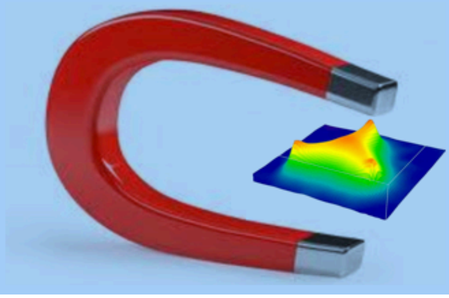

Workshop on QCD in strong magnetic fields

12-16 November 2012, Trento, Italy

Magnetized Vacuum & Matter: *from Magnetic Catalysis to Chiral Asymmetry**

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*Supported by NSF

Outline

- Magnetic catalysis (MC)
  - The essence of MC
  - MC in QCD
  - MC in graphene
- From MC to chiral shift
  - New order parameters in graphene
  - Chiral shift
- Chiral asymmetry in QED (preliminary)
- Outlook

Magnetic catalysis

- Massless fermions in a magnetic field

$$\mathcal{L} = \bar{\Psi} i \gamma^{\mu} D_{\mu} \Psi + \text{(interactions)}$$

(with *any* attractive particle-antiparticle interaction)

[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

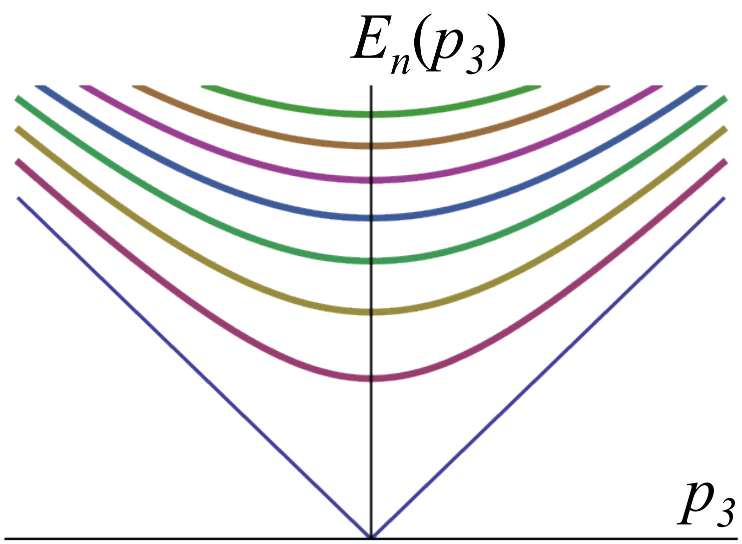
- Energy spectrum (0th order)

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where $n = k + s + \frac{1}{2}$

orbital
k = 0,1,2,…

spin
s = ± ½



Some references

- **Symmetry breaking + magnetic field**
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  - K. Klimenko, Z. Phys. C **54**, 323 (1992); Math. Phys. **89**, 1161 (1992)
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- **Magnetic catalysis**
  - V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994); Phys. Rev. D **52**, 4718 (1995); Phys. Rev. D **52,** 4747 (1995)
- **Review:**
  - I. A. Shovkovy, arXiv:1207.5081

Important Clues

- Lowest Landau level dominates low-energy dynamics $\left(|E|, |p_3| << \sqrt{|eB|}\right)$

$$E_0^{(3+1)}(p_3) = \pm p_3$$

- Dimensional reduction

$$D \Rightarrow D - 2$$

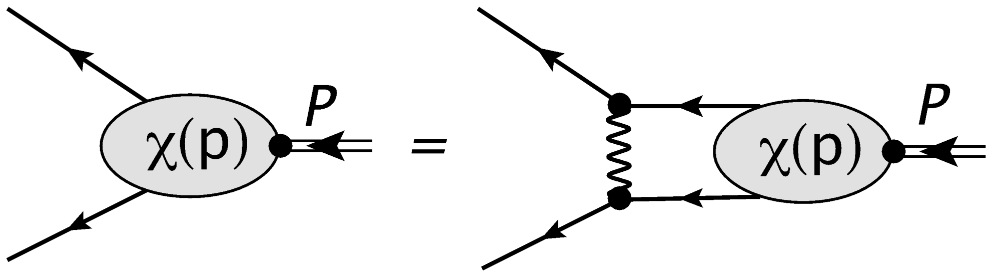
- Density of states at $E = 0$ is nonzero:

$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB| N_f}{4\pi^2}$$

(This may remind superconductivity…)

Revelation

- Particle-antiparticle pairing



- Massless bound state (pion)

$$\chi(r, P \to 0) \propto \Psi(r_{\parallel}), \quad \text{where} \quad r_{\parallel} = (it, z)$$

and

$$\left[-\nabla^2_{r_{\parallel}} + m^2_{\mathrm{dyn}} + V(r_{\parallel})\right]\Psi(r_{\parallel}) = 0$$

i.e., a Schrödinger equation for a bound state with

$$E_{\mathrm{b}} = -m^2_{\mathrm{dyn}}$$

[Gusynin, Miransky, Shovkovy, Phys. Rev. D **52** (1995) 4747]

MC universality

- Different interaction types
  - NJL: $V(r_\parallel) = \dfrac{G|eB|}{\pi}\delta_\Lambda^{(2)}(r_\parallel)$
  - QED: $V(r_\parallel) = \dfrac{\alpha|eB|}{\pi}\exp\left(\dfrac{1}{2}r_\parallel^2|eB|\right)\mathrm{Ei}\left(\dfrac{1}{2}r_\parallel^2|eB|\right) \cong -\dfrac{2\alpha}{\pi}\dfrac{1}{r_\parallel^2}, \quad r_\parallel \to \infty$
- Schrödinger problem in 2D $\left[V(r_\parallel) \equiv gU(r_\parallel)\right]$

when $\int |U(r_\parallel)|^{1+\varepsilon} d^2 r_\parallel < \infty$ and $\int (1+r_\parallel^2)^\varepsilon |U(r_\parallel)| d^2 r_\parallel < \infty$

there is a bound state at $g \to 0$ **if and only if**

$$\boxed{\int U(r_\parallel) d^2 r_\parallel \le 0}$$ (i.e., attractive at least on **average**)

