MAGNETIZED DENSE RELATIVISTIC MATTER

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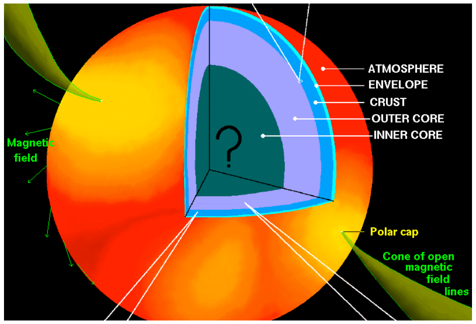
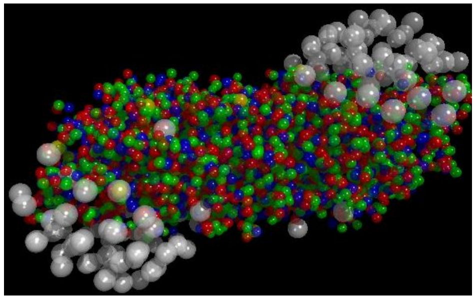
Relativistic Matter

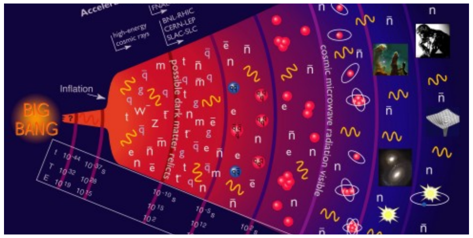
- Examples of relativistic matter
  - **Electrons, protons, quarks** inside compact stars (white dwarfs, neutron, hybrid or quark stars)



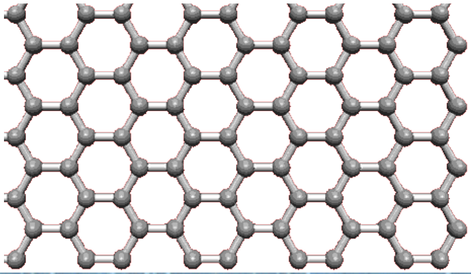
  - **Quark gluon plasma** in heavy ion collisions ($k_B T \sim 200$ MeV $\sim 10^{12}$ K)



  - **Hot matter** in the Early Universe ($k_B T \sim 100$ GeV at *EW* transition)



  - **Quasiparticles** in graphene (zero mass Dirac fermions)



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What Means "Relativistic"?

- **Relativistic matter** ($p \gg mc$)

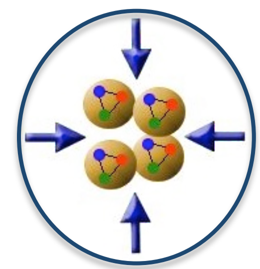
$$E = c\sqrt{p^2 + m^2c^2} \approx cp$$

- compare with nonrelativistic case ($p \ll mc$)

$$E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

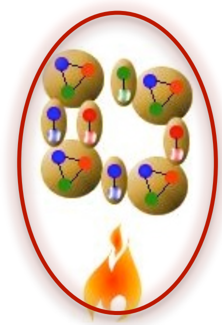
  – **High density** (e.g., in stars) leads to occupation of states with large momenta:

$$p \sim \hbar n^{1/3} \simeq 200 \left(\frac{n}{1\ \mathrm{fm}^3}\right)^{1/3}\ \mathrm{MeV/c}$$



  – **High temperature** (e.g., heavy ion collisions) means energetic particles,

$$p \sim k_B T/c \simeq 200 \left(\frac{k_B T}{200\ \mathrm{MeV}}\right)\ \mathrm{MeV/c}$$



  – **Vanishing mass** (e.g., graphene) works too…

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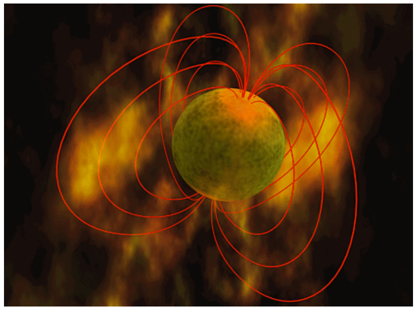
Magnetic Fields

