 16 generators read (*spin* ⊗ *pseudospin*)

$$\frac{\sigma^\alpha}{2} \otimes I_4, \quad \frac{\sigma^\alpha}{2i} \otimes \gamma^3, \quad \frac{\sigma^\alpha}{2} \otimes \gamma^5, \quad \text{and} \quad \frac{\sigma^\alpha}{2} \otimes \gamma^3\gamma^5.$$

- The Zeeman term breaks *U(4)* down to $U(2)_+ \times U(2)_-$
- Dirac mass breaks $U(2)_s$ down to $U(1)_s$

Energy scales in the problem

- Landau energy scale

$$\epsilon_B \equiv \sqrt{2\hbar|eB_\perp|v_F^2/c} \simeq 424\sqrt{|B_\perp[\mathrm{T}]|}\ \mathrm{K}$$

- Zeeman energy

$$Z \simeq \mu_B B = 0.67B[\mathrm{T}]\ \mathrm{K}$$

- Dynamical mass scales ($Z \ll A \leq M \ll \epsilon_B$)

$$A \equiv \frac{G_{\mathrm{int}}|eB_\perp|}{8\pi\hbar c} = \frac{\sqrt{\pi}\lambda\epsilon_B^2}{4\Lambda}$$

- In our calculations,

$$M = 4.84 \times 10^{-2}\epsilon_B \text{ and } A = 3.90 \times 10^{-2}\epsilon_B$$

Full propagator

- We use the following general ansatz:

$$iG_s = \Big[(i\hbar\partial_t + \mu_s + \tilde{\mu}_s \gamma^3\gamma^5)\gamma^0 - v_F(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \tilde{\Delta}_s + \Delta_s \gamma^3\gamma^5 \Big]^{-1}$$

Electron chemical potential (μ_s)

Pseudospin magnetic moment ($\tilde{\mu}_s$)

Dirac mass ($\tilde{\Delta}_s$)

***T*-odd mass** (Δ_s)

- Physical meaning of the order parameters

$$\Delta_s: \qquad \bar{\Psi}\gamma^3\gamma^5 P_s \Psi = \psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} + \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

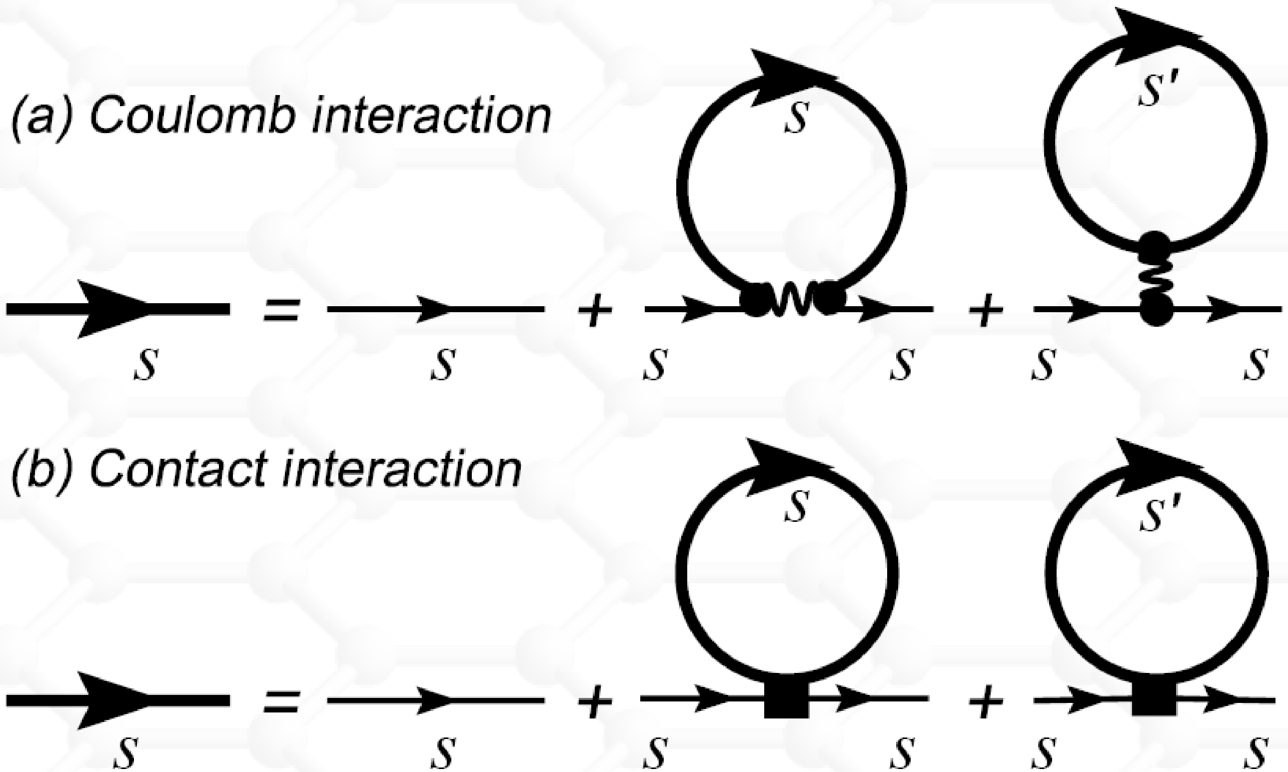
$$\tilde{\Delta}_s: \qquad \bar{\Psi}P_s\Psi = \psi^\dagger_{KAs}\psi_{KAs} + \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