[Simon, Annals Phys. **97** (1976) 279]

MC in QCD

- Screening

$$\Pi^{\mu\nu} \cong -\frac{\alpha_s}{\pi}\left(k_\parallel^\mu k_\parallel^\mu - g_\parallel^{\mu\nu} k_\parallel^2\right)\sum_f \frac{|e_f B|}{k_\parallel^2} \quad \text{for} \quad m_f^2 << |k_\parallel^2| << |eB|$$

leads to gluon mass

$$M_g^2 \cong \frac{\alpha_s}{\pi}\sum_f |e_f B|$$

- Dynamical fermion mass

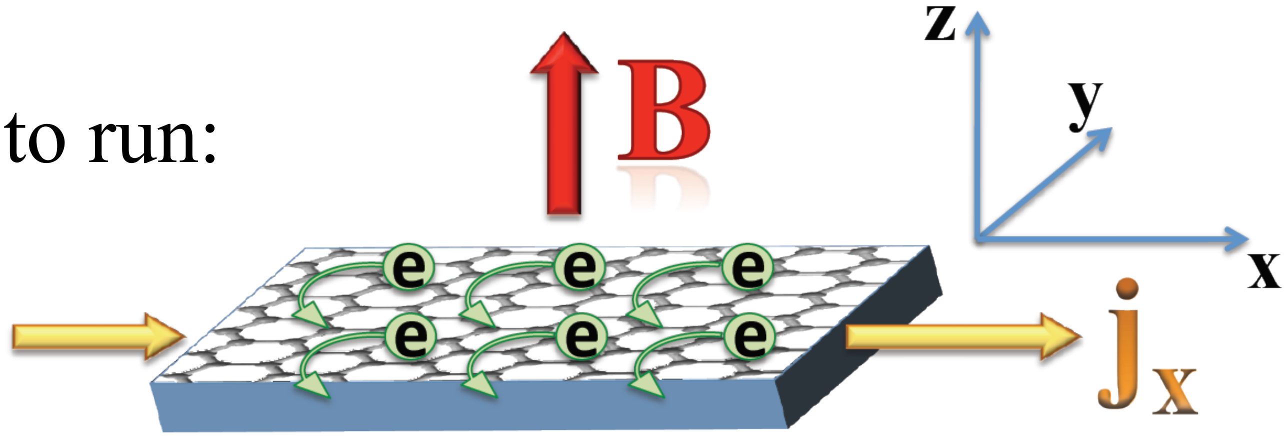
$$m_f^2 \propto |e_f B| \alpha_s^{2/3} \exp\left(-\frac{4\pi N_c}{\alpha_s (N_c^2 - 1)\ln(C/\alpha_s)}\right)$$

- Confinement scale is also reduced

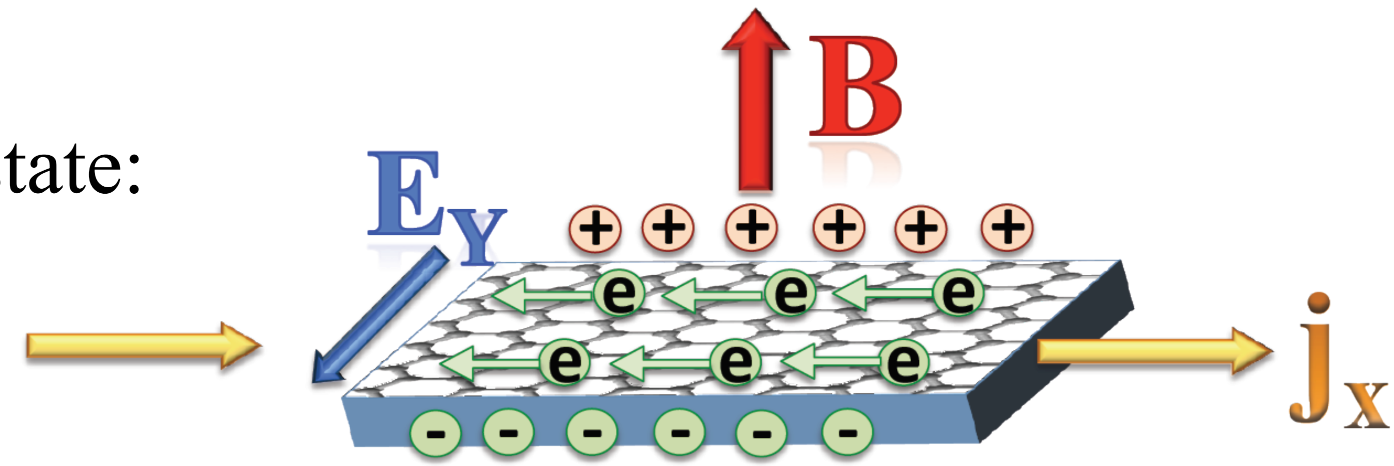
[Miransky, Shovkovy, Phys. Rev. D **66** (2002) 045006]

MC in graphene

- General setup
  - Current starts to run:



  - Steady state:

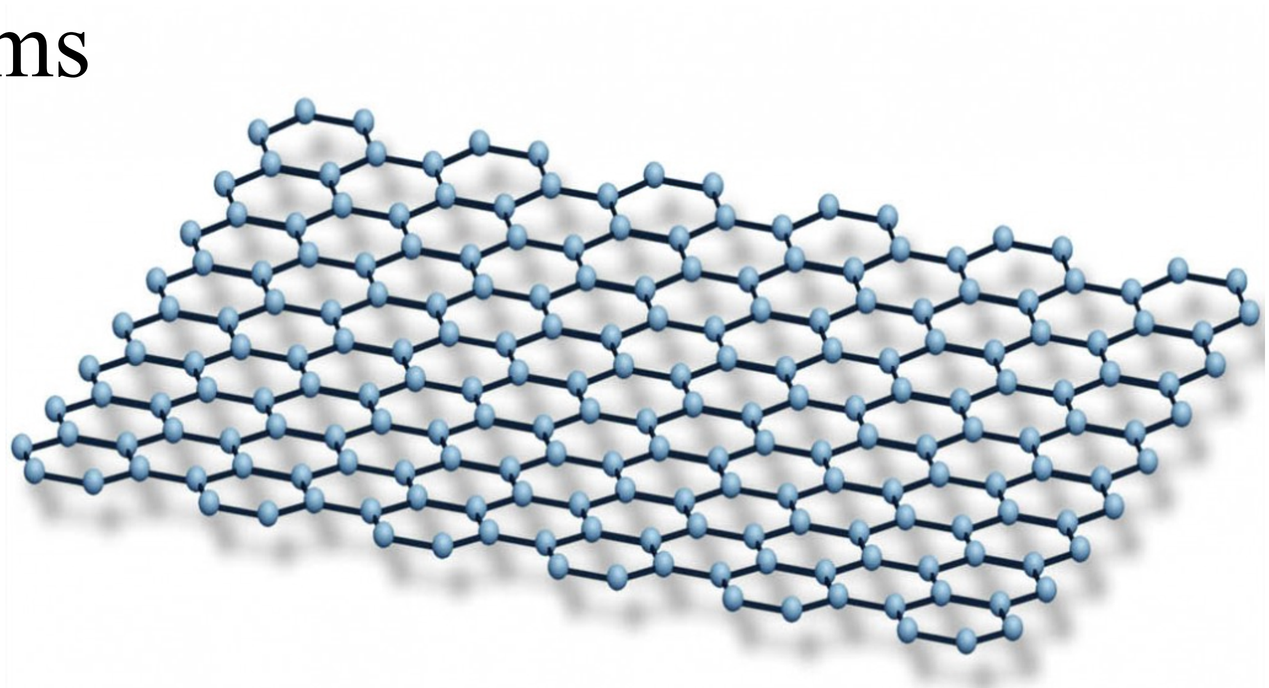


  - Hall conductivity:

$$j_x = \sigma_{xy} E_y$$

Graphene

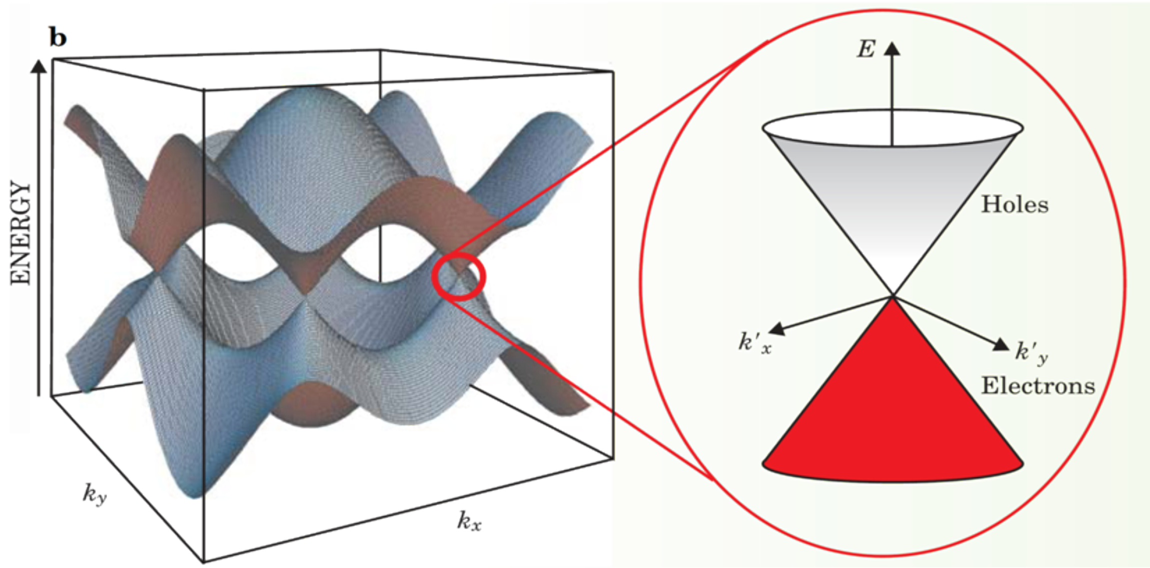
- It is a single atomic layer of graphite
  [Novoselov et al., Science **306**, 666 (2004)]
- 2D crystal with hexagonal lattice of carbon atoms



- Interesting basic physics
- Great promise for applied physics

Dirac Fermions in Graphene

- Low energy quasiparticles are **massless** Dirac fermions $(v_F = c/300)$



[Wallace, Phys. Rev. **71,** 622 (1947)]
[Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)]

- Spinor:

$$\Psi_s = \begin{pmatrix} \psi_{KAs} \\ \psi_{KBs} \\ \psi_{K'Bs} \\ \psi_{K'As} \end{pmatrix}$$

- Low-energy model with U(4) global symmetry:

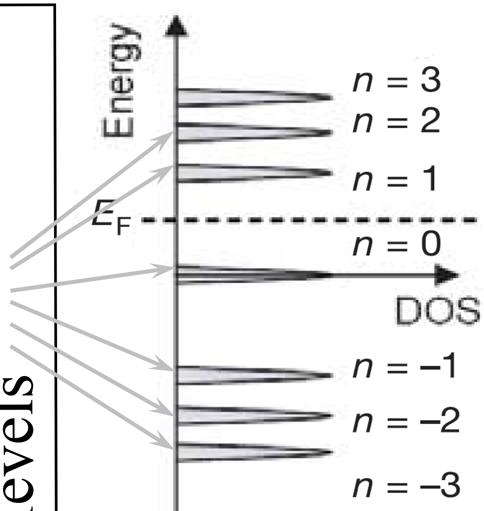
$$H_0 = v_F \int d^2 r\, \overline{\Psi}_s \left(\gamma^1 \pi_x + \gamma^2 \pi_y\right) \Psi_s$$

$$H_C = \frac{1}{2} \int d^2 r\, d^2 r' \Psi_s^+(r) \Psi_s(r) U\left(r - r'\right) \Psi_{s'}^+(r') \Psi_{s'}(r')$$

