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POLYTECHNIC CAMPUS

Kent State University
JAN 30 2014
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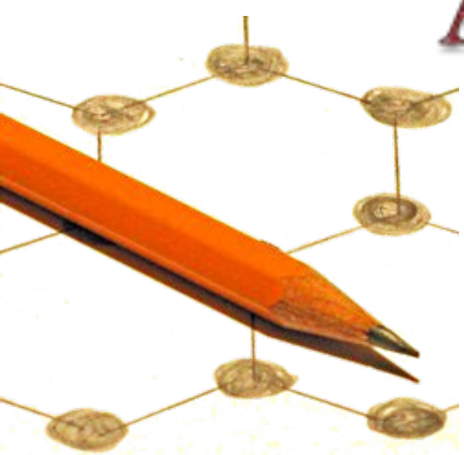
HIGH-ENERGY PHYSICS AT THE TIP OF A PENCIL



Igor Shovkovy

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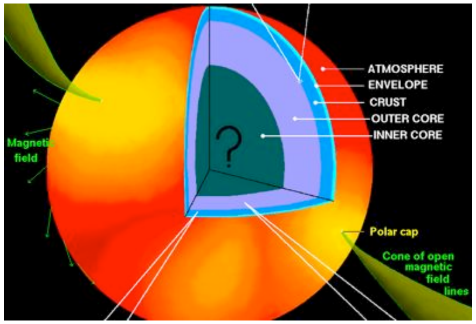
Part 1

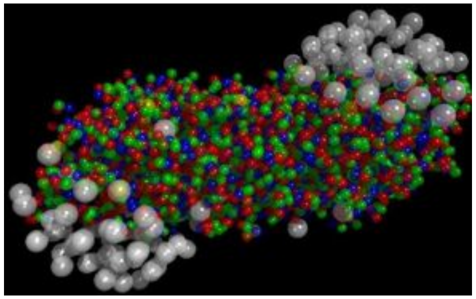
INTRODUCTION

EXTREME ENVIRONMENTS

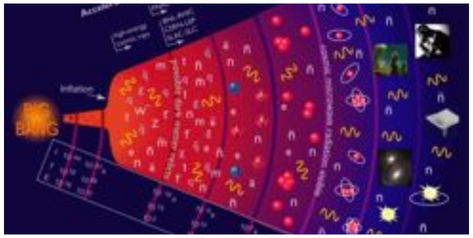
- Examples of relativistic matter
  - **Electrons, protons, quarks** inside compact stars (very dense matter)



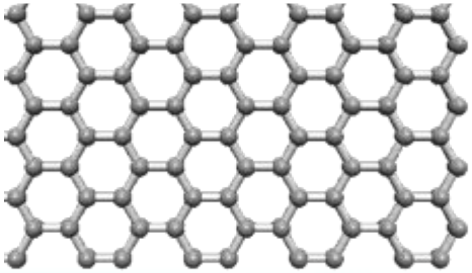
  - **Quark gluon plasma** in heavy ion collisions (a very hot fireball)



  - **Hot matter** in the Early Universe (an extremely hot world)



  - **Massless particles** in graphene (quasiparticles behave as massless particles)



RELATIVISTIC MATTER

- **Non-relativistic**
  - Particles move much slower than the speed of light
  - Kinetic energies are much smaller than the rest energy

$$E_{\text{kin}} << E_{\text{rest}} : \quad E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

- **Relativistic**
  - Particle velocities approach the speed of light
  - Kinetic energies are comparable to, or larger than E_{rest}

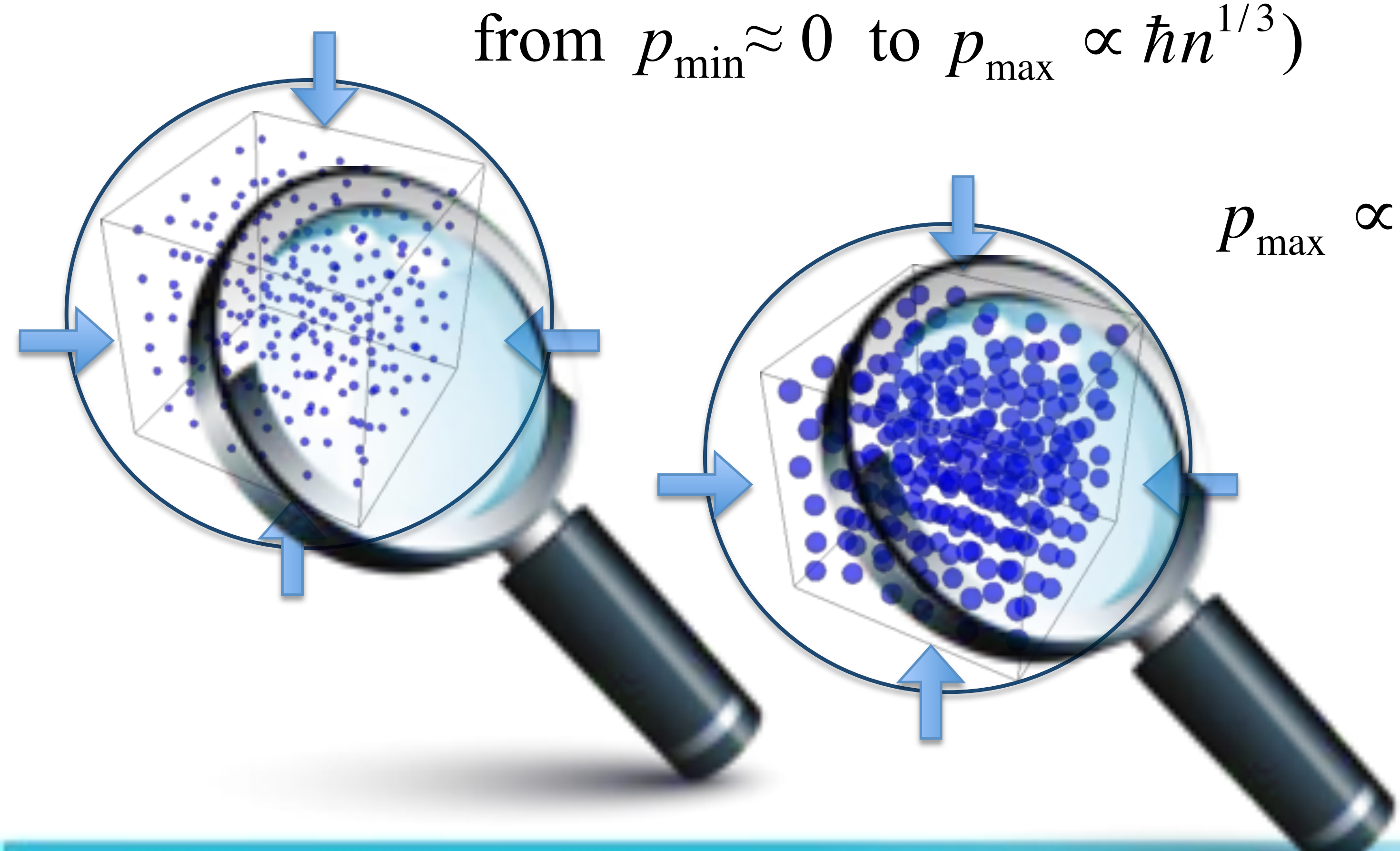
$$E_{\text{kin}} \geq E_{\text{rest}} : \quad E = c\sqrt{p^2 + m^2c^2} \approx c\,p$$

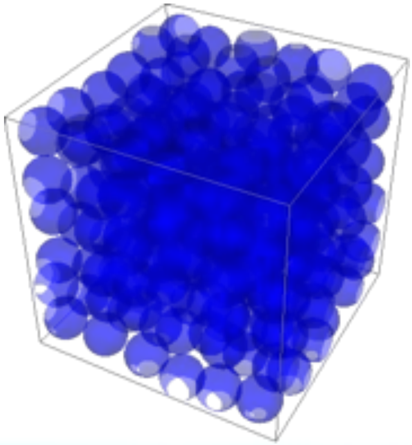
SUPER-DENSE MATTER

- **What happens when you squeeze matter to very high density?** (e.g., neutrons inside neutron stars)

Pauli exclusion principle: fermions cannot occupy same quantum states (they end up filling out all states from $p_{\min} \approx 0$ to $p_{\max} \propto \hbar n^{1/3}$)

