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RELATIVISTIC MATTER IN MAGNETIC FIELDS

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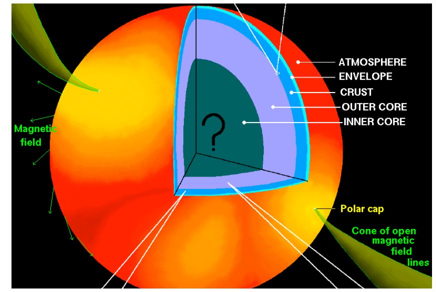
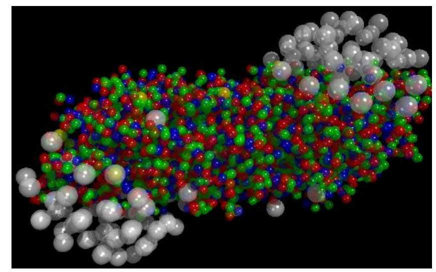
OUTLINE

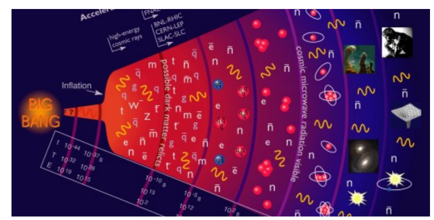
- **Introduction**
  - Relativistic matter; graphene
  - Extreme magnetic fields in nature
- **Magnetic catalysis**
  - Dimensional reduction; pairing dynamics
- **Quantum Hall effect in graphene**
  - Observable features; theoretical ideas
- **Chiral magnetic effect & beyond**
  - Anomalous symmetries; chirality; interactions
- **Summary**

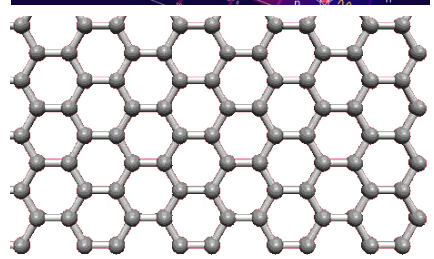
RELATIVISTIC MATTER

- Examples of relativistic matter
  - **Electrons, protons, quarks** inside compact stars (white dwarfs, neutron, hybrid or quark stars)
  - **Quark gluon plasma** in heavy ion collisions ($k_B T \sim 200$ MeV $\sim 10^{12}$ K)
  - **Hot matter** in the Early Universe ($k_B T \sim 100$ GeV at EW transition)
  - **Quasiparticles** in graphene (zero mass Dirac fermions)









WHAT MEANS "RELATIVISTIC"?

- **Relativistic matter** ($p \gg mc$)

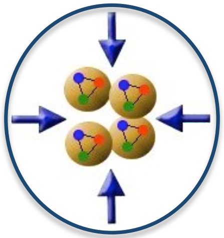
$$E = c\sqrt{p^2 + m^2c^2} \approx cp$$

- compare with nonrelativistic case ($p \ll mc$)

$$E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

  – **High density** (e.g., in stars) leads to occupation of states with large momenta:

$$p \sim \hbar n^{1/3} \simeq 200 \left(\frac{n}{1\ \mathrm{fm}^3}\right)^{1/3}\ \mathrm{MeV/c}$$



  – **High temperature** (e.g., heavy ion collisions) means energetic particles,

$$p \sim k_B T/c \simeq 200 \left(\frac{k_B T}{200\ \mathrm{MeV}}\right)\ \mathrm{MeV/c}$$

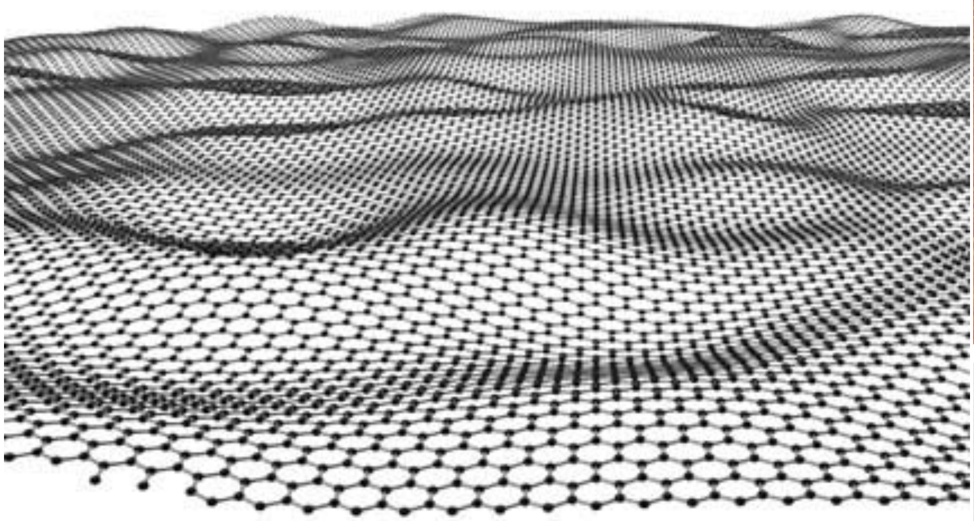
  – **Vanishing mass** (e.g., graphene) works too…

GRAPHENE

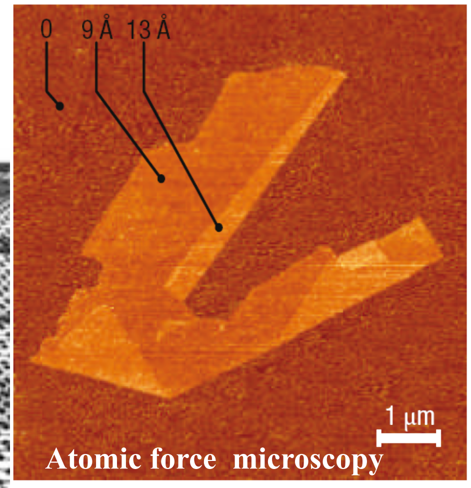
- It is a single atomic layer of graphite
  [Novoselov et al., Science **306**, 666 (2004)]

