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POLYTECHNIC CAMPUS

# Abnormal normal ground state of dense relativistic matter in magnetic field

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*E.V. Gorbar, V.A. Miransky, I.S., Phys. Rev. C 80 (2009), 032801(R)
arXiv:0904.2164 [hep-ph] + work in progress

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# Dense relativistic matter

- Dense relativistic matter is common inside compact stars
  - Electrons in white dwarfs

    $T \ll m \lesssim \mu$ ($i.e.$, $T \lesssim 1$ keV & $\mu \simeq 1$ MeV)
  - Neutrons of nuclear matter

    $T \ll m \lesssim \mu$ ($i.e.$, $T \lesssim 10$ MeV & $\mu \simeq 1$ GeV)
  - Electrons inside stellar nuclear matter

    $m \lesssim T \ll \mu$ ($i.e.$, $T \lesssim 10$ MeV & $\mu \simeq 100$ MeV)
  - Dense quark matter in stellar cores (if formed)

    $T \lesssim m \ll \mu$ ($i.e.$, $T \lesssim 10$ MeV & $\mu \gtrsim 400$ MeV)

# General idea

- Topological current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = \frac{eB}{2\pi^2}\mu$$ (free theory!)

[Metlitski, Zhitnitsky, PRD **72**, 045011 (2005)]

- Should there be a dynamical "mass" $\Delta$, associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \mathcal{L}_\Delta \simeq \Delta\bar{\psi}\gamma^3\gamma^5\psi$$

- Note: $\Delta$=0 is not protected by any symmetry

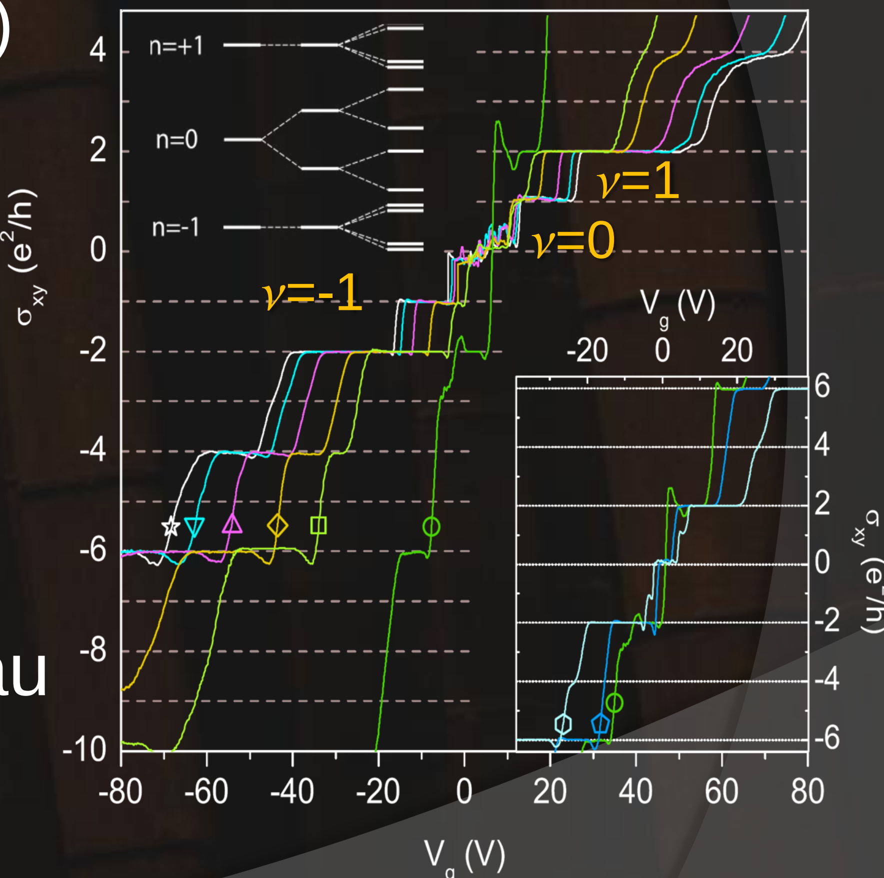
# Lesson from graphene

- Dynamics of Quantum Hall Effect in graphene (≈ QED in 2+1 dimensions)
  - Parity and time-reversal odd Dirac mass

$$\Delta \sim \langle \bar{\Psi}\gamma^3\gamma^5\Psi\rangle$$

[Gorbar, Gusynin, Miransky, I.S., PRB **78**, 085437 (2008)]