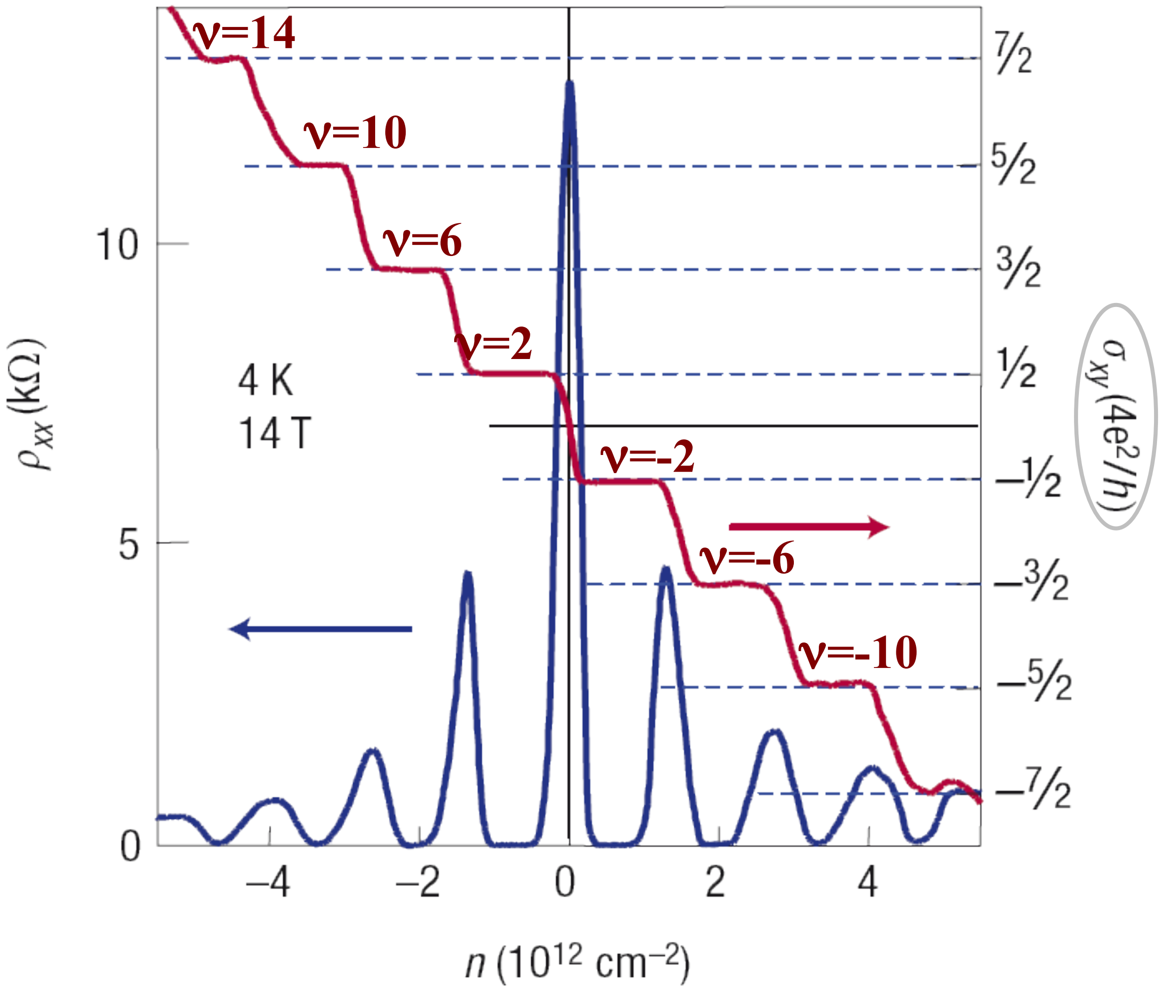
QHE in Graphene

$$E_n(p_3) = \pm\sqrt{2\hbar v_F^2 n|eB|}$$

4-fold degenerate levels



$$\sigma_{xy} = \nu \frac{e^2}{h} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$$



[Gusynin, Sharapov, PRL **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, PRB **73**, 125411 (2006)]
} **Theory (m=0)**

[Novoselov et al., Nature **438**, 197 (2005)]
[Zhang et al., Nature **438**, 201 (2005)]
} **Experiment**

