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Chiral asymmetry in relativistic matter in a magnetic field

*E.V. Gorbar, V.A. Miransky, I.S., arXiv:0904.2164 [hep-ph]

Motivation

- Dynamics of Quantum Hall Effect in graphene (2+1 dimensions)
  - Parity and time-reversal odd Dirac mass

$$\Delta \sim \langle \bar{\Psi} \gamma^3 \gamma^5 \Psi \rangle$$

[Gorbar, Gusynin, Miransky, I.S., PRB **78**, 085437 (2008)]

- Topological current in relativistic matter in a magnetic field (3+1 dimensions)
  - $\langle \bar{\psi} \gamma^3 \gamma^5 \psi \rangle = \frac{eB}{2\pi^2}\mu$ (free theory!)

[Metlitski, Zhitnitsky, PRD **72**, 045011 (2005)]

Model

- Lagrangian density:

$$\mathcal{L} = \bar{\psi}\left(iD_\nu + \mu_0 \delta^0_\nu\right)\gamma^\nu \psi + \frac{G_{\rm int}}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

- The dimensionless coupling is weak,

$$g \equiv G_{\rm int}\Lambda^2/(4\pi^2) \ll 1$$

- Magnetic field is inside $D_\nu = \partial_\nu - ieA_\nu$ where $A_\nu = xB\delta^2_\nu$

- Gap equation in mean-field approximation:

Vacuum state

- Magnetic catalysis (spontaneous generation of a nonzero Dirac mass even at $g \ll 1$):

$$m_0^2 = \frac{1}{\pi l^2} \exp\left(-\frac{\Lambda^2 l^2}{g}\right) \quad \text{where} \quad l = 1/\sqrt{|eB|}$$

(along with $\mu = \mu_0$)

[Gusynin, Miransky, I.S., PRL **73**, 3499 (1994); PLB **349**, 477 (1995)]

- This solution exists for $\mu_0 < m_0$, but it is less stable than the normal state ($m = 0$) already for $\mu_0 \gtrsim m_0/\sqrt{2}$ [Clogston, PRL **9**, 266 (1962)]

“Abnormal” normal ground state

- Gap equation allows another solution,

$$\mu \simeq \mu_0 \text{ and } \Delta \simeq -g\mu_0 eB/\Lambda^2$$

- This solution is almost independent of temperature when $T \ll \mu$
- This is the *normal* ground state since its symmetry is same as in the Lagrangian
- Besides, there is no solution with $\Delta=0$…

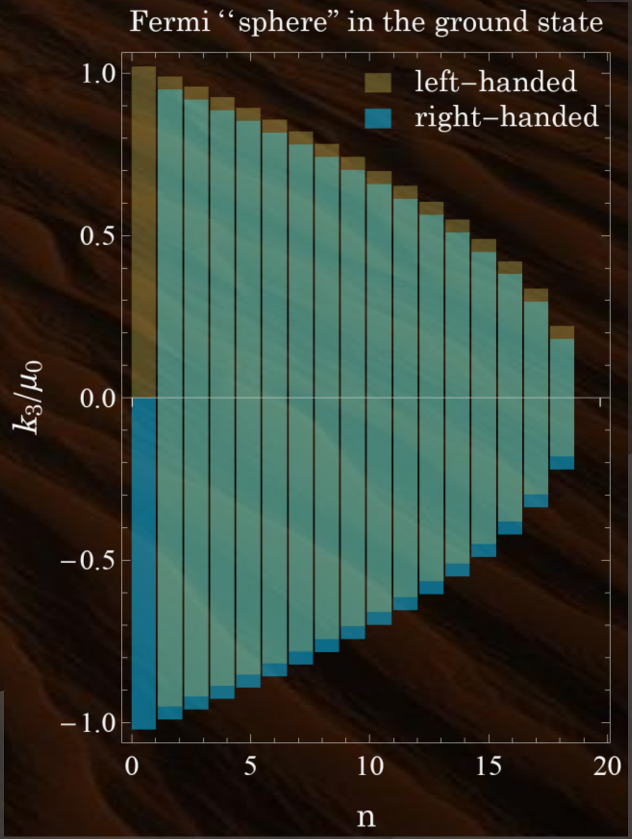
Physical meaning of Δ

- The dispersion relation of quasiparticles:

$$\omega_{n,\sigma} = -\mu \pm \sqrt{[k_3 + \sigma\Delta]^2 + 2n|eB|}$$

where $\sigma = \pm 1$ is the chirality

- Longitudinal momenta of opposite chirality fermions are *shifted*, *i.e.*, $k_3 \to k_3 \pm \Delta$
- All Landau levels ($n \geq 0$) are affected by Δ



Induced axial current

- The axial current in the ground state is

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = \frac{eB}{2\pi^2}\mu - \frac{|eB|}{2\pi^2}\Delta - \frac{|eB|}{\pi^2}\Delta\sum_{n=1}^{\infty}\kappa(\sqrt{2n|eB|},\Lambda)$$

- In addition to the topological contribution, $\frac{eB}{2\pi^2}\mu$ there are dynamical ones $\propto \Delta$
- The cutoff function:

$$\kappa(x,\Lambda) \simeq \begin{cases} 1 & , x \ll \Lambda \\ 0 & , x \gg \Lambda \end{cases}$$

- An equivalent result is also obtained in the Pauli-Villars regularization

Potential implications

- Physical properties to be affected
  - transport
  - emission

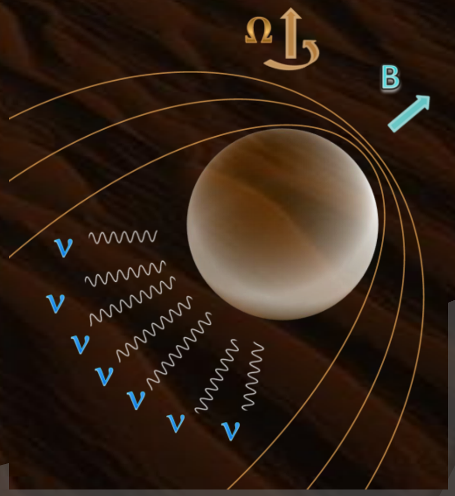
    (if sensitive to anisotropy and/or CP violation)
- Specific physical systems
  - Compact stars
    - Quark stars (quarks)
    - Hybrid stars (quarks, electrons)
    - Neutron stars (electrons)
    - White dwarfs (electrons)
  - Heavy ion collisions [Kharzeev & Zhitnitsky, NPA **797**, 67(2007), Kharzeev, McLerran & Warringa, NPA 803, 227 (2008), …]

Pulsar kicks

- The dynamical chiral shift parameter is nonzero even at moderate temperatures ($T \ll \mu$):

$$\Delta \simeq -g\mu_0 eB/\Lambda^2$$

- This creates a strong anisotropy in the distribution of left-handed electrons/quarks
- The anisotropy is transferred to neutrinos by elastic scattering
- Pulsar gets a kick when neutrinos escape



Supernova explosions

- Because of the robustness of the axial currents at finite temperatures, even supernova explosions may be affected

- A small early-time neutrino asymmetry may *facilitate* explosions and give a kick at the same time, e.g., see

  [Fryer & Kusenko, Astrophys. J. Supp. **163**, 335 (2006)]

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Summary

- $\mu < \mu_c$: Chiral symmetry is broken in the ground state (magnetic catalysis)
- $\mu > \mu_c$: Normal ground state of dense relativistic matter in a magnetic field is characterized by
  - Axial current along the field (topological and dynamical contributions)
  - Chiral shift parameter
- No solution with vanishing Δ exists

Outlook

- Detailed analysis of normal ground state in models with explicitly broken chiral symmetry (work in progress)
- Calculation of neutrino emission/diffusion in the state with axial currents
- Transport properties of the normal state with nonzero axial currents
- Modification of the "chiral magnetic effect" due to Δ in heavy ion collisions

  [Fukushima, Kharzeev & Warringa, PRD **78**, 074033 (2008)]