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QUANTUM MAGNETIC WORLD: SYMMETRY BREAKING & MORE

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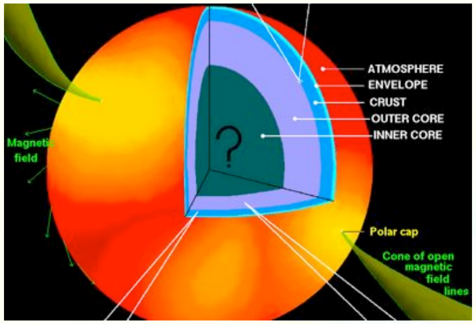
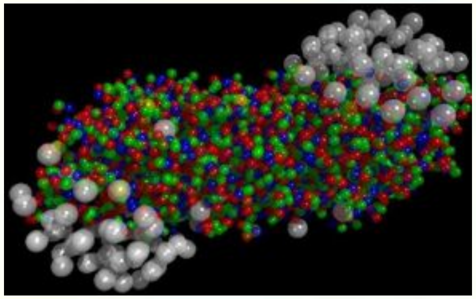
OUTLINE

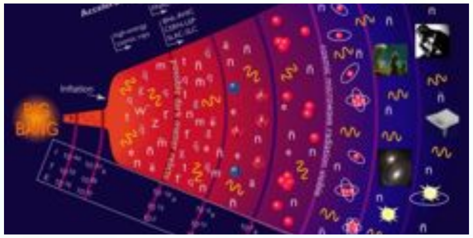
- **Introduction**
  - Relativistic matter
  - Strong magnetic fields in nature
- **Magnetic catalysis**
  - Dimensional reduction
  - Symmetry breaking
- **Magnetic catalysis in graphene**
  - Theoretical ideas & observable features
- **Chiral magnetic effect & beyond**
  - Chirality, currents & chiral asymmetry
- **Summary**

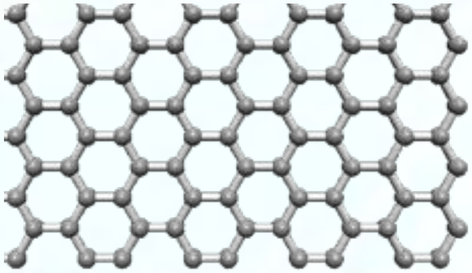
RELATIVISTIC MATTER

- Examples of relativistic matter
  - **Electrons, protons, quarks** inside compact stars (white dwarfs, neutron, hybrid or quark stars)
  - **Quark gluon plasma** in heavy ion collisions ($k_B T \sim 200$ MeV $\sim 10^{12}$ K)
  - **Hot matter** in the Early Universe ($k_B T \sim 100$ GeV at EW transition)
  - **Quasiparticles** in graphene (zero mass Dirac fermions)









WHAT MEANS "RELATIVISTIC"?

- **Relativistic matter** $(p \gg mc)$

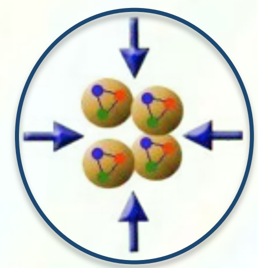
$$E = c\sqrt{p^2 + m^2c^2} \approx cp$$

compare with nonrelativistic case $(p \ll mc)$

$$E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

  – **High density** (e.g., in stars) leads to occupation of states with large momenta:

$$p \sim \hbar n^{1/3} \simeq 200 \left(\frac{n}{1\ \mathrm{fm}^3}\right)^{1/3}\ \mathrm{MeV/c}$$



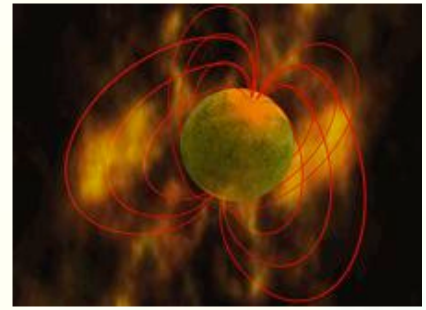
  – **High temperature** (e.g., heavy ion collisions) means energetic particles,

$$p \sim k_B T/c \simeq 200 \left(\frac{k_B T}{200\ \mathrm{MeV}}\right)\ \mathrm{MeV/c}$$

  – **Vanishing mass** (e.g., graphene) works too…

Magnetic Fields

- Strong magnetic fields exist inside *compact stars*
  - **10^{10} to 10^{15} Gauss**



- In *heavy ion collisions*, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

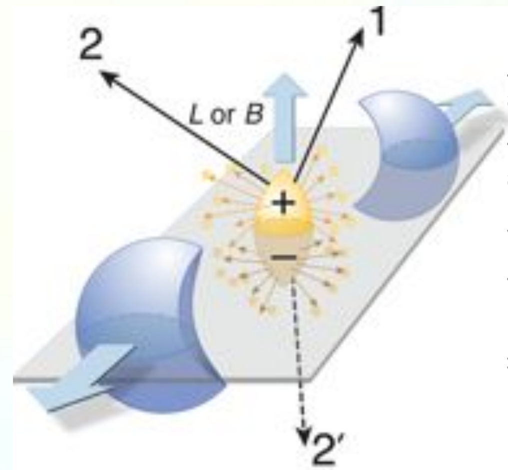


Illustration by Carin Cain

- *Early Universe*
  - **up to 10^{24} Gauss**
- *Graphene* (High Magnetic Field Laboratory)
  - **4.5×10^5 Gauss**

Part 1

MAGNETIC CATALYSIS

Review: arXiv:1207.5081

LANDAU LEVELS

- Fermions in magnetic field

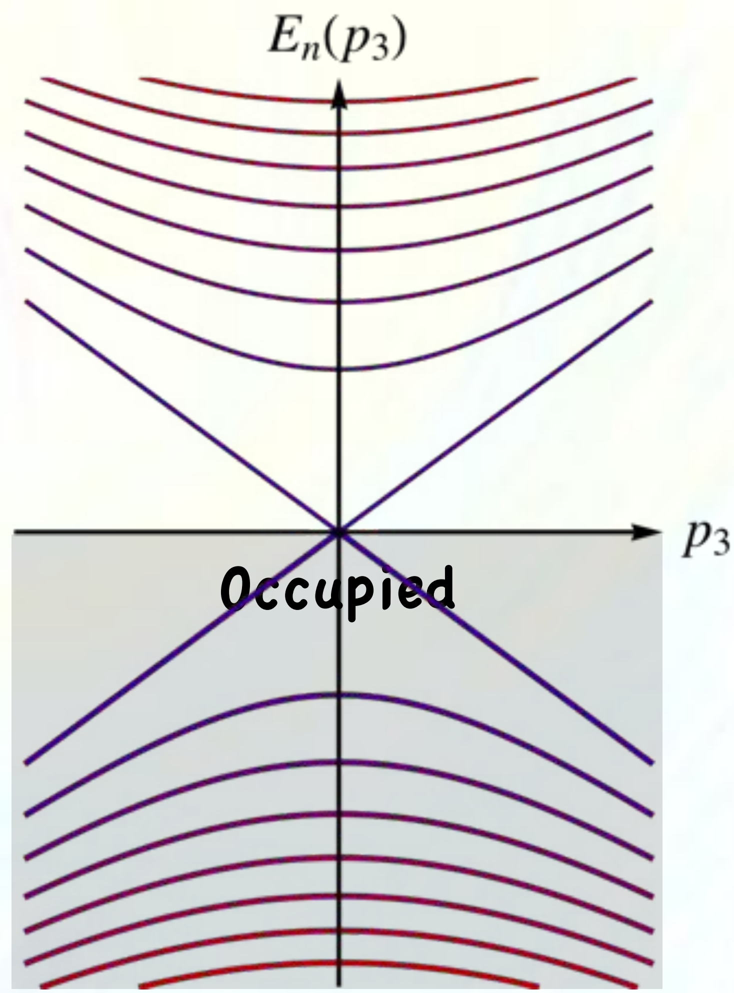
$$\mathcal{L} = \overline{\Psi} i \gamma^\mu D_\mu \Psi + (\text{interactions})$$