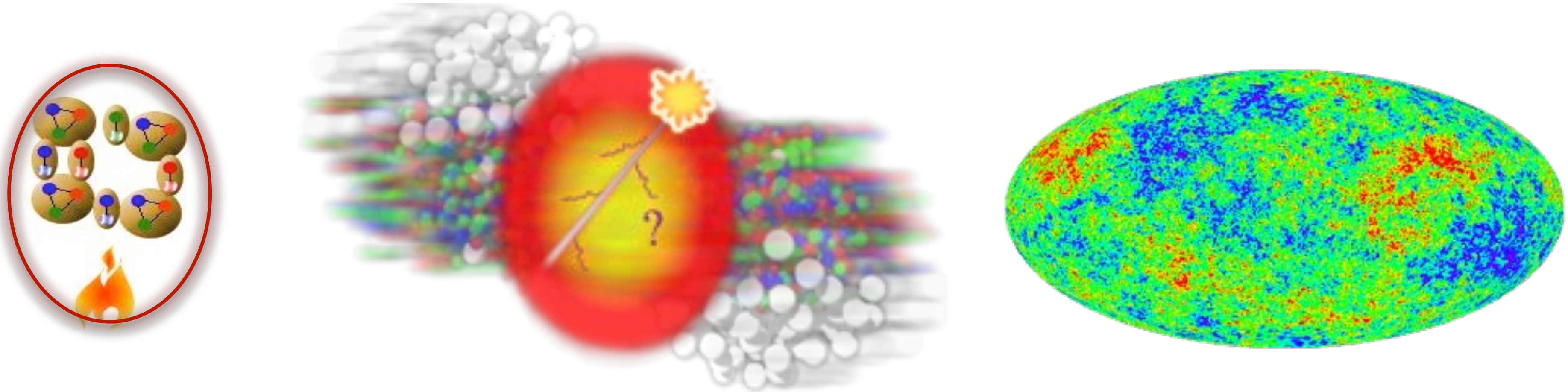
$$p_{\max} \propto 200 \left(\frac{n}{1\,\mathrm{fm}^3} \right)^{1/3} \text{MeV/c}$$





SUPER-HOT MATTER

- **What happens when you heat matter to very high temperature?** (e.g., matter in heavy ion collisions)



Heat is equivalent to **kinetic energy**: average kinetic energy of particles is proportional to temperature:

$$p \propto k_B T / c \sim 200\left(\frac{k_B T}{200\ \text{MeV}}\right)\text{MeV/c}\quad(\text{assuming } p >> mc)$$

MASSLESS PARTICLES

- **Can matter be made of massless particles?**

  Yes! Electron quasiparticles masquerade as massless particles in some materials (no rest mass energy)

- **Examples:**
  - Graphene

  - $Bi_{1-x}Sb_x$ alloy with $x \approx 0.03$
  - cadmium arsenide Cd_3As_2
  - potassium bismuthide Na_3Bi

2D (planar) materials

$$E = v_F \sqrt{k_x^2 + k_y^2}$$

3D materials

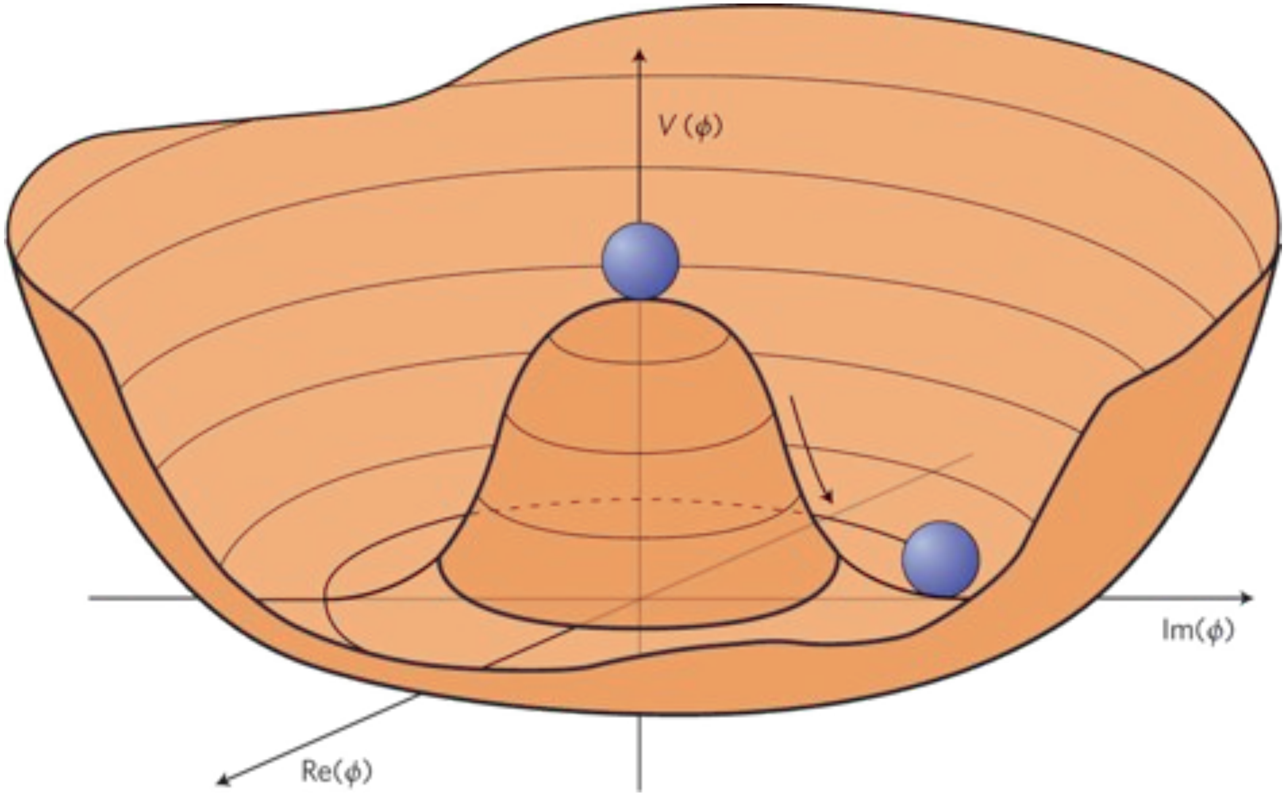
$$E = v_x k_x + v_y k_y + v_z k_z$$

Part 2

CHIRAL SYMMETRY

SYMMETRY BREAKING

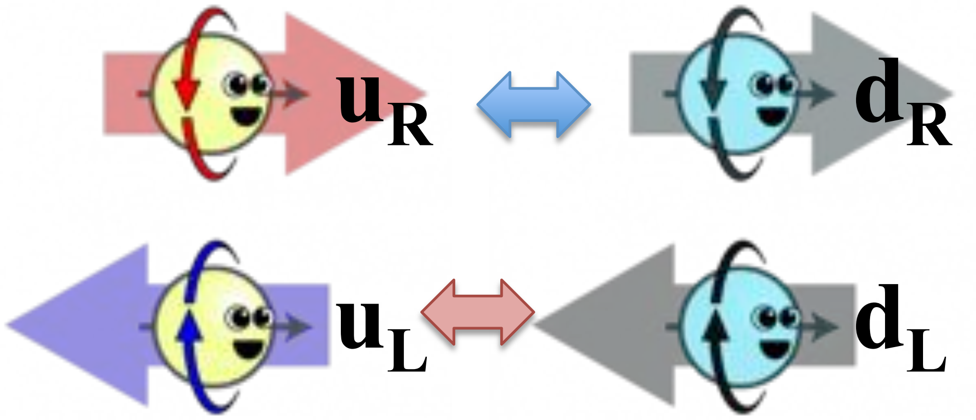
- Underlying laws are symmetric, but the system/ground state changes under a symmetry transformation



- Symmetry may refer to “internal” symmetries (e.g. rotations in color/flavor spaces, rescaling, etc.)