- $\Delta$ describes the $0^{\mathrm{th}}$ plateau in Quantum Hall effect in graphene




Zhang et al., PRL **96,** 136806 (2006)

# Model

- Lagrangian density:

$$\mathcal{L} = \bar{\psi}\left(iD_\nu + \mu_0 \delta_\nu^0\right)\gamma^\nu \psi + \frac{G_{\rm int}}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i \gamma^5 \psi\right)^2\right]$$

- The dimensionless coupling is

$$g \equiv G_{\rm int}\Lambda^2/(4\pi^2) \ll 1$$

- Magnetic field is inside $D_\nu = \partial_\nu - ieA_\nu$ where $A_\nu = xB\delta_\nu^2$ (Landau gauge)

# Approximation

- Gap equation in mean-field approximation:

$$G^{-1}(u,u') = S^{-1}(u,u') - iG_{\rm int}\left\{G(u,u) - \gamma^5 G(u,u)\gamma^5 \right.$$
$$\left. - \ \mathrm{tr}[G(u,u)] + \gamma^5\,\mathrm{tr}[\gamma^5 G(u,u)]\right\}\delta^4(u-u')$$

where

$$iG^{-1}(u,u') = \Big[(i\partial_t + \mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3$$
$$+ \ i\tilde{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m\Big]\delta^4(u-u')$$

and

$$iS^{-1}(u,u') = \left[(i\partial_t + \mu_0)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3\right]\delta^4(u-u')$$

# Vacuum state

- Magnetic catalysis (spontaneous generation of a nonzero Dirac mass even at $g \ll 1$):

  $$m_0^2 = \frac{1}{\pi l^2} \exp\left(-\frac{\Lambda^2 l^2}{g}\right) \quad \text{where} \quad l = 1/\sqrt{|eB|}$$

  (along with $\mu = \mu_0$)

  [Gusynin, Miransky, I.S., PRL **73**, 3499 (1994); PLB **349**, 477 (1995)]

- The solution exists for $\mu_0 < m_0$, although it will be less stable than the normal state ($m = 0$) already for $\mu_0 \gtrsim m_0/\sqrt{2}$ [Clogston, PRL **9**, 266 (1962)]

# "Abnormal" normal ground state

- The gap equation allows another solution,

$$\mu \simeq \mu_0 \text{ and } \Delta \simeq g\mu_0 eB/\Lambda^2$$

- This solution is almost independent of temperature when $T \ll \mu$
- This is the normal ground state since its symmetry is same as in the Lagrangian
- Besides, there is no trivial solution $\Delta$=0

# Change of ground state

- The free energy in the state with $m \neq 0$ (broken symmetry)

$$\Omega_m \simeq -\frac{m_0^2}{2(2\pi l)^2}\left(1+(m_0 l)^2 \ln|\Lambda l|\right)$$

- The free energy in the normal state, $\Delta \neq 0$

$$\Omega_\Delta \simeq -\frac{\mu_0^2}{(2\pi l)^2}\left(1-g\frac{|eB|}{\Lambda^2}\right)$$