- Free energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

$$n = \overbrace{s}^{s = \pm 1/2 \ \text{(spin)}} + \underbrace{k + \frac{1}{2}}_{k = 0, 1, 2, \ldots \ \text{(orbital)}}$$

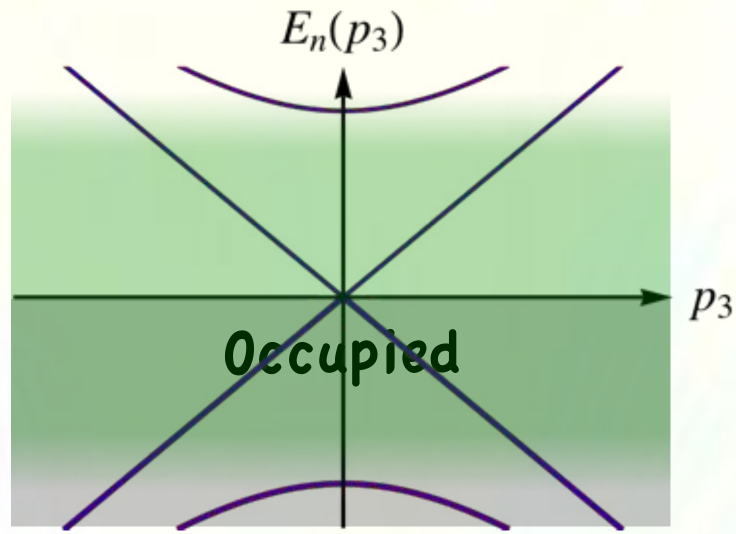


DIMENSIONAL REDUCTION

- Low-energy is due to n=0 Landau level

$$n = 0: \qquad E_0^{(3+1)}(p_3) = \pm p_3$$

$$\left(k = 0, s = -\frac{1}{2}\right)$$



- This is (1+1)D spectrum!

- Propagator looks (1+1)D as well:

$$S(p_{\|}) \approx i\, e^{-p_\perp^2 \ell^2} \frac{\hat{p}_{\|} + m}{\hat{p}_{\|} + m} \underbrace{\left(1 - i\gamma^1\gamma^2\right)}_{s = -\frac{1}{2} \text{ spin projector}}, \text{where } \hat{p}_{\|} = p_0\gamma^0 - p_3\gamma^3$$

MAGNETIC CATALYSIS (CLUES)

- Low-energy regime is dimensionally reduced

$$D \Rightarrow D-2$$

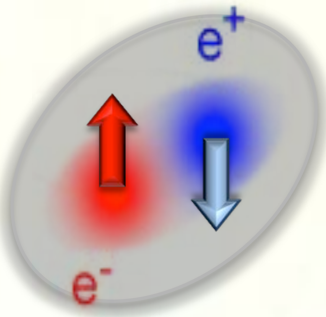
- Density of states at $E = 0$

$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB|N_f}{4\pi^2}$$

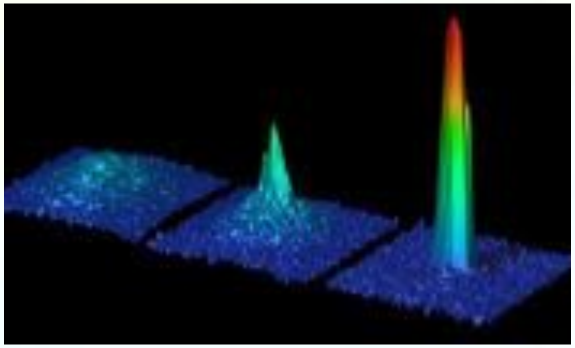
(This may remind superconductivity…)

Magnetic Catalysis (physics)

- n=0: particles & anti-particles
- Bound states are energetically favorable (an energy gain of E_b per pair)



- Bound states are bosons
- Bosons can (and will) occupy same zero momentum quantum state



- Bose condensate forms
- Symmetry breaking ➔ energy (mass) gap

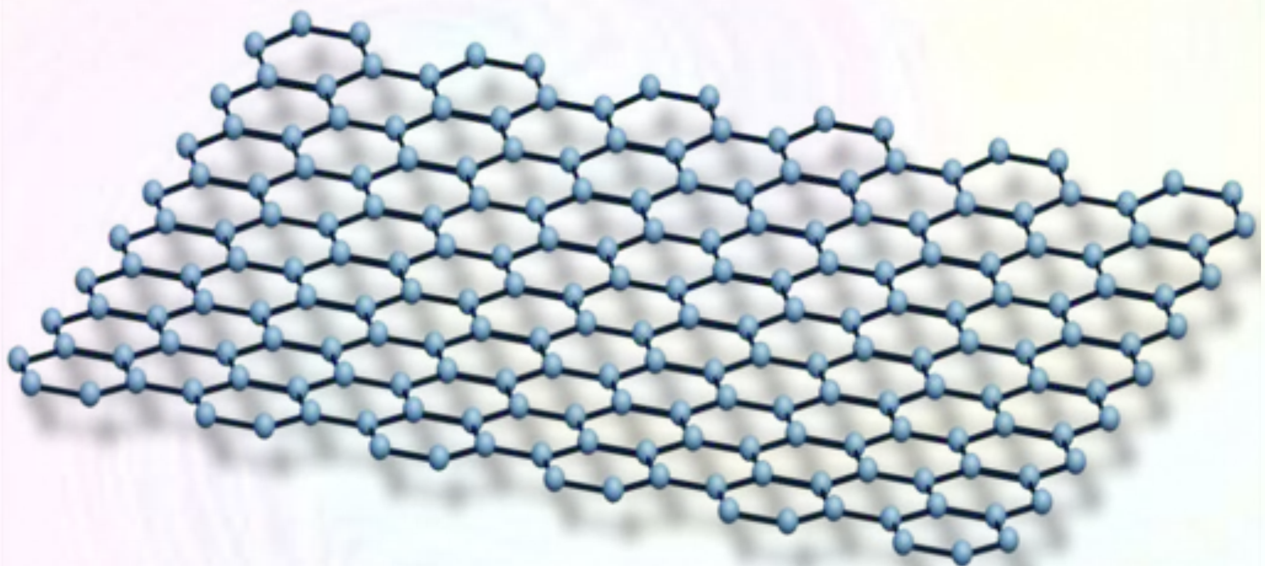
[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

Part 2

GRAPHENE

GRAPHENE

- It is a single atomic layer of graphite
  [Novoselov et al., Science **306**, 666 (2004)]
- 2D crystal with hexagonal lattice of carbon atoms

