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Chiral asymmetry and axial anomaly in magnetized relativistic matter

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Matter in magnetic field

- Relativistic matter & strong magnetic fields ($10^8 - 10^{15}$ G) are common inside compact stars
  - **Electrons** in white dwarfs

    $T \ll m \lesssim \mu$ (i.e., $T \lesssim 1$ keV & $\mu \lesssim 1$ MeV)
  - **Protons** in dense neutron matter

    $T \ll m \lesssim \mu$ (i.e., $T \lesssim 10$ MeV & $\mu \simeq 1$ GeV)
  - **Electrons** in dense neutron matter

    $m \lesssim T \ll \mu$ (i.e., $T \lesssim 10$ MeV & $\mu \lesssim 100$ MeV)
  - **Quark matter** in stellar cores (if formed)

    $T \lesssim m \ll \mu$ (i.e., $T \lesssim 10$ MeV & $\mu \lesssim 500$ MeV)

General idea

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2}\mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

- Is there a dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \mathcal{L}_\Delta \simeq \Delta \bar{\psi}\gamma^3\gamma^5\psi$$

- Note: Δ=0 is not protected by any symmetry

Axial anomaly

- Does the chiral shift modify the axial anomaly relation?
- Using point splitting method, one derives

$$\langle \partial_\mu j_5^\mu(u) \rangle = -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right)$$

$$\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \quad \text{for} \quad \epsilon \to 0$$

[E. V. Gorbar & V. A. Miransky, I.A. Shovkovy, Phys. Lett. B 695 (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- Does the chiral shift give any contribution to the axial current?
- In point splitting method, one derives

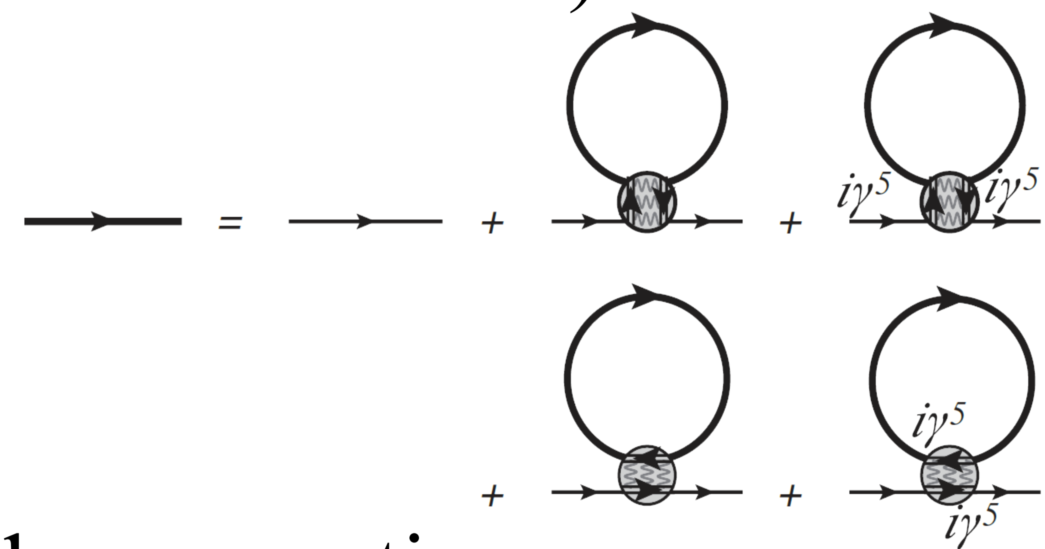
$$\langle j_5^\mu \rangle_{\mathrm{singular}} = -\frac{\Delta}{2\pi^2\epsilon^2}\delta_3^\mu \sim \frac{\Lambda^2\Delta}{2\pi^2}\delta_3^\mu$$

[E. V. Gorbar & V. A. Miransky, I.A. Shovkovy, Phys. Lett. B 695 (2011) 354]

- This is consistent with the NJL calculations
- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the axial current is finite

Chiral shift in NJL model

- NJL model (local interaction)



- This leads to three equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle \quad \text{(“effective” chemical potential)}$$

$$m = m_0 - G_{\text{int}} \langle \bar{\psi}\psi \rangle \quad \text{(dynamical mass)}$$

$$\Delta = -\frac{1}{2} G_{\text{int}} \langle j_5^3 \rangle \quad \text{(chiral shift parameter)}$$

