Radiative corrections to chiral separation effect in QED

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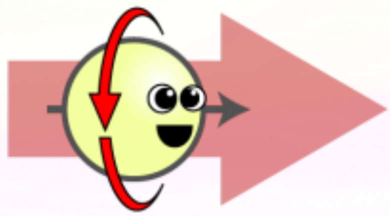
*Gorbar, Miransky, Shovkovy & Xinyang Wang, arXiv:1304.4606

Outline

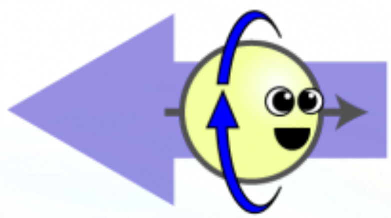
- Introduction
- Chiral magnetic & separation effects
- CSE in NJL model
  - Chiral shift
  - Axial current
  - Axial anomaly relation
- CSE in QED
  - Radiative corrections to axial current
  - Chiral shift
- Discussion & Summary

Helicity/Chirality

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved



Righ-handed



Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)
- At quantum level, however, such symmetry is anomalous

Chiral magnetic effect

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\left\langle j^3 \right\rangle_{\text{free}} = -\frac{e^2 B}{2\pi^2}\mu_5 \qquad \text{(free theory!)}$$

[Kharzeev, McLerran, Warringa, Nucl. Phys. A 803, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]

- Exact result (is it?), following from

$$\partial_\mu j^\mu = e\frac{e^2}{16\pi^2}\left(F_L^{\mu\nu}\tilde{F}_{L,\mu\nu} - F_R^{\mu\nu}\tilde{F}_{R,\mu\nu}\right)$$

after $eA_0^5 \to \mu_5$ and integration…

CME in collisions

- Chiral charge is produced by topological QCD configurations

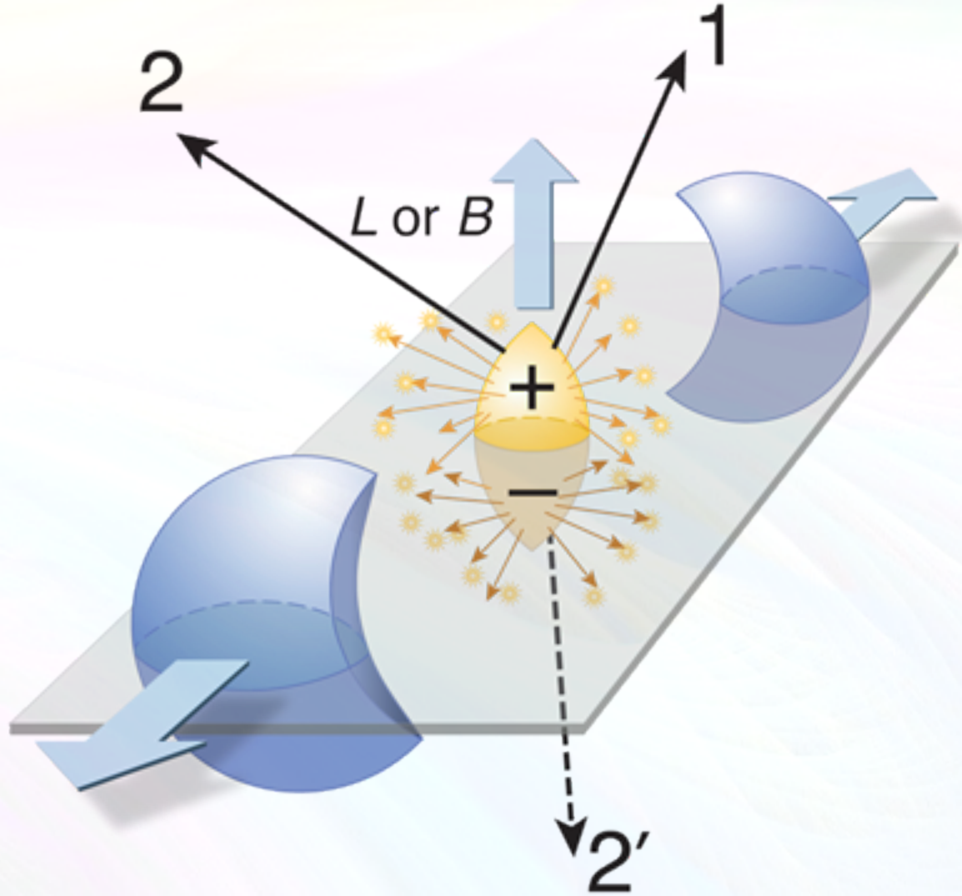
$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_a^{\mu\nu} \tilde{F}^a_{\mu\nu}$$