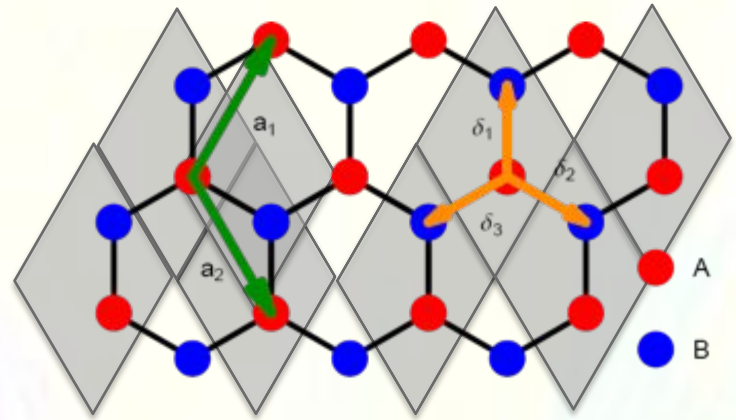


- Interesting basic physics
- Great promise for applied physics

EMERGENCE OF DIRAC FERMIONS

- Translation vectors of the lattice

$$\mathbf{a}_1 = a\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right),\quad \mathbf{a}_2 = a\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$$

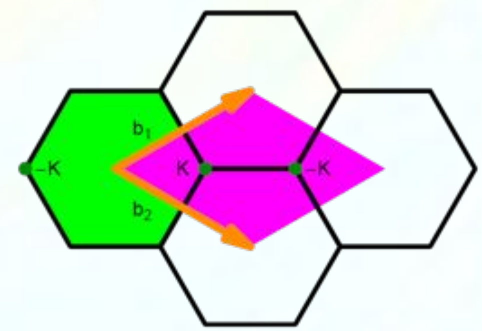


Lattice constant: $a \approx 1.42\ \AA$

- Two carbon atoms per primitive cell

- Reciprocal lattice

$$\mathbf{b}_1 = \frac{2\pi}{a}\left(1,\frac{1}{\sqrt{3}}\right),\quad \mathbf{b}_2 = \frac{2\pi}{a}\left(1,-\frac{1}{\sqrt{3}}\right)$$



- Two Dirac points in the Brillouin zone

TIGHT BINDING MODEL

- There are strong covalent sigma-bonds between nearest neighbors
- Hamiltonian

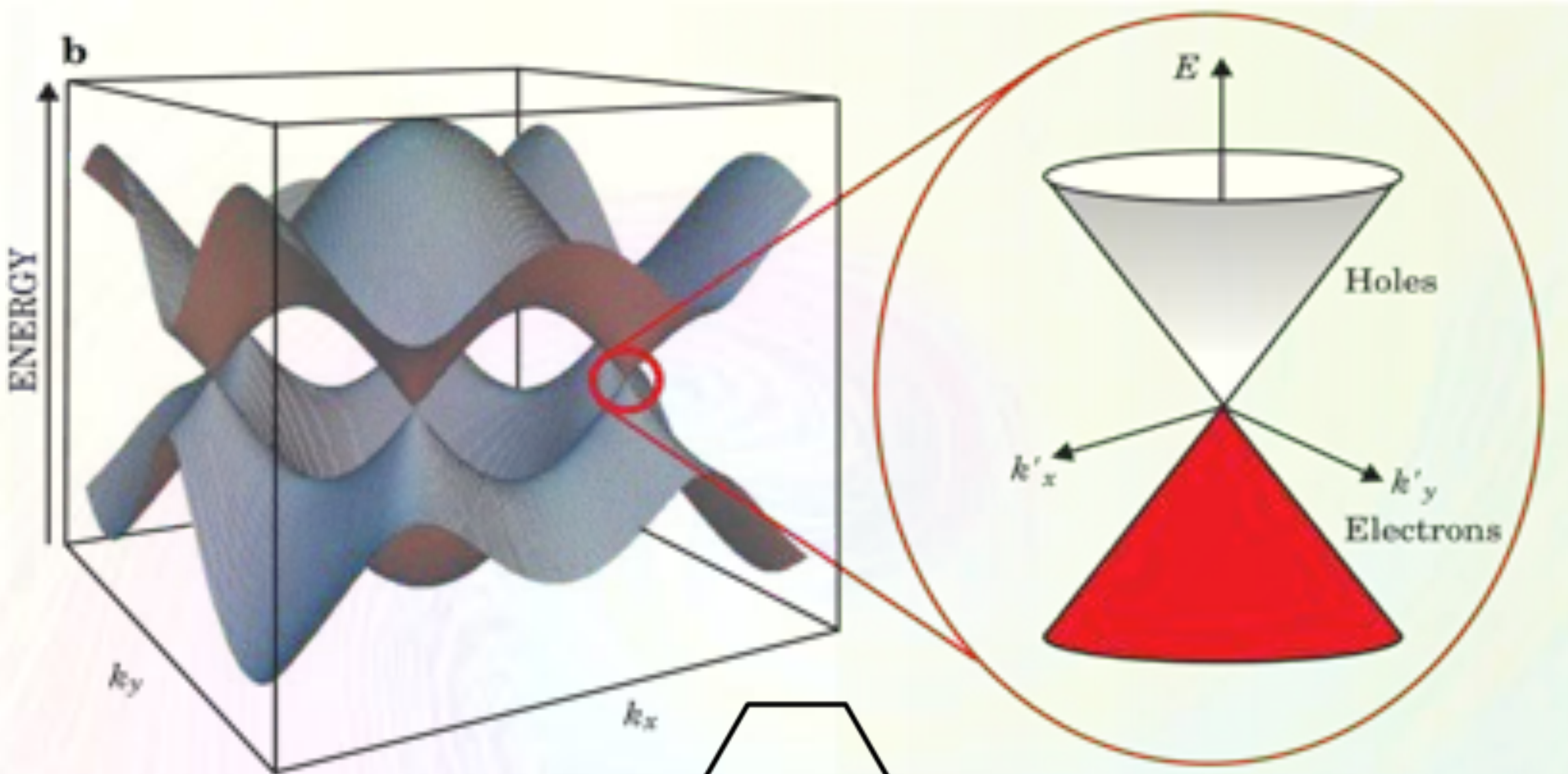
$$H = -t \sum_{\mathbf{n},\delta_i,\sigma} \left[a^{+}_{\mathbf{n},\sigma} \exp\left(\frac{ie}{\hbar c} \mathbf{A} \cdot \delta_i \right) b^{+}_{\mathbf{n}+\delta,\sigma} + c.c. \right]$$

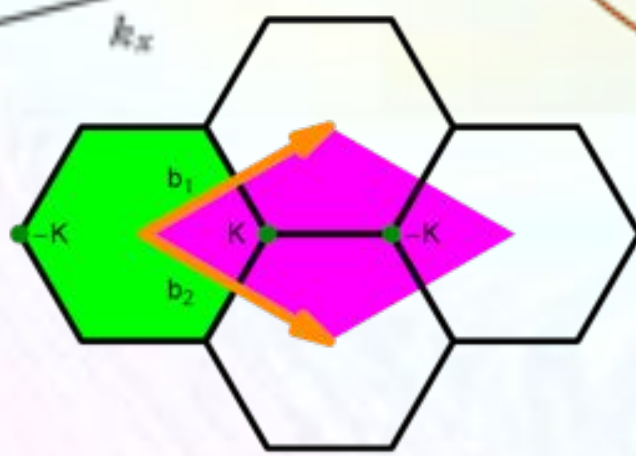
$a_{\mathbf{n},\sigma}/b_{\mathbf{n}+\delta,\sigma}$ are annihilation operators in A/B-sublattice & spin $\sigma=\uparrow,\downarrow$

- The nearest neighbor vectors are

$$\delta_1 = \frac{\mathbf{a}_1 - \mathbf{a}_2}{3}, \quad \delta_2 = \frac{\mathbf{a}_1 + 2\mathbf{a}_2}{3}, \quad \delta_3 = -(\delta_1 + \delta_2)$$

