

Chiral separation effect: Recent developments

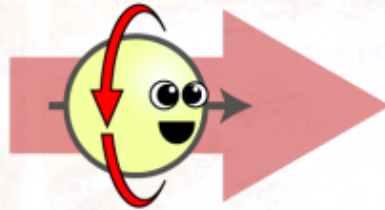
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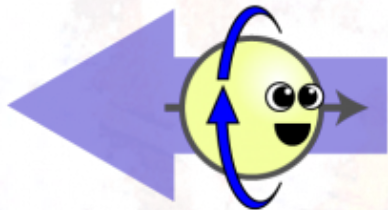
*E. Gorbar, V. Miransky, I. Shovkovy, and Xinyang Wang, arXiv:1304.4606



- Helicities of (ultra-relativistic) massless particles are (approximately) conserved



Right-handed



Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)
- The symmetry is anomalous at quantum level

- Chiral charge is produced by topological QCD configurations

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$

- Random fluctuations with nonzero chirality in each event

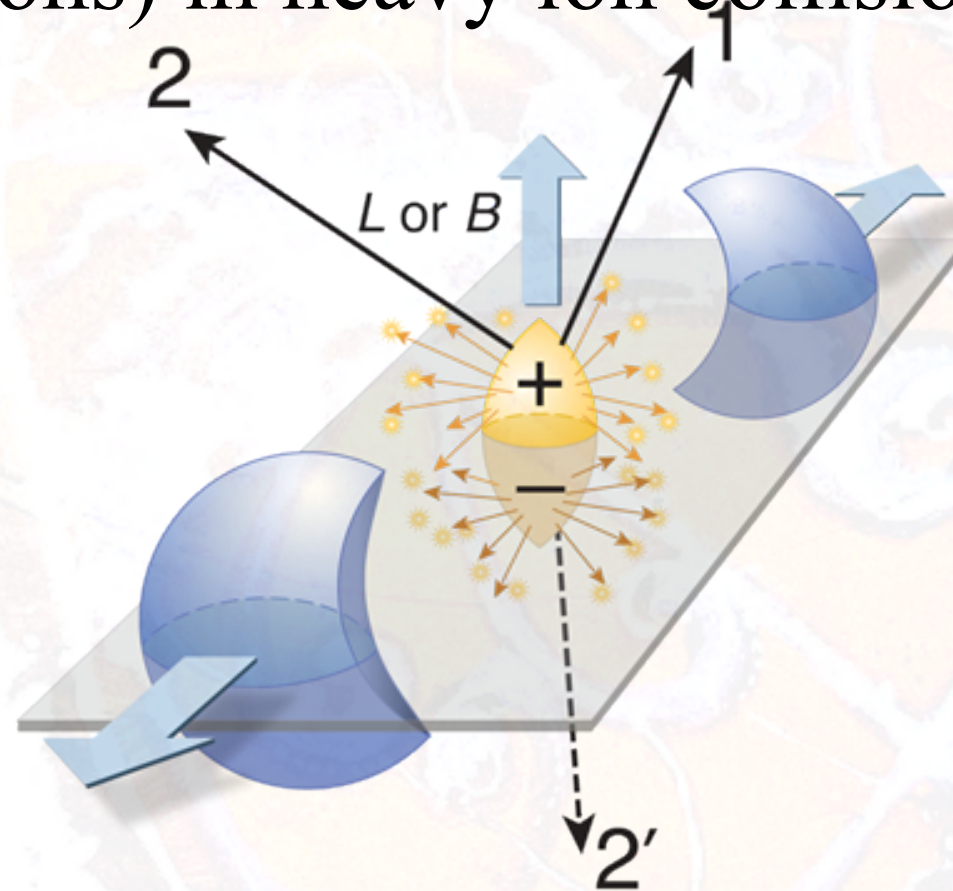
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- Driving electric current

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

Heavy ion collisions

- Dipole pattern of electric currents (charge correlations) in heavy ion collisions