- Fluctuations are random, but chirality is nonzero in each event

$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

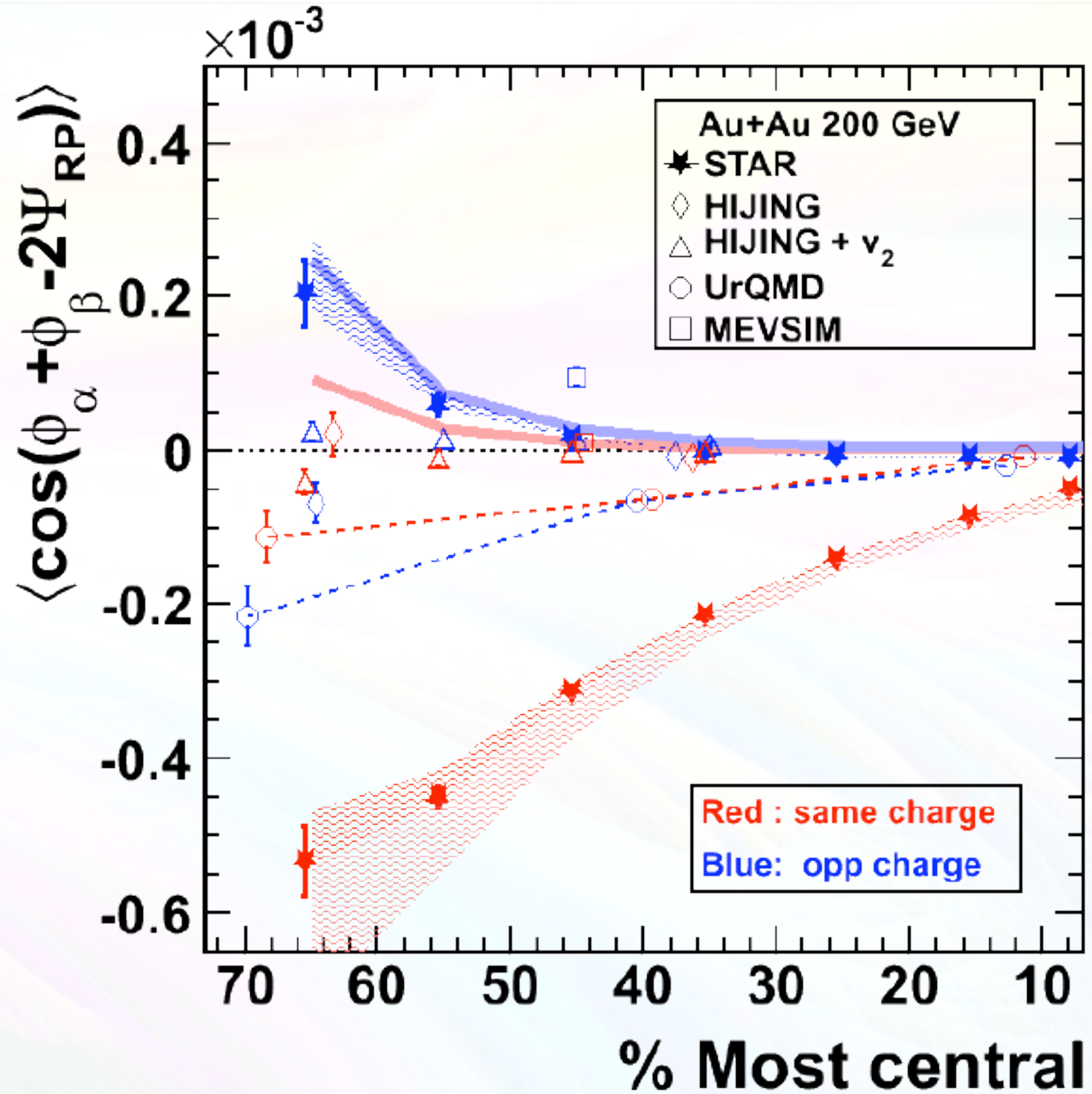
Heavy ion collisions

- Dipole pattern of electric currents (charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Experimental evidence



[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]
[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]

Chiral separation effect

- Electric current induced by axial chemical potential

$$\langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Exact result (is it?), which follows from chiral anomaly relation

- No radiative corrections expected…

Possible implication

- Seed chemical potential (μ) induces axial current

$$\langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2}\mu$$

- Leading to separation of chiral charges:

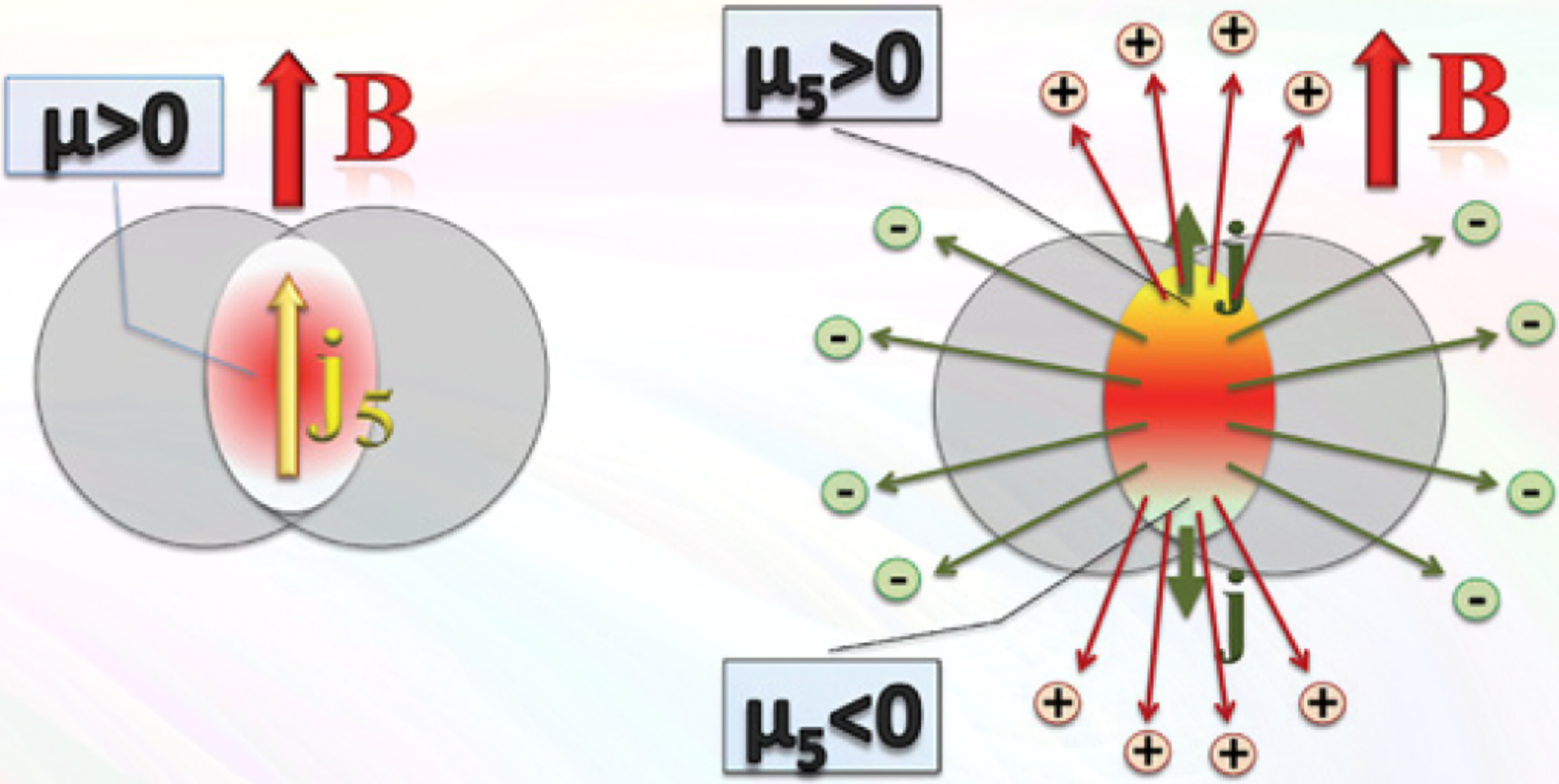
μ_5>0 (one side) & μ_5>0 (another side)

- In turn, chiral charges induce back-to-back electric currents through

$$\langle j^3 \rangle_{\text{free}} = -\frac{e^2 B}{2\pi^2}\mu_5$$

Quadrupole CME

- Start from a small baryon density and B≠0



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Motivation

- Any additional consequences of the CSE relation?

$$\langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2}\mu \qquad \text{(free theory!)}$$