Anomalous QHE

- New plateaus at

  ν=0

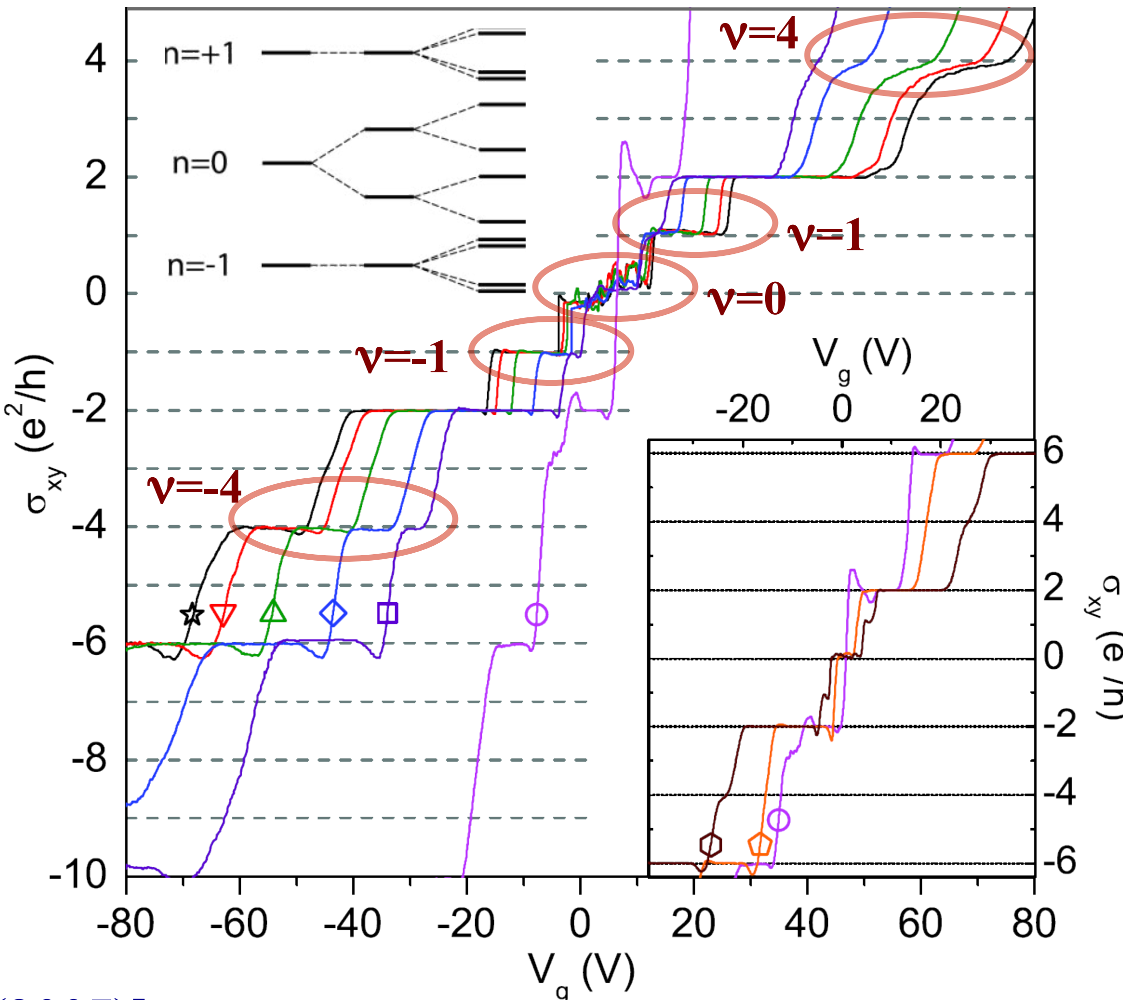
  ν=±1

  ν=±3

  ν=±4

- Some Landau level degeneracy is lifted

Zhang et al., PRL **96,** 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]
[Abanin et al., PRL **98**, 196806 (2007)]
[Checkelsky et al., PRL 100, 206801 (2008)]
[Xu Du et al., Nature **462**, 192 (2009)]

Order parameters

- Several different order parameters may be generated (pairing from different valleys/sublattices)
- Dirac and Haldane masses

$$\tilde{\Delta}_s: \quad \overline{\Psi} P_s \Psi = \psi^+_{KAs}\psi_{KAs} - \psi^+_{KBs}\psi_{KBs} + \psi^+_{K'As}\psi_{K'As} - \psi^+_{K'Bs}\psi_{K'Bs}$$

$$\Delta_s: \quad \overline{\Psi} \gamma^3\gamma^5 P_s \Psi = \psi^+_{KAs}\psi_{KAs} - \psi^+_{KBs}\psi_{KBs} - (\psi^+_{K'As}\psi_{K'As} - \psi^+_{K'Bs}\psi_{K'Bs})$$

+ spin & pseudo-spin densities

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **78** (2008) 085437]
[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scr. T **146** (2012) 014018]

- Any analog of the Haldane mass in 3D?

Initial ideas

- A new dynamical parameter Δ ("chiral shift") similar to Haldane mass?

$$\mathcal{L}_{\text{eff}} = \Delta \overline{\psi} \gamma^3 \gamma^5 \psi$$

(Δ=0 is not protected by symmetry at $B \neq 0$)

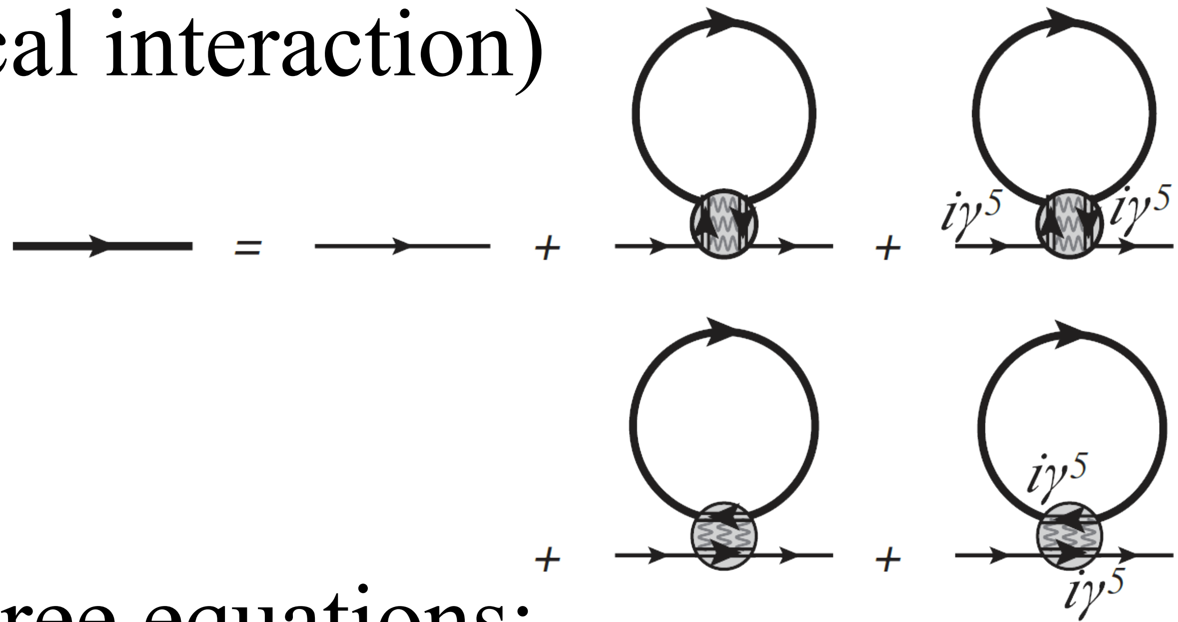
- Δ is associated with the axial current density (spin density):

$$\left\langle j_5^3 \right\rangle_0 = \frac{eB}{2\pi^2} \mu \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

Chiral shift in NJL model

- NJL model (local interaction)