- So, indeed symmetry is restored for $\mu > \mu_c$ ,

$$\mu_c \simeq m_0/\sqrt{2}$$

# Physical meaning of Δ

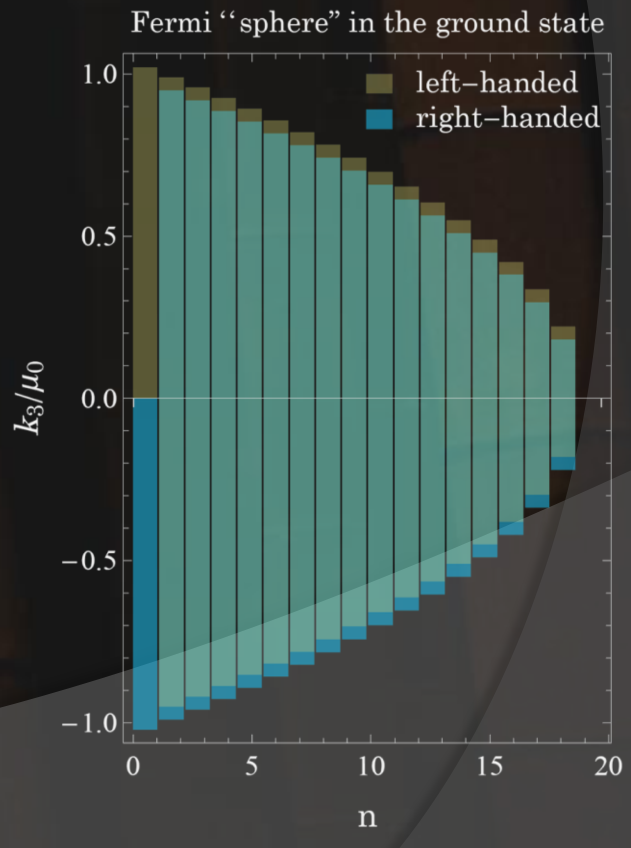
- The dispersion relation of quasiparticles:

$$\omega_{n,\sigma} = -\mu \pm \sqrt{[k_3 + \sigma\Delta]^2 + 2n|eB|}$$

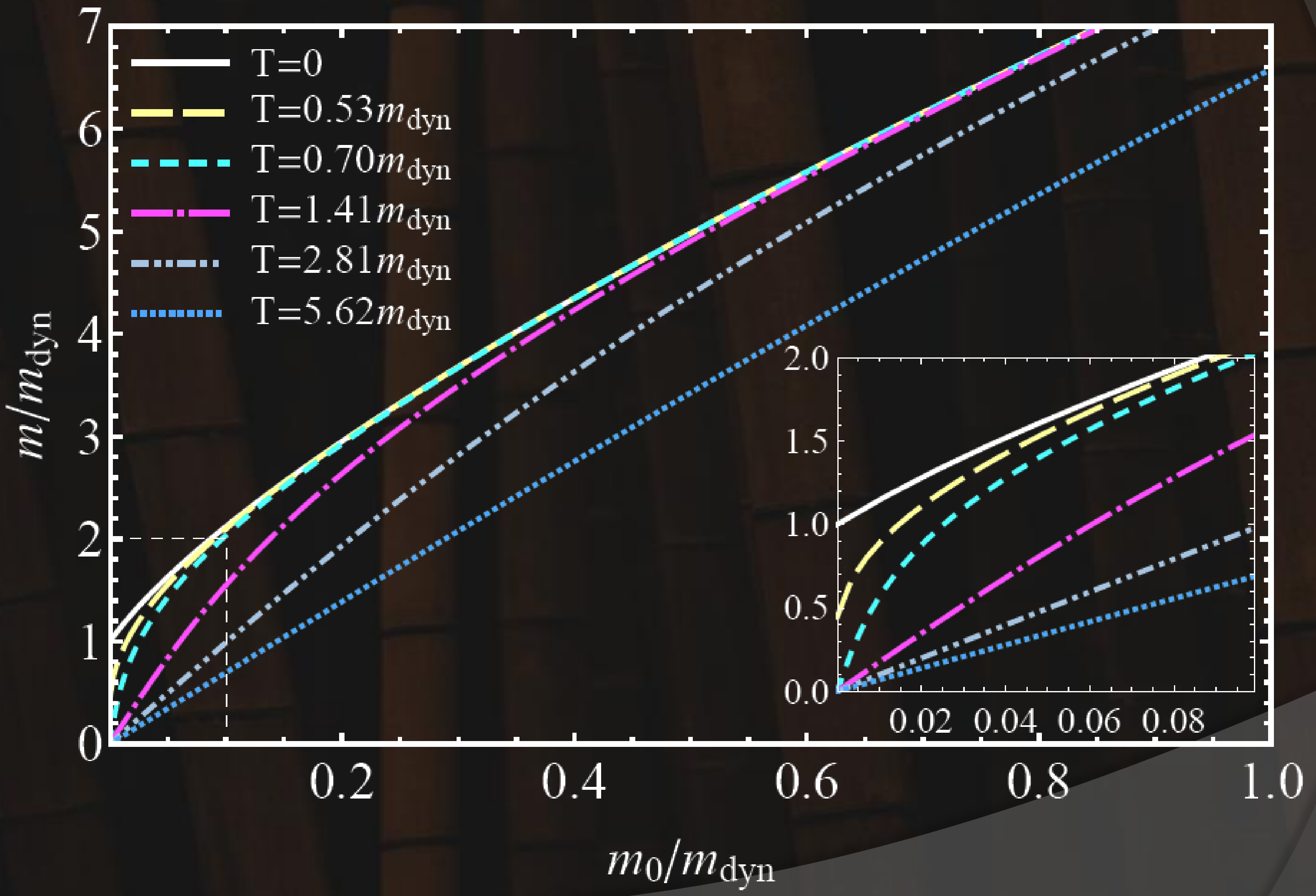
where $\sigma = \pm 1$ is the chirality

- Longitudinal momenta of opposite chirality fermions are *shifted, i.e.*, $k_3 \to k_3 \pm \Delta$

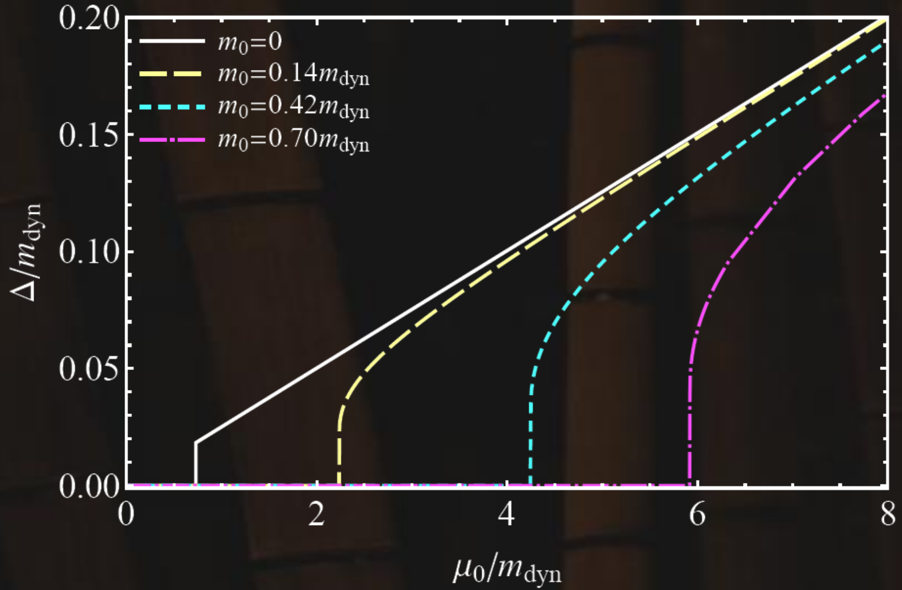
- All Landau levels ($n \geq 0$) are affected by Δ