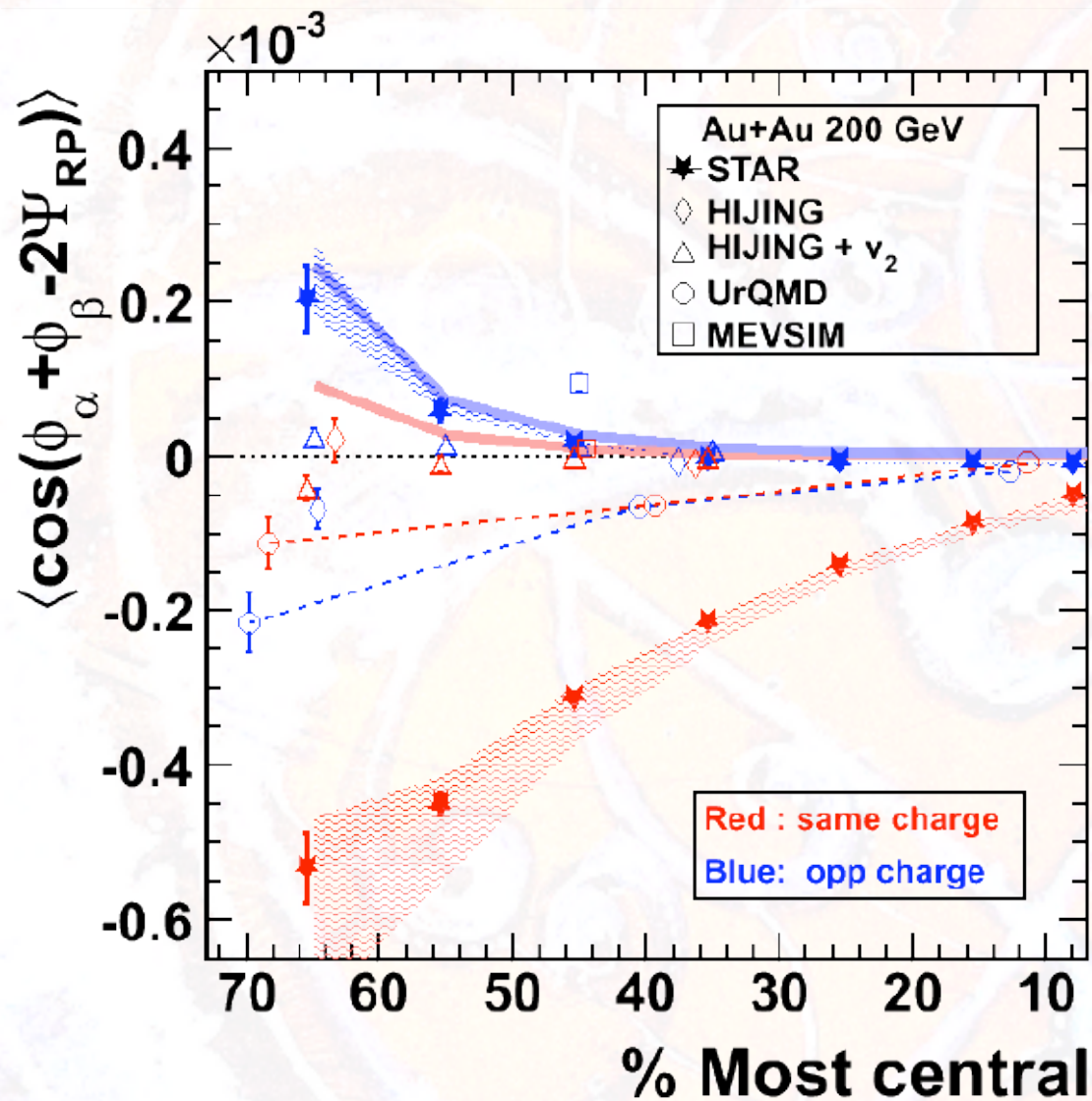


[Kharzeev, Zhitnitsky, Nucl. Phys. A **797**, 67 (2007)]

[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Experimental evidence



[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]

[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]

- Axial current density induced by electric chemical potential

$$\langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu \quad (\text{free theory!})$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Exact result (is it?), which follows from chiral anomaly relation
- No radiative correction expected...

- Seed chemical potential (μ) induces axial current

$$\langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu$$

- Leading to separation of chiral charges:

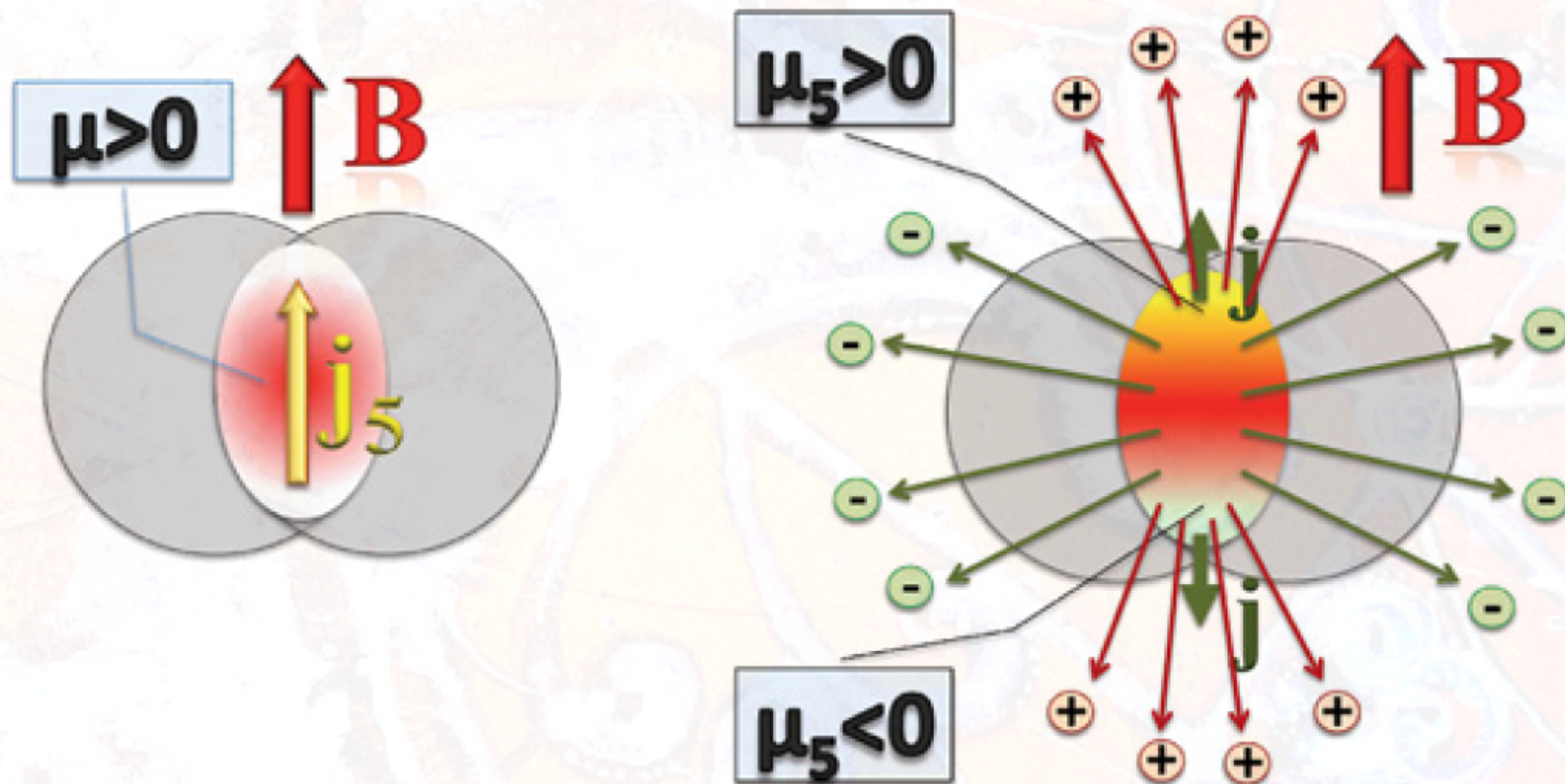
$$\mu_5 > 0 \text{ (one side)} \quad \& \quad \mu_5 < 0 \text{ (another side)}$$