- This leads to three equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \left\langle j^0 \right\rangle$$ ("effective" chemical potential)

$$m = m_0 - G_{\text{int}} \left\langle \bar{\psi}\psi \right\rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\text{int}} \left\langle j_5^3 \right\rangle$$ (chiral shift parameter)

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83** (2011) 085003]

Chiral shift @ Fermi surface

- Chirality is $\approx$ well-defined at Fermi surface ($|\boldsymbol{k}^3| \gg \boldsymbol{m}$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

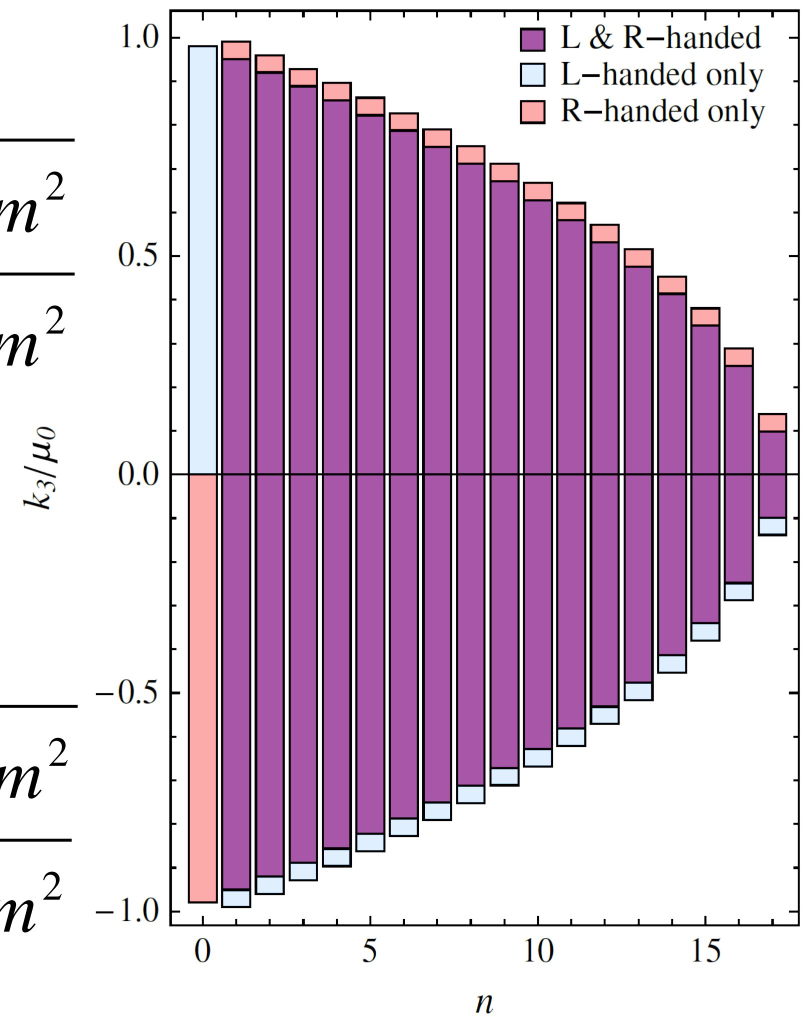
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Chiral shift vs. axial anomaly

- Does the chiral shift modify the axial anomaly relation?
- Using point splitting method, one derives

$$\begin{aligned}\langle \partial_\mu j_5^\mu(u)\rangle &= -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right) \\ &\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \to 0\end{aligned}$$

[E. V. Gorbar & V. A. Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- Does the chiral shift give any contribution to the axial current?

- In the point splitting method, one has

$$\langle j_5^\mu \rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\varepsilon^2}\delta_\mu^3 \cong \frac{\Lambda^2\Delta}{2\pi^2}\delta_\mu^3$$

[E. V. Gorbar & V. A. Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

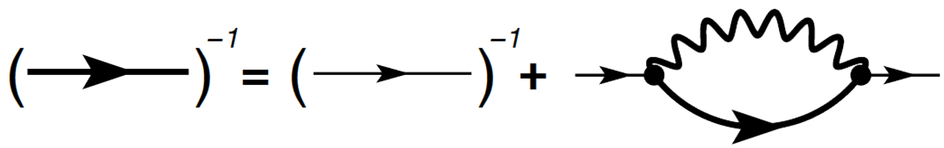
- This is consistent with the NJL calculations

- Since $\Delta \propto g\mu eB/\Lambda^2$, the correction to the axial current could be finite

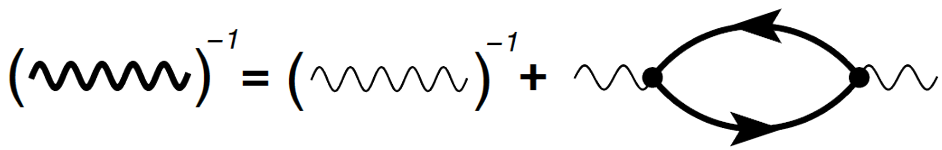
Chiral shift in QED (preliminary)

[Gorbar, Miransky, Shovkovy, Wang, in progress]

- Chiral shift is also generated in QED



HDL:



- Fermion propagator

$$G(\boldsymbol{k}_{\parallel}, \mathbf{r}, \mathbf{r}') = e^{i\Phi(\mathbf{r},\mathbf{r}')} \bar{G}(\boldsymbol{k}_{\parallel}, \mathbf{r} - \mathbf{r}')$$

$$\bar{G}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = i e^{-k^2 \ell^2} \sum_{n=0}^{\infty} (-1)^n D_n(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) \frac{1}{\mathcal{M} - 2n|eB|}$$

A well-defined function of the parameters in the n^{th} Landau level

- Gauge boson propagator

$$D_{\mu\nu}(q) \simeq \frac{|\boldsymbol{q}|}{|\boldsymbol{q}|^3 + \frac{\pi}{4} m_D^2 |q_4|} O_{\mu\nu}^{(\mathrm{mag})} + \frac{O_{\mu\nu}^{(\mathrm{el})}}{q_4^2 + |\boldsymbol{q}|^2 + m_D^2}$$

QED: Weak field limit (preliminary)