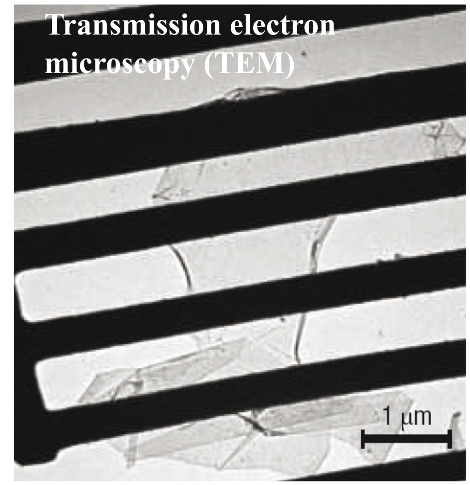
2010

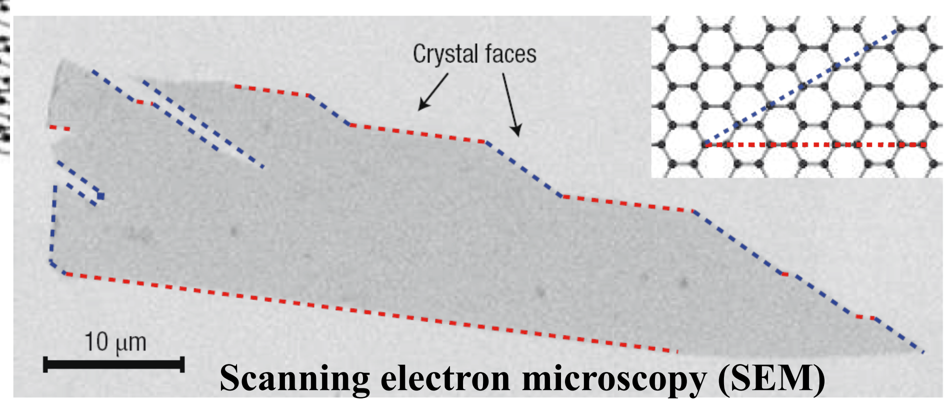
- Great promise for basic & applied physics



2D crystal with hexagonal lattice of carbon atoms



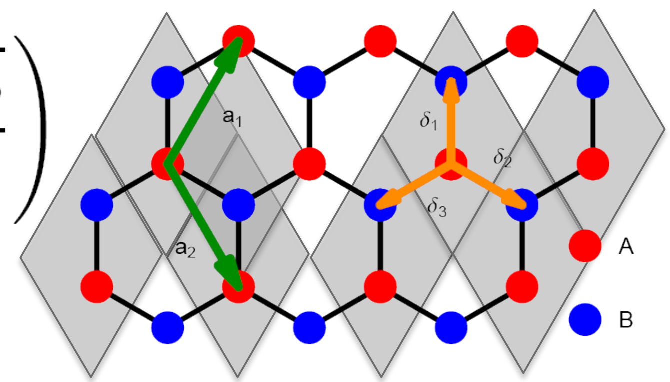




EMERGENCE OF DIRAC FERMIONS

- Translation vectors of the hexagonal lattice

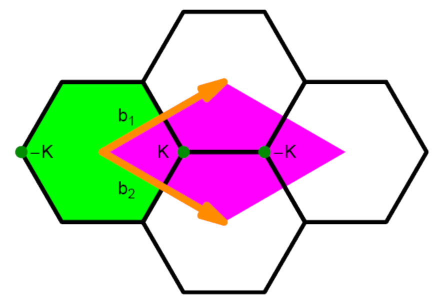
$$\mathbf{a}_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad \mathbf{a}_2 = a\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$



Lattice constant: $a \approx 1.42\ \text{Å}$

- Two carbon atoms per primitive cell

- Reciprocal lattice



$$\mathbf{b}_1 = 2\pi/a(1, 1/\sqrt{3}),\ \mathbf{b}_2 = 2\pi/a(1, -1/\sqrt{3})$$

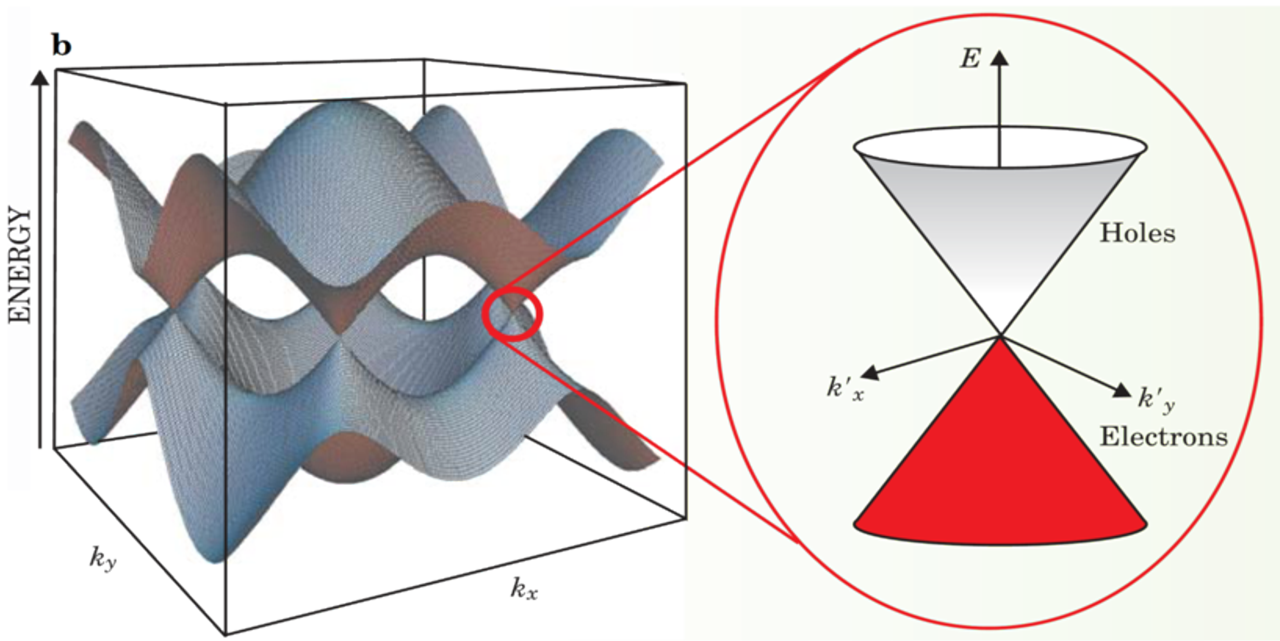
- Two (K & K′) Dirac points in the Brillouin zone