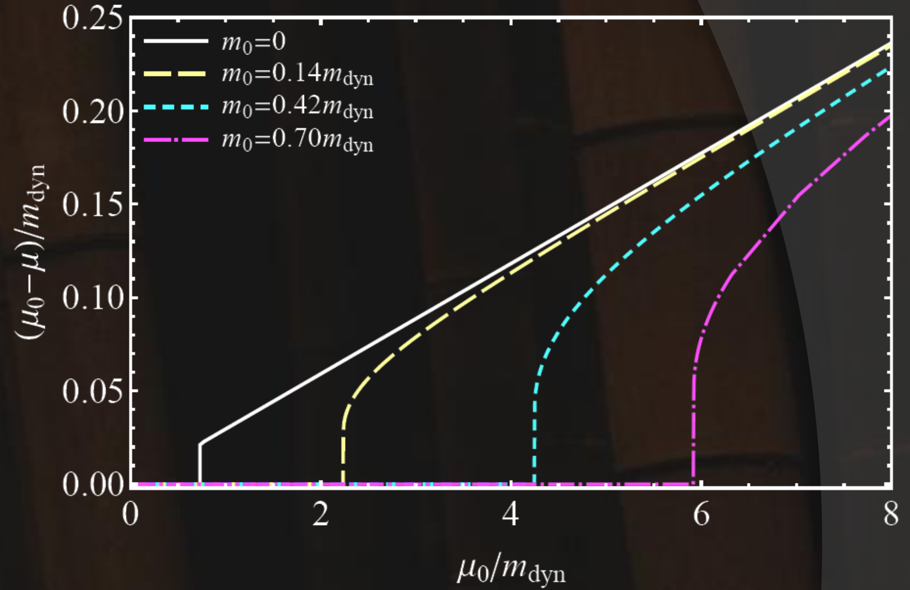
# Magnetic catalysis at $\mu_0$=0



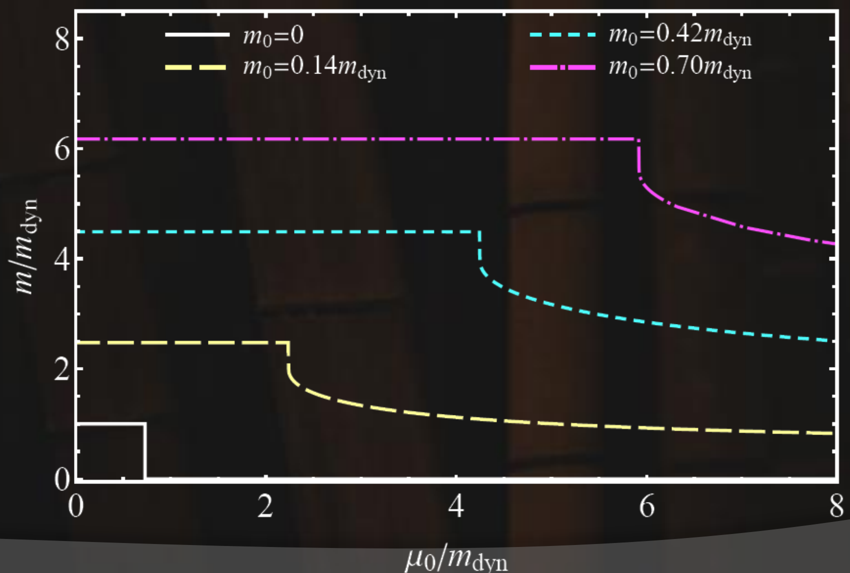
# T=0 results



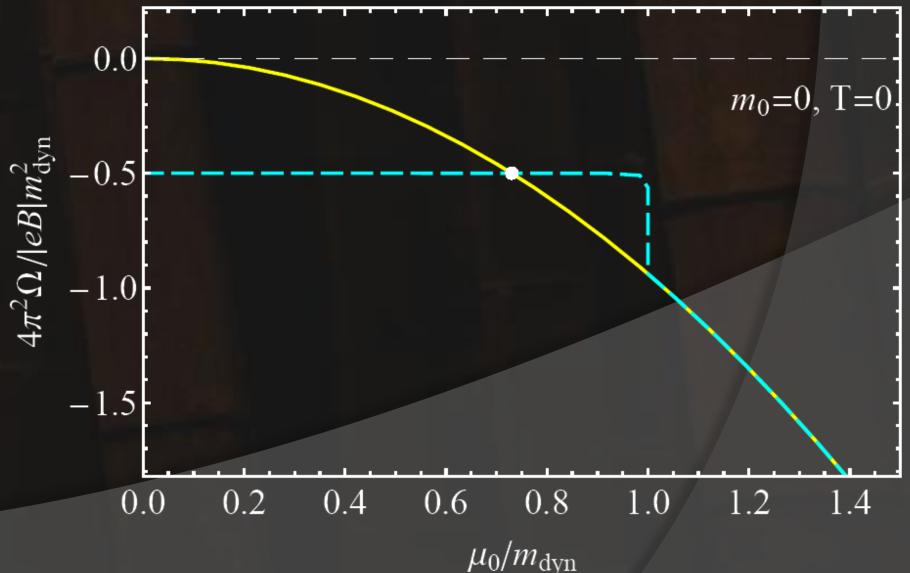
$m_0=0$
$m_0=0.14m_{\rm dyn}$
$m_0=0.42m_{\rm dyn}$
$m_0=0.70m_{\rm dyn}$
$\Delta/m_{\rm dyn}$
$\mu_0/m_{\rm dyn}$