- Weak field expansion

$$\bar{S}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = \bar{S}^{(0)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) + \bar{S}^{(1)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) + \cdots$$

$$\bar{S}^{(0)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = i\frac{(\omega+\mu_0)\gamma^0 + m_0 - \boldsymbol{k}\cdot\boldsymbol{\gamma}}{(\omega+\mu_0)^2 - m_0^2 - k^2},$$

$$\bar{S}^{(1)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = -\gamma^1\gamma^2 eB\frac{(\omega+\mu_0)\gamma^0 + m_0 - k_3\gamma^3}{[(\omega+\mu_0)^2 - m_0^2 - k^2]^2}$$

- Perturbative (linear in B) result for chiral shift:

$$\Delta^{(1)} \simeq -\frac{\alpha eB}{8\pi\mu_0}\left[\underbrace{\frac{2}{3}\sin^2\theta_{Bp}\ln\frac{2\mu_0}{|\omega'_E|}}_{\text{magnetic modes}} - \underbrace{\left(\frac{2}{3}\sin^2\theta_{Bp}+1\right)\ln\frac{C_1}{\alpha}}_{\text{electric modes}}\right]$$

- Estimate

$$\Delta \cong 0.5\,\text{keV}\left(\frac{400\text{ MeV}}{\mu}\right)\left(\frac{B}{10^{16}\,\text{G}}\right)\ln\frac{\mu}{\alpha T} \quad \text{at} \quad T \approx 1\text{keV}$$

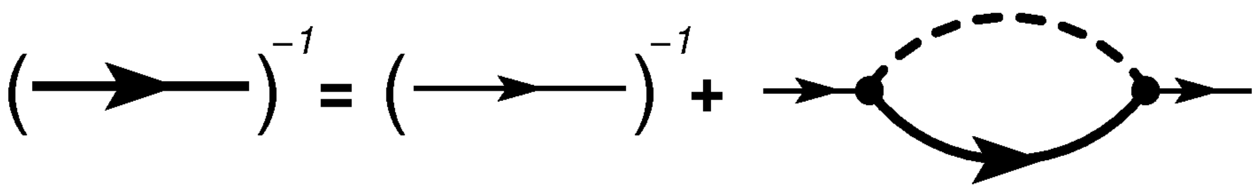
Yukawa model (preliminary)

- One can define a model without IR problems,

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} + m_0\right)\psi + \frac{1}{2}\partial_{\mu}\phi_{\sigma}\partial^{\mu}\phi_{\sigma} + \frac{1}{2}\partial_{\mu}\phi_{\pi}\partial^{\mu}\phi_{\pi}$$

$$-\frac{1}{2}m_{\phi}^2\left(\phi_{\sigma}^2 + \phi_{\pi}^2\right) + g\bar{\psi}\left(\phi_{\sigma} + i\gamma^5\phi_{\pi}\right)\psi$$

- Chiral shift is also generated,



- The value of the shift will be finite

$$\Delta \propto g^2 \frac{eB}{\mu_0} \ln\left(\frac{\mu_0}{m_{\phi}}\right)^2$$

QED: Strong field limit (preliminary)

- Lowest Landau level approximation

$$\Delta = \pi\alpha \int \frac{d^2k_{\parallel} d^2k_{\perp}}{(2\pi)^4} \mathrm{Tr}\left[\gamma^3\gamma^5\gamma^\mu \, \bar{S}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp})\gamma^\nu\right] D_{\mu\nu}(p-k)$$

where the LLL propagator is used

$$\bar{S}^{(\mathrm{LLL})}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = 2ie^{-k_{\perp}^2 \ell^2} \frac{(\omega+\mu_0)\gamma^0 + m_0 - k^3\gamma^3}{(\omega+\mu_0)^2 - m_0^2 - (k^3)^2} P_-$$

- The value of the shift:

$$\Delta \cong \frac{\alpha}{2}\sqrt{|eB|}$$

- Estimate:

$$\Delta \cong 30\ \mathrm{keV}\sqrt{\frac{B}{10^{16}\,\mathrm{G}}}$$

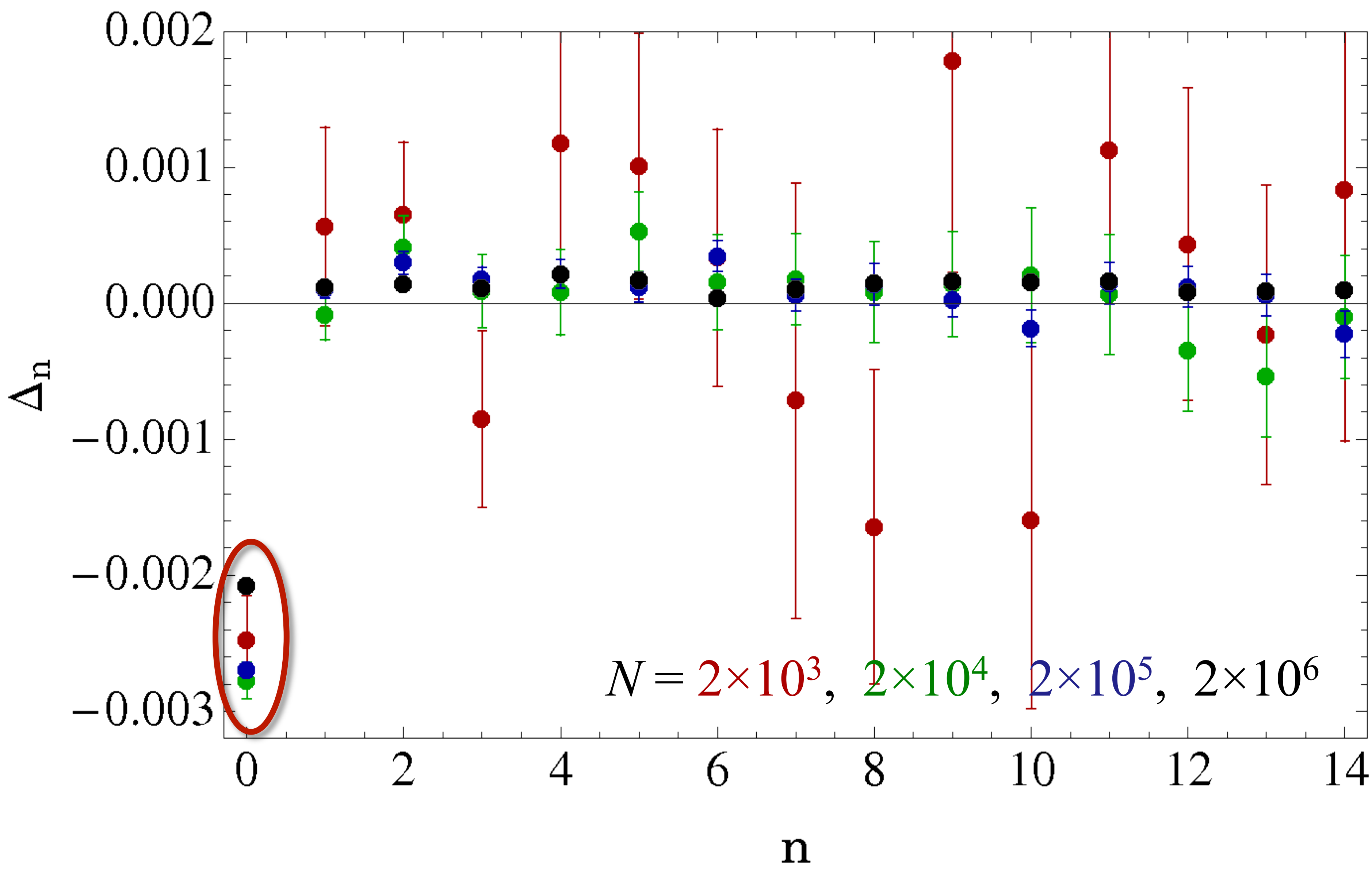
Brute force approach (preliminary)