MASS VS. SYMMETRY

- Massless fermions enjoy chiral symmetry (rotation of left-handed and right-handed particles in flavor space)



Righ-handed

Left-handed

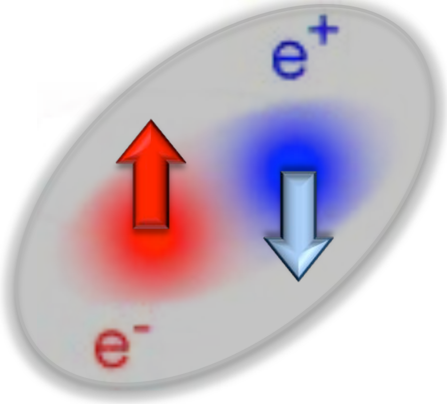
- Massive fermions (e.g., quarks, nucleons, etc.) “break” chiral symmetry

$$|m\rangle \propto C_1|L\rangle + C_2|R\rangle$$

- Hadron physics reveals traces of the original chirally symmetric laws, which are broken in the ground state

HOW SYMMETRY BREAKS (1)

- Particles & anti-particles form bound states



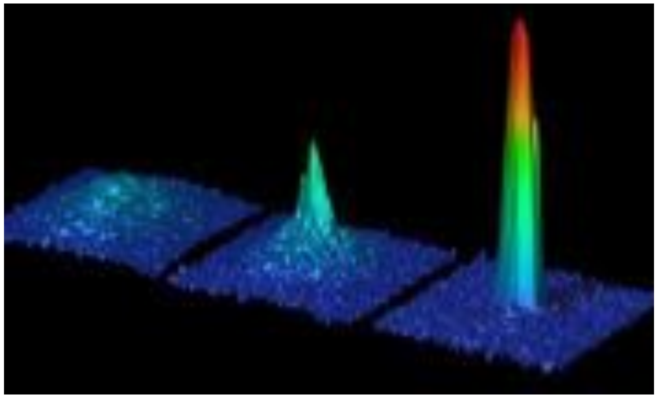
- Let's assume that the binding energy is E_b per pair

- Bound states are bosons with mass $M=2m_{\mathrm{dyn}}-E_b$

- If binding sufficiently strong $M=0$

HOW SYMMETRY BREAKS (2)

- Bosons can (and will) occupy the same lowest energy quantum state (with $p=0$ and $E=0$)
- The result is a Bose condensation in ground state



- The properties of the ground state change (and its fermionic excitations become massive)

$$m_{\mathrm{dyn}} = E_b/2$$

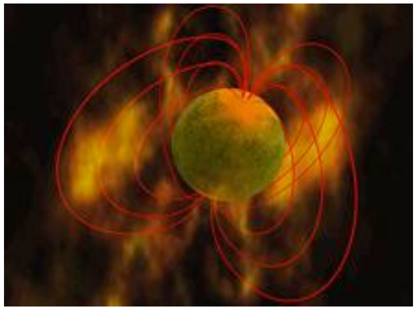
Part 3

MAGNETIC CATALYSIS

Review: arXiv:1207.5081

MAGNETIC FIELDS IN NATURE

- Strong magnetic fields are common inside compact stars
  - **10^{10} to 10^{15} Gauss**



- In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

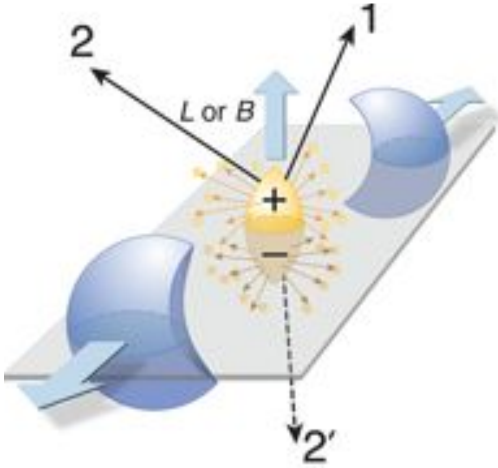


Illustration by Carin Cain

- Early Universe
  - **up to 10^{24} Gauss**
- Graphene (High Magnetic Field Laboratory)
  - **4.5×10^5 Gauss**

IDEA OF MAGNETIC CATALYSIS

- Magnetic field constrains perpendicular motion of charged particles
- Particle-antiparticle binding becomes easy
- Even *arbitrarily* weak attractive interaction is sufficient to form bound states
- Condensate forms and symmetry breaks down
- Fermions become massive

This is **MAGNETIC CATALYSIS**

[I.S. arXiv:1207.5081]

LANDAU LEVELS

- Fermions in a magnetic field in $(3_{\text{space}}+1_{\text{time}})$D

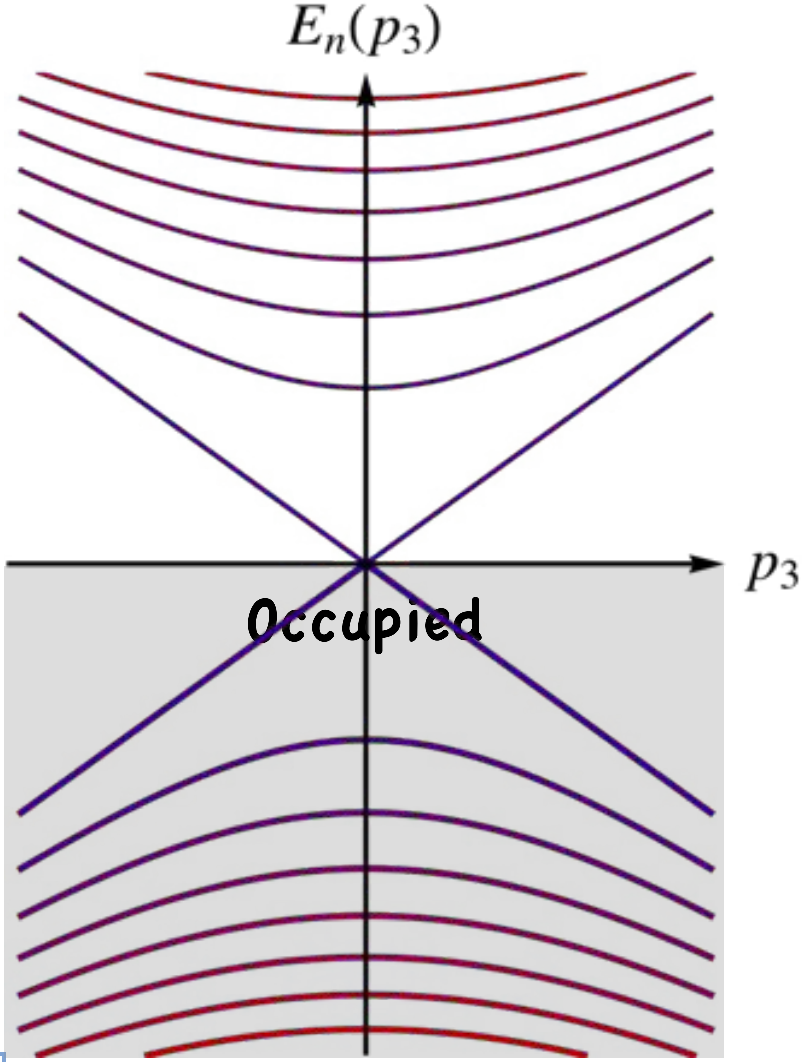
$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla}+ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

$$n = \overbrace{s}^{\boxed{s=\pm 1/2 \quad \text{(spin)}}} + \underbrace{k + \frac{1}{2}}_{\boxed{k=0,1,2,\ldots \quad \text{(orbital)}}}$$

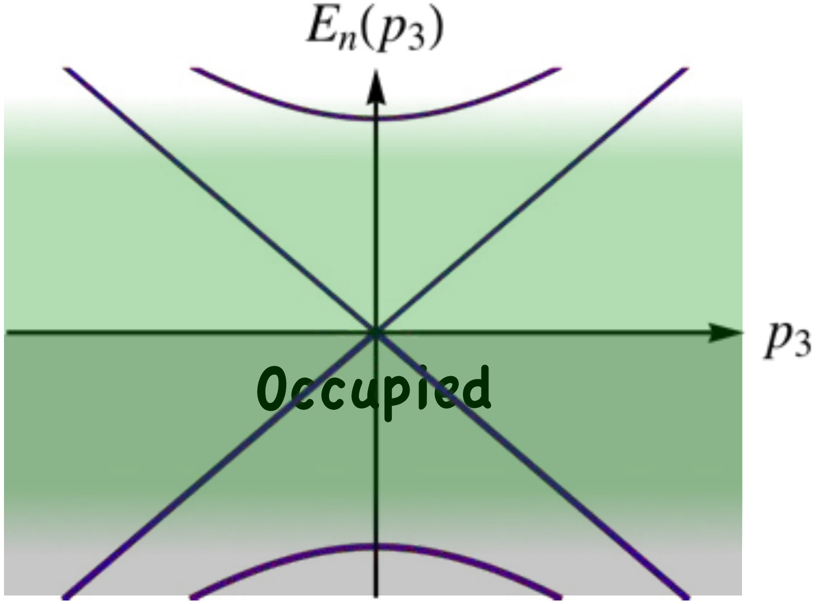


DIMENSIONAL REDUCTION

- At low energies, only n=0 (highly degenerate) Landau level is relevant

$$n = 0: \quad E_0^{(3+1)}(p_3) = \pm p_3$$

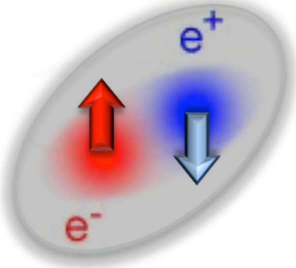
$$\left(k = 0, s = -\frac{1}{2}\right)$$