$$\mu_3: \qquad \Psi^\dagger\sigma^3\Psi = \frac{1}{2}\sum_{\kappa=K,K'}\sum_{a=A,B}\left(\psi^\dagger_{\kappa a+}\psi_{\kappa a+} - \psi^\dagger_{\kappa a-}\psi_{\kappa a-}\right)$$

$$\tilde{\mu}_s: \qquad \Psi^\dagger\gamma^3\gamma^5 P_s\Psi = \psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} + \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

Schwinger Dyson equation

- Hartree-Fock (mean field) approximation:



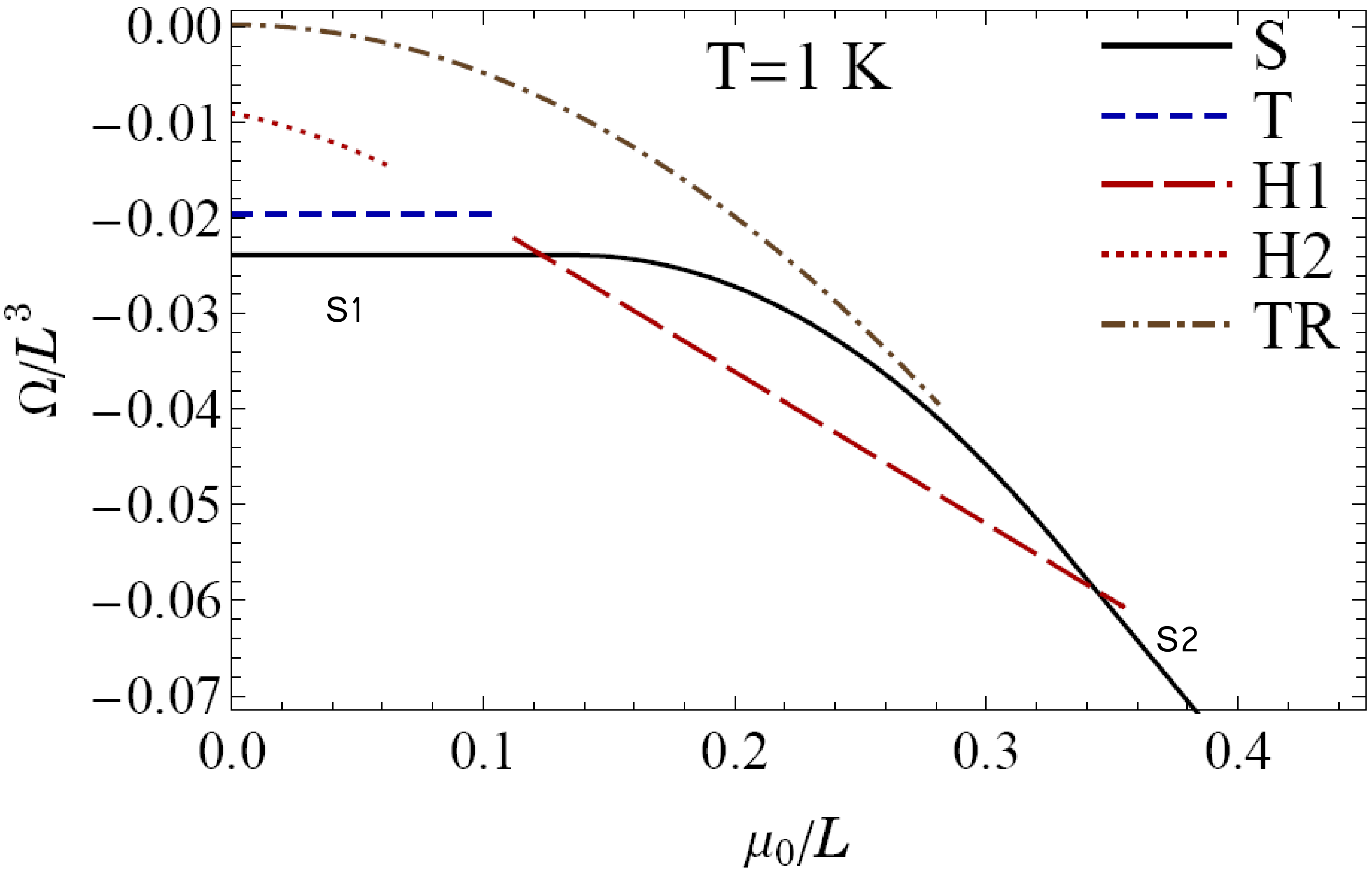
Three types of solutions

i. ***S*** (*singlet* with respect to $U(2)_s$ where s=↑,↓)
