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Chiral shift in renormalizable theories in magnetic field

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Motivation

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2} \mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

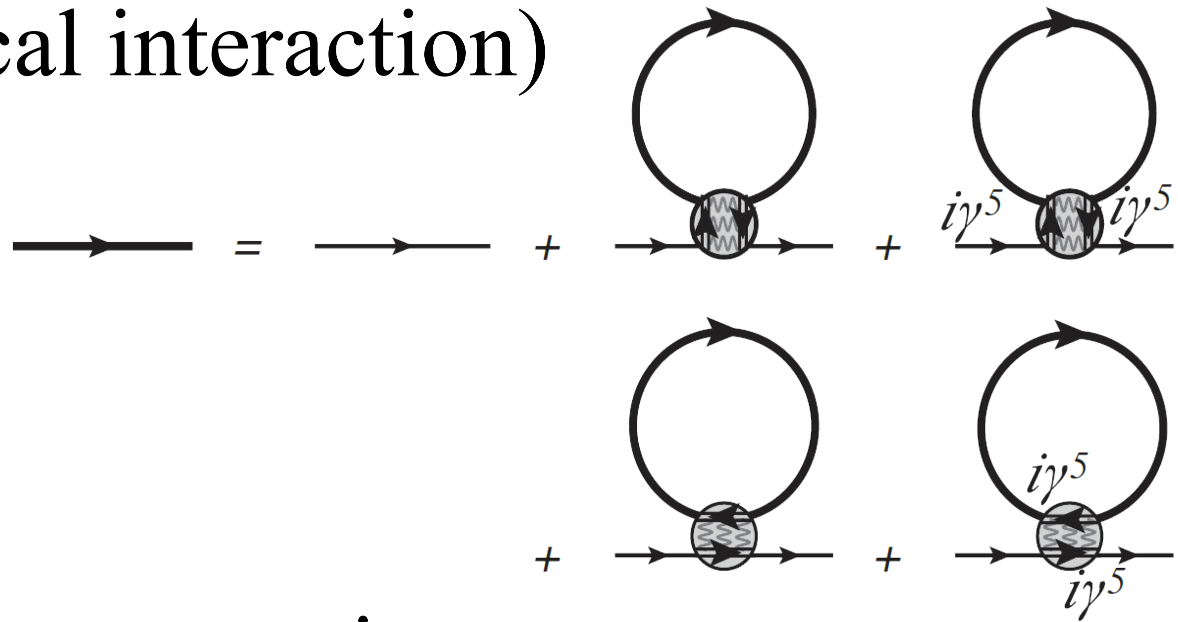
- Is there a dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \mathcal{L}_\Delta \simeq \Delta \bar{\psi} \gamma^3 \gamma^5 \psi$$

- Note: Δ=0 is not protected by any symmetry

Chiral shift in NJL model

- NJL model (local interaction)



- This leads to three equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle$$ (“effective” chemical potential)

$$m = m_0 - G_{\text{int}} \langle \bar{\psi}\psi \rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\text{int}} \langle j_5^3 \rangle$$ (chiral shift parameter)

Solutions

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\rm int}|eB|}\right) \qquad \left(|\mu_0| \lesssim \frac{m}{\sqrt{2}}\right)$$

$$\Delta = 0 \quad \& \quad \mu = \mu_0$$

- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2} \qquad \left(|\mu_0| \gtrsim \frac{m}{\sqrt{2}}\right)$$

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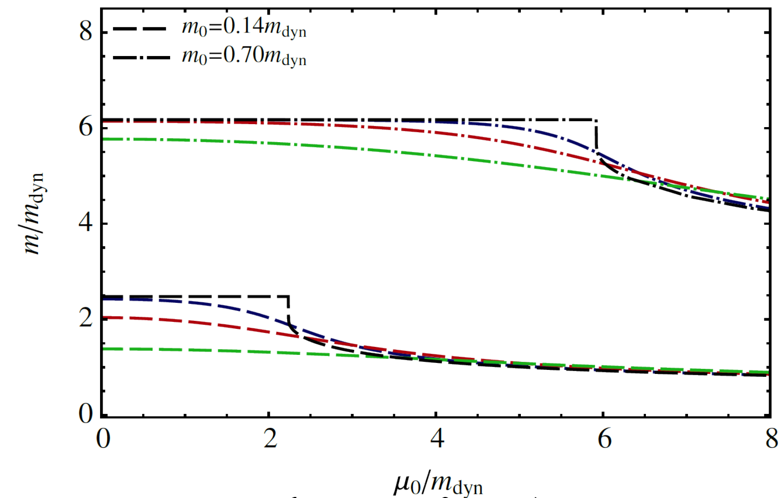