DISPERSION RELATION





DIRAC FERMIONS IN GRAPHENE

- Low energy quasiparticles are **massless** Dirac fermions $(\upsilon_F = c/300)$

- Spinor:

$$\Psi_s = \begin{pmatrix} \psi_{KAs} \\ \psi_{KBs} \\ \psi_{K'Bs} \\ \psi_{K'As} \end{pmatrix}$$

- Low-energy model with U(4) global symmetry:

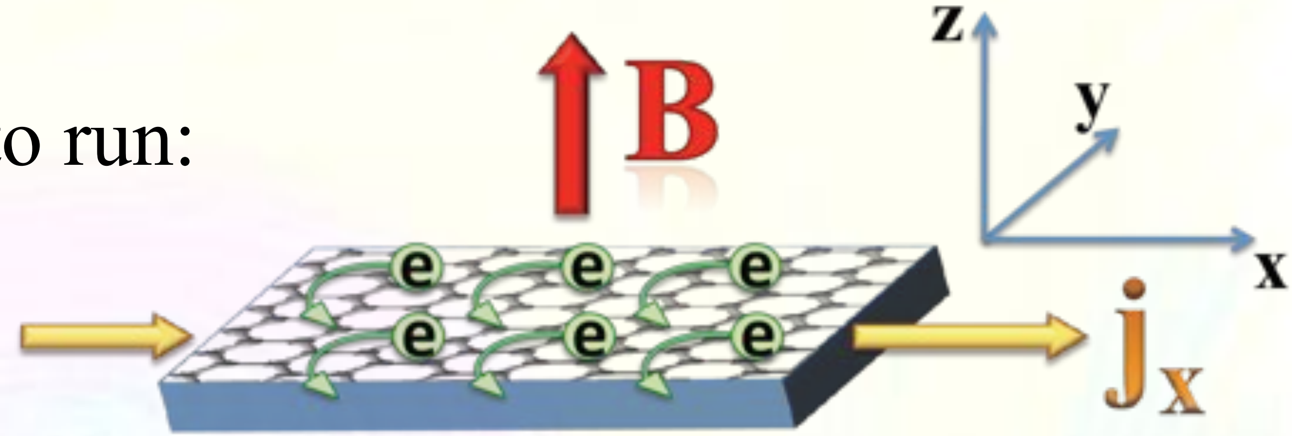
$$H_0 = \upsilon_F \int d^2 r \, \overline{\Psi}_s \left(\gamma^1 \pi_x + \gamma^2 \pi_y \right) \Psi_s$$

[Wallace, Phys. Rev. **71,** 622 (1947)]

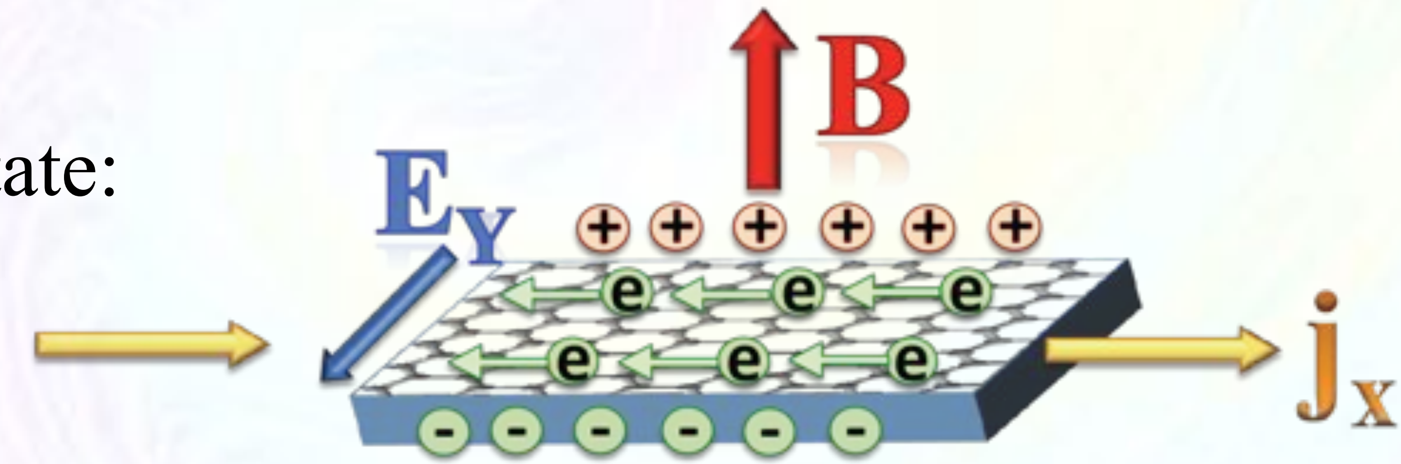
[Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)]

QUANTUM HALL EFFECT

- General setup
  - Current starts to run:



  - Steady state:

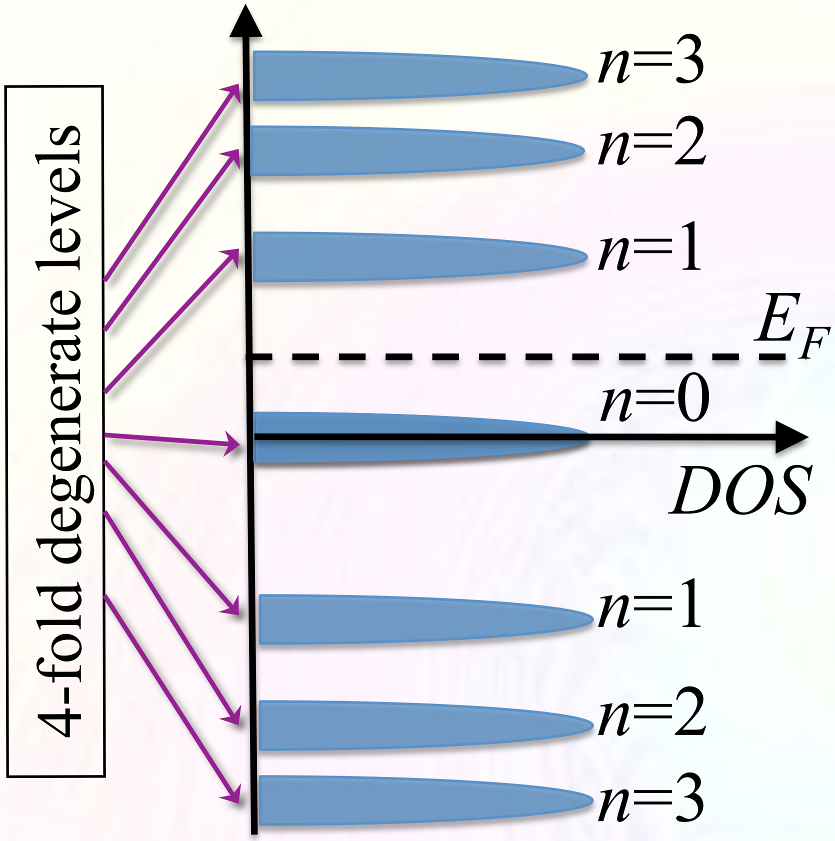


  - Hall conductivity:

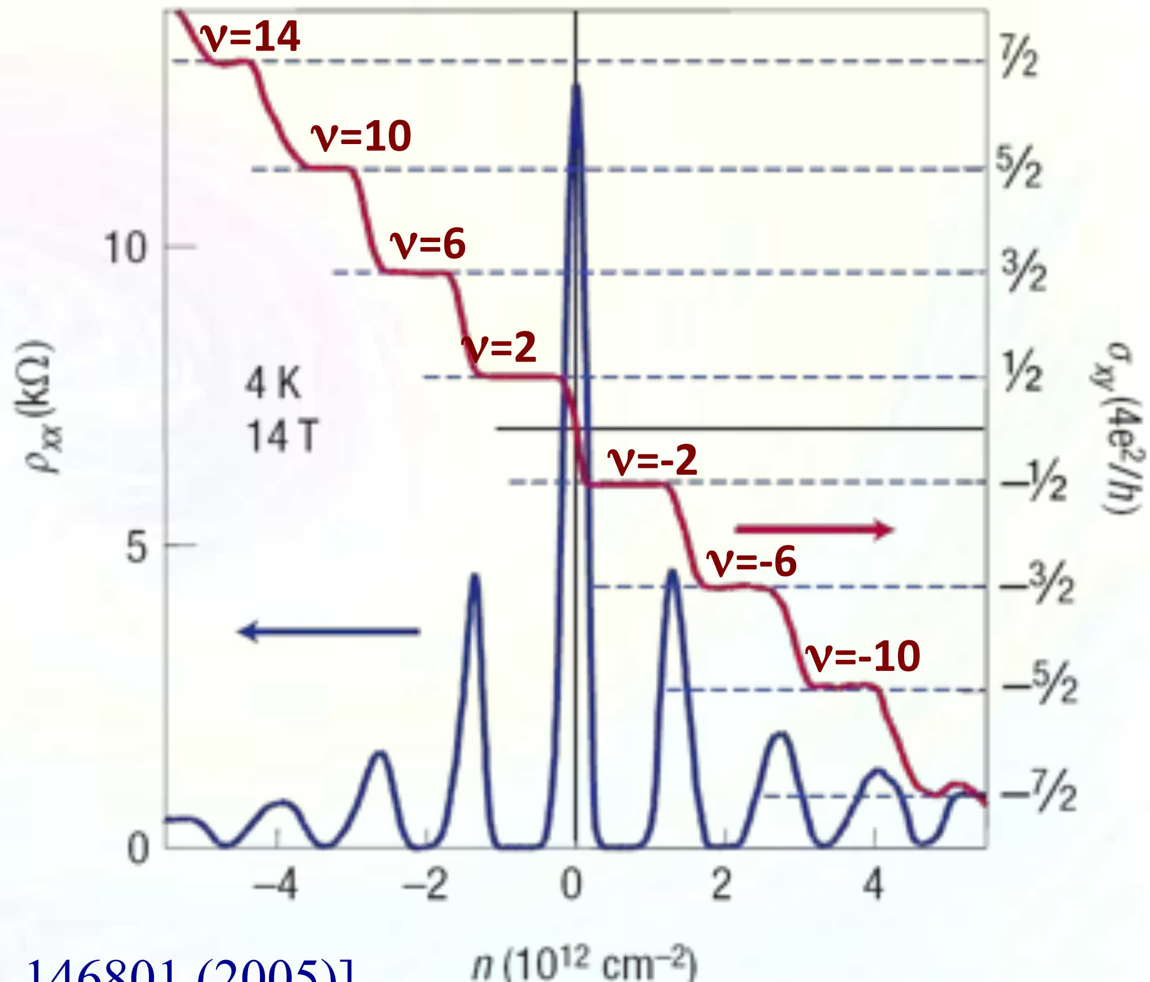
$$j_x = \sigma_{xy} E_y$$

QHE in Graphene

$$E_n = \pm\sqrt{2\hbar v_F^2 n|eB|}$$



$$\sigma_{xy} = \nu \frac{e^2}{h} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$$



[Gusynin, Sharapov, Phys. Rev. Lett. **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, Phys. Rev. B **73**, 125411 (2006)]
[Novoselov et al., Nature **438**, 197 (2005)], [Zhang et al., Nature **438**, 201 (2005)]

ANOMALOUS QHE

- New plateaus at

  ν=0

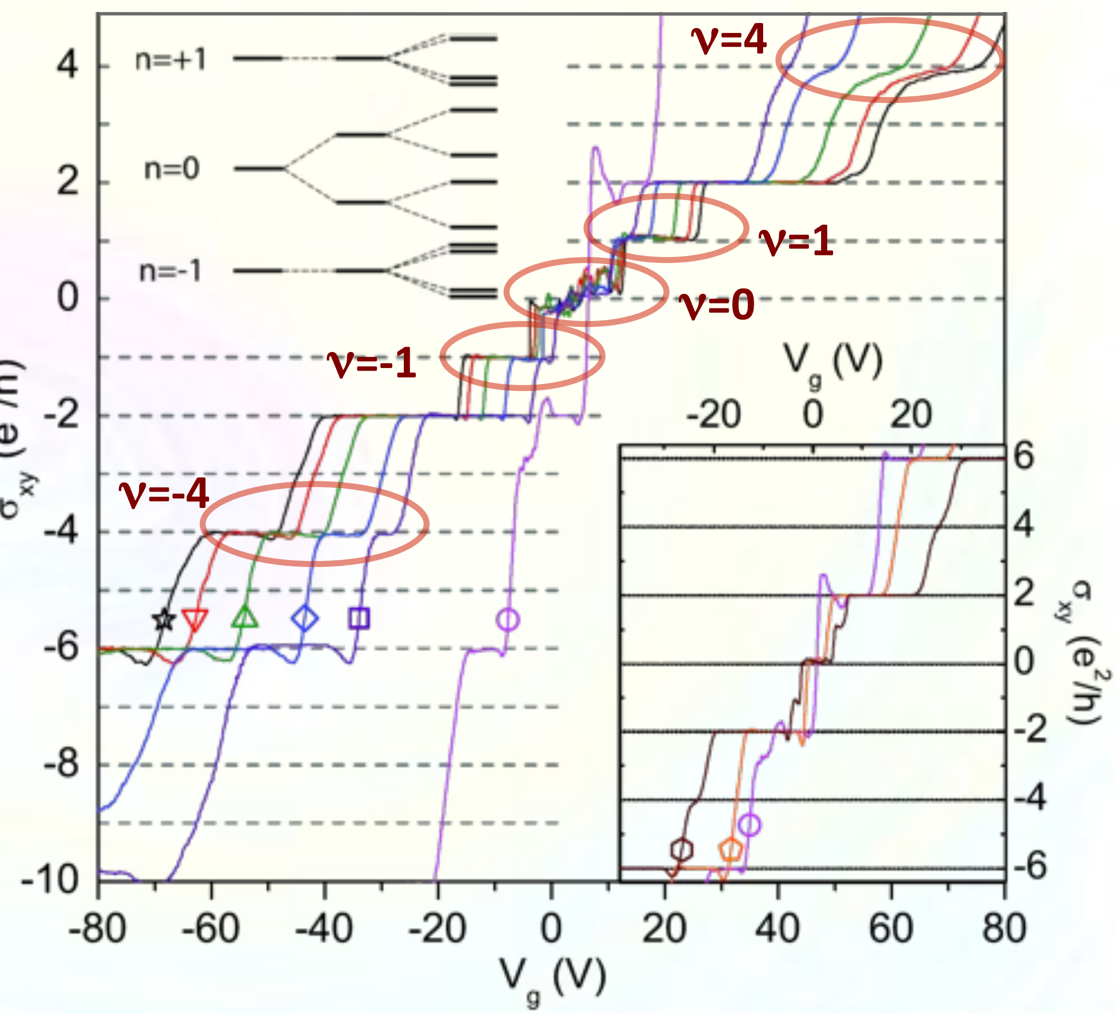
  ν=±1

  ν=±3

  ν=±4

- Some Landau level degeneracy is lifted

Zhang et al., PRL **96,** 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]
[Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)]
[Checkelsky et al., Phys. Rev. Lett. **100**, 206801 (2008)]
[Xu Du et al., Nature **462**, 192 (2009)]