- In turn, chiral charges induce back-to-back electric currents through

$$\langle j^3 \rangle_{\text{free}} = \frac{e^2 B}{2\pi^2} \mu_5$$

Quadrupole CME

- Start from a small baryon density and $B \neq 0$



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

- Any additional consequences of the CSE relation?

$$\langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu \quad (\text{free theory!})$$

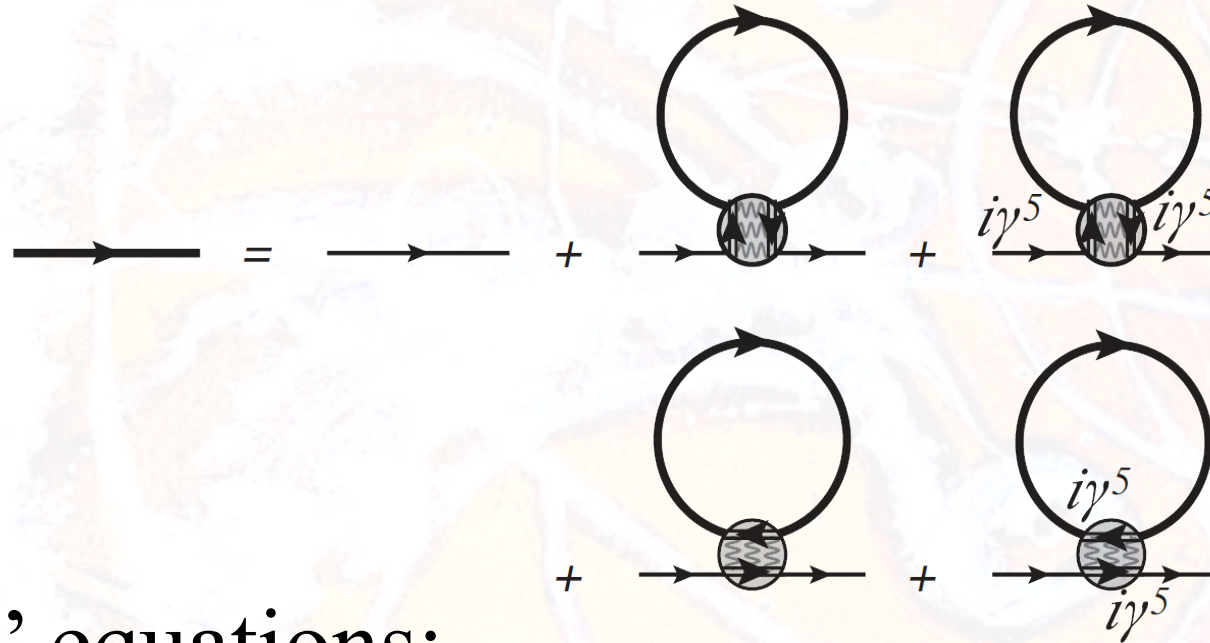
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

- Any dynamical parameter Δ (“chiral shift”) associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \Delta \bar{\psi} \gamma^3 \gamma^5 \psi$$

- Note: $\Delta=0$ is not protected by any symmetry

- NJL model (local interaction)



- “Gap” equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle \quad (\text{“effective” chemical potential})$$

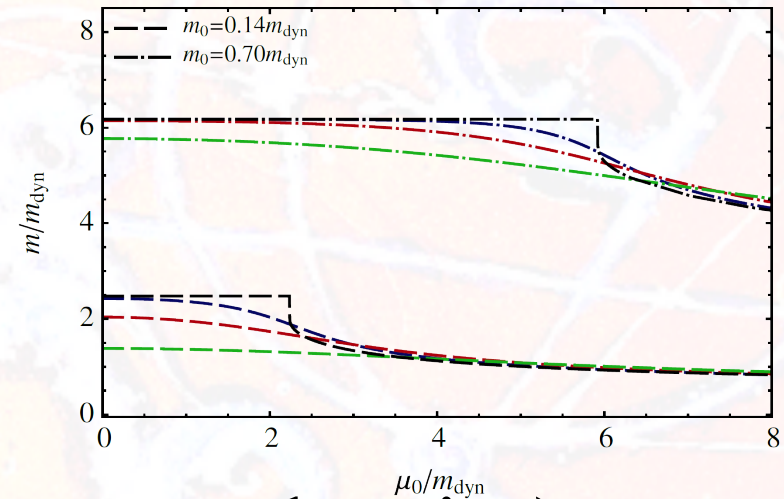
$$m = m_0 - G_{\text{int}} \langle \bar{\psi} \psi \rangle \quad (\text{dynamical mass})$$

$$\Delta = -\frac{1}{2} G_{\text{int}} \langle j_5^3 \rangle \quad (\text{chiral shift parameter})$$

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\text{int}}|eB|}\right)$$

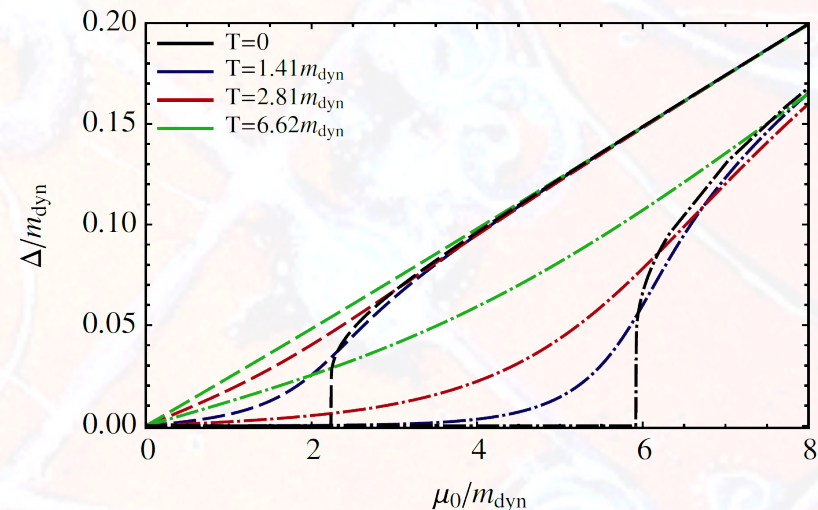
$$\Delta = 0 \quad \& \quad \mu = \mu_0$$



- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{gs_{\perp}\mu}{(\Lambda l)^2 + \frac{1}{2}g(\Lambda l)^2}$$



Chiral shift @ Fermi surface

- Chirality is \approx well defined at Fermi surface ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_{\perp} \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_{\perp} \Delta)^2 - m^2}$$

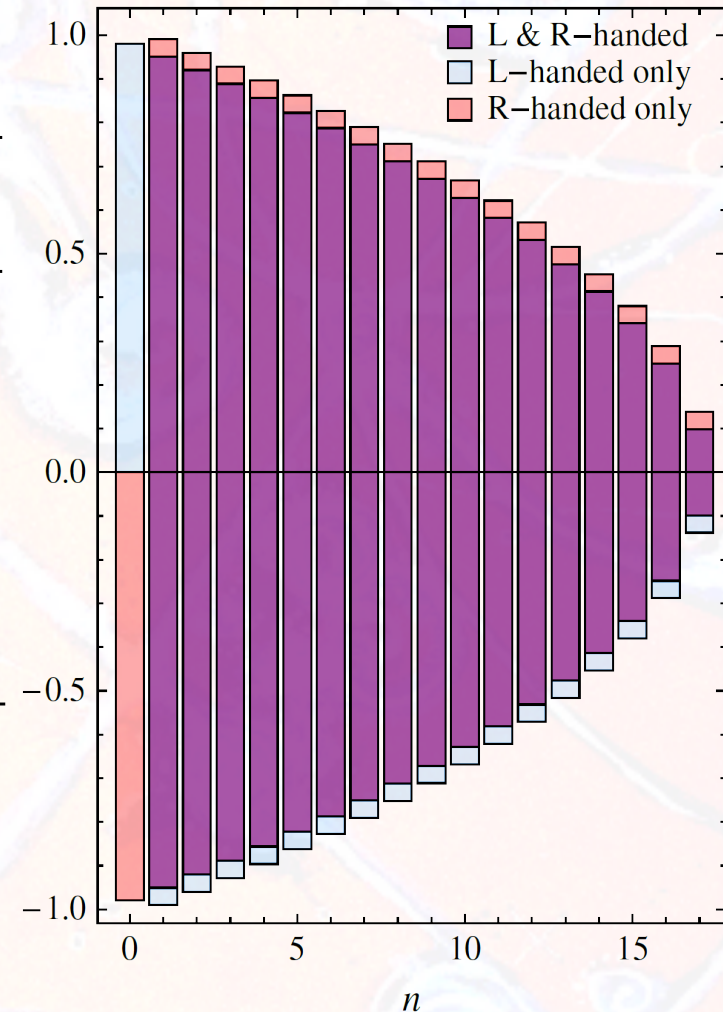
$$k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_{\perp} \Delta)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_{\perp} \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_{\perp} \Delta)^2 - m^2}$$

$$k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_{\perp} \Delta)^2 - m^2}$$



- Does the chiral shift modify the axial anomaly relation?
- Using point splitting method, one derives

$$\begin{aligned} \langle \partial_\mu j_5^\mu(u) \rangle &= -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right) \\ &\rightarrow -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \quad \text{for } \epsilon \rightarrow 0 \end{aligned}$$

[Gorbar, Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

- Does the chiral shift give any contribution to the axial current?
- In the point splitting method, one has

$$\left\langle j_5^\mu \right\rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2 \varepsilon^2} \delta_\mu^3 \cong \frac{\Lambda^2 \Delta}{2\pi^2} \delta_\mu^3$$

[Gorbar, Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- This is consistent with the NJL calculations
- Since $\Delta \sim g\mu eB/\Lambda^2$, the correction to the axial current should be finite

- Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} (i \gamma^\mu D_\mu + \mu \gamma^0 - m) \psi + (\text{counterterms})$$

- Axial current

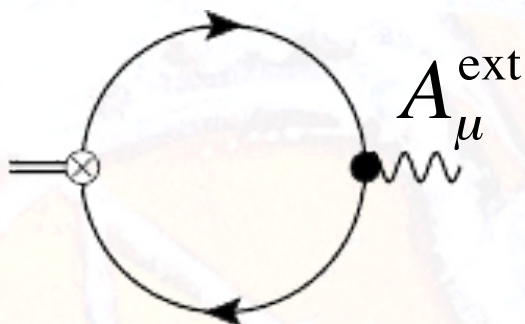
$$\langle j_5^3 \rangle = -Z_2 \text{tr}[\gamma^3 \gamma^5 G(x, x)]$$