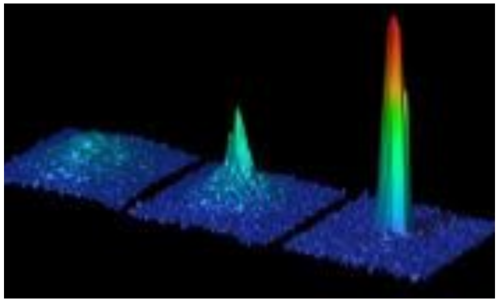
- Particles behave like ($1_{\text{space}}+1_{\text{time}}$)-dimensional
- Only motion in *z-direction* is unconstrained
- Motion in *xy-plane* is restricted

PAIRING & NEW GROUND STATE

- Particles & anti-particles in n=0 level form bound states



- Bound states (bosons) can (and will) occupy the same lowest energy quantum state
- Such a Bose condensate modifies the ground state (vacuum)



- Fermions are massive in the new vacuum

  [Gusynin, Miransky, Shovkovy, PRL **73** (1994) 3499]

DYNAMICAL MASS

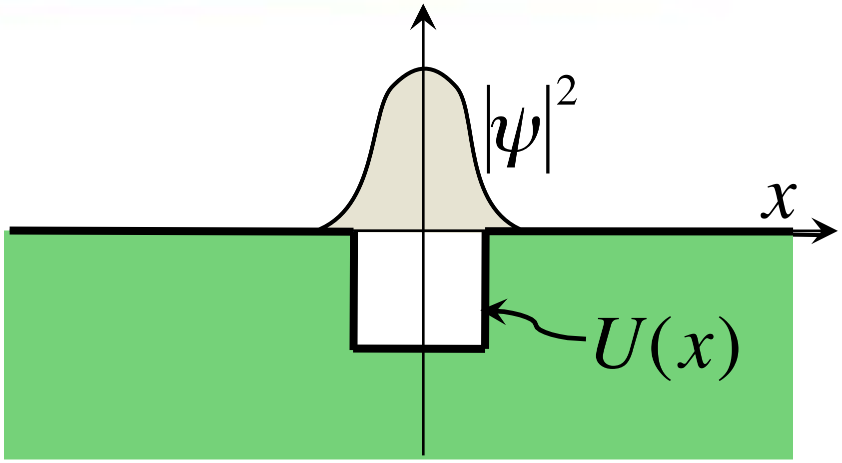
- While $m_0=0$ originally, a nonzero “dynamical” mass m_{dyn} is generated

$$m_{dyn}^{(2D)} \propto \sqrt{\alpha}\sqrt{|eB|}\,, \quad \text{and} \quad m_{dyn}^{(3D)} \propto \sqrt{|eB|}e^{-C/\alpha}$$

- This happens even at the *weakest* interaction (“catalysis”)
- The phenomenon is *universal* (model details are irrelevant)
- Dimensional *reduction* is the key ingredient (massless bound states form = symmetry breaking)

BOUND STATES IN 1D

- Bound state energy



$$\left|E_{1D}\right| \approx \frac{m_*}{2\hbar^2}\left(-\int_{-\infty}^{+\infty} U(x)dx\right)^2$$

- This is a perturbative result

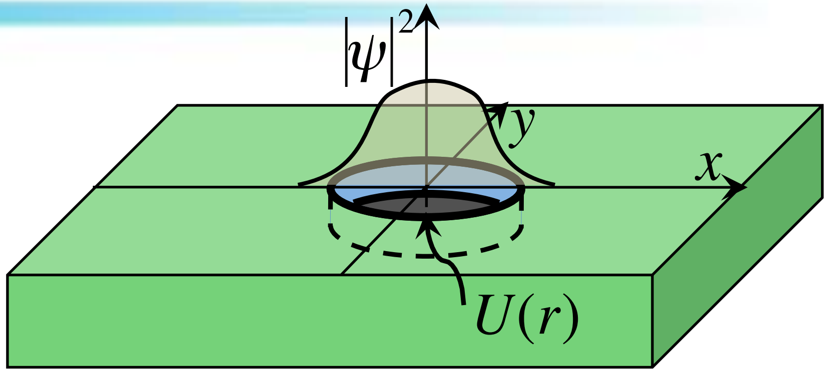
$$\left|E_{1D}\right| \propto g^2 , \quad \text{when} \quad U(x) \rightarrow gU(x)$$

- Bound state exists if

$$\int \left(1+|x|\right)|U(x)|\,dx < \infty \quad \& \quad \int U(x)\,dx \le 0$$

[B. Simon, Annals Phys. 97 (1976) 279]

BOUND STATES IN 2D



- Bound states energy

$$|E_{2D}| \approx \frac{\hbar^2}{a^2 m_*} \exp\left(-\frac{\hbar^2}{m_*} \left| \int_0^\infty rU(r)dr \right|^{-1} \right)$$

- This is a non-perturbative result

$$|E_{2D}| \propto \exp\left(-\frac{C}{g} \right), \quad \text{when} \quad U(x) \to gU(x)$$

- Bound state exists if

$$\int |U(x)|^{1+\varepsilon} \, d^2x < \infty, \quad \int (1+x^2)^\varepsilon |U(x)| \, d^2x < \infty \quad \& \quad \int U(x) \, d^2x \le 0$$

[B. Simon, Annals Phys. 97 (1976) 279]

BOUND STATES IN 3D

- Potential well in 3D

$$U(r) = \begin{cases} -g\dfrac{\pi^2\hbar^2}{8m_*a^2} & \text{for} \quad r \le a \\ 0 & \text{for} \quad r > a \end{cases}$$

- Bound state energy exists only when g>1

$$|E_{3D}| \approx \frac{\pi^4\hbar^2}{2^7 a^2 m_*}(g-1)^2 , \quad \text{assuming} \quad 0 < g-1 << 1$$

- No bound states when g<1

UNIVERSALITY OF MC

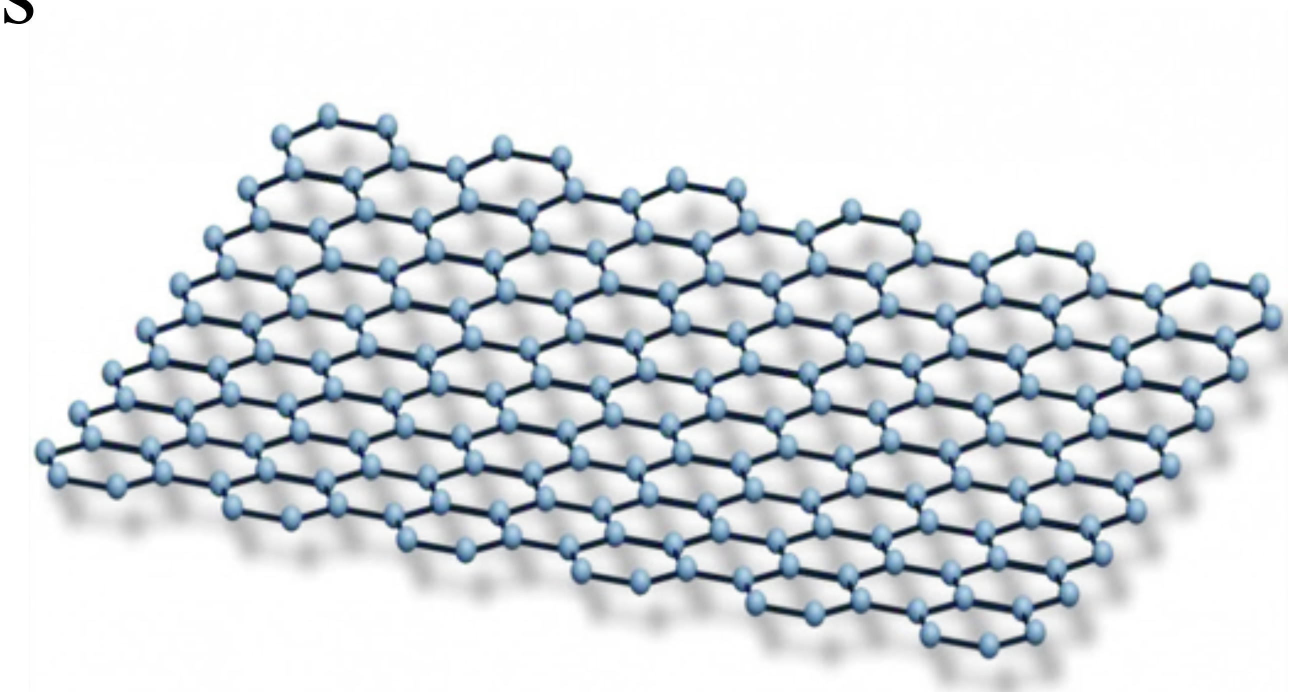
- **Input**
  - Spin-1/2 charged particles and B≠0
  - Attractive particle-antiparticle interaction
- **Output**
  - Dimensional reduction D->D-2 (low energies)
  - Bound state can and do form
  - Symmetry breaking happend
  - Dynamical mass is generated