- Order parameters: μ_3 and/or Δ_s
- Symmetry: $U(2)_+ \times U(2)_-$

ii. ***T*** (*triplet* with respect to $U(2)_s$)
- Order parameters: $\tilde{\mu}_s$ and/or $\tilde{\Delta}_s$
- Symmetry: $U(1)_+ \times U(1)_-$

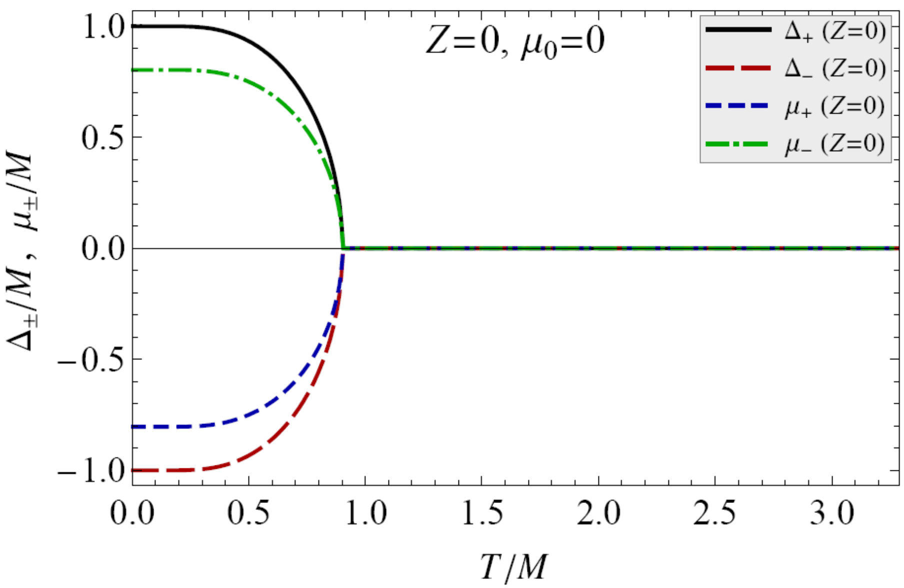
iii. ***H*** (*hybrid*, i.e., singlet + triplet)
- Order parameters: mixture of ***S*** and ***T*** types
- Symmetry: $U(2)_+ \times U(1)_-$ or $U(1)_+ \times U(2)_-$

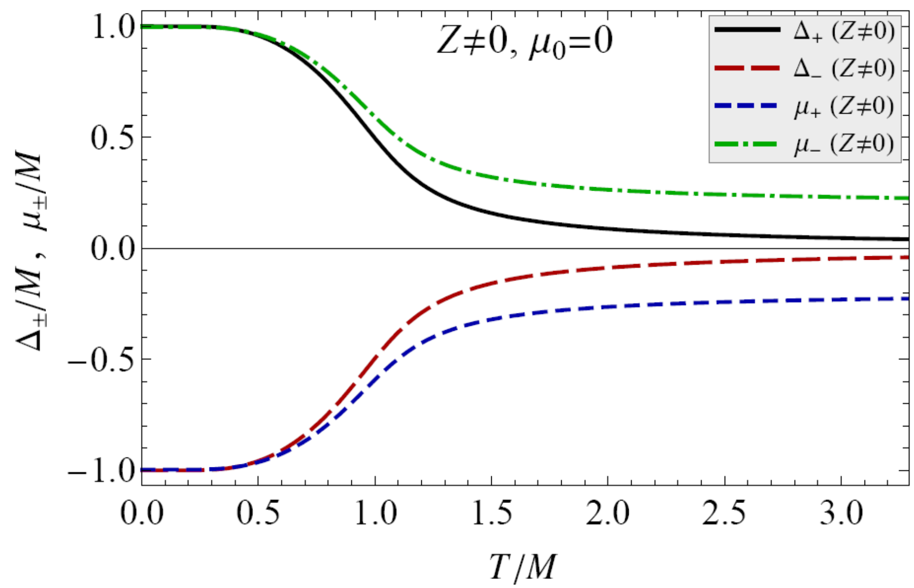
Solutions at LLL ($\mu_0 \ll \epsilon_B$)