Magnetic Catalysis in Graphene

- Charge carriers are massless Dirac fermions
- Spectrum in magnetic field:

$$E_n = \pm\sqrt{2\hbar v_F^2 n|eB|}$$

- Degenerate E=0 level with particles & holes
- Electron-hole (excitonic) pairing occurs
- $m_{dyn} \neq 0$ is generated
- In qualitative agreement with experiment

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **66** (2002) 045108]

MODEL

- Hamiltonian

  [Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **78** (2008) 085437]

$$H = H_0 + H_C + \int d^2r\left(\mu_B B \Psi^{+}\sigma^{3}\Psi - \mu\Psi^{+}\Psi\right)$$

where the Coulomb interaction term is

$$H_C = \frac{1}{2}\int d^2r\, d^2r' \Psi_s^{+}(r)\Psi_s(r)U\left(r-r'\right)\Psi_{s'}^{+}(r')\Psi_{s'}(r')$$

- Note:
  - H_C is invariant under flavor U(4)

ORDER PARAMETERS

- Many order parameters may be generated (pairing from different valleys/sublattices)

- Dirac masses [triplet under $U(2)_s$]

$$\tilde{\Delta}_s: \quad \overline{\Psi} P_s \Psi = \psi^{+}_{KAs}\psi_{KAs} - \psi^{+}_{KBs}\psi_{KBs} + \psi^{+}_{K'As}\psi_{K'As} - \psi^{+}_{K'Bs}\psi_{K'Bs}$$

(charge-density wave)

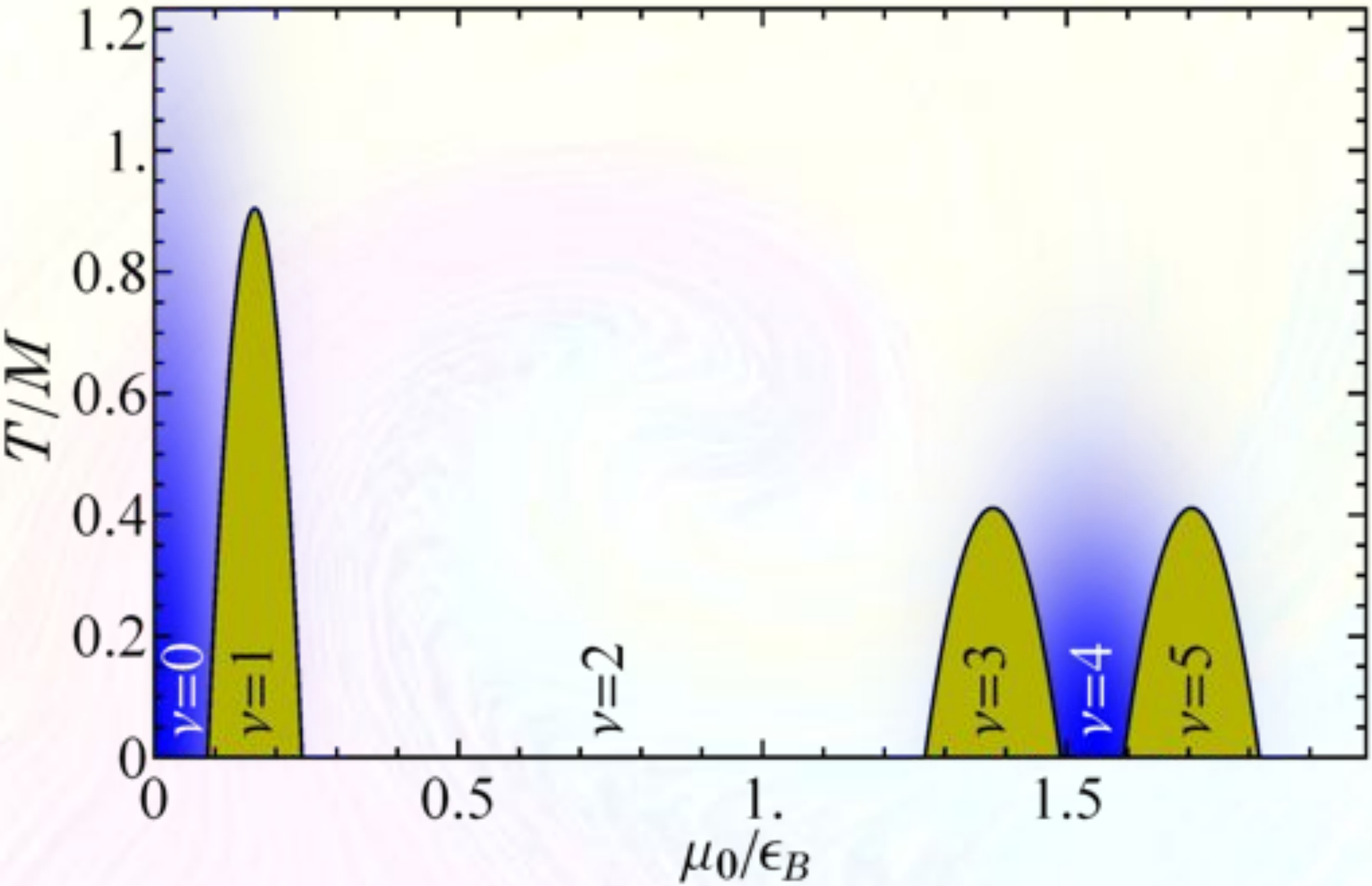
- Haldane masses [singlet under $U(2)_s$]

$$\Delta_s: \quad \overline{\Psi}\gamma^3\gamma^5 P_s \Psi = \psi^{+}_{KAs}\psi_{KAs} - \psi^{+}_{KBs}\psi_{KBs} - (\psi^{+}_{K'As}\psi_{K'As} - \psi^{+}_{K'Bs}\psi_{K'Bs})$$

- + spin & pseudo-spin densities

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **78** (2008) 085437]
[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scr. T **146** (2012) 014018]

PHASE DIAGRAM



Theoretical complications

- Competition between Dirac & Haldane masses is subtle
- Symmetry breaking lattice effects
- Dynamical screening effects
- Competition with quantum Hall ferromagnetism
- Nonzero electron/hole density (ν>0)
- impurities, lattice defects, ripples, etc.