Solutions

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\rm int}|eB|}\right) \qquad \left(|\mu_0| \lesssim \frac{m}{\sqrt{2}}\right)$$

$$\Delta = 0 \quad \& \quad \mu = \mu_0$$

- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2} \qquad \left(|\mu_0| \gtrsim \frac{m}{\sqrt{2}}\right)$$

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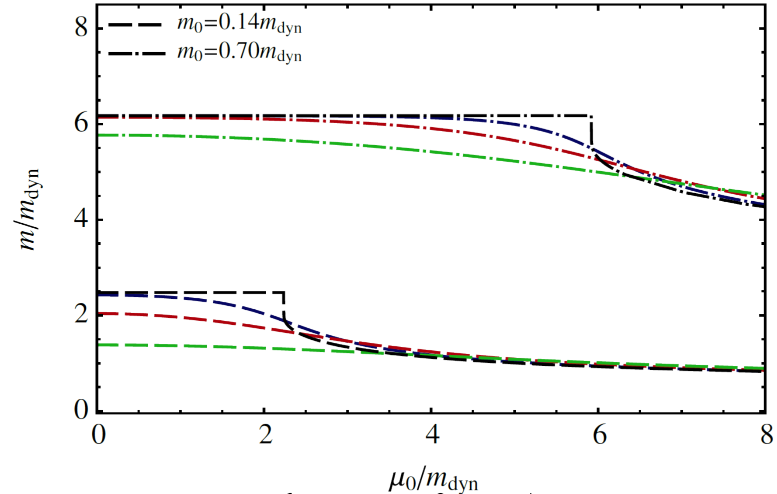

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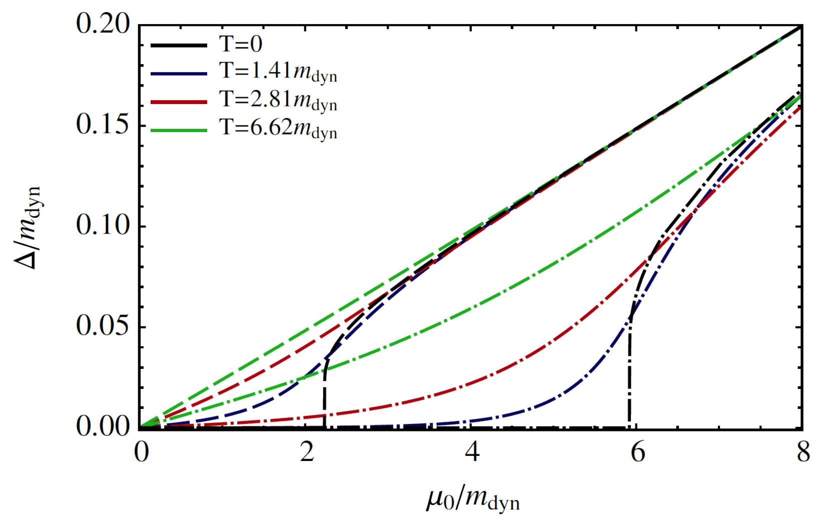


Chiral shift and Fermi surface

- Chirality is approx. well defined at Fermi surface ($|\boldsymbol{k}^3| \gg \boldsymbol{m}$)
- L-handed Fermi surface:

$$n = 0: \; k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \; k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