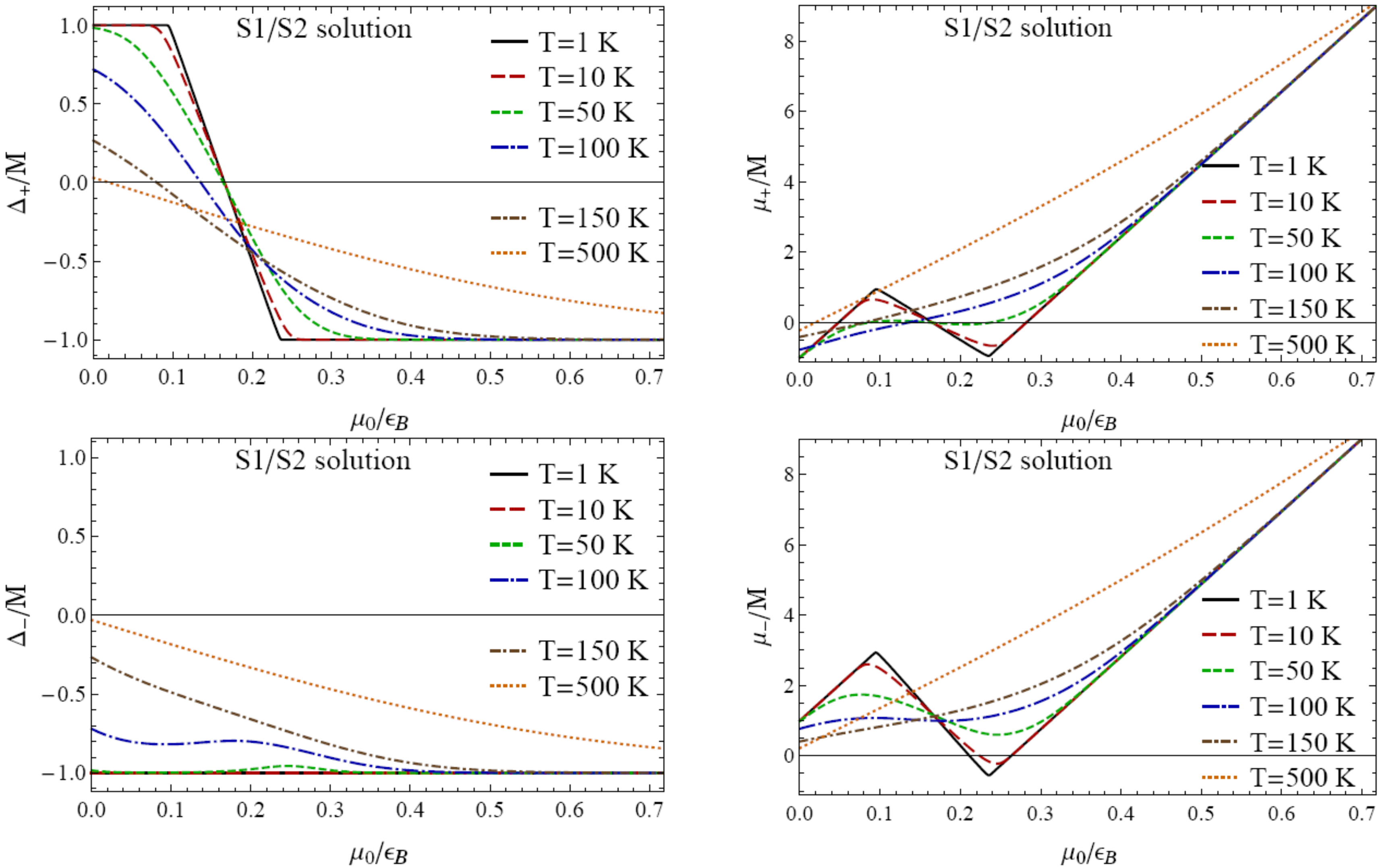
Singlet solution vs. T (ν=0 QHE state)

$$\tilde{\Delta}_+ = \tilde{\mu}_+ = 0, \qquad \mu_+ = \bar{\mu}_+ - A, \qquad \Delta_+ = s_\perp M,$$
$$\tilde{\Delta}_- = \tilde{\mu}_- = 0, \qquad \mu_- = \bar{\mu}_- + A, \qquad \Delta_- = -s_\perp M.$$