Part 4

APPLICATIONS

GRAPHENE

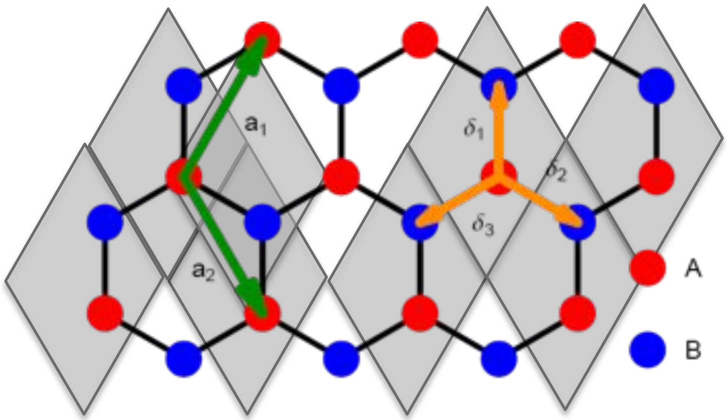
- It is a single atomic layer of graphite
  [Novoselov et al., Science **306**, 666 (2004)]
- 2D crystal with hexagonal lattice of carbon atoms



- Interesting basic physics
- Great promise for applied physics

EMERGENCE OF DIRAC FERMIONS

- Translation vectors of the lattice

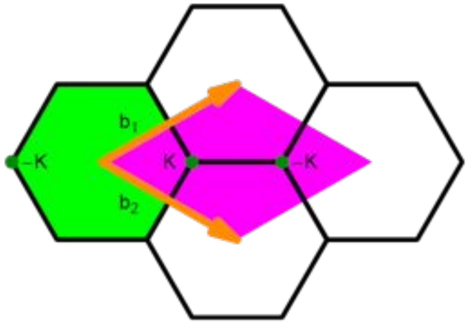


$$\mathbf{a}_1 = a\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right),\quad \mathbf{a}_2 = a\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$$

Lattice constant: $a \approx 1.42\ \mathring{A}$

- Two carbon atoms per primitive cell

- Reciprocal lattice



$$\mathbf{b}_1 = \frac{2\pi}{a}\left(1,\frac{1}{\sqrt{3}}\right),\quad \mathbf{b}_2 = \frac{2\pi}{a}\left(1,-\frac{1}{\sqrt{3}}\right)$$

- Two Dirac points in the Brillouin zone

TIGHT BINDING MODEL

- There are strong covalent sigma-bonds between nearest neighbors
- Hamiltonian

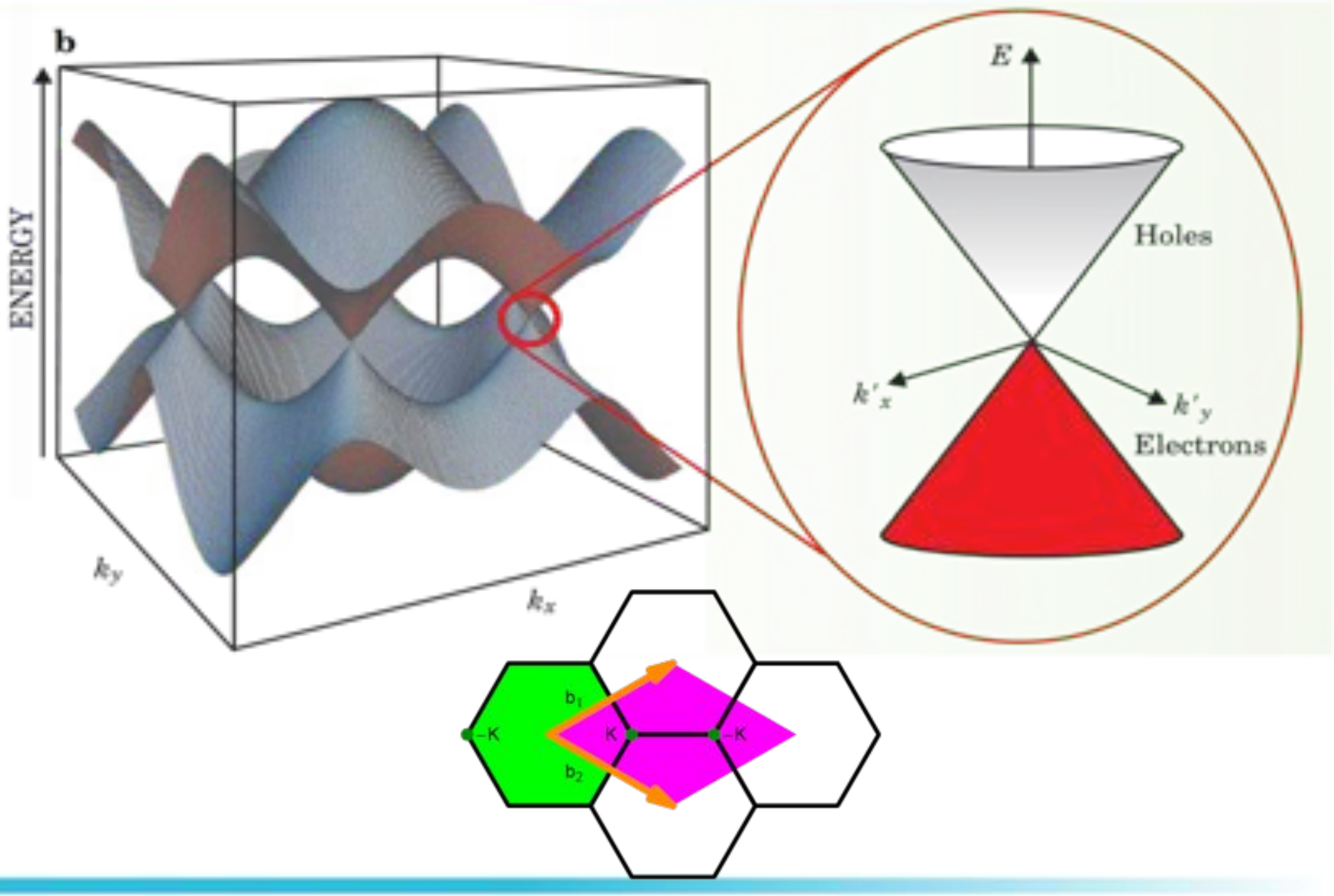
$$H = -t \sum_{\mathbf{n},\delta_i,\sigma} \left[a^{+}_{\mathbf{n},\sigma} \exp\left(\frac{ie}{\hbar c} \mathbf{A} \cdot \delta_i \right) b^{+}_{\mathbf{n}+\delta,\sigma} + c.c. \right]$$

$a_{\mathbf{n},\sigma}/b_{\mathbf{n}+\delta,\sigma}$ are annihilation operators in A/B-sublattice & spin $\sigma=\uparrow,\downarrow$

- The nearest neighbor vectors are

$$\delta_1 = \frac{\mathbf{a}_1 - \mathbf{a}_2}{3}, \quad \delta_2 = \frac{\mathbf{a}_1 + 2\mathbf{a}_2}{3}, \quad \delta_3 = -(\delta_1 + \delta_2)$$

DISPERSION RELATION



MAGNETIC CATALYSIS IN GRAPHENE

- Charge carriers are spin-½ fermions with m=0
- $m_{dyn} \neq 0$ is expected in a strong magnetic field

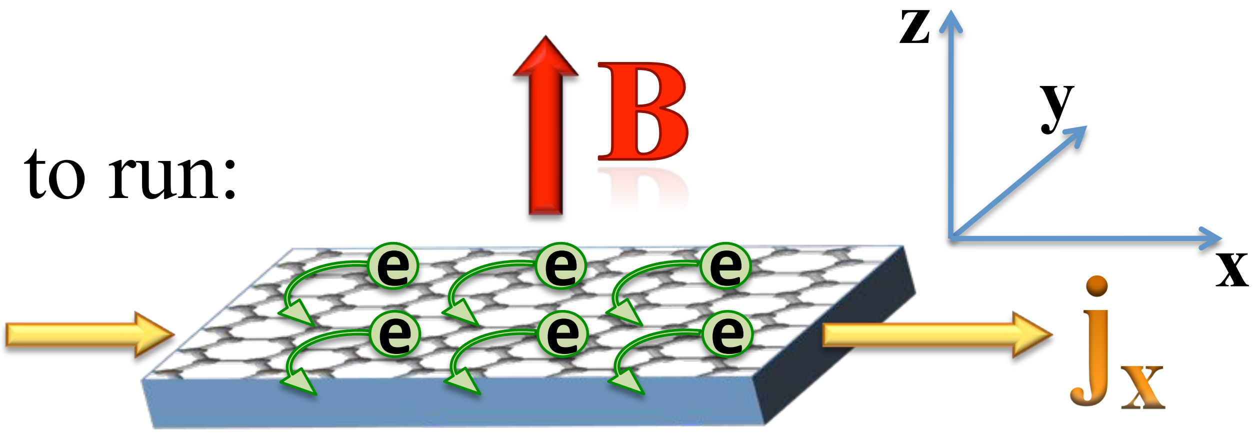
  [Khveshchenko, PRL **87**, 206401 (2001)]