- To leading order in coupling $\alpha=e^2/(4\pi)$

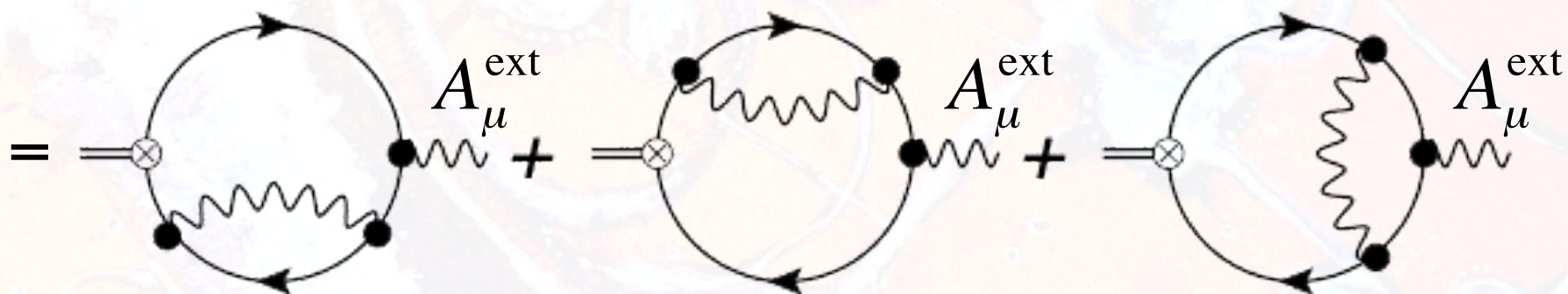
$$G(x, y) = S(x, y) + i \int d^4 u d^4 v S(x, u) \Sigma(u, v) S(v, y)$$

Expansion in external field

- Use expansion of $S(x,y)$ in powers of A_μ^{ext}
- To leading order in coupling,

$$\langle j_5^3 \rangle_0 = \text{diagram}$$


- The radiative correction is

$$\langle j_5^3 \rangle_\alpha = \text{diagram 1} + \text{diagram 2} + \text{diagram 3}$$


- Expand $S(x, y) = e^{i\Phi(x, y)} \bar{S}(x - y)$ in field

$$S(x, y) = \underbrace{\bar{S}^{(0)}(x - y) + \bar{S}^{(1)}(x - y)}_{\text{Translation invariant part}} + \underbrace{i\Phi(x, y) S^{(0)}(x - y)}_{\text{Schwinger phase}}$$

- The Schwinger phase (in Landau gauge)

$$\Phi(x, y) = -\frac{eB}{2}(x_1 + y_1)(x_2 - y_2)$$

- Note: the phase is not translation invariant

- Fourier transforms

$$\bar{S}^{(0)}(k) = i \frac{(k_0 + \mu)\gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} + m}{(k_0 + \mu + i\varepsilon \text{sign}(k_0))^2 - \mathbf{k}^2 - m^2}$$

$$\bar{S}^{(1)}(k) = -\gamma^1 \gamma^2 eB \frac{(k_0 + \mu)\gamma^0 - k_3 \gamma^3 + m}{\left[(k_0 + \mu + i\varepsilon \text{sign}(k_0))^2 - \mathbf{k}^2 - m^2 \right]^2}$$

- Note the singularity near the Fermi surface...

- “Vacuum” + “matter” parts

$$\frac{1}{\left[(k_0 + \mu + i\varepsilon \operatorname{sign}(k_0))^2 - \mathbf{k}^2 - m^2 \right]^n} = \text{"Vac."} + \text{"Mat."}$$

where

$$\text{"Vac."} = \frac{1}{\left[(k_0 + \mu)^2 - \mathbf{k}^2 - m^2 + i\varepsilon \right]^n}$$

$$\text{"Mat."} = \frac{2\pi i (-1)^{n-1}}{(n-1)!} \theta(|\mu| - |k_0|) \theta(-k_0 \mu) \delta^{(n-1)} \left[(k_0 + \mu)^2 - \mathbf{k}^2 - m^2 \right]$$

- From definition

$$\langle j_5^3 \rangle_0 = - \int \frac{d^4 k}{(2\pi)^4} \text{tr} [\gamma^3 \gamma^5 \bar{S}^{(1)}(k)]$$

- After integrating over energy

$$\langle j_5^3 \rangle_0 = - \frac{eB \text{sign}(\mu)}{4\pi^3} \int d^3 \mathbf{k} \underbrace{\delta(\mu^2 - \mathbf{k}^2 - m^2)}_{\text{Matter part}}$$

and finally

$$\langle j_5^3 \rangle_0 = - \frac{eB \text{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2}$$

- Note the role of the Fermi surface (!)

- Only the lowest ($n=0$) Landau level contributes

$$\langle j_5^3 \rangle_0 = \frac{eB}{4\pi^2} \int dk_3 \left[\theta\left(-\mu - \sqrt{k_3^2 + m^2}\right) - \theta\left(\mu - \sqrt{k_3^2 + m^2}\right) \right]$$

giving same answer

$$\langle j_5^3 \rangle_0 = -\frac{eB \operatorname{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2}$$

- There are no contributions from higher Landau levels ($n \geq 1$)
- There is a connection with the index theorem