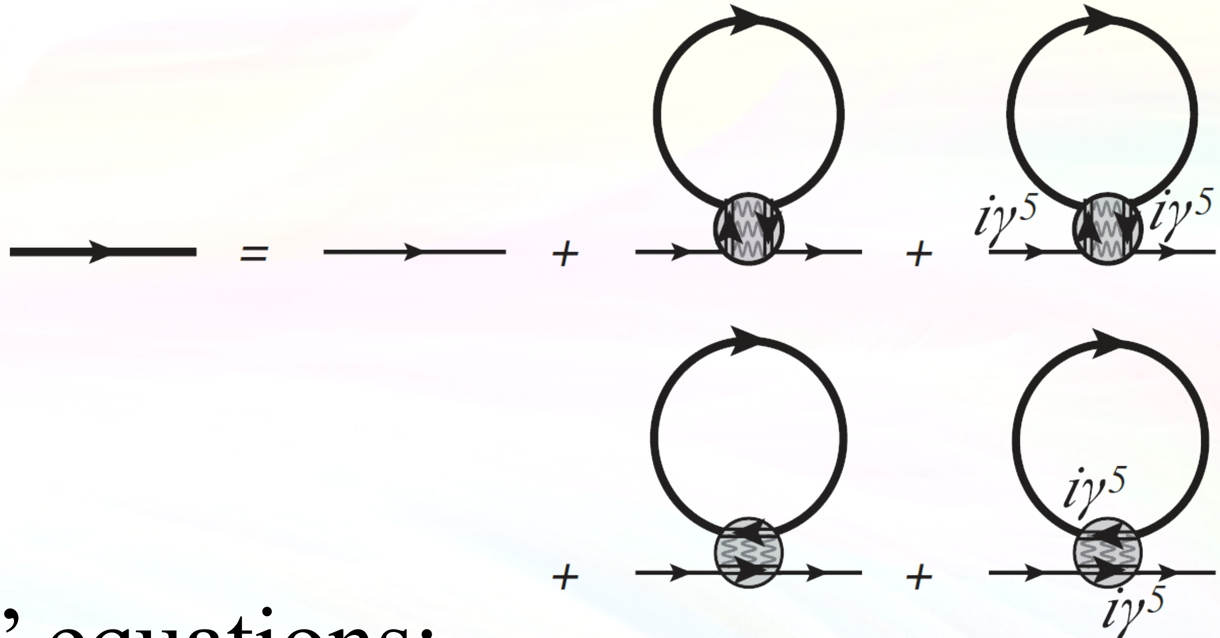
- Nonzero dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \Delta \overline{\psi} \gamma^3 \gamma^5 \psi$$

- Note: Δ=0 is not protected by any symmetry

Chiral shift in NJL model

- NJL model (local interaction)



- "Gap" equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle$$ ("effective" chemical potential)

$$m = m_0 - G_{\text{int}} \langle \bar{\psi} \psi \rangle$$ (dynamical mass)

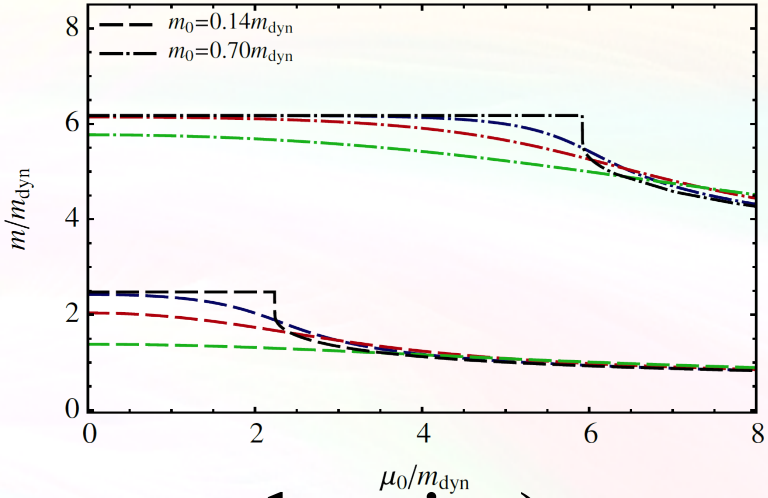
$$\Delta = -\frac{1}{2} G_{\text{int}} \langle j_5^3 \rangle$$ (chiral shift parameter)

Solutions

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\rm int}|eB|}\right)$$

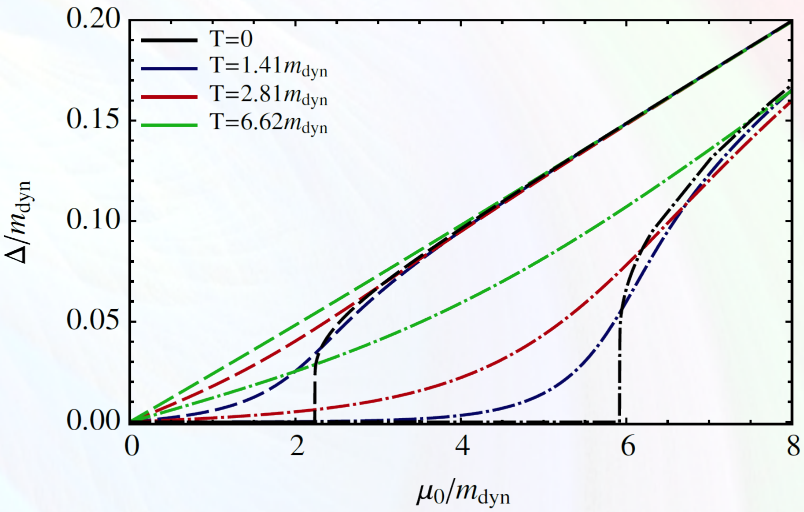
$$\Delta = 0 \quad \& \quad \mu = \mu_0$$



- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2}$$



Chiral shift @ Fermi surface

- Chirality is $\approx$ well defined at Fermi surface ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