  [Gorbar, Gusynin, Miransky, Shovkovy, PRB **66** (2002) 045108]

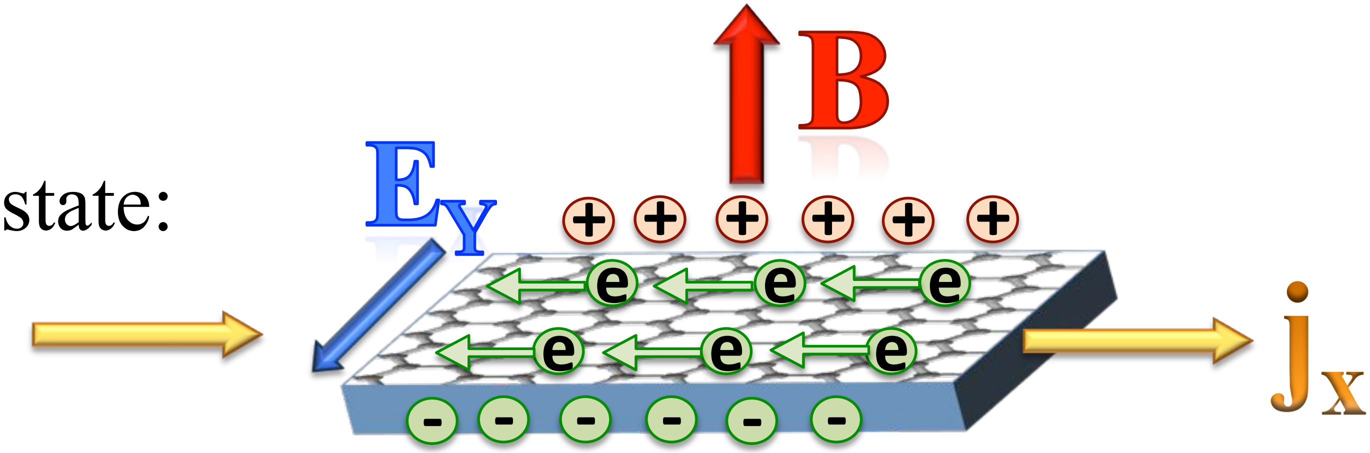
- Possible complications:
  - many types of "Dirac" masses in **2D**
  - competition with quantum Hall ferromagnetism
  - nonzero electron/hole density
  - impurities, lattice defects, ripples, etc.
- How to test this experimentally?

QUANTUM HALL EFFECT

- General setup
  - – Current starts to run:



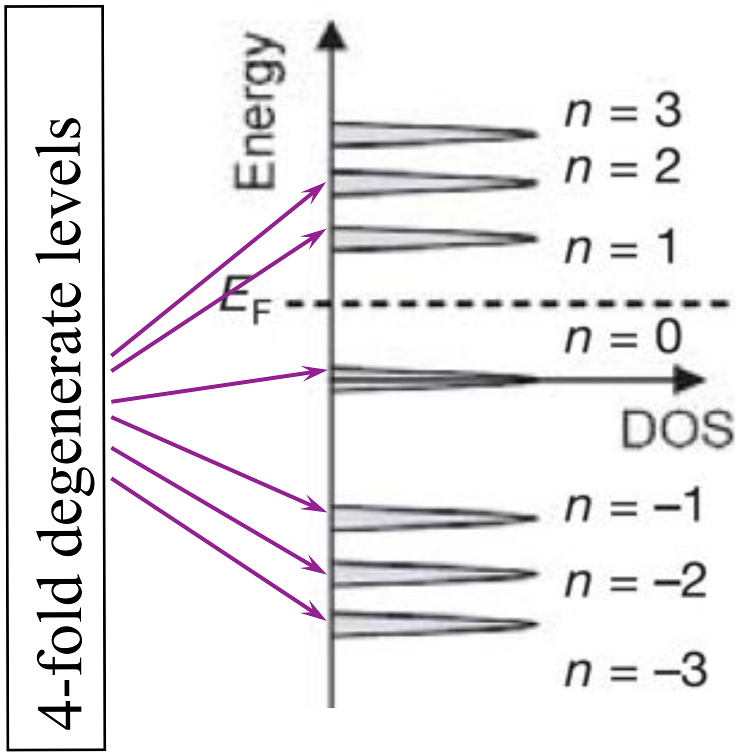
  - – Steady state:

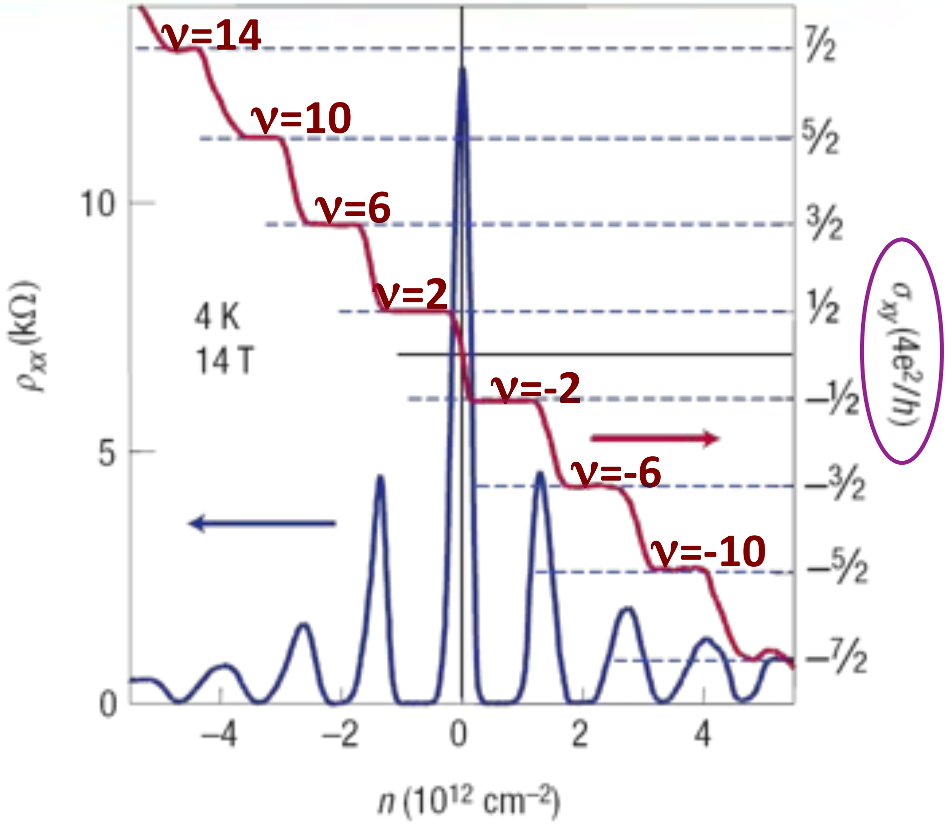


  - – Hall conductivity σ_{xy}:

$$j_x = \sigma_{xy} E_y$$

QHE IN GRAPHENE





$$E_n = \mathrm{sgn}(n)\sqrt{2e\hbar v_F^2|n|B}$$

$$\sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{h}\left(n + \frac{1}{2}\right)$$

[Gusynin, Sharapov, PRL **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, PRB **73**, 125411 (2006)]