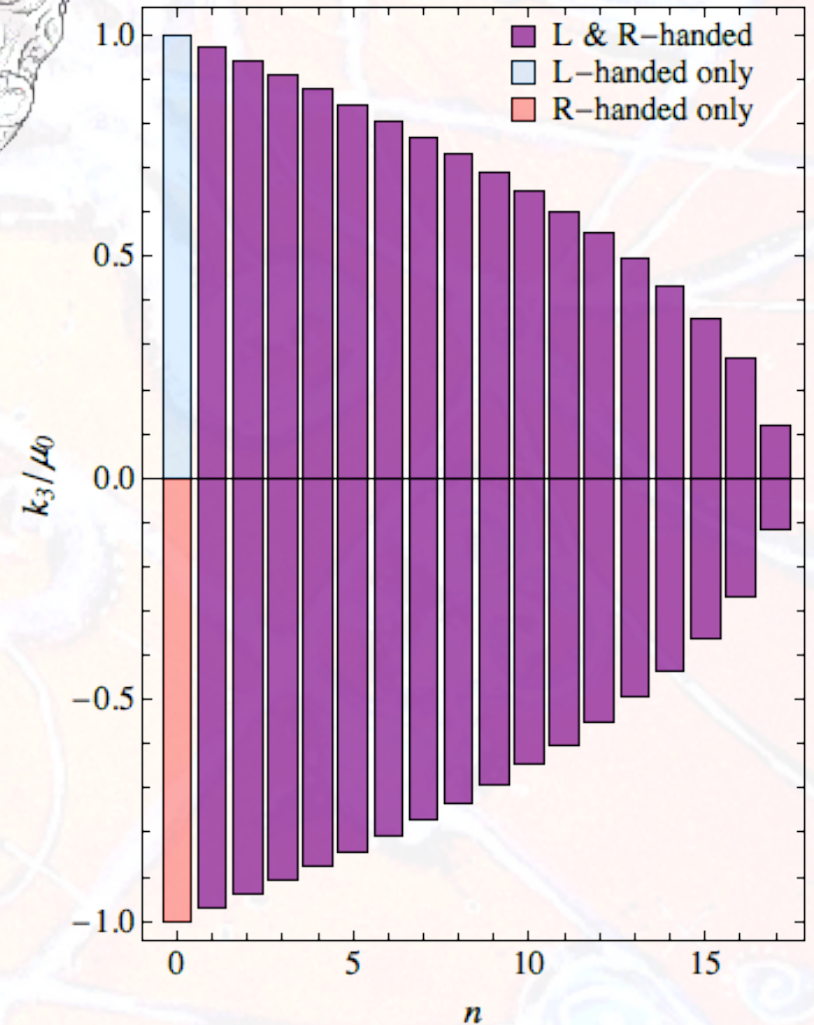
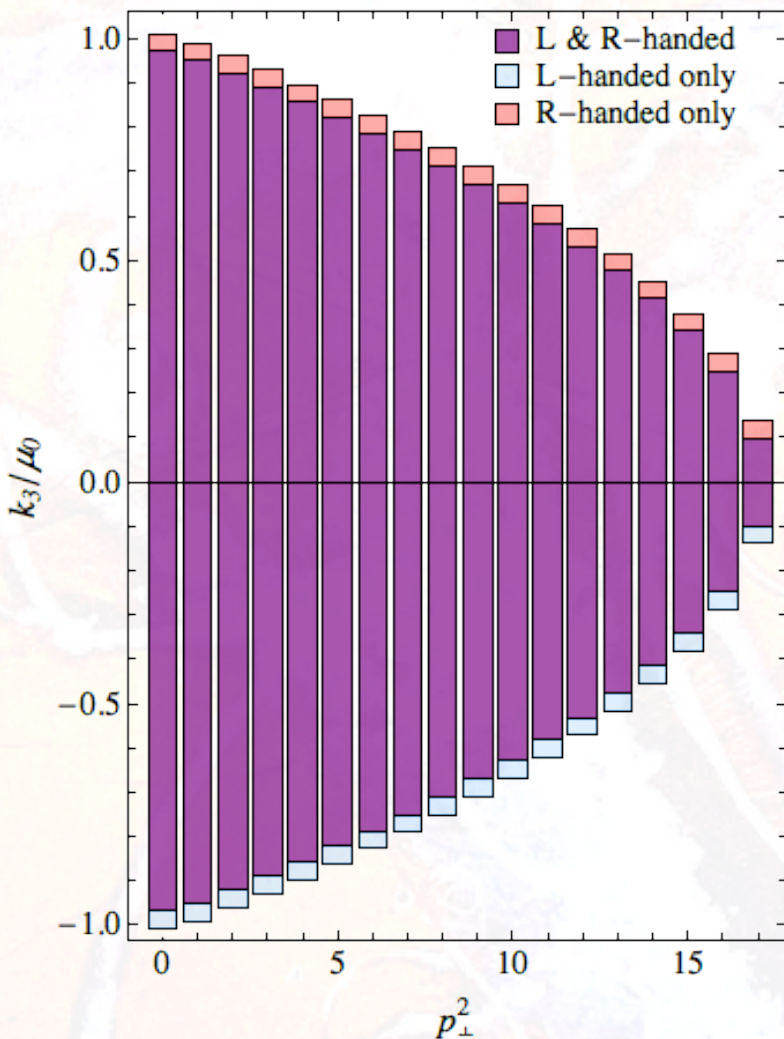
Two facets

- Two ways to look at the same result

$B \rightarrow 0$



$B \neq 0$



- Original two-loop expression

$$\langle j_5^3 \rangle_\alpha = 32\pi\alpha e B \int \frac{d^4 p d^4 k}{(2\pi)^8} \frac{1}{(P-K)_\Lambda^2} \left[\frac{(k_0 + \mu)[3(p_0 + \mu)^2 + \mathbf{p}^2 + m^2] - 4(p_0 + \mu)(\mathbf{p} \cdot \mathbf{k} + 2m^2)}{(P^2 - m^2)^3 (K^2 - m^2)} - \frac{(k_0 + \mu)[3(p_0 + \mu)^2 - \mathbf{p}^2 + 3m^2] - 2(p_0 + \mu)(\mathbf{p} \cdot \mathbf{k})}{3(P^2 - m^2)^2 (K^2 - m^2)^2} \right] + \langle j_5^3 \rangle_{\text{ct.}}$$

- After integration by parts

$$\langle j_5^3 \rangle_\alpha = 64i\pi^2 \alpha e B \int \frac{d^4 p d^4 k}{(2\pi)^8} \left[\frac{(k_0 + \mu)(p_0 + \mu) - \mathbf{p} \cdot \mathbf{k} - 2m^2}{(P-K)_\Lambda^2 (K^2 - m^2)} \delta'[\mu^2 - m^2 - \mathbf{p}^2] \delta(p_0) + \frac{3(p_0 + \mu)^2 - 3(k_0 + \mu)(p_0 + \mu) + \mathbf{p}^2 - \mathbf{p} \cdot \mathbf{k} + 3m^2}{3(P-K)_\Lambda^2 (P^2 - m^2)^2} \delta(\mu^2 - m^2 - \mathbf{k}^2) \delta(k_0) \right] + \langle j_5^3 \rangle_{\text{ct.}}$$

- Loop contribution

$$f_1 + f_2 + f_3 = \frac{\alpha e B \mu}{2\pi^3} \left(\ln \frac{\Lambda}{2\mu} + \frac{11}{12} \right) + \frac{\alpha e B m^2}{2\pi^3 \mu} \left(\ln \frac{\Lambda}{2^{3/2} \mu} + \frac{1}{6} \right)$$