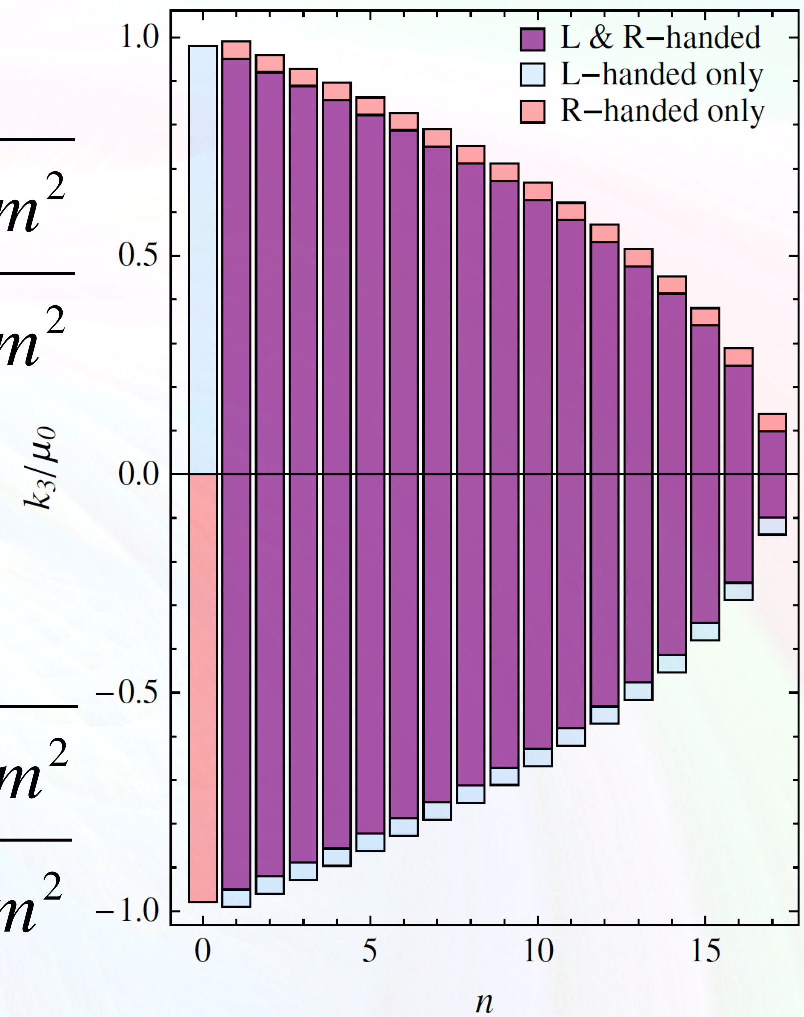
$$k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

$$k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$



Chiral shift vs. axial anomaly

- Does the chiral shift modify the axial anomaly relation?

- Using point splitting method, one derives

$$\langle \partial_\mu j_5^\mu(u) \rangle = -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right)$$

$$\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \to 0$$

[Gorbar, Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- Does the chiral shift give any contribution to the axial current?
- In the point splitting method, one has

$$\left\langle j_5^{\mu} \right\rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\varepsilon^2}\delta_{\mu}^{3} \cong \frac{\Lambda^2\Delta}{2\pi^2}\delta_{\mu}^{3}$$

[Gorbar, Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- This is consistent with the NJL calculations
- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the correction to the axial current should be finite

Axial current in QED

- Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \overline{\psi}\left(i\gamma^{\mu} D_{\mu} + \mu\gamma^{0} - m \right)\psi + (\text{counterterms})$$

- Axial current

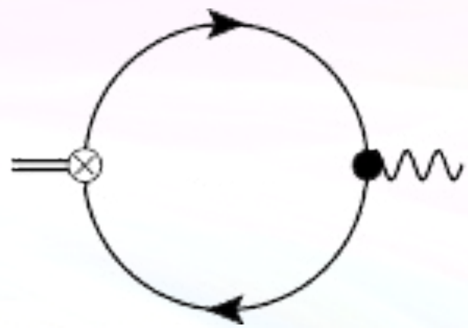
$$\left\langle j_5^3 \right\rangle = -Z_2 \, \mathrm{tr}\left[\gamma^3 \gamma^5 G(x,x) \right]$$