DIRAC FERMIONS IN GRAPHENE

- At low energies, quasiparticles in graphene are **massless** Dirac fermions ($v_F \approx c/300$)

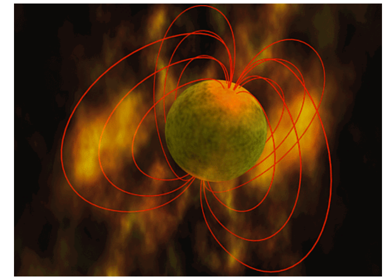
$$\Psi_s = \begin{pmatrix} \psi_{KAs} \\ \psi_{KBs} \\ \psi_{K'Bs} \\ \psi_{K'As} \end{pmatrix}$$

[Wallace, Phys. Rev. **71,** 622 (1947)]
[Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)]



Magnetic Fields

- Strong magnetic fields are common inside compact stars
  - **10^{10} to 10^{15} Gauss**



- In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

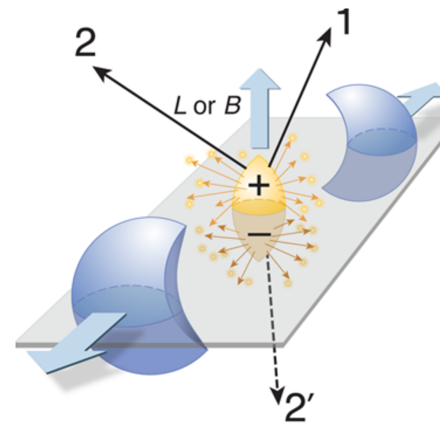


Illustration by Carin Cain

- Early Universe
  - **up to 10^{24} Gauss**
- Graphene (High Magnetic Field Laboratory)
  - **4.5×10^{5} Gauss**

MAGNETIC CATALYSIS

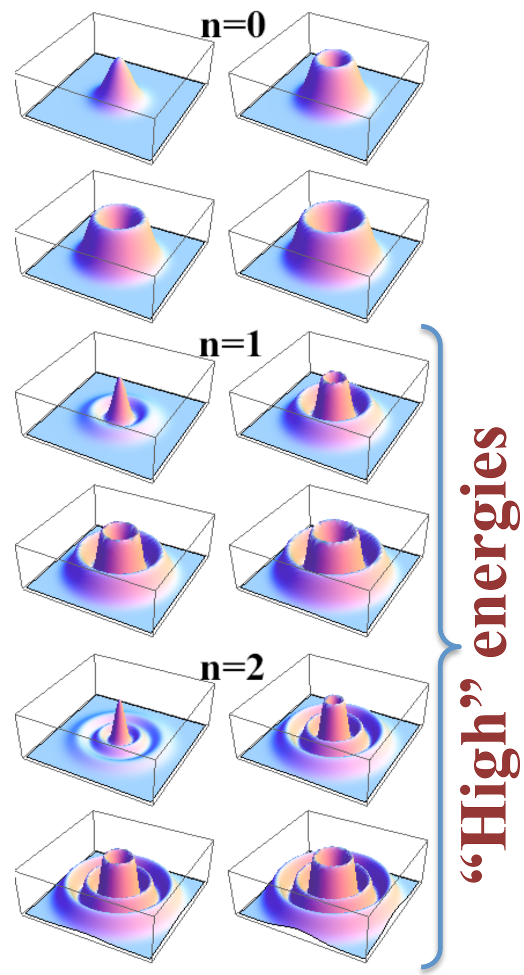
- Electron wave functions in magnetic field are localized [Gusynin, Miransky, Shovkovy, PRL **73** (1994) 3499]
- Effective dimensional reduction

  D space directions → ***D-2*** space directions
- Quantized energy levels:

$$E_n = \pm\sqrt{m^2c^4 + 2n\,\hbar c^2|eB|}$$
$$\simeq \pm\sqrt{2n\,\hbar c^2|eB|} \quad \text{for} \quad m \simeq 0$$

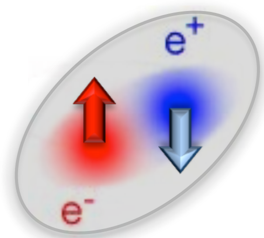
  – ***n* = *0***: zero energy for all states

  – ***n* ≥ *1*** states have "high" energies

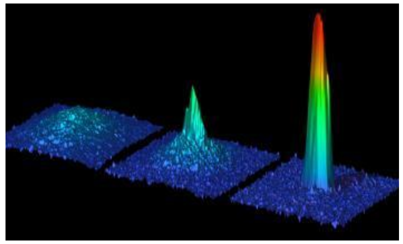


Pairing & Gap Generation

- Particles & anti-particles live in ***n=0*** level
- Forming bound states is energetically favorable (an energy gain of E_b per pair)