$m_0=0$
$m_0=0.14m_{\rm dyn}$
$m_0=0.42m_{\rm dyn}$
$m_0=0.70m_{\rm dyn}$
$(\mu_0-\mu)/m_{\rm dyn}$
$\mu_0/m_{\rm dyn}$

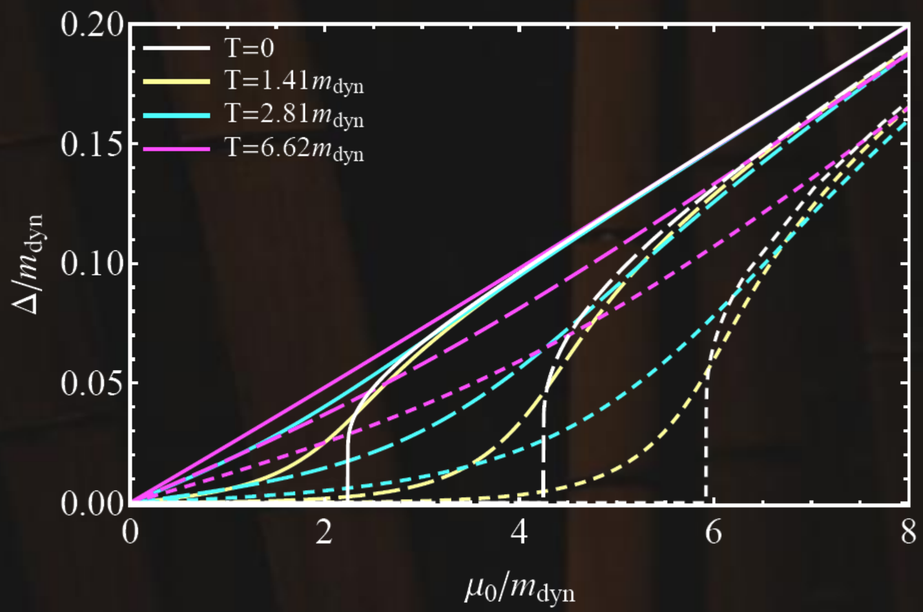



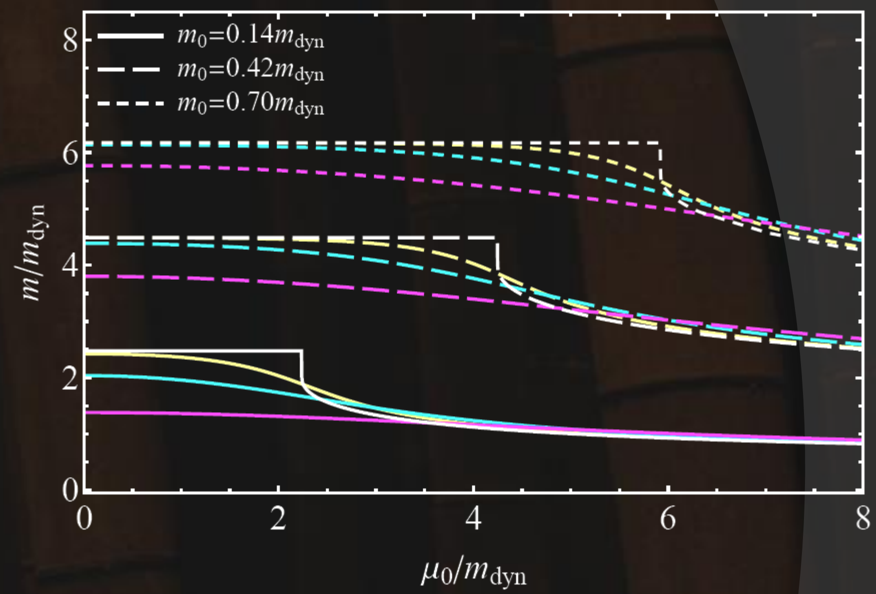
$m_0=0$
$m_0=0.14m_{\rm dyn}$
$m_0=0.42m_{\rm dyn}$
$m_0=0.70m_{\rm dyn}$
$m/m_{\rm dyn}$
$\mu_0/m_{\rm dyn}$