Part 3

CHIRAL EFFECTS

HELICITY/CHIRALITY

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved

Righ-handed

Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)

- The symmetry is anomalous at quantum level

Chiral Magnetic Effect

- Chiral charge is produced by topological QCD configurations

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$

- Random fluctuations with nonzero chirality in each event

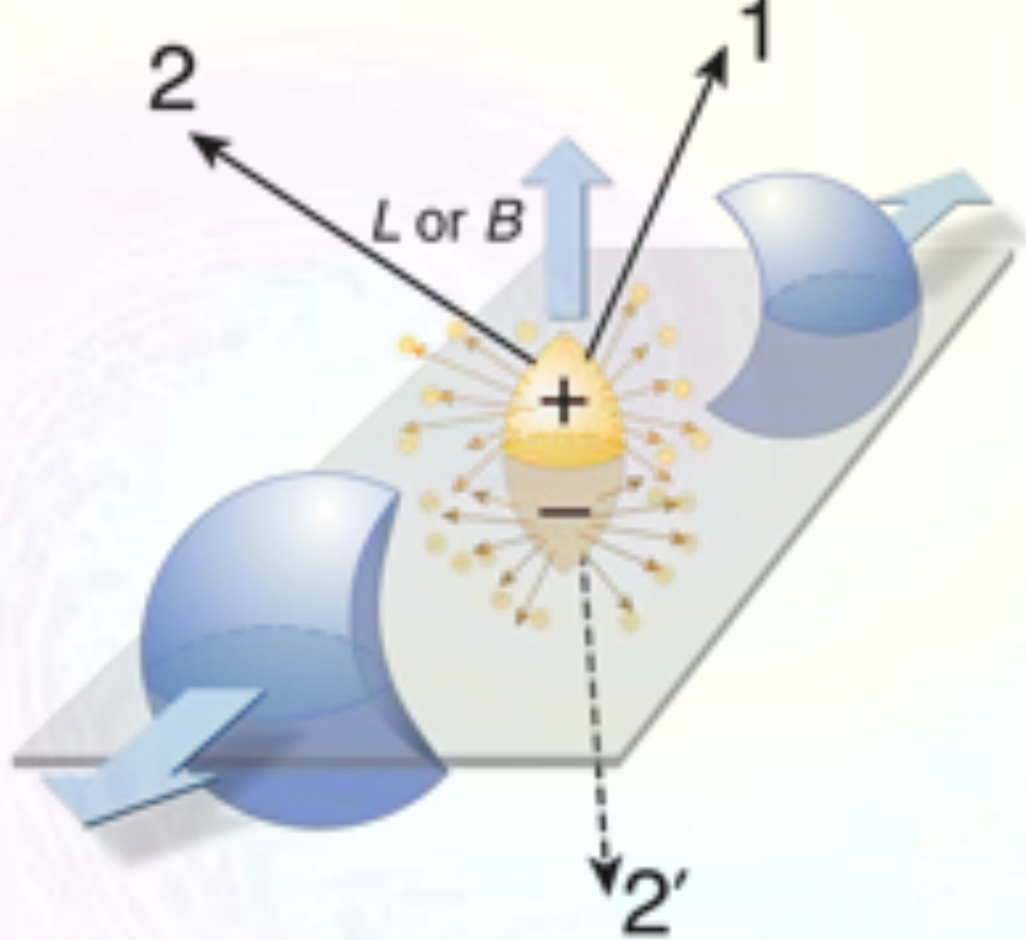
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- Driving electric current

$$\langle \vec{j} \rangle = -\frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

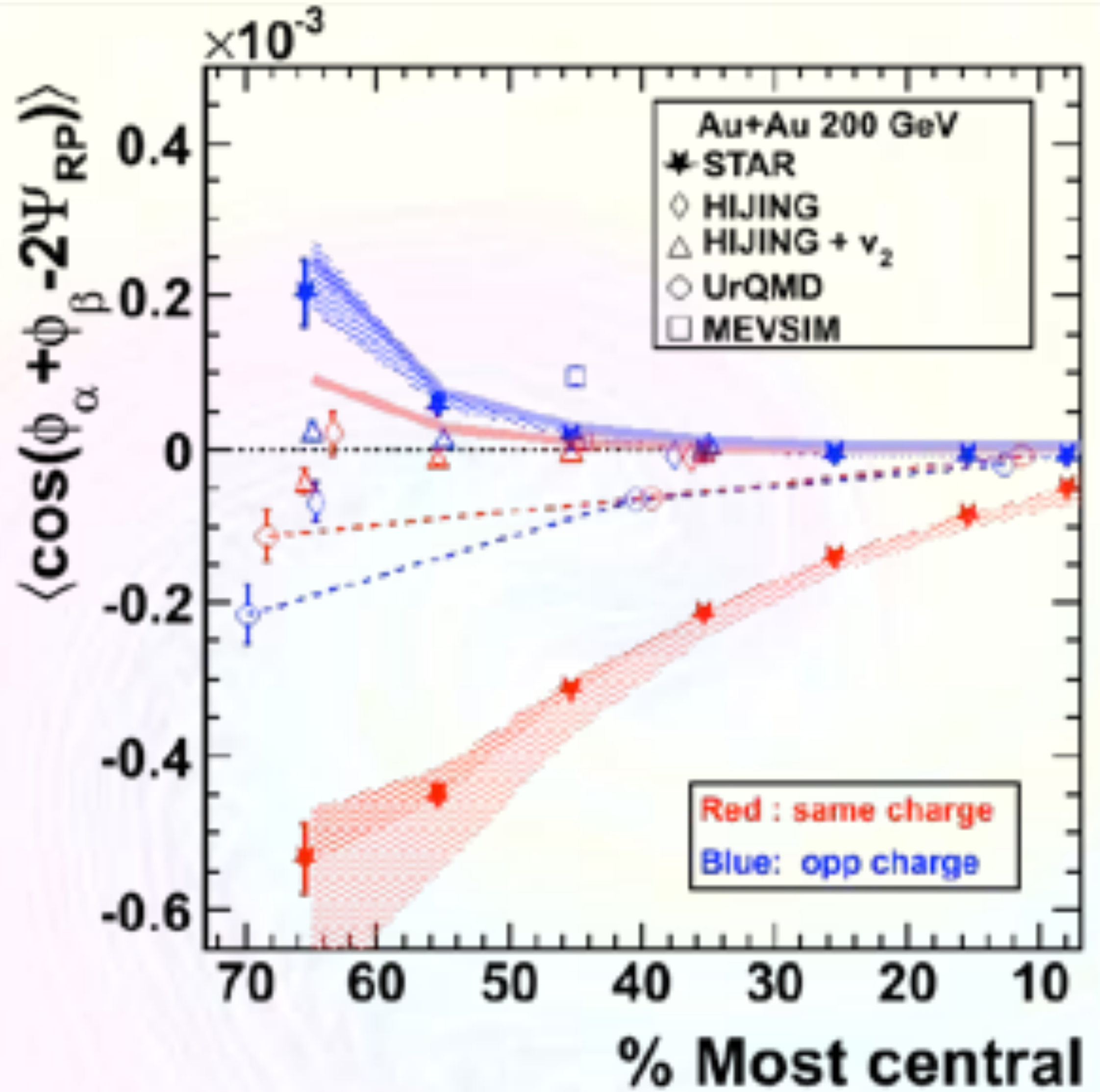
DIPOLE CME

- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Experimental evidence



[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]
[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]