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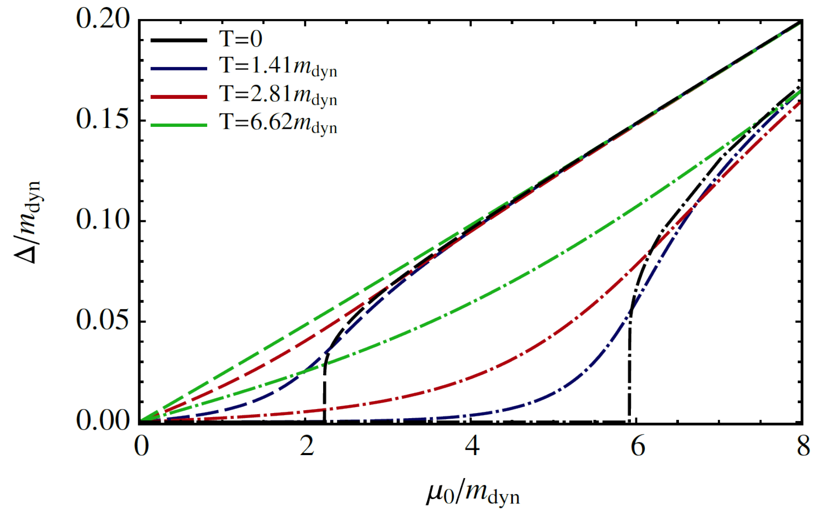


Chiral shift @ Fermi surface

- Chirality is $\approx$ well defined at Fermi surface ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

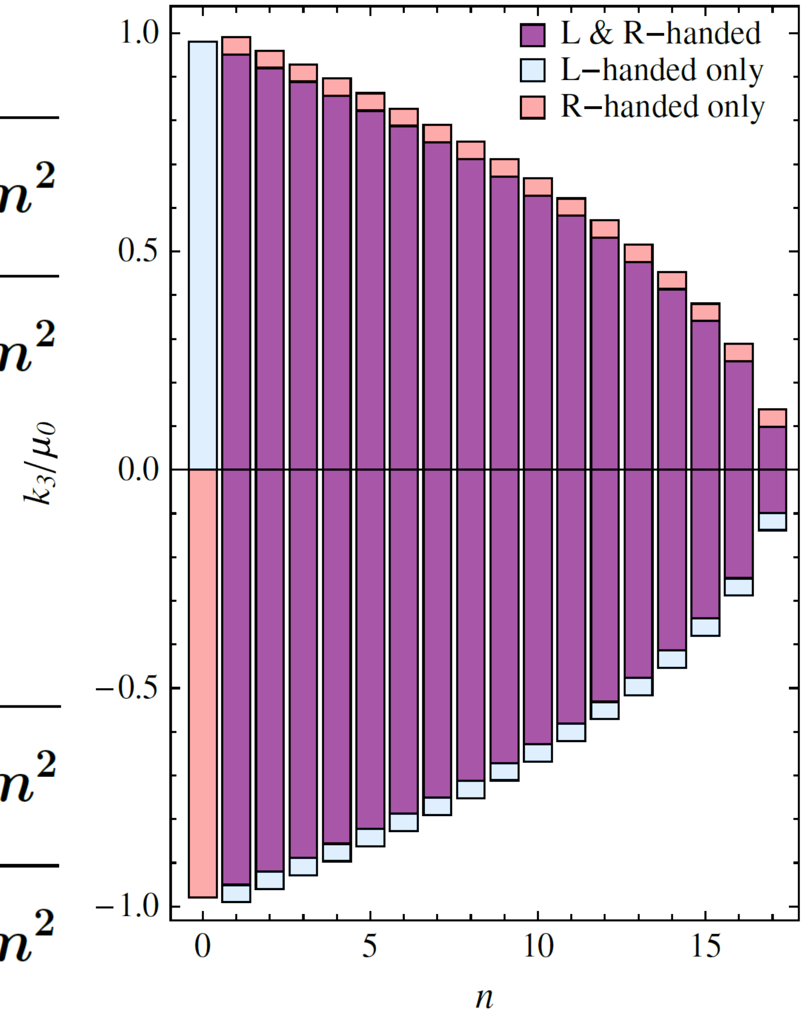
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Chiral shift vs. axial anomaly

