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Dynamics in the normal ground state of dense relativistic matter in magnetic field

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Collaborators: E. V. Gorbar & V. A. Miransky

Phys. Rev. C 80 (2009), 032801(R);
Phys. Lett. B 695 (2011) 354;
Phys. Rev. D 83 (2011) 085003.

General idea

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2}\mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

- Is there a dynamical parameter $\boldsymbol{\Delta}$ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \mathcal{L}_\Delta \simeq \Delta \bar{\psi} \gamma^3 \gamma^5 \psi$$

- Note: $\boldsymbol{\Delta}$=0 is not protected by any symmetry

Axial anomaly

- Does the chiral shift modify the axial anomaly relation?

- Using the point splitting method, one derives

$$\begin{aligned}\langle \partial_\mu j_5^\mu(u) \rangle &= -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right) \\ &\rightarrow -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \rightarrow 0\end{aligned}$$

[E. V. Gorbar & V. A. Miransky, I.A. Shovkovy, Phys. Lett. B 695 (2011) 354]

- Therefore, the chiral shift does not affect the conventional axial anomaly relation

Gap equation

- NJL model (local interaction)

- This leads to three equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle$$ ("effective" chemical potential)

$$m = m_0 - G_{\text{int}} \langle \bar{\psi}\psi \rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\text{int}} \langle j_5^3 \rangle$$ (chiral shift parameter)

Solutions

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\text{int}}|eB|}\right) \qquad \left(|\mu_0| \lesssim \frac{m}{\sqrt{2}}\right)$$

$$\Delta = 0 \quad \& \quad \mu = \mu_0$$

- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2} \qquad \left(|\mu_0| \gtrsim \frac{m}{\sqrt{2}}\right)$$

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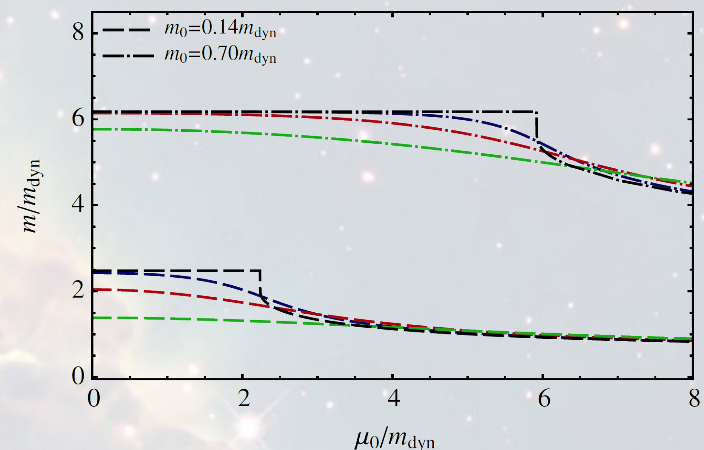

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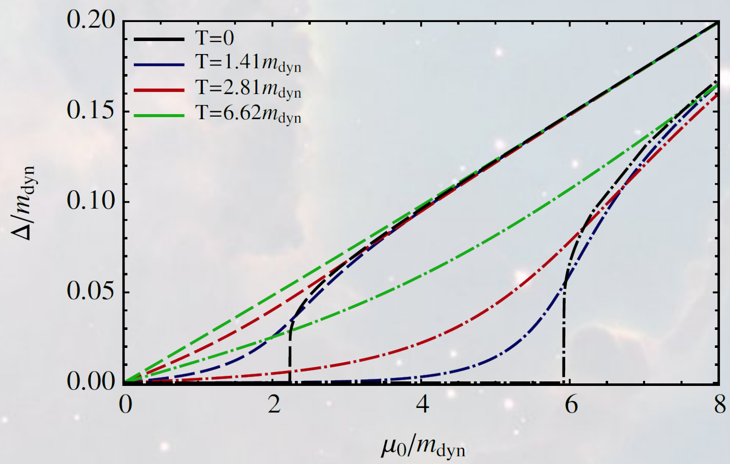


Chiral shift and Fermi surface

- Chirality is approx. well defined at Fermi surface ($|\boldsymbol{k}^3| \gg \boldsymbol{m}$)
- L-handed Fermi surface:

$$n = 0: \ k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \ k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

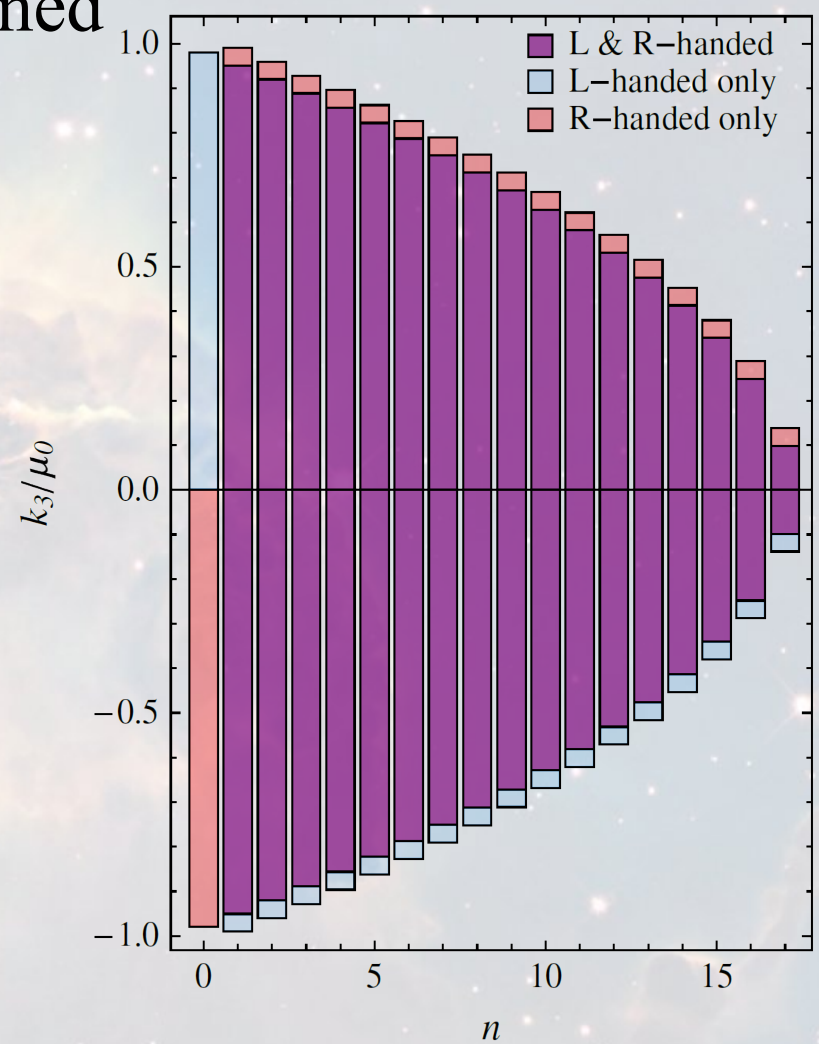
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \ k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \ k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Summary

- New dynamical parameter (chiral shift) is generated in magnetized dense matter
  - It leads to chiral asymmetry at the Fermi surface,
  - but axial anomaly relation is not modified
- Potential applications:
  - Pulsar kicks (?)
    - quark stars; neutron stars (?)
  - Facilitation of supernova explosions (?)
  - Axial current in QGP (at high temperature)
    - modified CME (chiral magnetic effect)