- The chiral shift becomes a function of *n*:

$$\Delta_n = -\text{sgn}(eB)\frac{e^2\ell^2}{\pi}\sum_{n'=0}^{\infty}\int\frac{d\omega_E dk_3 d^2k_\perp d^2p_\perp}{(2\pi)^4}\frac{(-1)^{n+n'}(i\omega_E+\mu_0)e^{-(p_\perp^2+k_\perp^2)\ell^2}}{(\omega_E-i\mu_0)^2+m_0^2+(k^3)^2+2n'|eB|}$$
$$\times\left\{\left[\frac{1}{q_4^2+|\boldsymbol{q}|^2+m_D^2}-\frac{q_\perp^2}{|\boldsymbol{q}|\left(|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|\right)}\right]\left[L_n(2p_\perp^2\ell^2)L_{n'}(2k_\perp^2\ell^2)-L_{n-1}(2p_\perp^2\ell^2)L_{n'-1}(2k_\perp^2\ell^2)\right]\right.$$
$$\left.-\frac{q_\perp^2+2q_3^2}{|\boldsymbol{q}|\left(|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|\right)}\left[L_{n-1}(2p_\perp^2\ell^2)L_{n'}(2k_\perp^2\ell^2)-L_n(2p_\perp^2\ell^2)L_{n'-1}(2k_\perp^2\ell^2)\right]\right\},$$

- There are no (obvious) divergences
- Two integrals (out of 6) can be done analytically
- The rest (including the sum) has to be done numerically

Δ_n @ Fermi surface (preliminary)



Summary (MC)

- The essence of magnetic catalysis:
  - LLL dynamics dominates at low energies
  - Dimensional reduction: $D \Rightarrow D-2$
  - Nonzero density of states at $E=0$
- Problem of a bound state ($E_{\mathrm{b}} = -m_{\mathrm{dyn}}^2$) in lower dimensional Schrödinger equation
- The mechanism is **universal** (e.g., graphene, QCD, Yukawa model, holographic models…)

Summary (shift)

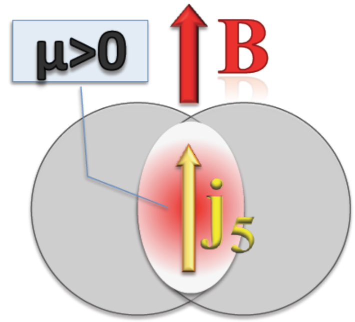
- Chiral shift is generated in magnetized matter
- No contradiction with the axial anomaly
- The magnitude of chiral shift scales as

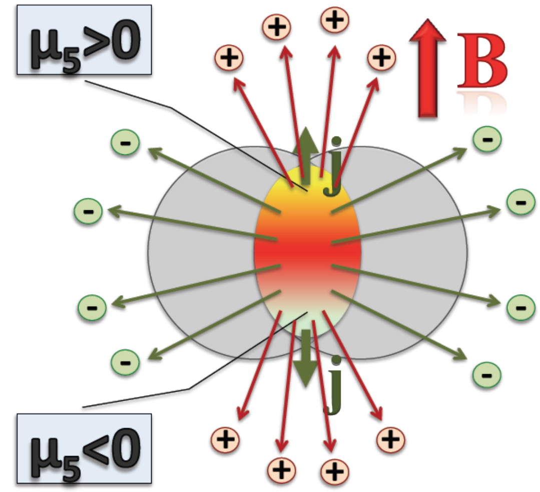
$$\Delta \propto \alpha \frac{eB}{\mu_0} \quad \text{or} \quad \Delta \propto \alpha \sqrt{eB}$$

- Chiral shift induces a chiral asymmetry at the Fermi surface
- Chiral shift may potentially contribute to the axial current density

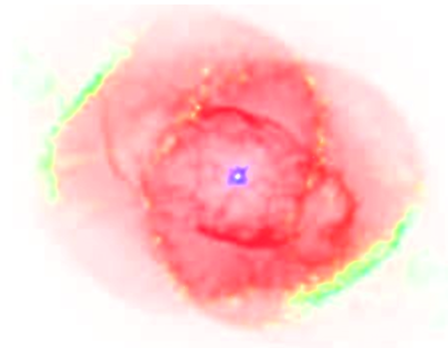
Summary (etc.)

- In search for applications:
  - RHIC physics





  - Compact stars



  - Graphene, topological insulators, etc.