- To leading order in coupling $\alpha = e^2/(4\pi)$

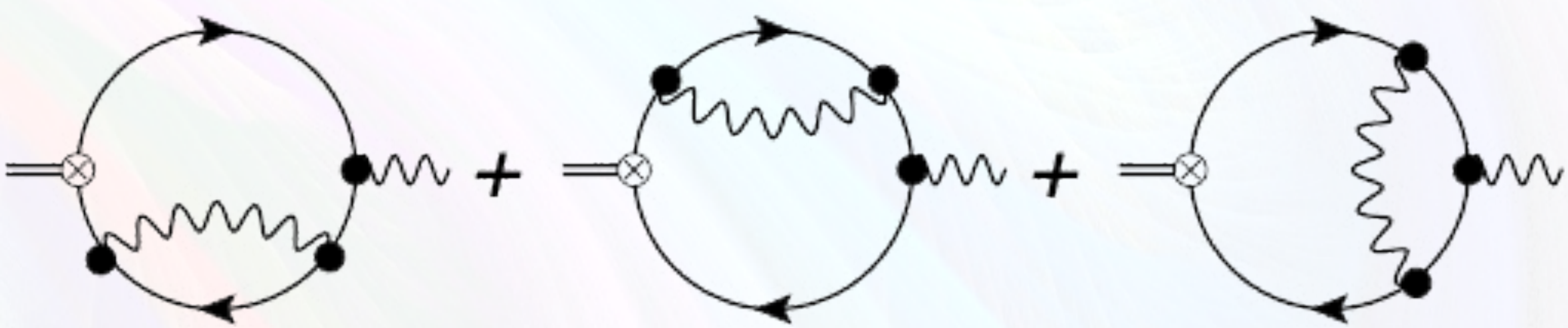
$$G(x,y) = S(x,y) + i \int d^4u \, d^4v \, S(x,u) \Sigma(u,v) S(v,y)$$

Leading order in the field

- After expanding $S(x,y)$ in powers of A_μ
- $\langle j_5^3 \rangle$ to leading order



- Radiative corrections to $\langle j_5^3 \rangle$



Alternative expansion

- Expanding $S(x,y)$ in powers of B

$$S(x,y) = \underbrace{\bar{S}^{(0)}(x-y) + \bar{S}^{(1)}(x-y)}_{\text{Translation invariant part}} + i\underbrace{\Phi(x,y)}_{\text{Schwinger phase}} S^{(0)}(x-y)$$

- The Schwinger phase (in Landau gauge)

$$\Phi(x,y) = -\frac{eB}{2}(x_1 + y_1)(x_2 - y_2)$$

- Note: this is not translation invariant

Translation invariant parts

- The Fourier transforms

$$\overline{S}^{(0)}(k) = i\frac{(k_0+\mu)\gamma^0 - \mathbf{k}\cdot\gamma + m}{\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2 - \mathbf{k}^2 - m^2}$$

$$\overline{S}^{(1)}(k) = -\gamma^1\gamma^2 eB\frac{(k_0+\mu)\gamma^0 - k_3\gamma^3 + m}{\left[\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2 - \mathbf{k}^2 - m^2\right]^2}$$

- Note the singularity near the Fermi surface…

Fermi surface singularity

- "Vacuum" + "matter" parts

$$\frac{1}{\left[\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2-\mathbf{k}^2-m^2\right]^n}=\text{"V"}+\text{"M"}$$

where

$$\text{"V"}=\frac{1}{\left[\left(k_0+\mu\right)^2-\mathbf{k}^2-m^2+i\varepsilon\right]^n}$$

$$\text{"M"}=\frac{2\pi i(-1)^{n-1}}{(n-1)!}\theta\left(|\mu|-|k_0|\right)\theta\left(-k_0\mu\right)\delta^{(n-1)}\left[\left(k_0+\mu\right)^2-\mathbf{k}^2-m^2\right]$$

Axial current (0th order)

- From definition

$$\langle j_5^3 \rangle_0 = -\int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\gamma^3 \gamma^5 \bar{S}^{(1)}(k)\right]$$

- After integrating over energy

$$\langle j_5^3 \rangle_0 = -\frac{eB\,\mathrm{sign}(\mu)}{4\pi^3} \int d^3\mathbf{k}\, \underbrace{\delta\left(\mu^2 - \mathbf{k}^2 - m^2\right)}_{\text{Matter part}}$$

and finally

$$\langle j_5^3 \rangle_0 = -\frac{eB\,\mathrm{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2}$$

- Note the role of the Fermi surface (!)

Conventional wisdom

- Only the lowest Landau level contributes

$$\langle j_5^3 \rangle_0 = \frac{eB}{4\pi^2} \int dk_3 \left[\theta\left(-\mu - \sqrt{k_3^2 + m^2}\right) - \theta\left(\mu - \sqrt{k_3^2 + m^2}\right) \right]$$

giving same answer

$$\langle j_5^3 \rangle_0 = -\frac{eB\,\mathrm{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2}$$