- Bound states are bosons
- Bosons can (and will) occupy the same lowest energy quantum state



- So, they form a Bose condensate
- The properties of the ground state change

[Gusynin, Miransky, Shovkovy, PRL **73** (1994) 3499]

NEW GROUND STATE

- Even if m_0=0 originally, a nonzero Dirac mass m_{dyn} is generated

2D: $m_{\rm dyn} \propto \sqrt{\alpha}\sqrt{|eB|}$ **3D:** $m_{\rm dyn} \propto \sqrt{|eB|}\, e^{-C_1/\alpha}$

- This happens even at the *weakest* interaction ("catalysis")
- The phenomenon is *universal* (specific details of interaction are mostly irrelevant)
- Dimensional *reduction* is the key ingredient (bound states form easier in low dimensions)

Magnetic Catalysis in Graphene

- Charge carriers are Dirac fermions with m=0
- Theoretically, $m_{dyn} \neq 0$ must be generated in a sufficiently strong magnetic field

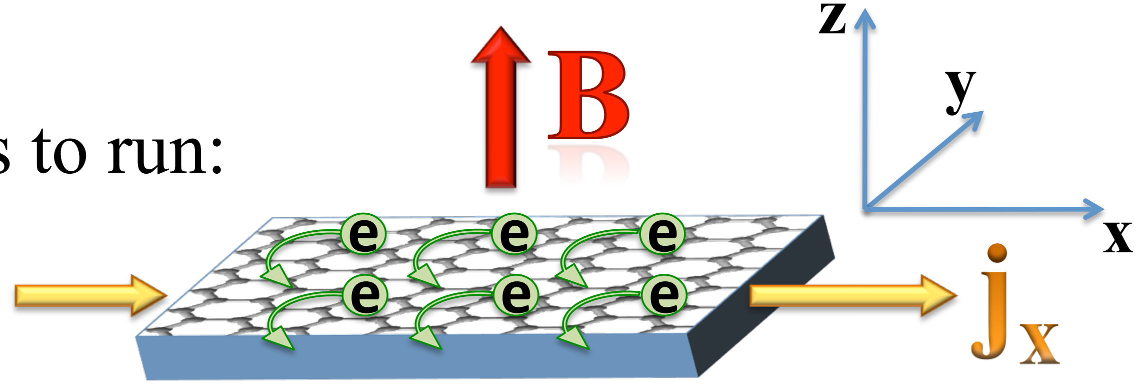
  [Khveshchenko, PRL **87**, 206401 (2001)]

  [Gorbar, Gusynin, Miransky, Shovkovy, PRB **66** (2002) 045108]

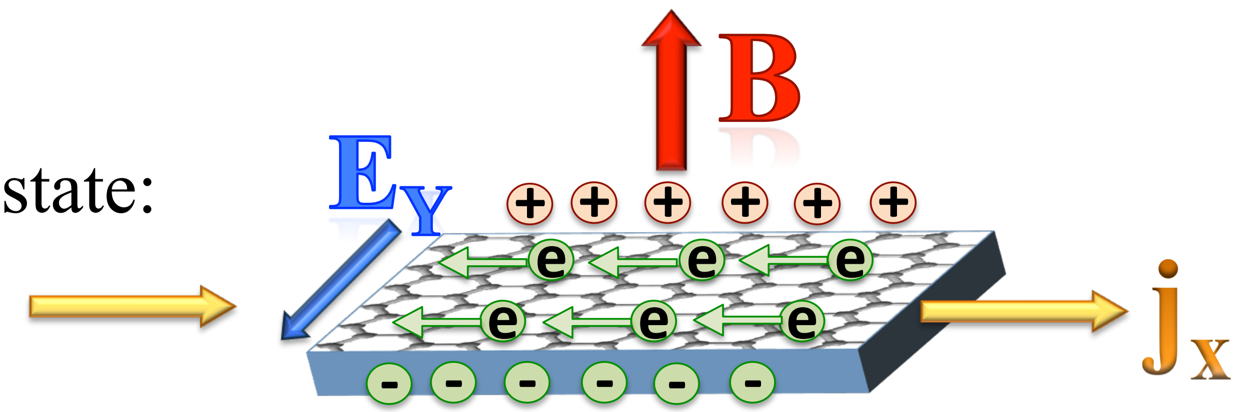
- Possible complications:
  - several types of Dirac masses may exist in **2D**
  - competition with quantum Hall ferromagnetism
  - nonzero electron/hole density
  - impurities, lattice defects, ripples, etc.
- How to test this experimentally?

QUANTUM HALL EFFECT

- General setup
  - Current starts to run:



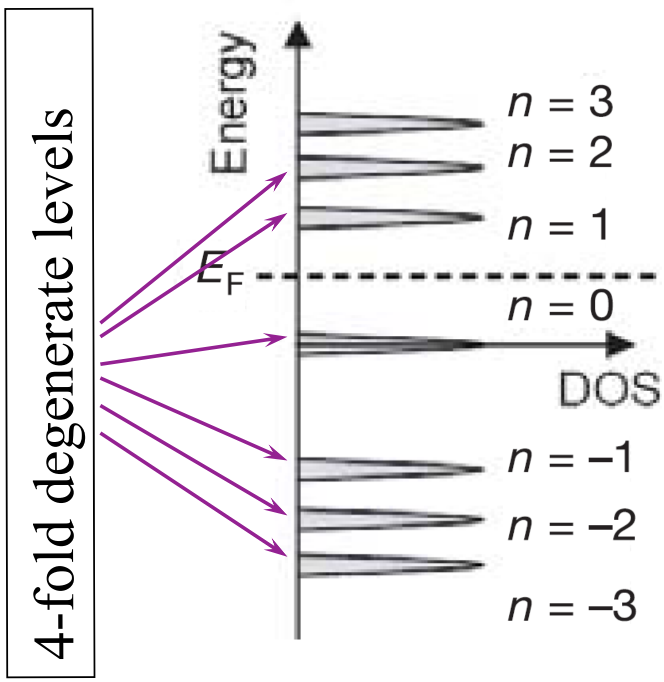
  - Steady state:

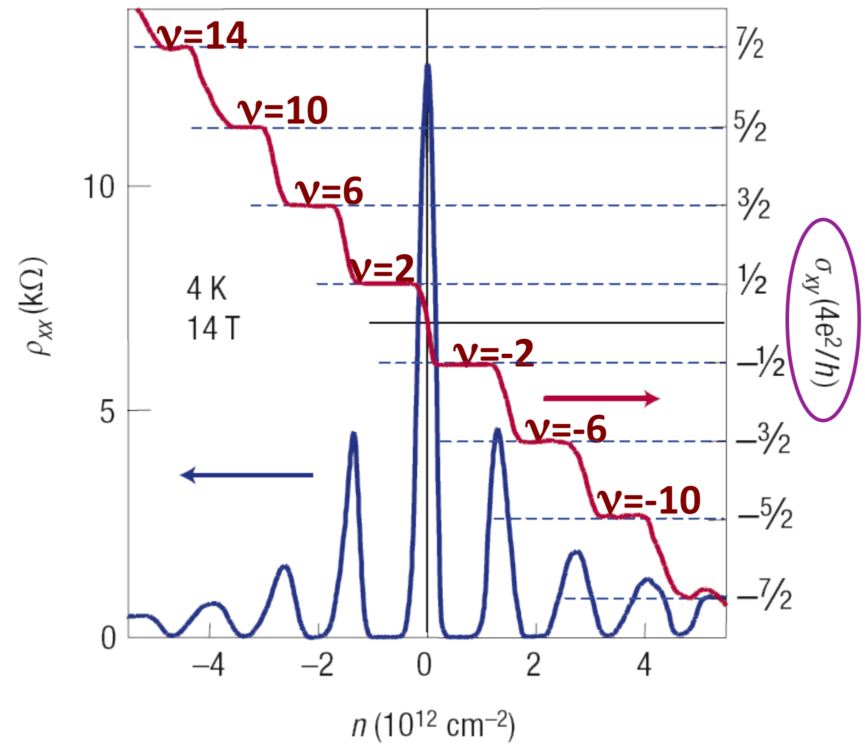


  - Hall conductivity $\boldsymbol{\sigma}_{xy}$:

$$\mathbf{j_x = \sigma_{xy}\, E_y}$$

QHE in Graphene





$$E_n = \mathrm{sgn}(n)\sqrt{2e\hbar v_F^2 |n| B}$$

$$\sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{h}\left(n + \frac{1}{2}\right)$$

[Gusynin, Sharapov, PRL **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, PRB **73**, 125411 (2006)]

} **Theory (m_0=0)**

[Novoselov et al., Nature **438**, 197 (2005)]
[Zhang et al., Nature **438**, 201 (2005)]

} **Experiment**

ANOMALOUS QHE

- New plateaus are observed at

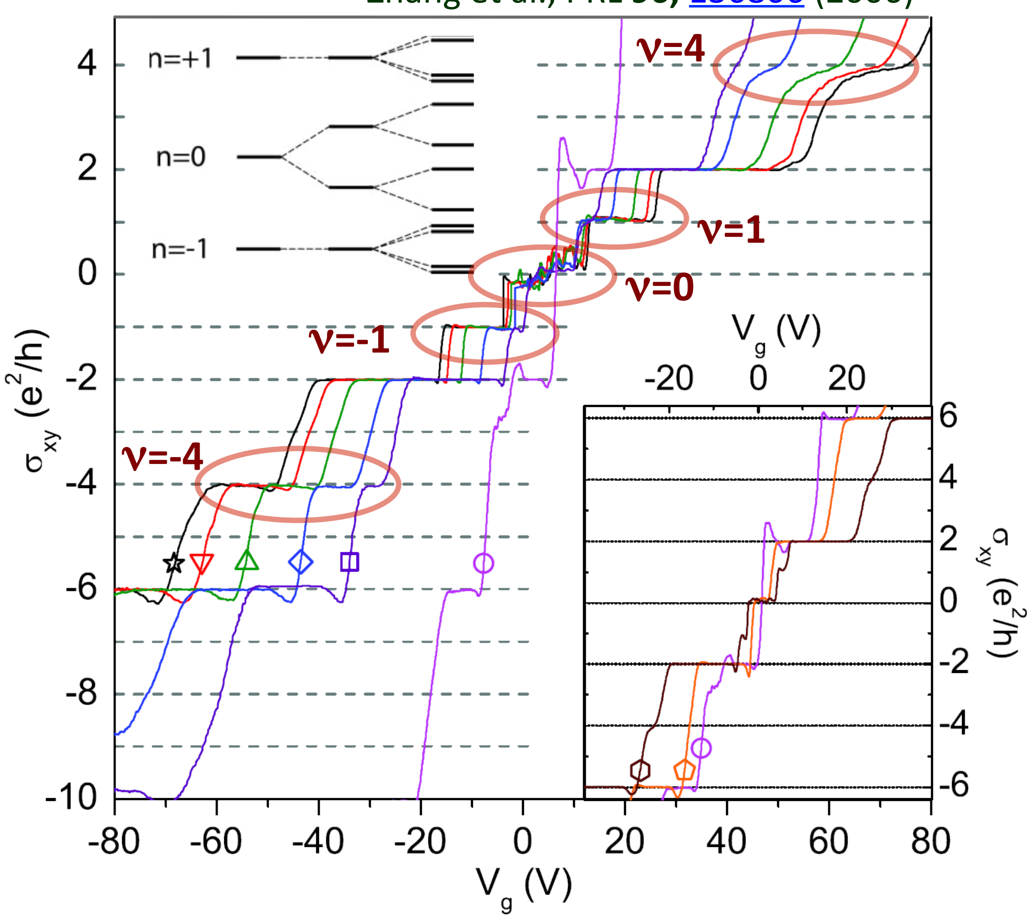
  ν=0

  ν=±1

  ν=±3

  ν=±4

Zhang et al., PRL **96**, 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]