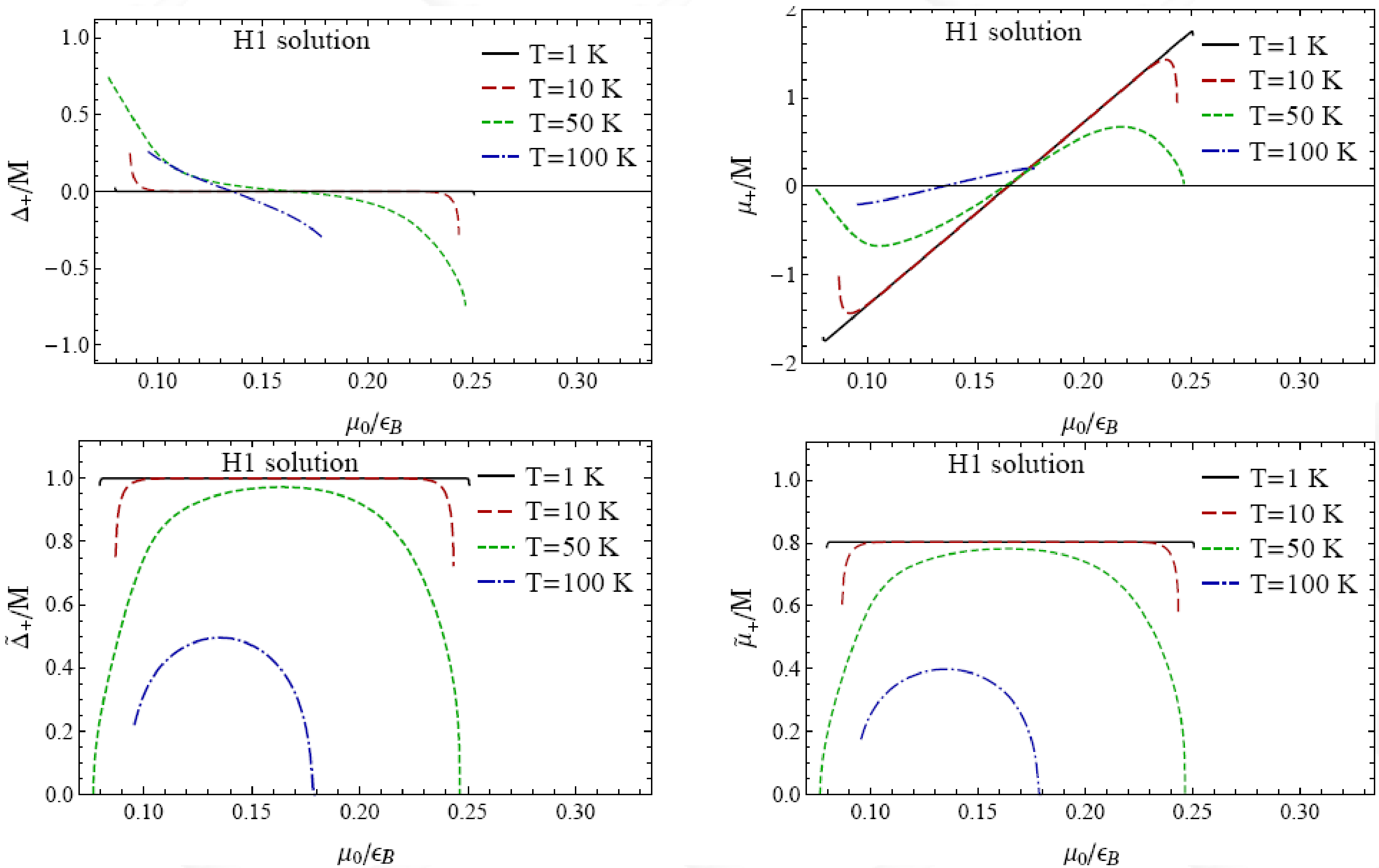
Singlet solution (ν=0 & 2 QHE states)