- Does the chiral shift modify the axial anomaly relation?
- Using point splitting method, one derives

$$\begin{aligned}\langle \partial_\mu j_5^\mu(u) \rangle &= -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right) \\ &\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \to 0\end{aligned}$$

[E. V. Gorbar & V. A. Miransky, I.A.S., Phys. Lett. B 695 (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- Does the chiral shift give any contribution to the axial current?
- In the point splitting method, one has

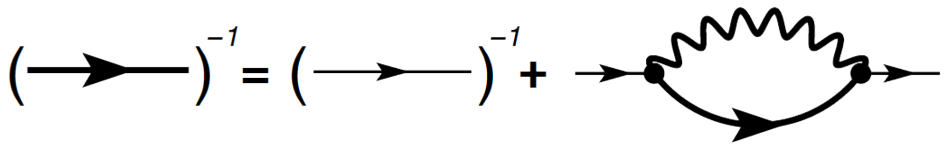
$$\langle j_5^\mu \rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\epsilon^2}\delta_3^\mu \sim \frac{\Lambda^2\Delta}{2\pi^2}\delta_3^\mu$$

[E. V. Gorbar & V. A. Miransky, I.A.S., Phys. Lett. B 695 (2011) 354]

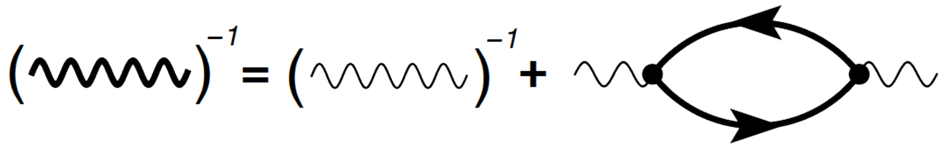
- This is consistent with the NJL calculations
- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the correction to the axial current should be finite

Chiral shift in QED (preliminary)

- Chiral shift is also generated in QED



HDL:



- Fermion propagator

A well-defined function of the parameters in the n^{th} Landau level

$$G(\boldsymbol{k}_{\parallel}, \mathbf{r}, \mathbf{r}') = e^{i\Phi(\mathbf{r},\mathbf{r}')}\bar{G}(\boldsymbol{k}_{\parallel}, \mathbf{r}-\mathbf{r}')$$

$$\bar{G}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = ie^{-k^2\ell^2}\sum_{n=0}^{\infty}(-1)^n D_n(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp})\frac{1}{\mathcal{M}-2n|eB|}$$

- Gauge boson propagator

$$D_{\mu\nu}(q) \simeq \frac{|\boldsymbol{q}|}{|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|}O_{\mu\nu}^{(\mathrm{mag})} + \frac{O_{\mu\nu}^{(\mathrm{el})}}{q_4^2+|\boldsymbol{q}|^2+m_D^2}$$

QED: Weak field limit (preliminary)

- Weak field expansion

$$\bar{S}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = \bar{S}^{(0)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) + \bar{S}^{(1)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) + \cdots$$

$$\bar{S}^{(0)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = i\frac{(\omega+\mu_0)\gamma^0 + m_0 - \boldsymbol{k}\cdot\boldsymbol{\gamma}}{(\omega+\mu_0)^2 - m_0^2 - k^2},$$

$$\bar{S}^{(1)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = -\gamma^1\gamma^2 eB\frac{(\omega+\mu_0)\gamma^0 + m_0 - k_3\gamma^3}{\left[(\omega+\mu_0)^2 - m_0^2 - k^2\right]^2}$$

- Perturbative (linear in B) result for chiral shift:

$$\Delta^{(1)} \simeq -\frac{\alpha e B}{8\pi\mu_0}\Bigg[\underbrace{\frac{2}{3}\sin^2\theta_{Bp}\ln\frac{2\mu_0}{|\omega'_E|}}_{\text{magnetic modes}} - \underbrace{\left(\frac{2}{3}\sin^2\theta_{Bp} + 1\right)\ln\frac{C_1}{\alpha}}_{\text{electric modes}}\Bigg]$$