- Strong magnetic fields are common inside compact stars
  - **10^{10} to 10^{15} Gauss**



- In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

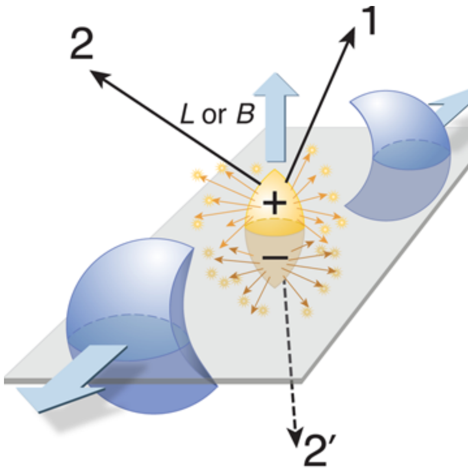
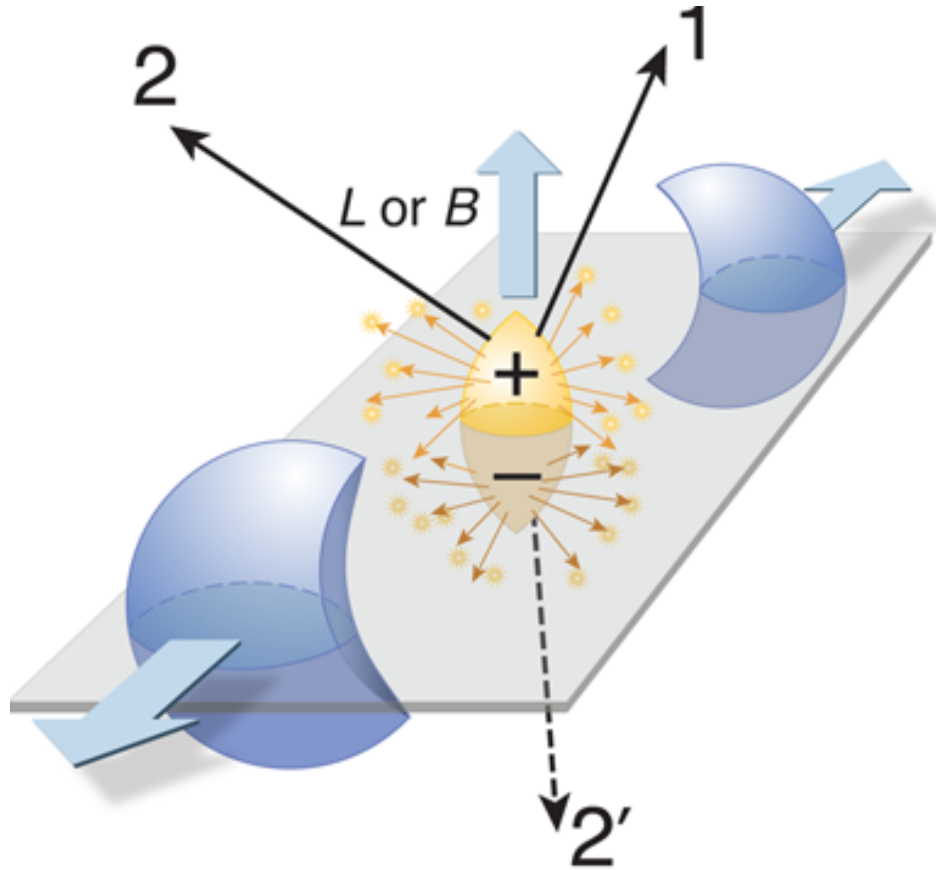