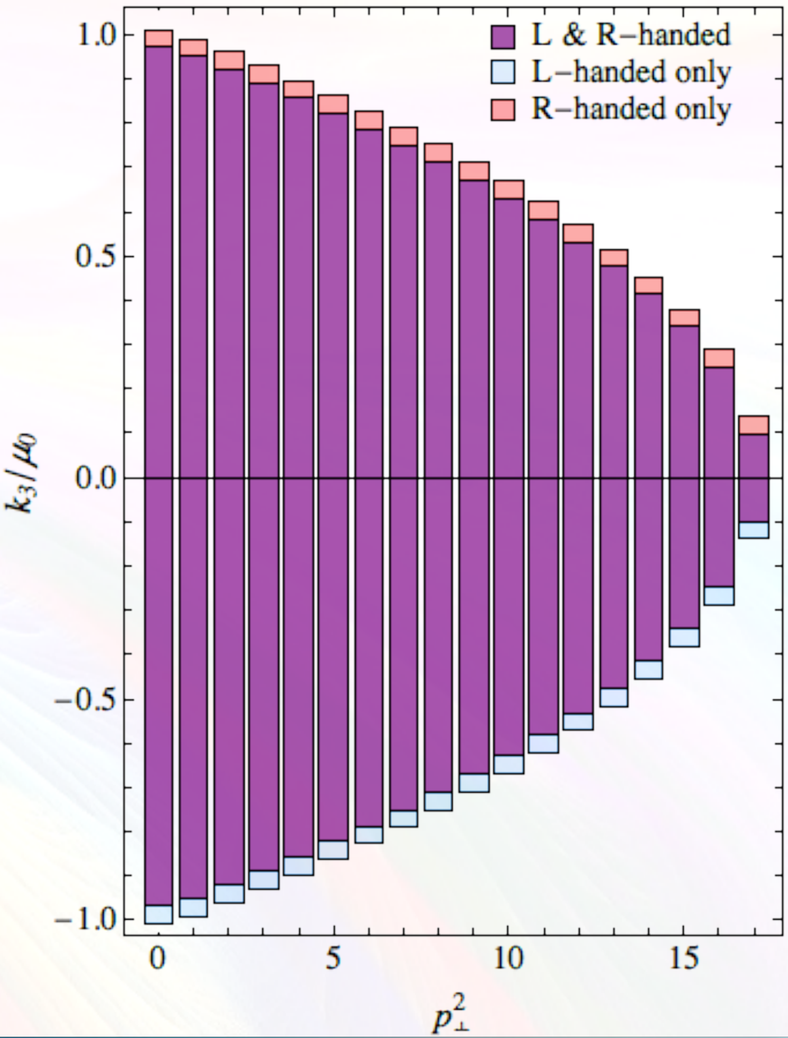
- There are no contributions from higher Landau levels (n≥1)

- There is a connection with the index theorem

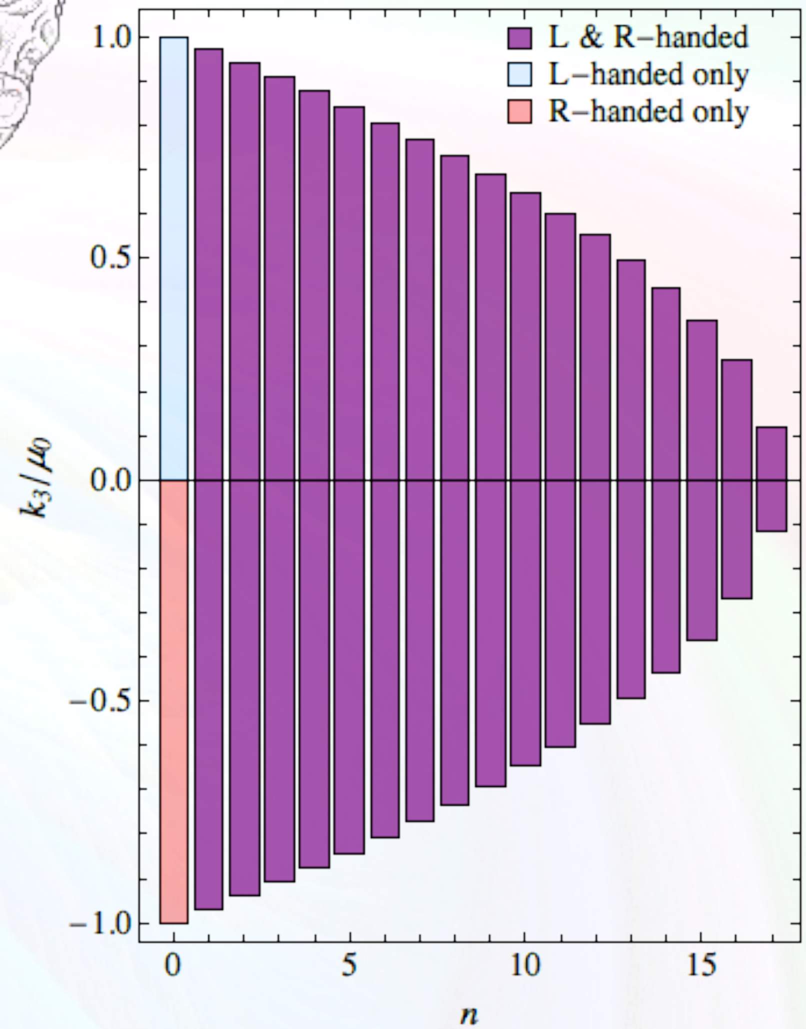
Two interpretations

- Two ways to look at the same result

$B \to 0$



$B \neq 0$



Radiative correction

- Original two-loop expression (too hard!)