- Estimate

$$\Delta^{(1)} \simeq 0.5\,\mathrm{keV}\left(\frac{400\,\mathrm{MeV}}{\mu_0}\right)\left(\frac{B}{10^{16}\,\mathrm{G}}\right)\ln\frac{\mu_0}{\alpha T} \qquad \text{at} \quad T \simeq 1\,\mathrm{keV}$$

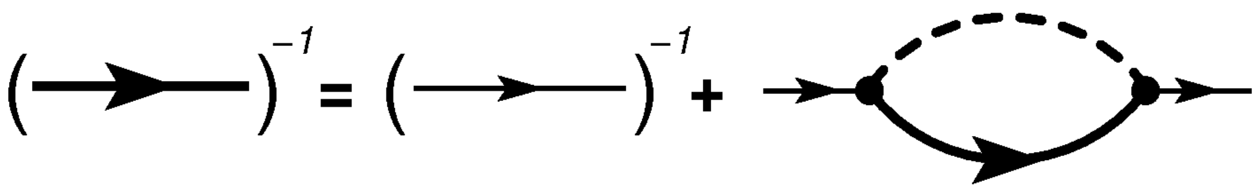
Yukawa model (preliminary)

- One can define a model without IR problems,

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu + m_0\right)\psi + \frac{1}{2}\partial_\mu\phi_\sigma\partial^\mu\phi_\sigma + \frac{1}{2}\partial_\mu\phi_\pi\partial^\mu\phi_\pi$$

$$-\frac{1}{2}m_\phi^2\left(\phi_\sigma^2 + \phi_\pi^2\right) + g\bar{\psi}\left(\phi_\sigma + i\gamma^5\phi_\pi\right)\psi.$$

- Chiral shift is also generated,



- The value of the shift will be finite

$$\Delta \propto g^2 \frac{eB}{\mu_0} \ln\left(\frac{\mu_0}{m_\phi}\right)^2$$

QED: Strong field limit (preliminary)

- Lowest Landau level approximation

$$\Delta = \pi\alpha \int \frac{d^2k_{\parallel} d^2k_{\perp}}{(2\pi)^4} \mathrm{Tr}\left[\gamma^3\gamma^5\gamma^\mu \bar{S}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp})\gamma^\nu\right] D_{\mu\nu}(p-k)$$

where the LLL propagator is used

$$\bar{S}^{(\mathrm{LLL})}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = 2ie^{-k_{\perp}^2\ell^2} \frac{(\omega+\mu_0)\gamma^0 + m_0 - k^3\gamma^3}{(\omega+\mu_0)^2 - m_0^2 - (k^3)^2} P_-$$

- The value of the shift:

$$\Delta \simeq \frac{s_{\perp}\alpha\,\mathrm{sgn}(\mu_0)}{2}\sqrt{|eB|}$$

- Estimate:

$$\Delta \simeq 30\,\mathrm{keV}\sqrt{\frac{B}{10^{16}\,\mathrm{G}}}$$

Brute force approach (preliminary)

- The chiral shift becomes a function of *n*:

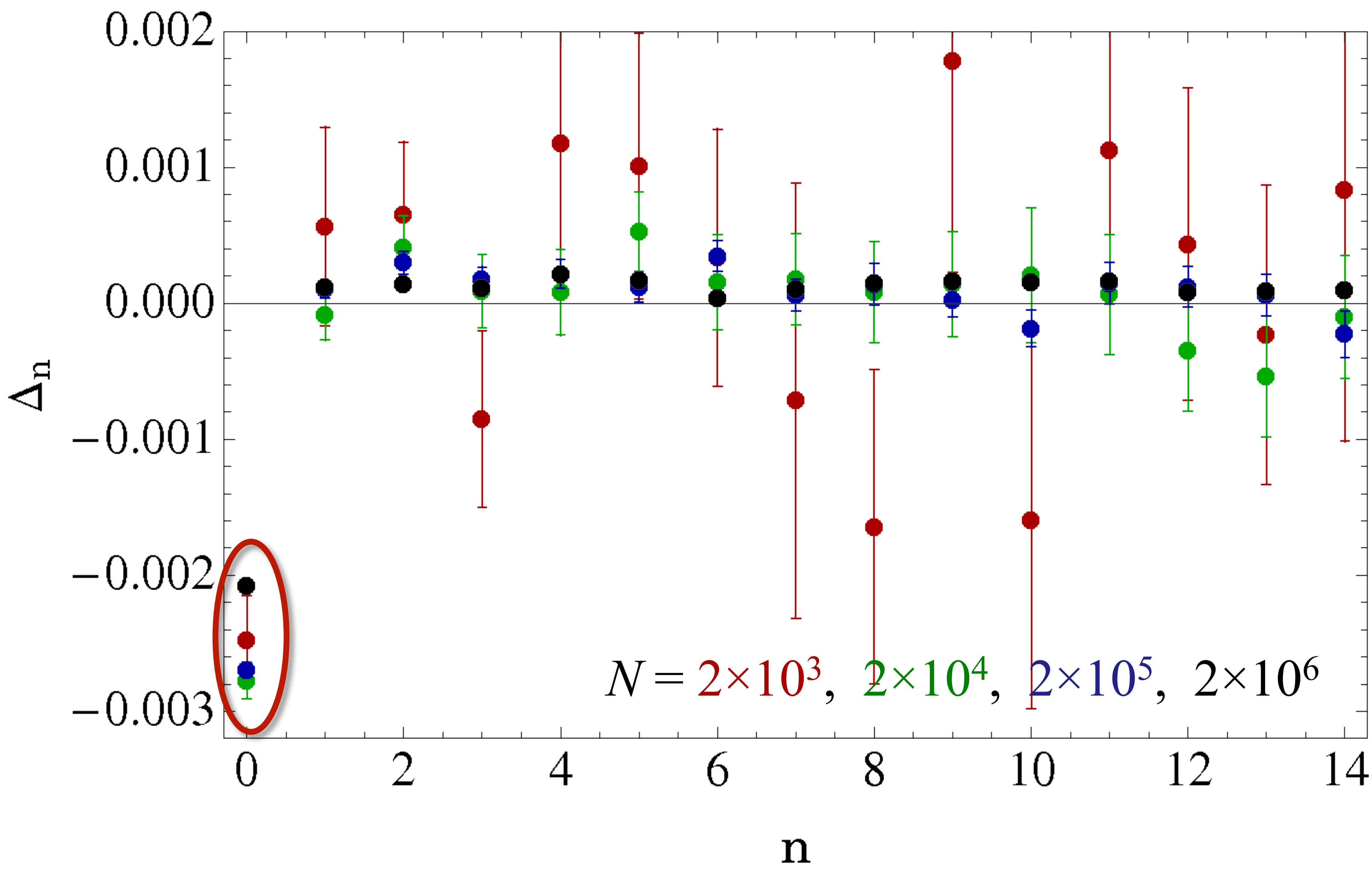
$$\Delta_n = -\text{sgn}(eB)\frac{e^2\ell^2}{\pi}\sum_{n'=0}^{\infty}\int\frac{d\omega_E dk_3 d^2k_\perp d^2p_\perp}{(2\pi)^4}\frac{(-1)^{n+n'}(i\omega_E+\mu_0)e^{-(p_\perp^2+k_\perp^2)\ell^2}}{(\omega_E-i\mu_0)^2+m_0^2+(k^3)^2+2n'|eB|}$$
$$\times\left\{\left[\frac{1}{q_4^2+|\boldsymbol{q}|^2+m_D^2}-\frac{q_\perp^2}{|\boldsymbol{q}|\left(|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|\right)}\right]\left[L_n(2p_\perp^2\ell^2)L_{n'}(2k_\perp^2\ell^2)-L_{n-1}(2p_\perp^2\ell^2)L_{n'-1}(2k_\perp^2\ell^2)\right]\right.$$
$$\left.-\frac{q_\perp^2+2q_3^2}{|\boldsymbol{q}|\left(|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|\right)}\left[L_{n-1}(2p_\perp^2\ell^2)L_{n'}(2k_\perp^2\ell^2)-L_n(2p_\perp^2\ell^2)L_{n'-1}(2k_\perp^2\ell^2)\right]\right\},$$