- Counterterm

$$\langle j_5^3 \rangle_{\text{ct}} = -\frac{\alpha e B \mu}{2\pi^3} \left(\ln \frac{\Lambda}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{9}{4} \right) - \frac{\alpha e B m^2}{2\pi^3 \mu} \left(\ln \frac{\Lambda}{m_\gamma} - \frac{3}{4} \right)$$

- Final result

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3} \left(\ln \frac{2\mu}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{4}{3} \right) - \frac{\alpha e B m^2}{2\pi^3 \mu} \left(\ln \frac{2^{3/2} \mu}{m_\gamma} - \frac{11}{12} \right)$$

- Unphysical dependence on photon mass

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB \mu}{2\pi^3} \left(\ln \frac{2\mu}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{4}{3} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left(\ln \frac{2^{3/2} \mu}{m_\gamma} - \frac{11}{12} \right)$$

- Infrared physics with

$$m_\gamma \leq |k_0|, |k_3| \leq \sqrt{|eB|}$$

not captured properly

- Note: similar problem exists in calculation of Lamb shift

Nonperturbative effects (?)

- Perpendicular momenta cannot be defined with accuracy better than

$$|\Delta\mathbf{k}_\perp|_{\min} \sim \sqrt{|eB|}$$

(In contrast to the tacit assumption in using expansion in powers of B -field)

- Screening effects provide a natural infrared regulator

$$m_\gamma \Rightarrow \sqrt{\alpha\mu}$$

(Formally, this goes beyond the leading order in coupling)

- Conjectured nonperturbative modification

(1) If non-conservation of momentum dominates

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB \mu}{2\pi^3} \left(\ln \frac{\mu |eB|}{m^3} + O(1) \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left(\ln \frac{\mu}{\sqrt{|eB|}} + O(1) \right)$$

(2) If photon screening is more important

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB \mu}{2\pi^3} \left(\ln \frac{\alpha \mu^3}{m^3} + O(1) \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left(\ln \frac{1}{\sqrt{\alpha}} + O(1) \right)$$

- Weak B -field limit: **new interpretation** of the topological contribution to CSE relation
- Radiative corrections are **nonzero**
- Radiative corrections vanish without “matter” part with **singularity on Fermi surface**
- **Nonperturbative** physics complicates the infrared contribution
- With **logarithmic accuracy**, the result can be conjectured

- Self-energy

$$\Sigma(x, y) = -4i\pi \gamma^\mu S(x, y) \gamma^\nu D_{\mu\nu}(x - y)$$

- General structure

$$\Sigma(x, y) = \exp(i\Phi(x, y)) \bar{\Sigma}(x - y)$$

- Translation invariant part:

$$\bar{\Sigma}(p) = -4i\pi \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \bar{S}(k) \gamma^\nu D_{\mu\nu}(k - p)$$

$$\bar{\Sigma}^{(1)}(p) = -4i\pi \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \bar{S}^{(1)}(k) \gamma^\nu D_{\mu\nu}(k-p)$$

- The result has the form

$$\bar{\Sigma}^{(1)}(p) = \gamma^3 \gamma^5 \Delta + \gamma^0 \gamma^5 \mu_5(p)$$

where

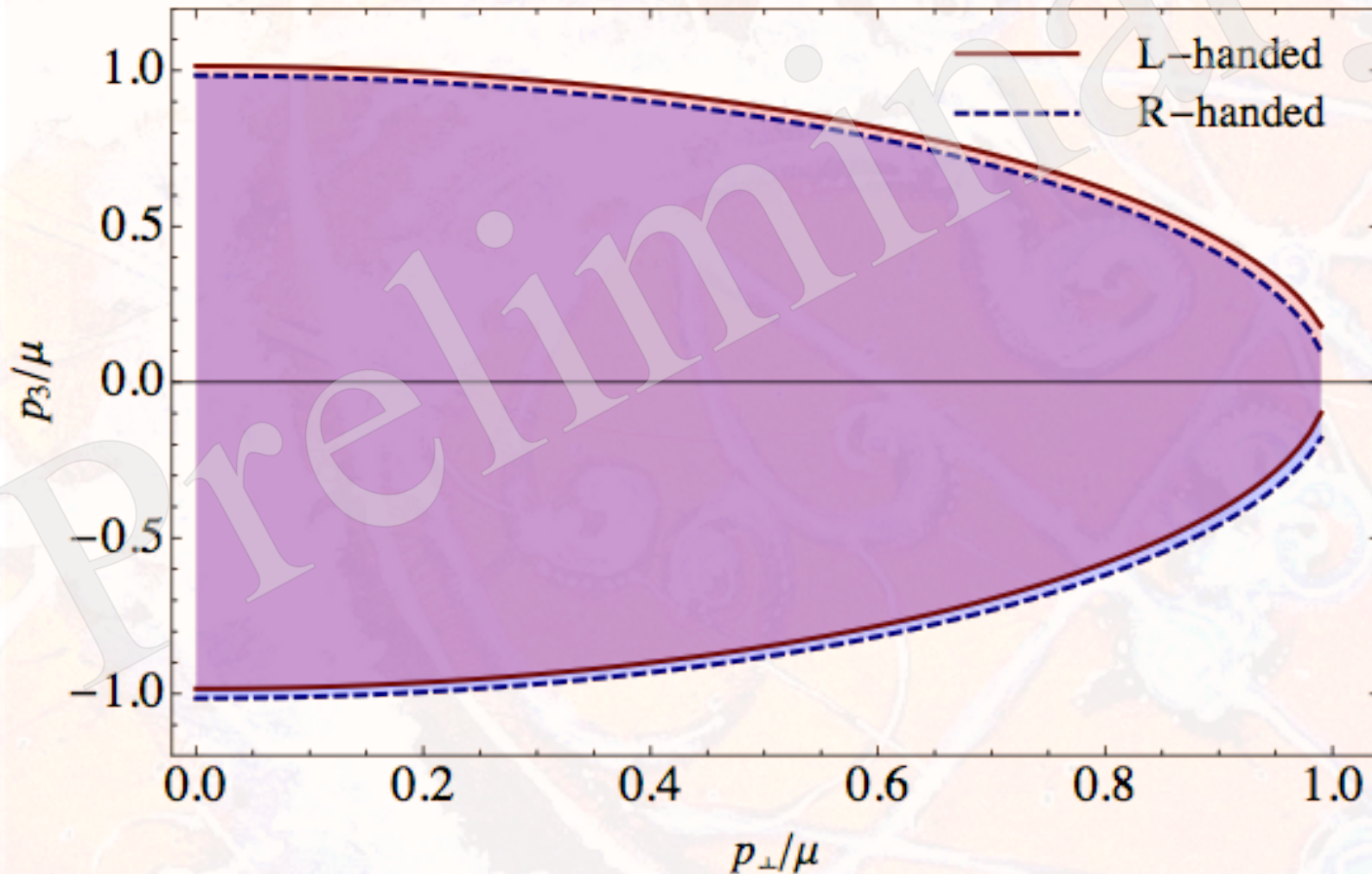
$$\Delta \approx \frac{\alpha e B \mu}{\pi m^2} \left(\ln \frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1 \right)$$