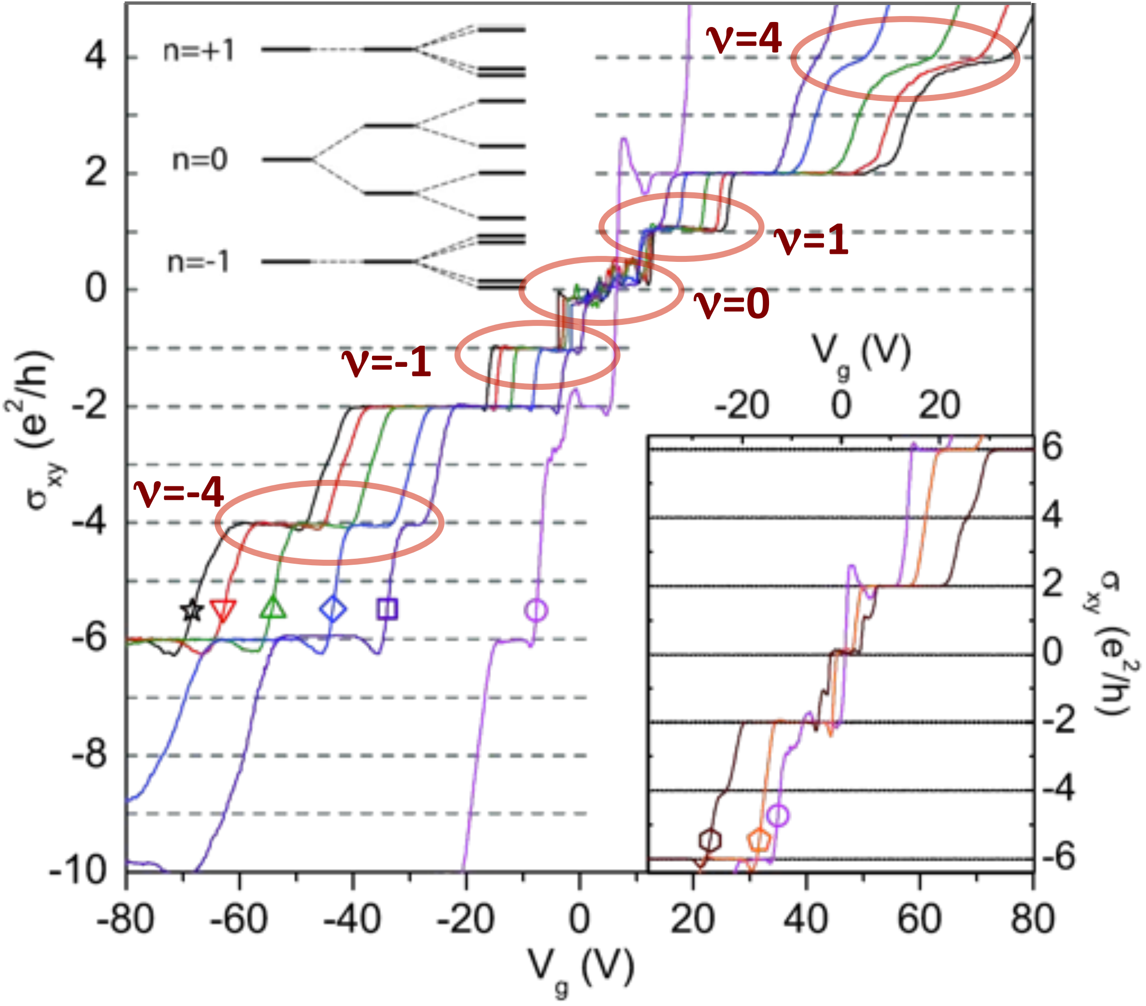
Theory (m_0=0)

[Novoselov et al., Nature **438**, 197 (2005)]
[Zhang et al., Nature **438**, 201 (2005)]

Experiment

ANOMALOUS QHE

- New plateaus are observed at
  ν=0
  ν=±1
  ν=±3
  ν=±4



Zhang et al., PRL **96,** 136806 (2006)

[Novoselov et al., Science **315**, 1379 (2007)]

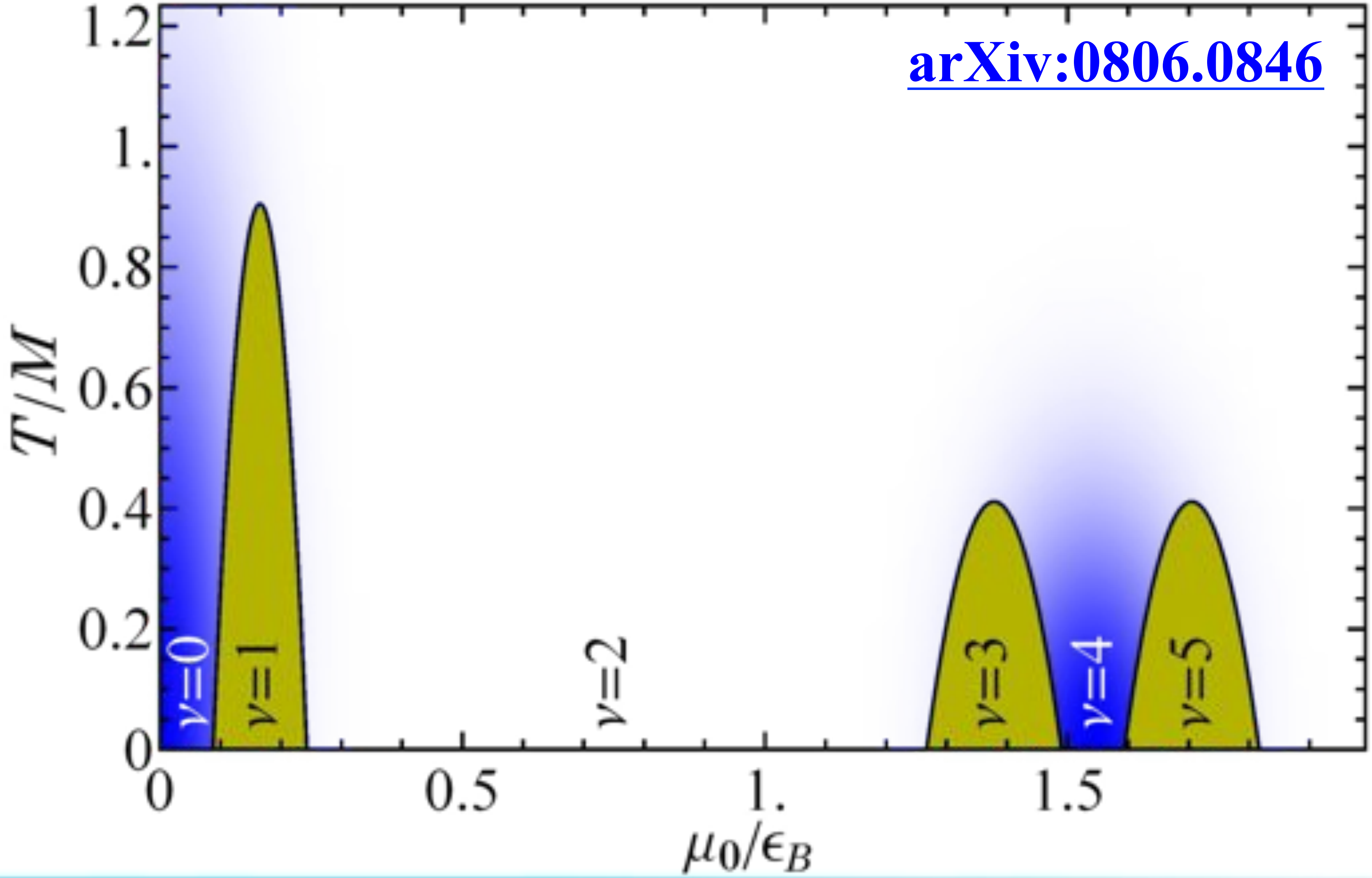
[Abanin et al., PRL **98**, 196806 (2007)]

[Checkelsky et al., PRL 100, 206801 (2008)]

[Xu Du et al., Nature **462**, 192 (2009)]

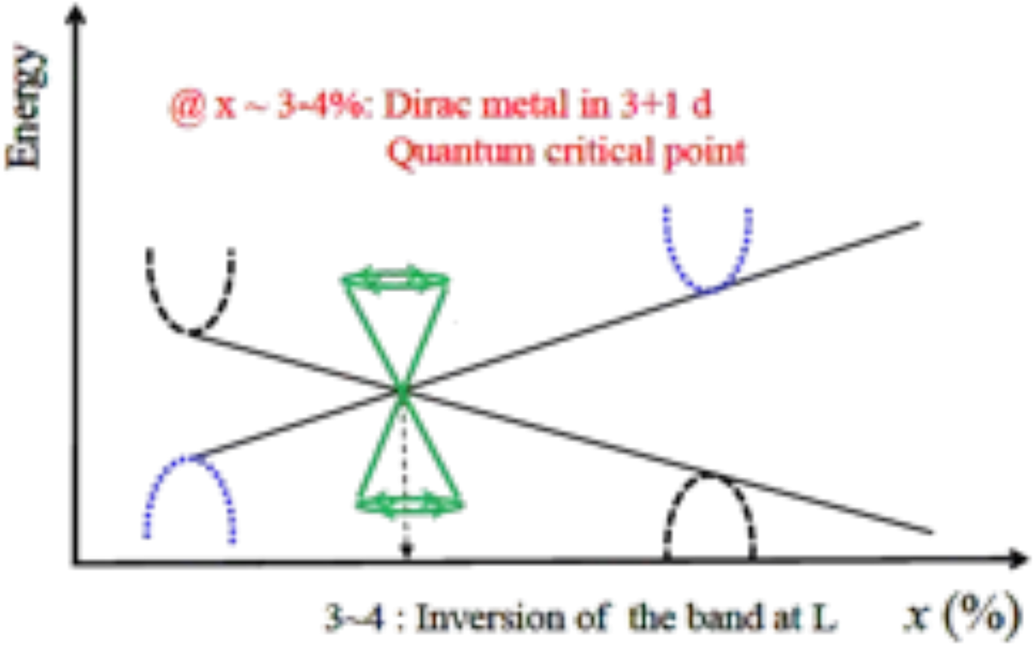
EXPLANATION OF QHE

- Generation of different dynamical “masses”: all integer plateaus are possible!