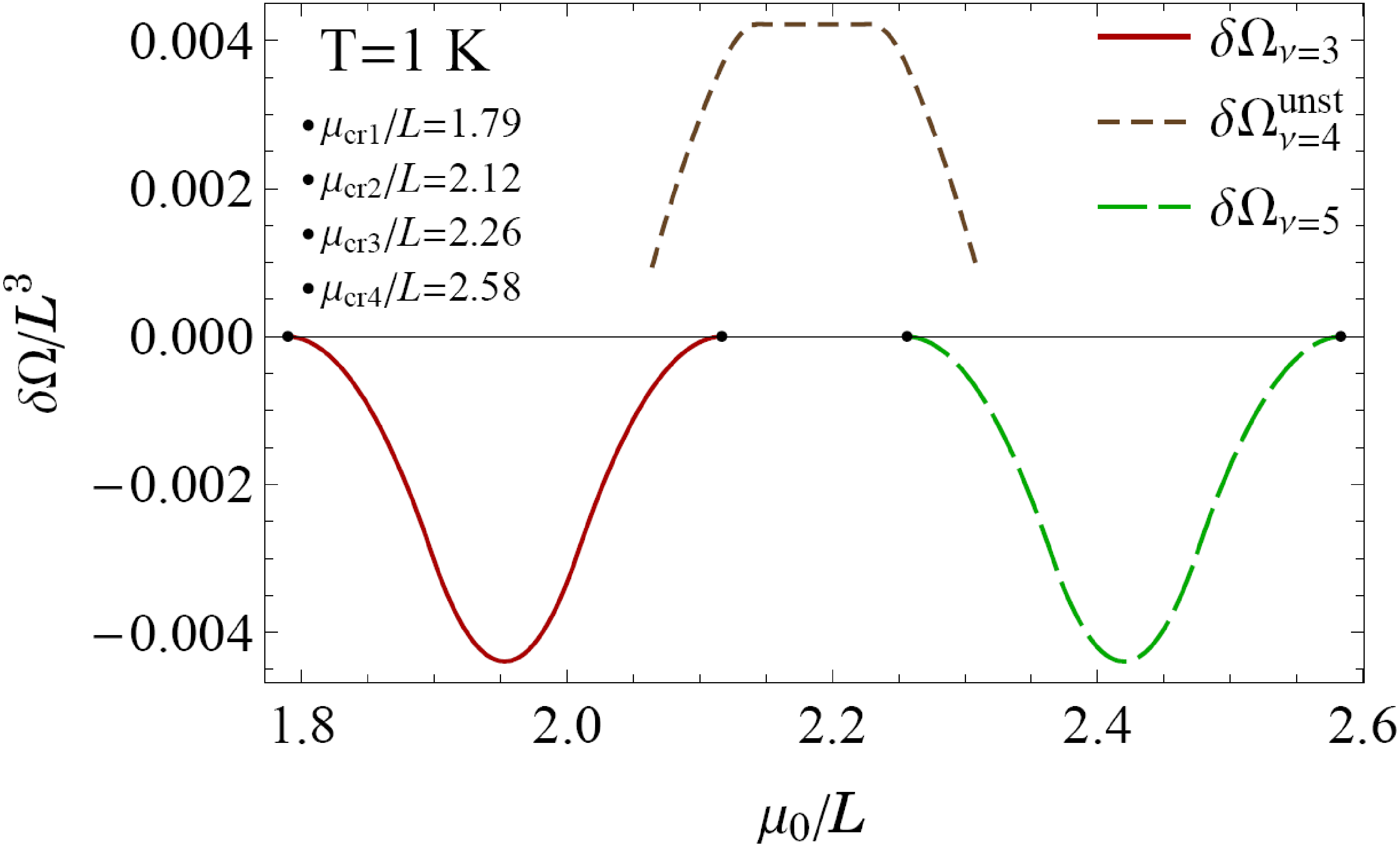
Hybrid solution (ν=1 QHE state)

$$\tilde{\Delta}_+ = M, \qquad \tilde{\mu}_+ = A s_\perp, \qquad \mu_+ = \bar{\mu}_+ - 4A, \qquad \Delta_+ = 0,$$