Illustration by Carin Cain

- Early Universe
  - **up to 10^{24} Gauss**
- Graphene (High Magnetic Field Laboratory)
  - **4.5×10^5 Gauss**

Chiral Magnetic Effect

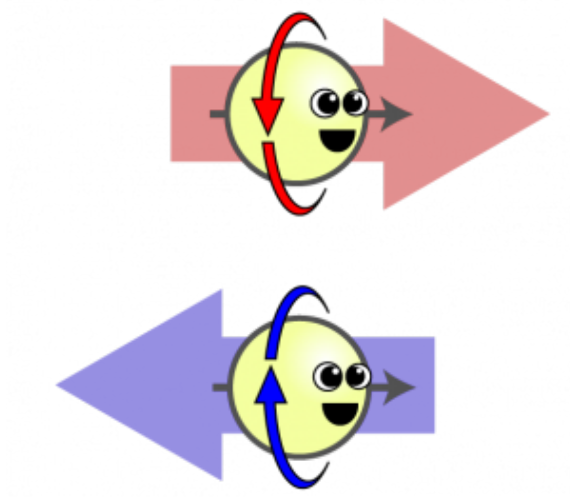
- A specific spatial pattern of electric currents (or charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

HELICITY/CHIRALITY

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved



Righ-handed

Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)
- At quantum level, however, such symmetry is anomalous

“CONTINUITY” EQUATION

- Continuity equation for the chiral charge

$$\frac{\partial \rho_5}{\partial t} - \vec{\nabla}\vec{j}_5 = -\frac{e^2}{4\pi^2}(\vec{E}\cdot\vec{B})$$

which is of topological nature and exact

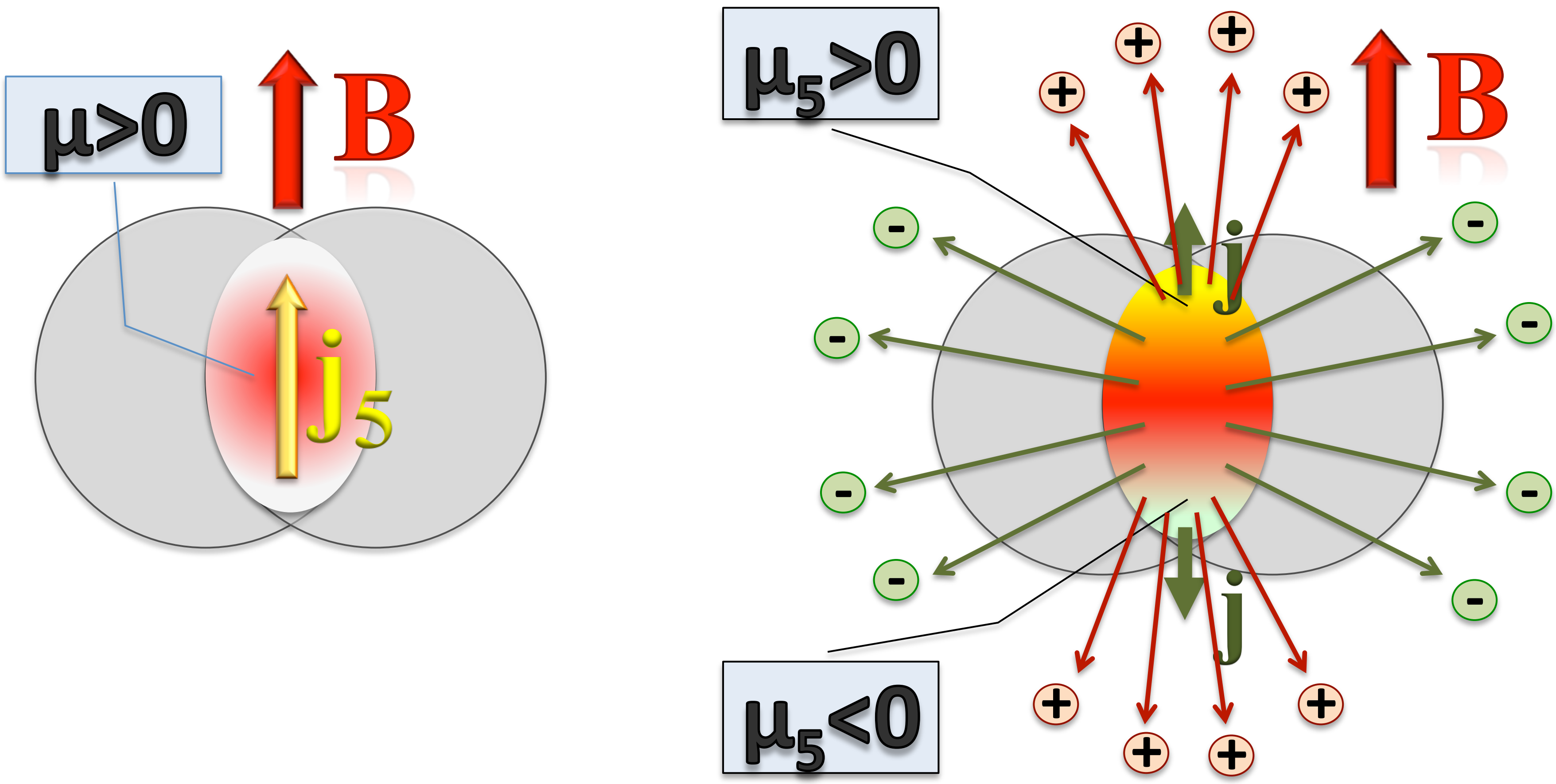
- Among its consequences are the relations:

$$\vec{j}_5 = \frac{eB}{2\pi^2}\mu \qquad \vec{j} = \frac{eB}{2\pi^2}\mu_5$$

- These relations are the key relations leading to the *chiral magnetic effect*

CME: CHARGE CORRELATIONS

- Start from a small baryon density and B≠0



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

BEYOND CME

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2} \mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

- Is it possible that interactions modify this relation?

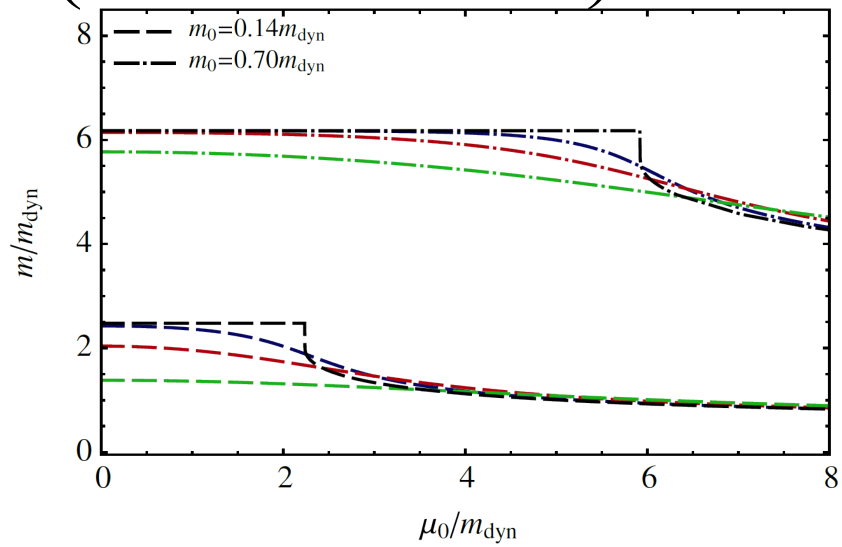
- Is there a dynamical generation of a “chiral shift” Δ? (Δ=0 is not protected by any symmetry)