[Abanin et al., PRL **98**, 196806 (2007)]

[Checkelsky et al., PRL 100, 206801 (2008)]

[Xu Du et al., Nature **462**, 192 (2009)]

EXPLANATION OF QHE

- Several different order parameters may be generated (pairing from different valleys/sublattices)

- Physical meaning of order parameters

$$\Delta_s: \quad n_{KAs} - n_{K'As} - n_{KBs} + n_{K'Bs} \quad \text{for} \quad s = \uparrow, \downarrow$$

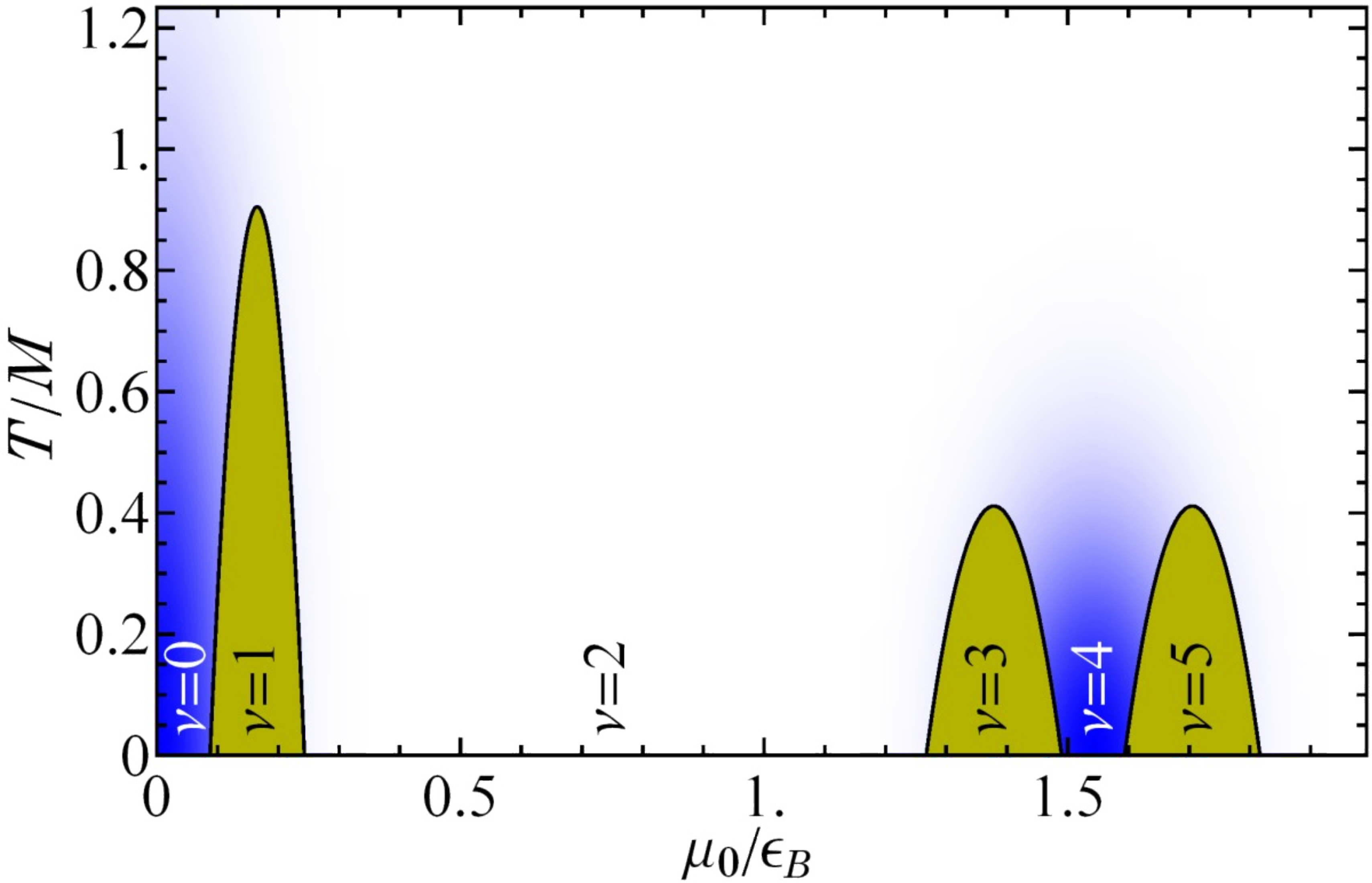
$$\tilde{\Delta}_s: \quad n_{KAs} + n_{K'As} - n_{KBs} - n_{K'Bs} \quad \text{for} \quad s = \uparrow, \downarrow$$

$$\tilde{\mu}_s: \quad n_{KAs} - n_{K'As} + n_{KBs} - n_{K'Bs} \quad \text{for} \quad s = \uparrow, \downarrow$$

$$\mu_3: \quad \frac{1}{2} \sum_{\kappa=K,K'} \sum_{a=AB} (n_{\kappa a\uparrow} - n_{\kappa a\downarrow})$$