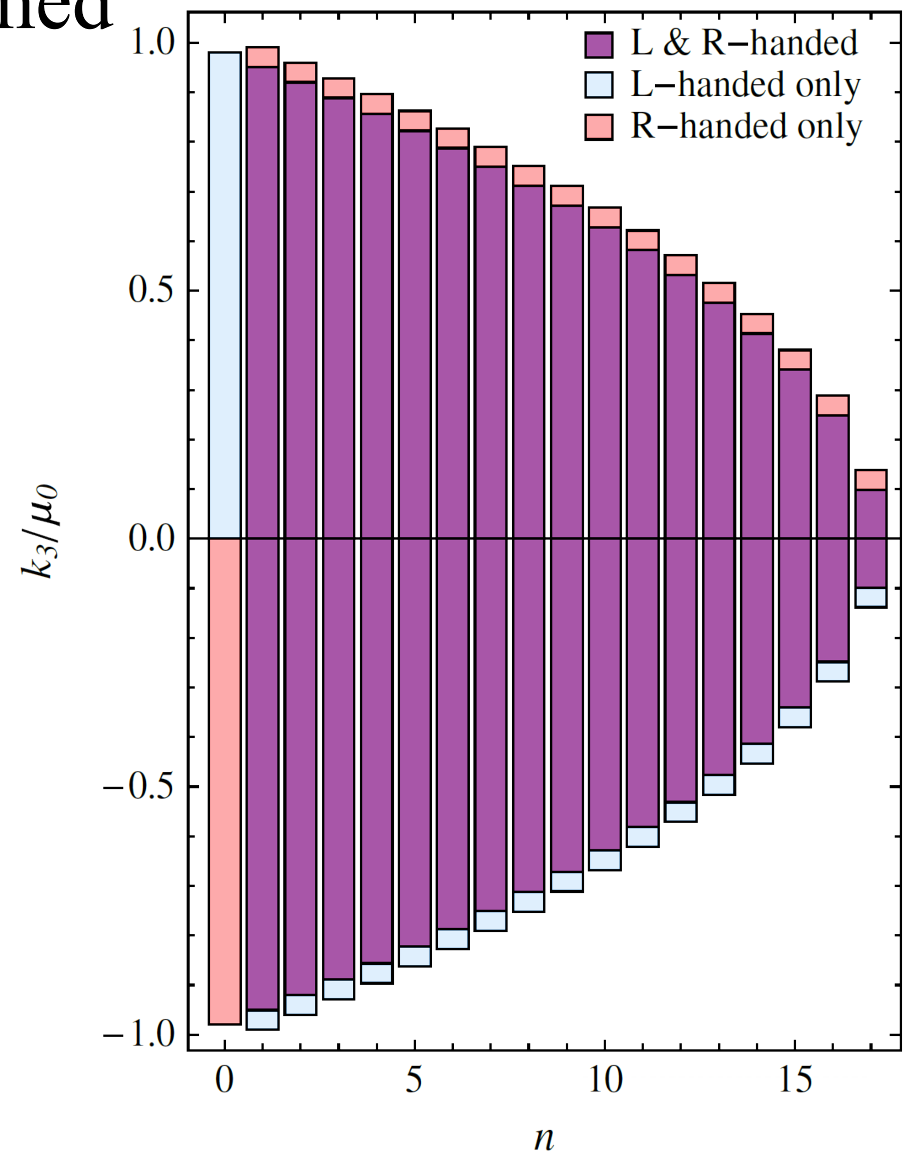
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \; k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

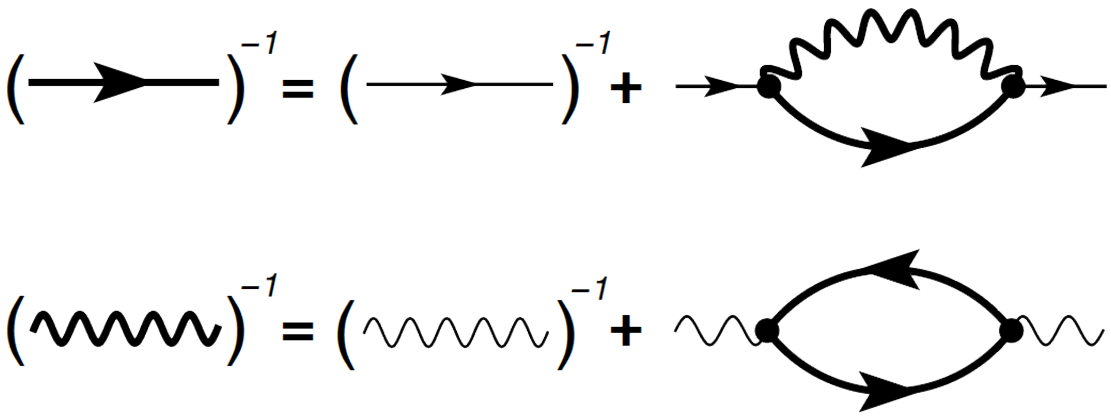
$$n > 0: \; k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Preliminary results in QED

- Chiral shift is also generated in QED



- Screening effects are important
- Large density estimate for the chiral shift:

$$\Delta^{QED} \simeq -\frac{\alpha}{\pi}\frac{eB}{M_D^2}\mu\,, \quad \text{where} \quad M_D = \frac{\alpha\mu}{2\pi}$$

- Numerically, $\Delta \simeq 1 - 100\ \mathrm{MeV} \quad (B \simeq 10^{13} - 10^{16}\ \mathrm{G})$

Summary

- New dynamical parameter (chiral shift) is generated in magnetized dense matter
- Chiral shift induces a chiral asymmetry at the Fermi surface
- Potential applications:
  - Pulsar kicks (?)
    - Quark stars
  - Facilitation of supernova explosions (?)
  - Axial current in QGP
    - modified CME (chiral magnetic effect)