Results in NJL model

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\text{int}}|eB|}\right)$$

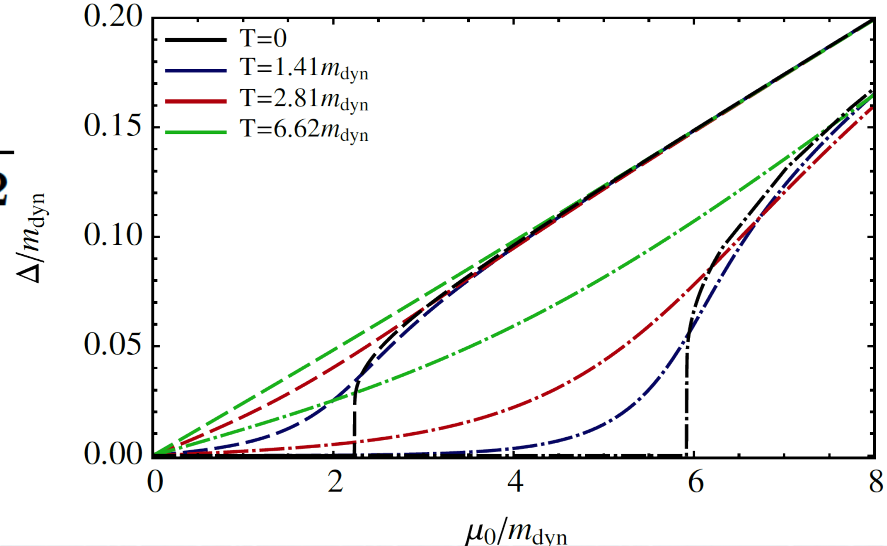
$$\Delta = 0 \quad \& \quad \mu = \mu_0$$



- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2}$$



AXIAL ANOMALY

- Does the chiral shift modify the axial anomaly relation?
- Using point splitting method, one derives

$$\langle \partial_\mu j_5^\mu(u) \rangle = -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right)$$

$$\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \to 0$$

[Gorbar, Miransky, Shovkovy, Phys. Lett. B 695 (2011) 354]

- Therefore, the chiral shift does not affect the conventional axial anomaly relation

AXIAL CURRENT

- Does the chiral shift give any contribution to the axial current?
- In point splitting method, one derives

$$\langle j_5^\mu \rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\epsilon^2}\delta_3^\mu \sim \frac{\Lambda^2\Delta}{2\pi^2}\delta_3^\mu$$

[Gorbar, Miransky, Shovkovy, Phys. Lett. B 695 (2011) 354]

- This is consistent with the NJL calculations
- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the axial current is finite

Chiral shift and Fermi surface

- Chirality is a "good" concept at large density ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n=0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n>0: \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