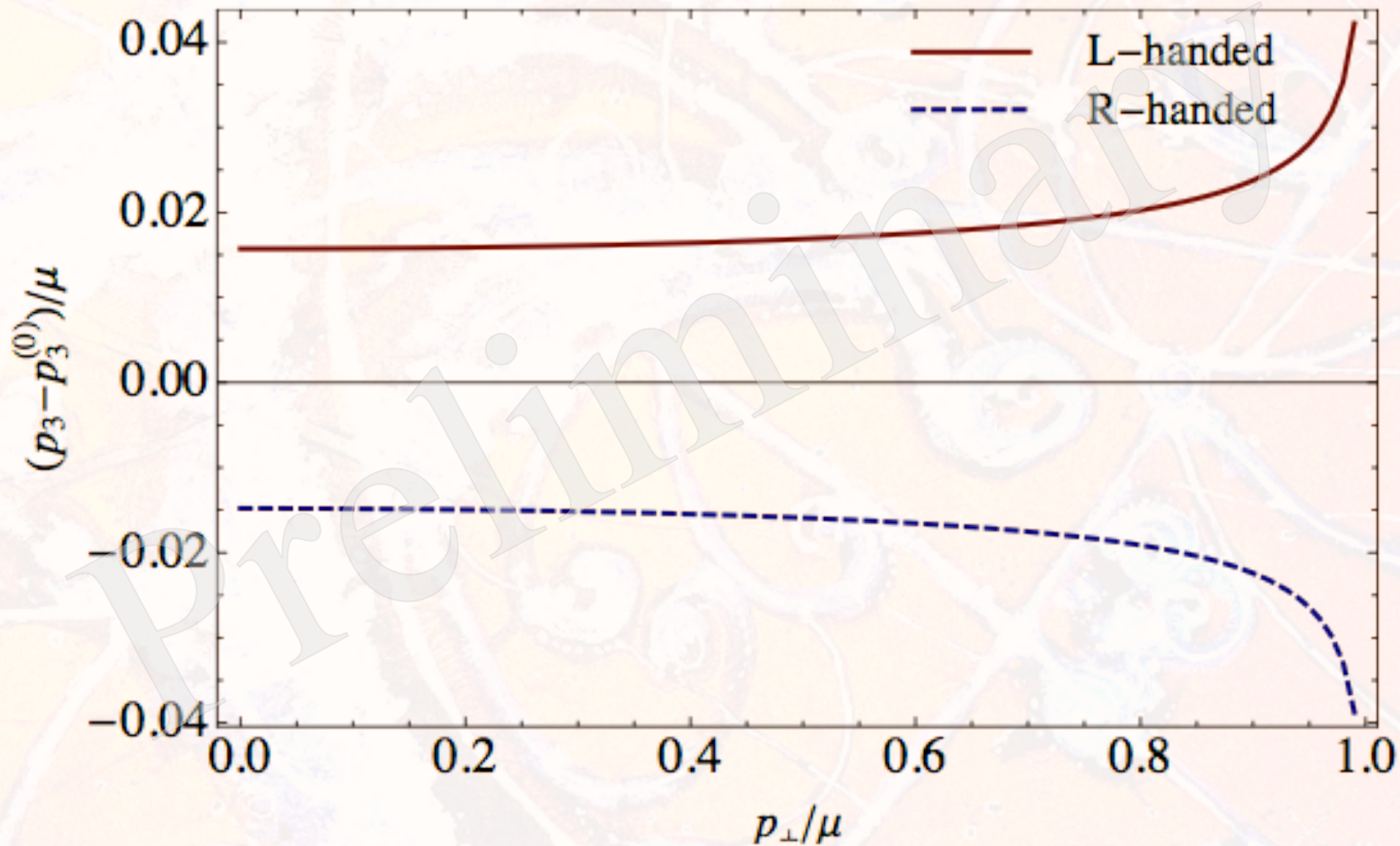
$$\mu_5(p) \approx -\frac{\alpha e B \mu}{\pi m^2} \frac{p_3}{p_F} \left(\ln \frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1 \right)$$

Dispersion relations

- Let us use the condition

$$\text{Det}\left[i\bar{S}^{-1}(p) + \bar{\Sigma}^{(1)}(p)\right] = 0$$





Summary (2)

- Chiral shift is generated in magnetized matter (evidence from renormalizable model now)
- The magnitude of chiral shift scales as

$$\Delta \propto \frac{\alpha e B \mu}{m^2} \ln \alpha$$

- Chiral shift induces a chiral asymmetry at the Fermi surface
- Chiral shift contributes to the axial current