$$\langle j_5^3 \rangle_\alpha = 32\pi\alpha e B \int \frac{d^4p\, d^4k}{(2\pi)^8} \frac{1}{(P-K)^2_\Lambda} \left[\frac{(k_0+\mu)[3(p_0+\mu)^2+\mathbf{p}^2+m^2] - 4(p_0+\mu)(\mathbf{p}\cdot\mathbf{k}+2m^2)}{(P^2-m^2)^3(K^2-m^2)} - \frac{(k_0+\mu)[3(p_0+\mu)^2-\mathbf{p}^2+3m^2] - 2(p_0+\mu)(\mathbf{p}\cdot\mathbf{k})}{3(P^2-m^2)^2(K^2-m^2)^2} \right] + \langle j_5^3 \rangle_{\mathrm{ct}}.$$

- After integration by parts (only matter parts contribute)

$$\langle j_5^3 \rangle_\alpha = 64 i\pi^2 \alpha e B \int \frac{d^4p d^4k}{(2\pi)^8} \left[\frac{(k_0+\mu)(p_0+\mu) - \mathbf{p}\cdot\mathbf{k} - 2m^2}{(P-K)^2_\Lambda (K^2-m^2)} \delta'\left[\mu^2-m^2-\mathbf{p}^2\right]\delta(p_0) + \frac{3(p_0+\mu)^2 - 3(k_0+\mu)(p_0+\mu) + \mathbf{p}^2 - \mathbf{p}\cdot\mathbf{k} + 3m^2}{3(P-K)^2_\Lambda (P^2-m^2)^2} \delta\left(\mu^2-m^2-\mathbf{k}^2\right)\delta(k_0) \right] + \langle j_5^3 \rangle_{\mathrm{ct}}$$

Result (m<<μ)

- Loop contributions

$$f_1+f_2+f_3=\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{\Lambda}{2\mu}+\frac{11}{12}\right)+\frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{\Lambda}{2^{3/2}\mu}+\frac{1}{6}\right)$$

- Counterterms

$$\left\langle j_5^3\right\rangle_{\mathrm{ct}}=-\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{\Lambda}{m}+\ln\frac{m_\gamma^2}{m^2}+\frac{9}{4}\right)-\frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{\Lambda}{m_\gamma}-\frac{3}{4}\right)$$

- Final result

$$\left\langle j_5^3\right\rangle_{\alpha}=-\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m}+\ln\frac{m_\gamma^2}{m^2}+\frac{4}{3}\right)-\frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma}-\frac{11}{12}\right)$$

Sign of nonperturbative physics

- Unphysical dependence on photon mass

$$\left\langle j_5^3 \right\rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha eB m^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

- Infrared physics with

$$m_\gamma \leq |k_0|, |k_3| \leq \sqrt{|eB|}$$

not captured properly

- Note: similar problem exists in calculation of Lamb shift

Nonperturbative effects (?)

- Perpendicular momenta cannot be defined with accuracy better than

$$\left|\Delta \mathbf{k}_{\perp}\right|_{\min} \sim \sqrt{|eB|}$$

(In contrast to the tacit assumption in using expansion in powers of *B*-field)

- Screening effects provide a natural infrared regulator

$$m_{\gamma} \Rightarrow \sqrt{\alpha}\,\mu$$

(Formally, this goes beyond the leading order in coupling)

Nonperturbative result (?)

- One may conjecture nonpertubative result

(1) If non-conservation of momentum dominates

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha\, eB\, \mu}{2\pi^3}\left(\ln\frac{\mu|eB|}{m^3} + O(1)\right) - \frac{\alpha\, eB\, m^2}{2\pi^3 \mu}\left(\ln\frac{\mu}{\sqrt{|eB|}} + O(1)\right)$$

(2) If photon screening is more important

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha\, eB\, \mu}{2\pi^3}\left(\ln\frac{\alpha\, \mu^3}{m^3} + O(1)\right) - \frac{\alpha\, eB\, m^2}{2\pi^3 \mu}\left(\ln\frac{1}{\sqrt{\alpha}} + O(1)\right)$$

Summary

- Weak *B*-field limit: **new interpretation** of the topological contribution to CSE relation
- Radiative corrections are **nonzero**
- Radiative corrections vanish without "matter" part with **singularity on Fermi surface**
- **Nonperturbative** physics complicates the infrared contribution
- With **logarithmic accuracy**, the result can be conjectured