$m_0=0$, T=0
$4\pi^2\Omega/|eB|m_{\rm dyn}^2$
$\mu_0/m_{\rm dyn}$

# T≠0 results




- These are smoothed versions of the T=0 results
- The dependence $\mu$-$\mu_0$ versus $\mu_0$ (not shown) at T≠0 is similar to Δ versus $\mu_0$ (shown)

# Induced axial current

- The axial current in the ground state is

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = \boxed{\frac{eB}{2\pi^2}\mu} - \frac{|eB|}{2\pi^2}\Delta - \frac{|eB|}{\pi^2}\Delta\sum_{n=1}^{\infty}\kappa(\sqrt{2n|eB|},\Lambda)$$

- In addition to the topological contribution, $\frac{eB}{2\pi^2}\mu$ there are dynamical ones $\propto \Delta$
- An equivalent result is also obtained in the Pauli-Villars regularization
- Note: on the solution to the gap equation:

$$\langle j_5^3(u)\rangle = \frac{2\Delta}{G_{\text{int}}} = \frac{\Delta}{2\pi^2}\frac{\Lambda^2}{g}$$

# Potential implications

- Physical properties to be affected
  - transport
  - emission

    (must be sensitive to anisotropy and/or CP violation)
- Specific physical systems
  - Compact stars
    - Quark stars (quarks)
    - Hybrid stars (quarks, electrons)
    - Neutron stars (electrons)
    - White dwarfs (electrons)
  - Heavy ion collisions [Kharzeev & Zhitnitsky, NPA **797**, 67(2007), Kharzeev, McLerran & Warringa, NPA 803, 227 (2008), …]

# Pulsar kicks

- The dynamical chiral shift parameter is driven by chemical potential ($T \ll \mu$)

$$\Delta \simeq g\mu_0 eB/\Lambda^2$$

  and is almost independent of temperature


- This creates an anisotropy in the distribution of left-handed quarks/electrons
- The anisotropy is transferred to left-handed neutrinos by elastic scattering
- Pulsar gets a kick when neutrinos escape

# Supernova explosions

- Because of the robustness of the axial currents at finite temperatures, even supernova explosions may be affected

- A small early-time neutrino asymmetry may *facilitate* explosions and give a kick at the same time, e.g., see

  [Fryer & Kusenko, Astrophys. J. Supp. **163**, 335 (2006)]

# Summary

- $\mu < \mu_c$: Chiral symmetry is broken in the ground state (magnetic catalysis)

- $\mu > \mu_c$: Normal ground state of dense relativistic matter in a magnetic field is characterized by
  - Chiral shift parameter (may have dramatic implications for stars)
  - Axial current along the field (physical effects are not obvious)
  - No solution with vanishing $\Delta$ exists

# Outlook

- Calculation of neutrino emission/diffusion in the state with a chiral shift parameter (work in progress)
- Transport properties of the normal state with nonzero chiral shift parameter
- The fate of the induced axial current in the renormalized models (work in progress)
- Modification of the chiral magnetic effect due to “vector-like” $\Delta$ in heavy ion collisions

  [Fukushima, Kharzeev & Warringa, PRD **78**, 074033 (2008)]

# Thank you