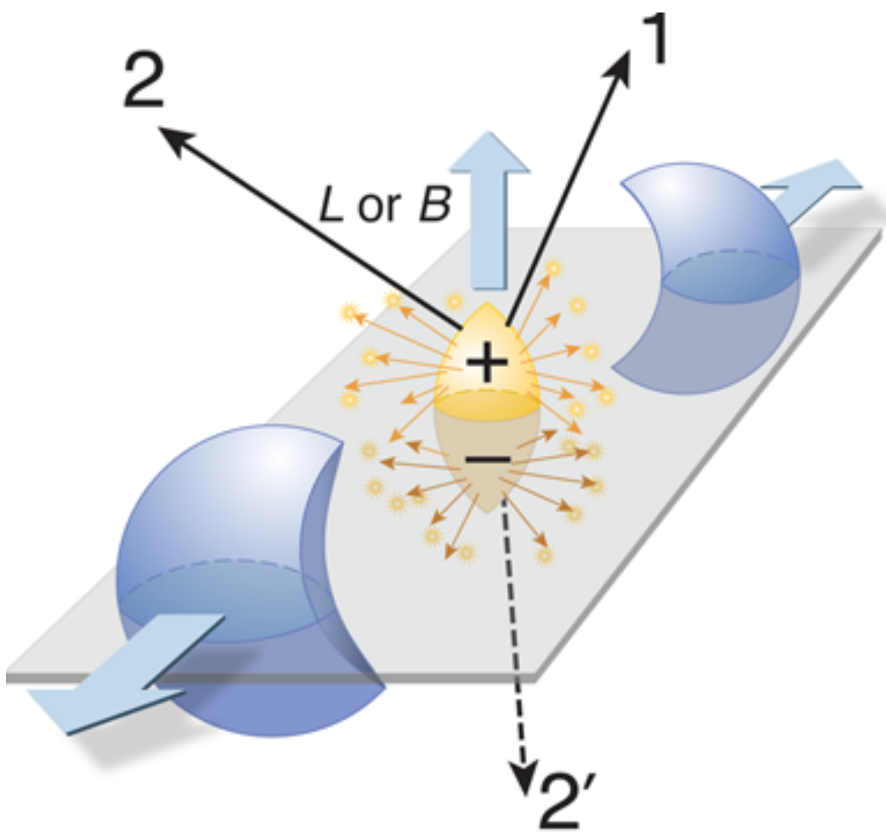
[Gorbar, Gusynin, Miransky, Shovkovy, PRB **78** (2008) 085437]
[Gorbar, Gusynin, Miransky, Shovkovy, arXiv:1105.1360]

PHASE DIAGRAM



Chiral Magnetic Effect

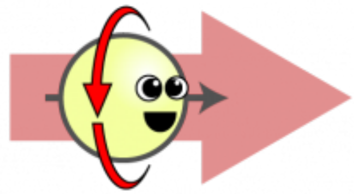
- A specific spatial pattern of electric currents (or charge correlations) in heavy ion collisions



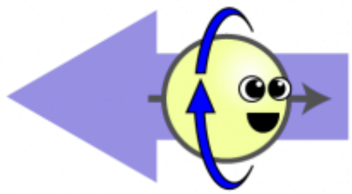
[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

HELICITY/CHIRALITY

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved



Righ-handed



Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)
- At quantum level, however, such symmetry is anomalous

“CONTINUITY” EQUATION

- Continuity equation for the chiral charge

$$\frac{\partial \rho_5}{\partial t} - \vec{\nabla}\vec{j}_5 = -\frac{e^2}{4\pi^2}(\vec{E}\cdot\vec{B})$$

  which is of topological nature and exact

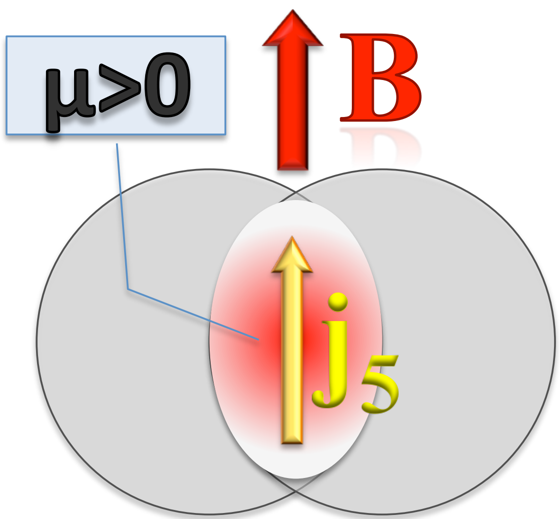
- Among its consequences are the relations:

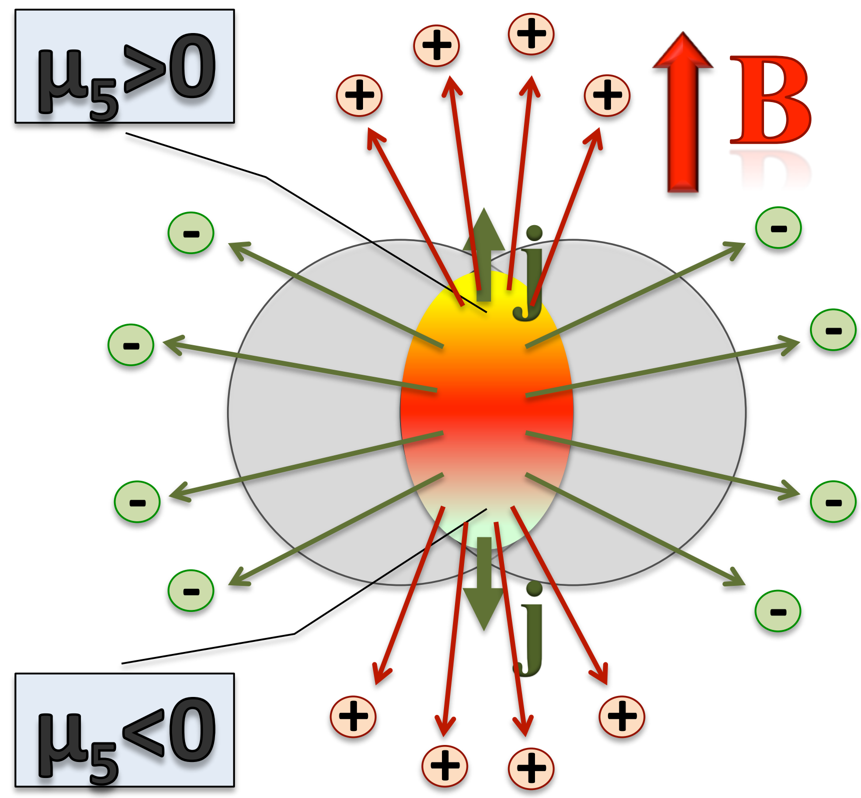
$$\vec{j}_5 = \frac{eB}{2\pi^2}\mu \qquad\qquad \vec{j} = \frac{eB}{2\pi^2}\mu_5$$