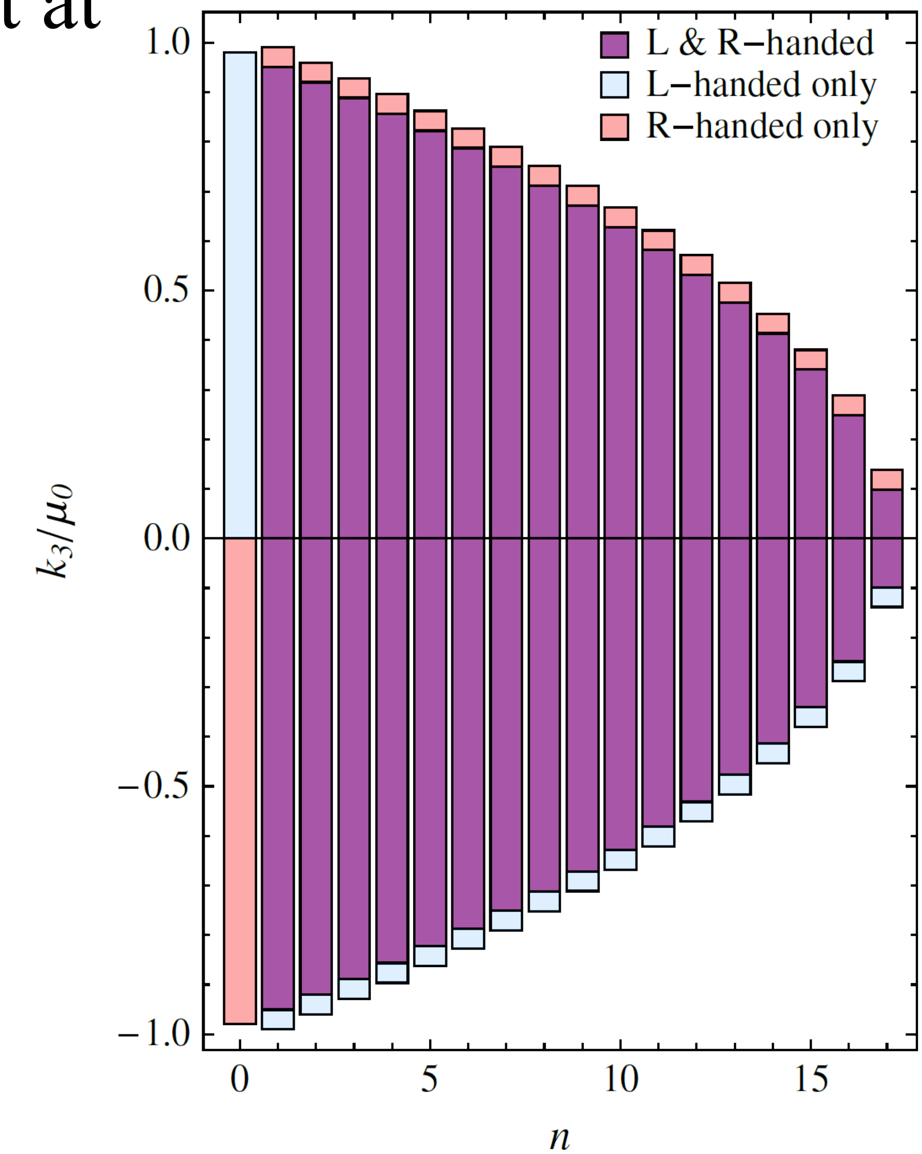
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n=0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

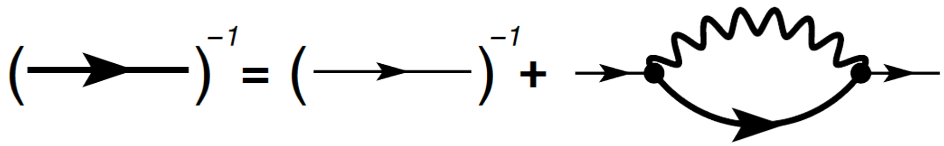
$$n>0: \quad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

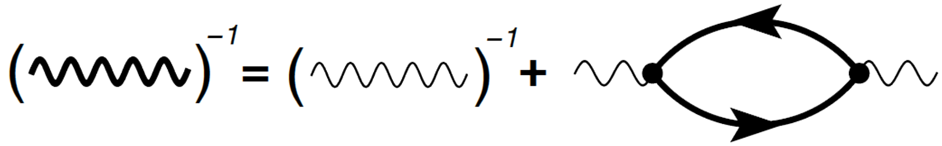
$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



CHIRAL SHIFT IN QED (PRELIMINARY)

- Chiral shift is also generated in QED





- Fermion propagator

A well-defined function of the parameters in the n^{th} Landau level

$$G(\boldsymbol{k}_{\parallel}, \mathbf{r}, \mathbf{r}') = e^{i\Phi(\mathbf{r},\mathbf{r}')} \bar{G}(\boldsymbol{k}_{\parallel}, \mathbf{r}-\mathbf{r}')$$

$$\bar{G}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = i e^{-k^2\ell^2} \sum_{n=0}^{\infty} (-1)^n D_n(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) \frac{1}{\mathcal{M} - 2n|eB|}$$