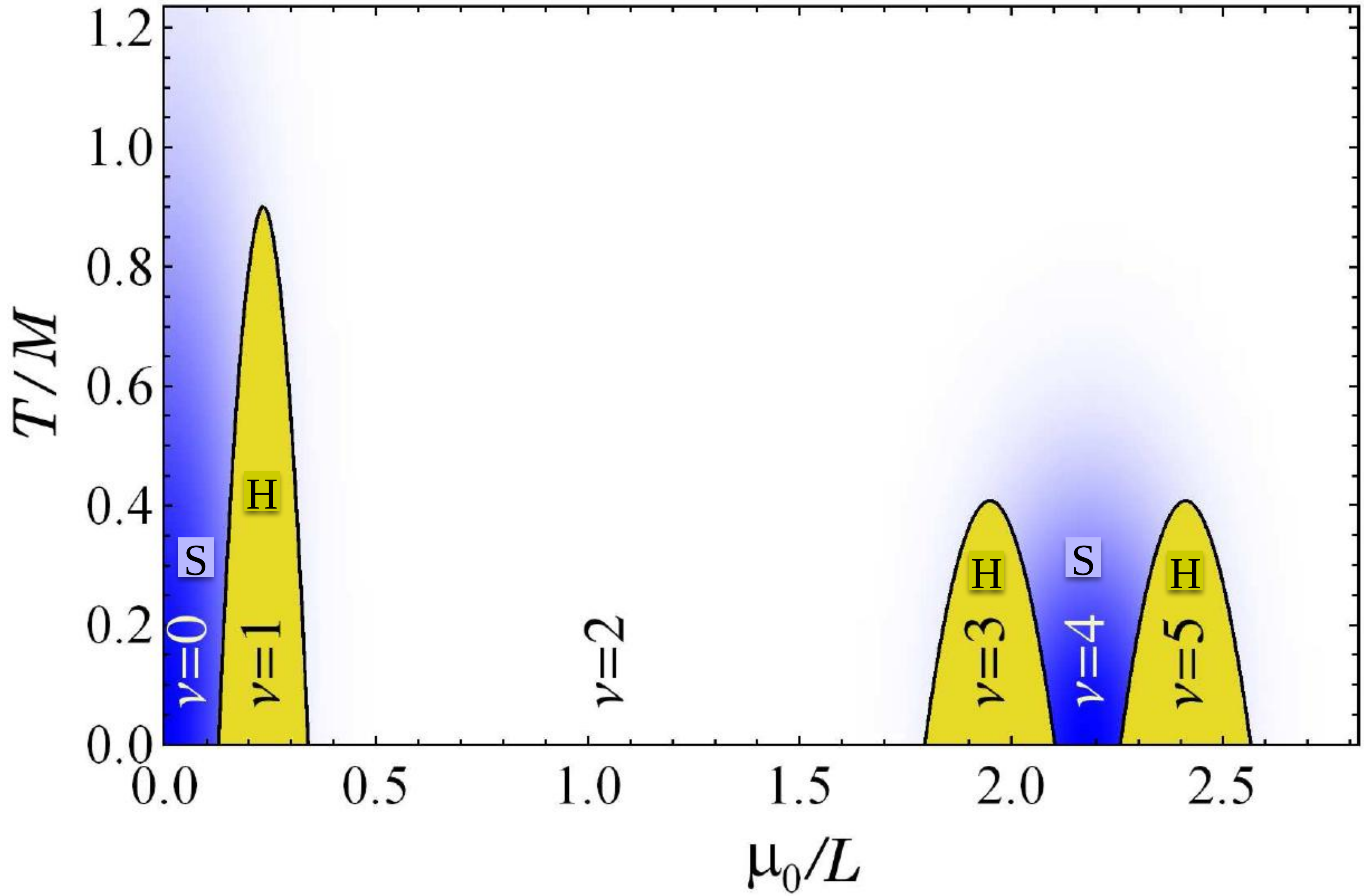
$$\tilde{\Delta}_- = \tilde{\mu}_- = 0, \qquad \mu_- = \bar{\mu}_- - 3A, \qquad \Delta_- = -s_\perp M.$$