- These relations are the key relations leading to the *chiral magnetic effect*

CME: CHARGE CORRELATIONS

- Start from a small baryon density and B≠0





- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

BEYOND CME

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2} \mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

- Is it possible that interactions modify this relation?

- Is there a dynamical generation of a "chiral shift" Δ? (resembling m_{dyn} in the magnetic catalysis)

Axial anomaly

- The main result was derived from the axial anomaly relation…

  which is an **exact** result (even with interactions!)

- Recent developments
  - Axial anomaly relation is robust
  - Chiral shift is generated
  - Axial current may be modified

  [Gorbar, Miransky, Shovkovy, Phys. Lett. B 695 (2011) 354]

- The chiral shift Δ can be of great importance

Chiral shift and Fermi surface

- Chirality is a "good" concept at large density ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

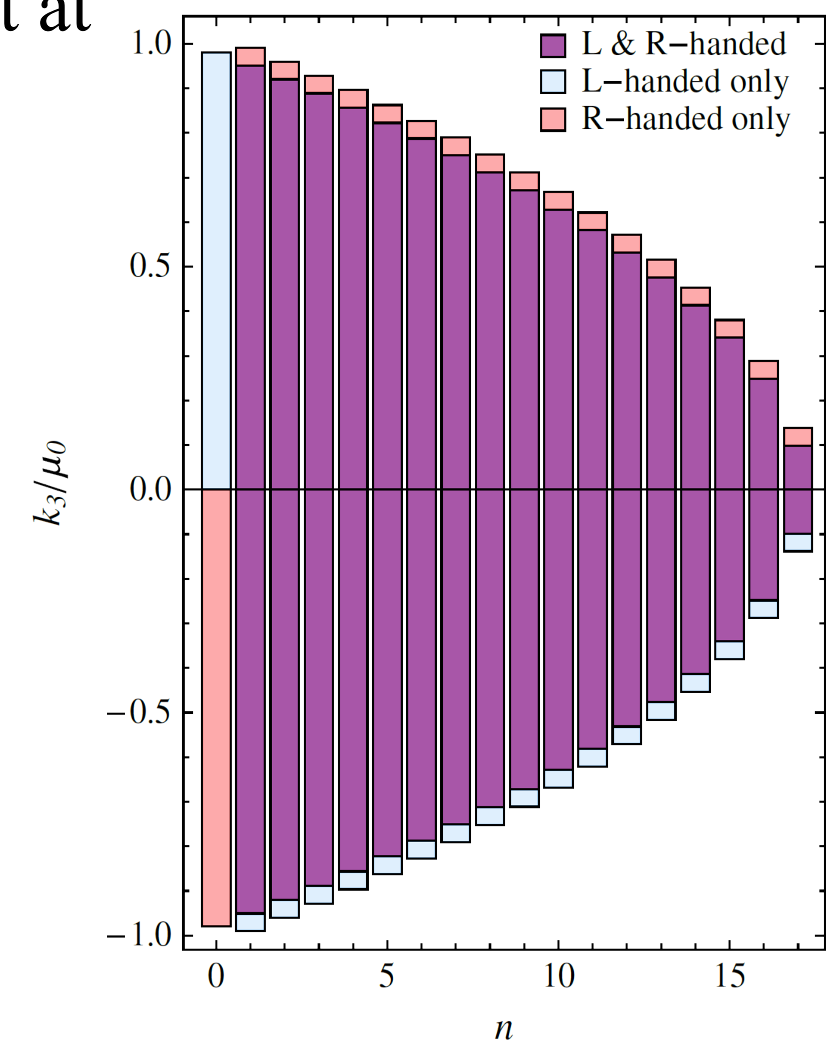
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Physics due to Chiral Shift

- Chiral shift induces a chiral asymmetry at the Fermi surface
- Chiral shift modifies axial current (≈ spin polarization)
- Potential applications:
  - Pulsar kicks
  - Facilitation of supernova explosions
  - modified Chiral magnetic effect

SUMMARY

- Studies of relativistic matter in magnetic field are relevant for many branches of physics
- The underlying physics is conceptually rich
- Recent developments include
  - Magnetic catalysis [Gusynin, Miransky, Shovkovy, PRL **73**, 3499 (1994)]
  - Chiral magnetic effect [Fukushima, Kharzeev, Warringa, PRD **78**, 074033 (2008)]
  - Chiral shift [Gorbar, Miransky, Shovkovy, PRC **80**, 032801 (R) (2009)]
  - Chiral magnetic spiral [Basar, Dunne, Kharzeev, PRL 104, 232301 (2010)]
  - Paraelectricity in QED [Ferrer, Incera, Sanchez, PRL **107**, 041602 (2011)]
  - Magnetic color superconductors […]
  - and many others […]