- There are no (obvious) divergences

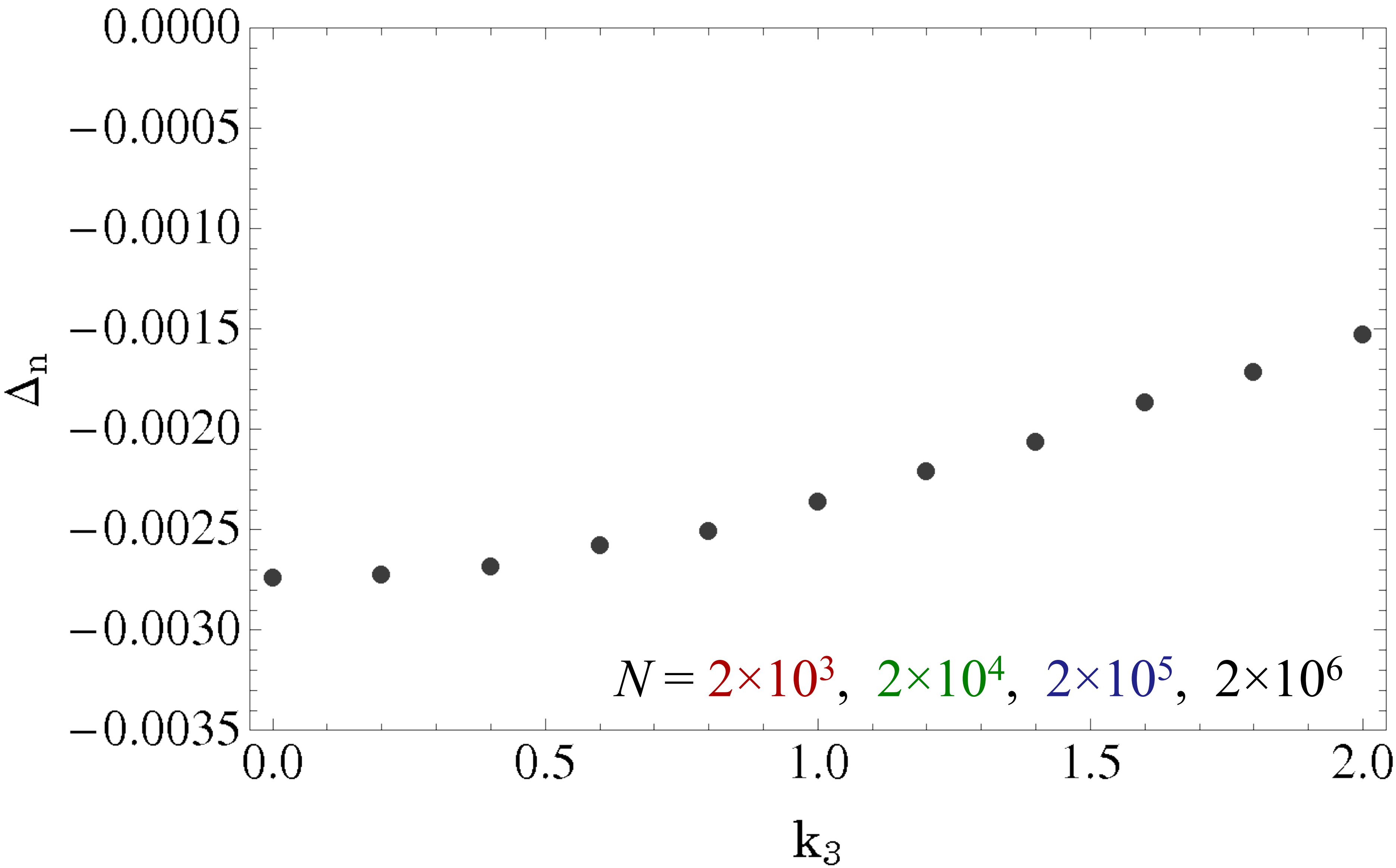
- Two integrals (out of 6) can be done analytically

- The rest (including the sum) has to be done numerically

Δ_n @ Fermi surface (preliminary)



Δ_0 vs. k_3 (preliminary)



Summary (1)

- Chiral shift is generated in magnetized matter (evidence from renormalizable models)
- The magnitude of chiral shift scales as

$$\Delta \propto \alpha \frac{eB}{\mu_0} \quad \text{or} \quad \Delta \propto \alpha \sqrt{eB}$$

- Chiral shift induces a chiral asymmetry at the Fermi surface
- Chiral shift may potentially contribute to the axial current density

Summary (2)

- Potential applications:
  - Pulsar kicks (?)
    - Quark/hybrid stars
  - Facilitation of supernova explosions (?)
  - Axial current in hot QGP
    - Quadrupole CME