Hybrid solutions at 1st Landau level



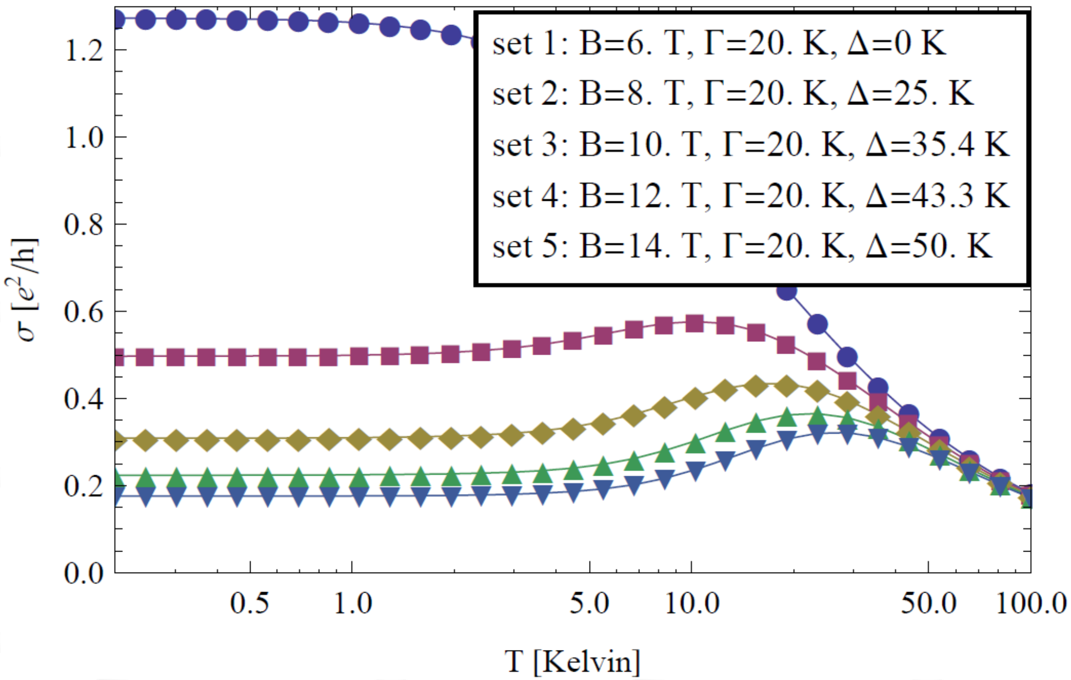
Phase diagram



Theory vs. experiment (1)

- Theory predicts all “new” plateaus observed in a strong magnetic field (i.e., $\nu=0$, $\nu=\mp1$, $\nu=\mp4$)
- The plateaus $\nu=\mp3$, $\nu=\mp5$, which are not observed yet, are also predicted
- This might be in a qualitative agreement with a suggested large width of higher Landau levels [Giesbers et al., PRL **99**, 206803 (2007)]

Theory vs. experiment (2)

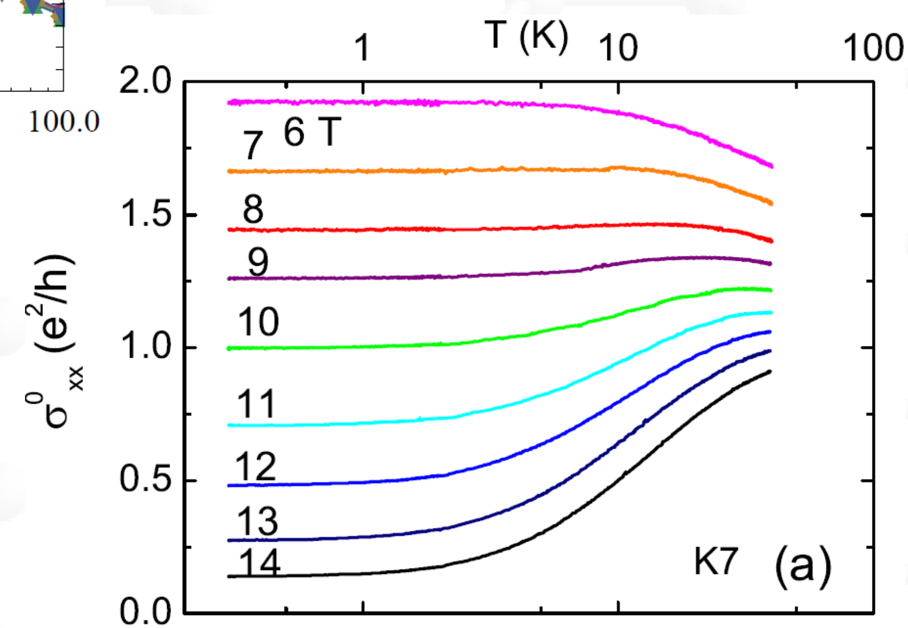


⇐Theory

First try with a "reasonable" set of parameters

Experiment ⇒

Checkelsky, Li, Ong, PRL **100**, 206801 (2008)



Summary

- A rich phase diagram in the T-μ plane is proposed
- Both MC and QHF are responsible for dynamical symmetry breaking and lifting the degeneracy of Landau levels in graphene
- Qualitative agreement with experiment is evident, but details remain to be worked out