- Gauge boson propagator

$$D_{\mu\nu}(q) \simeq \frac{|\boldsymbol{q}|}{|\boldsymbol{q}|^3 + \frac{\pi}{4} m_D^2 |q_4|} O_{\mu\nu}^{(\mathrm{mag})} + \frac{O_{\mu\nu}^{(\mathrm{el})}}{q_4^2 + |\boldsymbol{q}|^2 + m_D^2}$$

WEAK FIELD LIMIT (PRELIMINARY)

- Weak field expansion

$$\bar{S}(\boldsymbol{k}_{\|},\boldsymbol{k}_{\perp}) = \bar{S}^{(0)}(\boldsymbol{k}_{\|},\boldsymbol{k}_{\perp}) + \bar{S}^{(1)}(\boldsymbol{k}_{\|},\boldsymbol{k}_{\perp}) + \cdots$$

$$\bar{S}^{(0)}(\boldsymbol{k}_{\|},\boldsymbol{k}_{\perp}) = i\frac{(\omega+\mu_0)\gamma^0 + m_0 - \boldsymbol{k}\cdot\boldsymbol{\gamma}}{(\omega+\mu_0)^2 - m_0^2 - k^2},$$

$$\bar{S}^{(1)}(\boldsymbol{k}_{\|},\boldsymbol{k}_{\perp}) = -\gamma^1\gamma^2 eB\frac{(\omega+\mu_0)\gamma^0 + m_0 - k_3\gamma^3}{\left[(\omega+\mu_0)^2 - m_0^2 - k^2\right]^2}$$

- Perturbative (linear in B) result for chiral shift:

$$\Delta^{(1)} \simeq -\frac{\alpha eB}{8\pi\mu_0}\left[\underbrace{\frac{2}{3}\sin^2\theta_{Bp}\ln\frac{2\mu_0}{|\omega'_E|}}_{\text{magnetic modes}} - \underbrace{\left(\frac{2}{3}\sin^2\theta_{Bp}+1\right)\ln\frac{C_1}{\alpha}}_{\text{electric modes}}\right]$$

- Estimate

$$\Delta^{(1)} \simeq 0.5\,\mathrm{keV}\left(\frac{400\,\mathrm{MeV}}{\mu_0}\right)\left(\frac{B}{10^{16}\,\mathrm{G}}\right)\ln\frac{\mu_0}{\alpha T} \qquad \text{at} \quad T\simeq 1\,\mathrm{keV}$$

STRONG FIELD LIMIT (PRELIMINARY)

- Lowest Landau level approximation

$$\Delta = \pi\alpha \int \frac{d^2k_{\parallel} d^2k_{\perp}}{(2\pi)^4} \mathrm{Tr}\left[\gamma^3\gamma^5\gamma^\mu \bar{S}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp})\gamma^\nu\right] D_{\mu\nu}(p-k)$$

where the LLL propagator is used

$$\bar{S}^{(\mathrm{LLL})}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = 2ie^{-k_\perp^2\ell^2} \frac{(\omega+\mu_0)\gamma^0 + m_0 - k^3\gamma^3}{(\omega+\mu_0)^2 - m_0^2 - (k^3)^2} P_-$$

- The value of the shift:

$$\Delta \simeq \frac{s_\perp \alpha \,\mathrm{sgn}(\mu_0)}{2}\sqrt{|eB|}$$

- Estimate:

$$\Delta \simeq 30\,\mathrm{keV}\sqrt{\frac{B}{10^{16}\,\mathrm{G}}}$$

PHYSICS DUE TO CHIRAL SHIFT

- Symmetry arguments & dynamics suggest that chiral shift is induced in magnetized matter
- High density: chiral asymmetry of Fermi surface
- High temperature: modified chiral magnetic effect
- Potential applications:
  - pulsar kicks
  - facilitation of supernova explosions
  - modified Chiral Magnetic Effect