Chiral separation effect

- Electric current induced by axial chemical potential

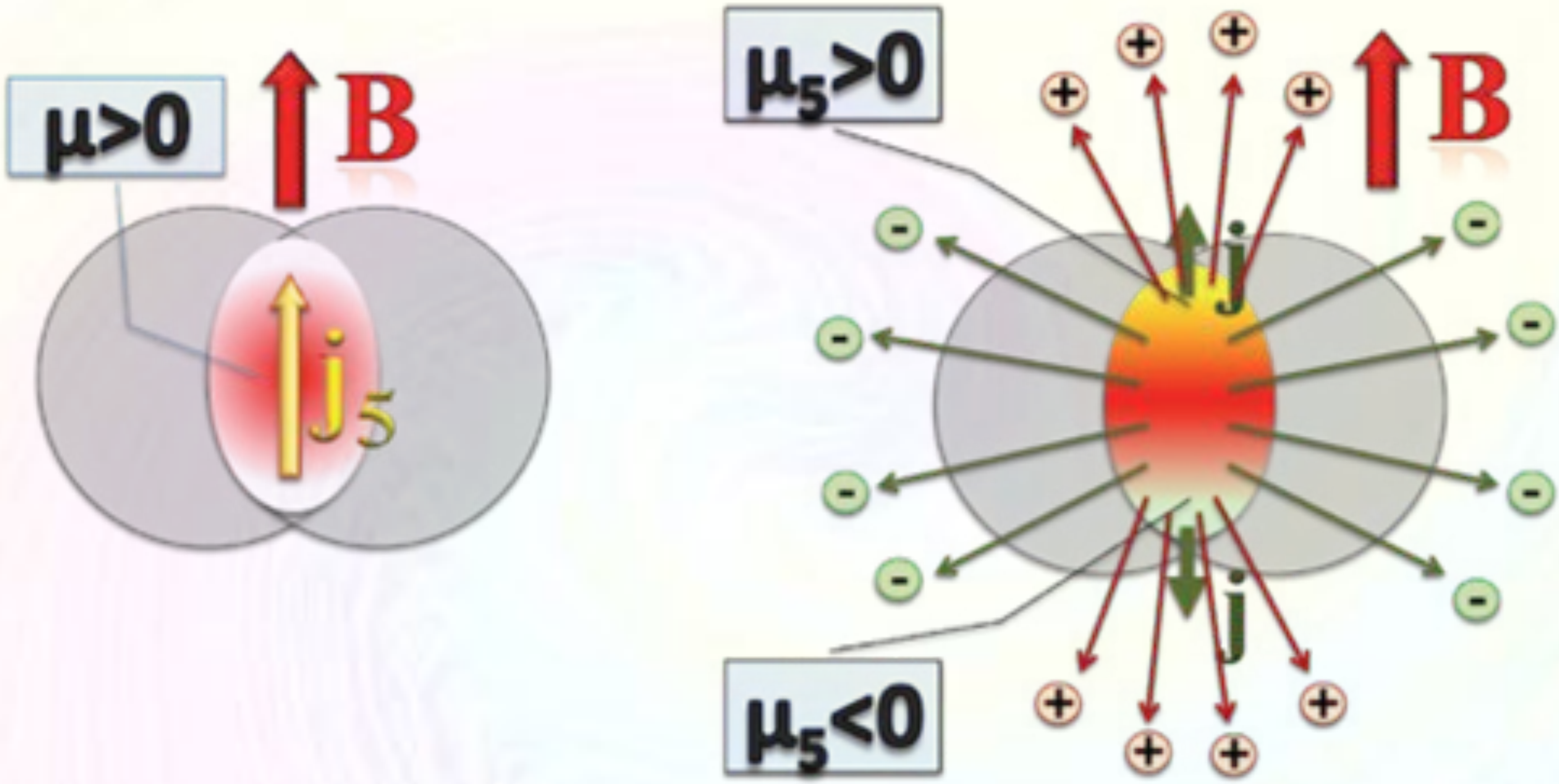
$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Exact result (is it?), which follows from chiral anomaly relation
- No radiative corrections expected…

QUADRUPOLE CME

- Start from a small baryon density and B≠0



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

BEYOND CSE/CME

- Any radiative corrections to CSE?

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu + \ldots \qquad \text{(yes)}$$

[Gorbar, Miransky, Shovkovy, Wang, arXiv:1304.4606]

- Any dynamical parameter Δ (“chiral shift”) associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \Delta\overline{\psi}\gamma^3\gamma^5\psi \qquad \text{(yes)}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83 (2011) 085003]

- Note: Δ=0 is not protected by any symmetry

CHIRAL SHIFT & FERMI SURFACE

- Chirality is $\approx$ well defined at Fermi surface $\left(|k_3| >> m\right)$
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

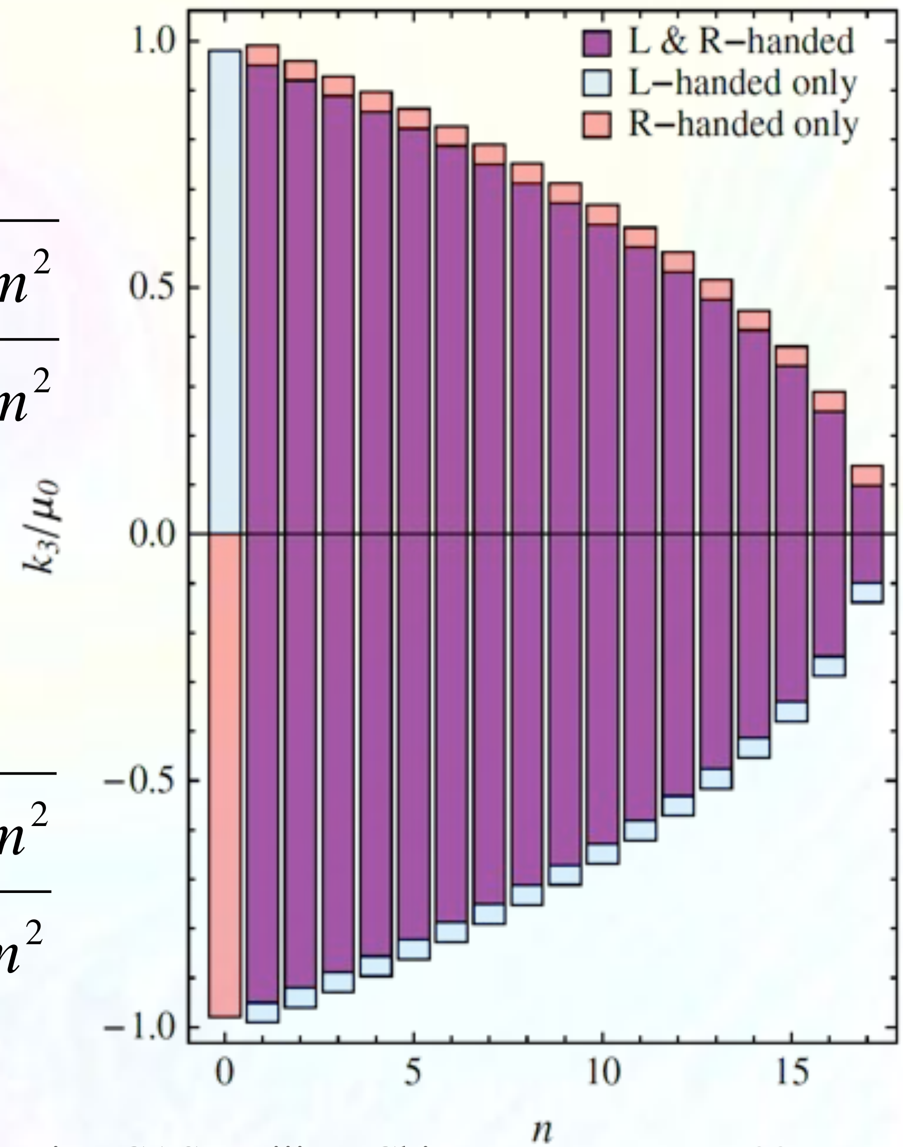
$$k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

$$k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$



Physics due to Chiral Shift

- Chiral shift induces a chiral asymmetry at the Fermi surface
- Chiral shift modifies axial current (≈ spin polarization)
- Potential applications:
  - Pulsar kicks
  - Facilitation of supernova explosions
  - modified Chiral magnetic effect

SUMMARY

- Studies of relativistic matter in magnetic field are relevant for many branches of physics
- The underlying physics is conceptually rich
- Recent developments include
  - Magnetic catalysis
  - Chiral magnetic effect
  - Chiral shift
  - Chiral magnetic spiral
  - Paraelectricity in magnetized QED
  - Magnetic color superconductors
  - and many others