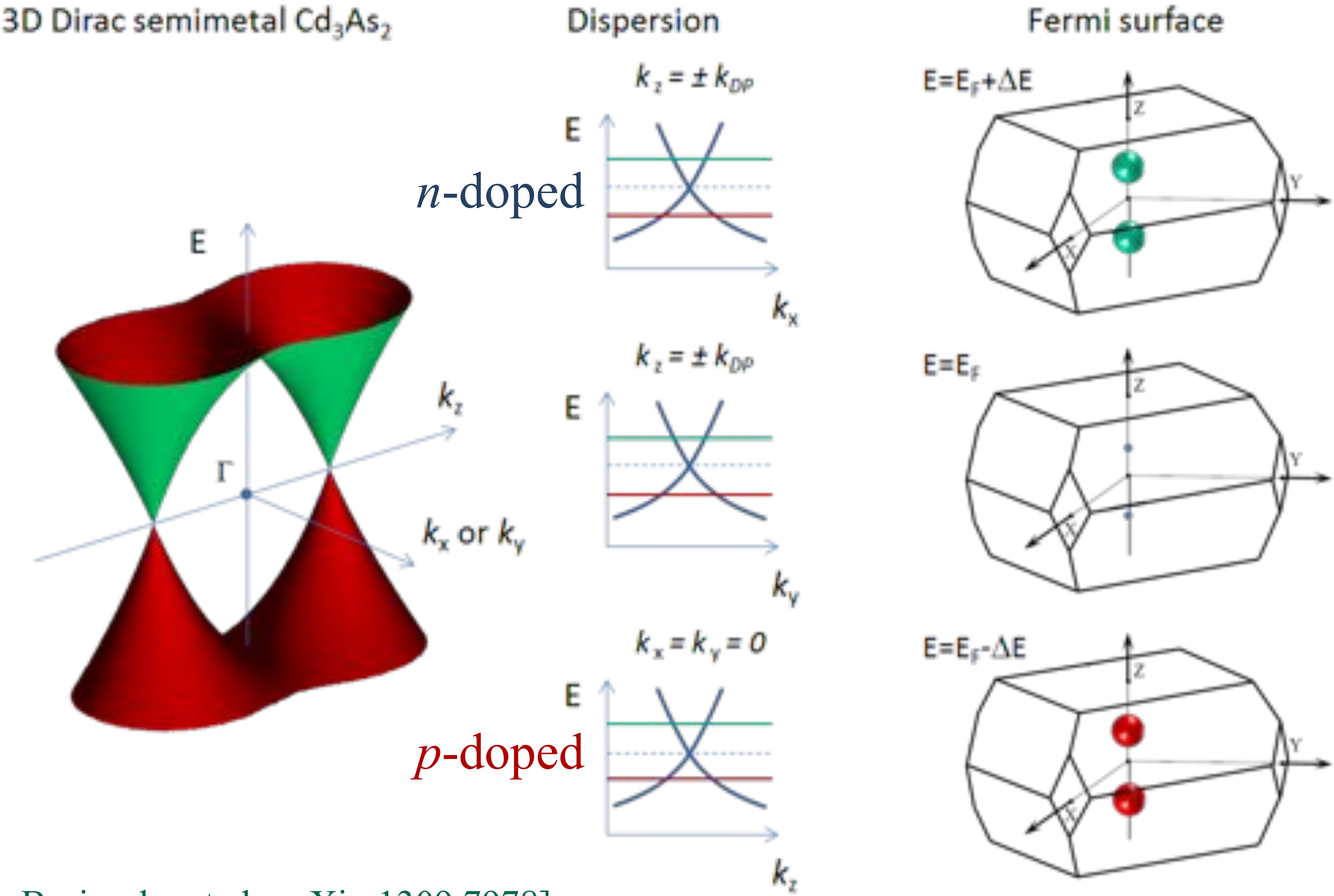
Is “3D Graphene” Possible?

- 3D materials with Dirac quasiparticles:
  - – $Bi_{1\text{-}x}Sb_x$ alloy



- “New” 3D Dirac materials (ARPES):
  - – Na_3Bi [Z. K. Liu et al., arXiv:1310.0391]
  - – Cd_3As_2 [M. Neupane et al., arXiv:1309.7892] [S. Borisenko et al., arXiv:1309.7978]

CADMIUM ARSENIDE

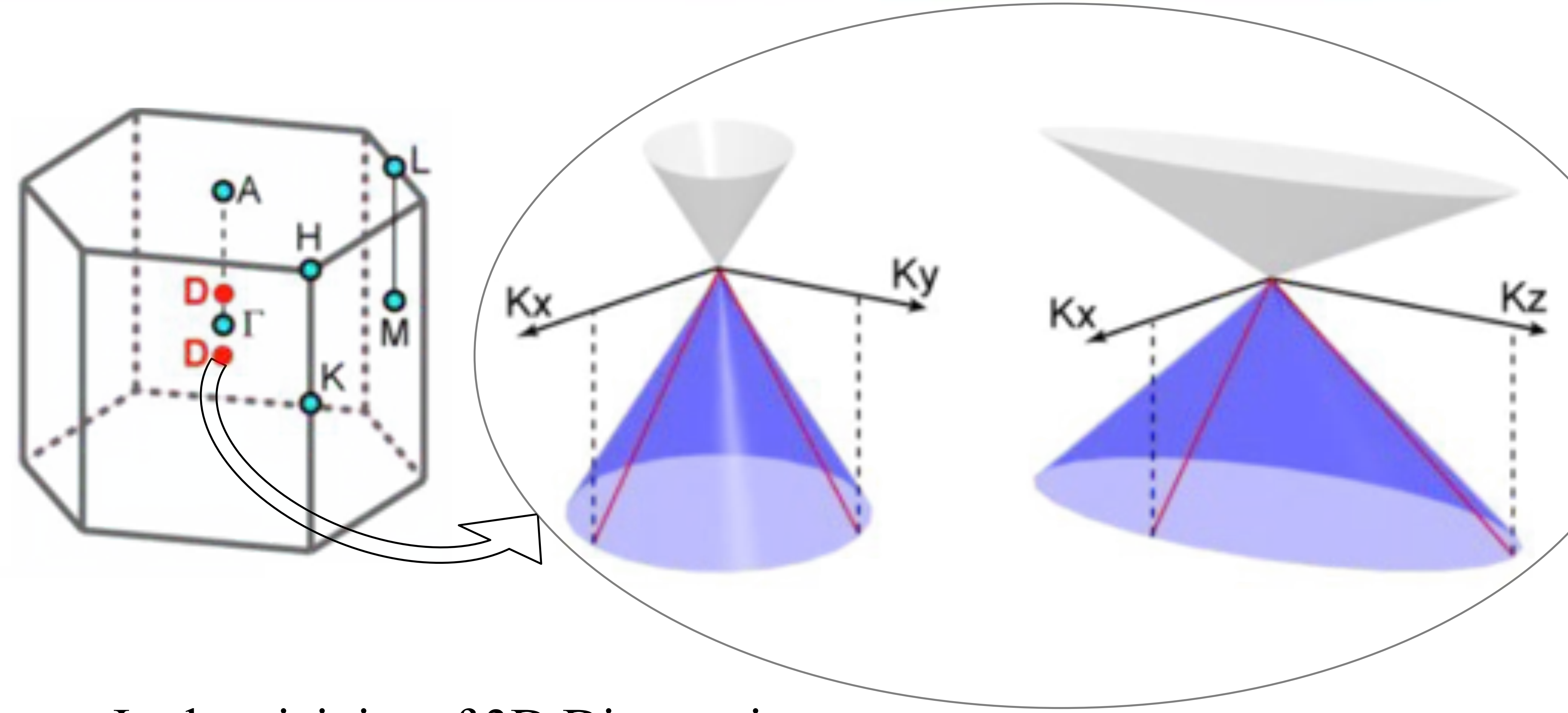


n-doped

p-doped

[S. Borisenko et al., arXiv:1309.7978]

POTASSIUM BISMUTHIDE



In the vicinity of 3D Dirac points:

$$E = v_x k_x + v_y k_y + v_z k_z$$

[Z. K. Liu et al., arXiv:1310.0391]

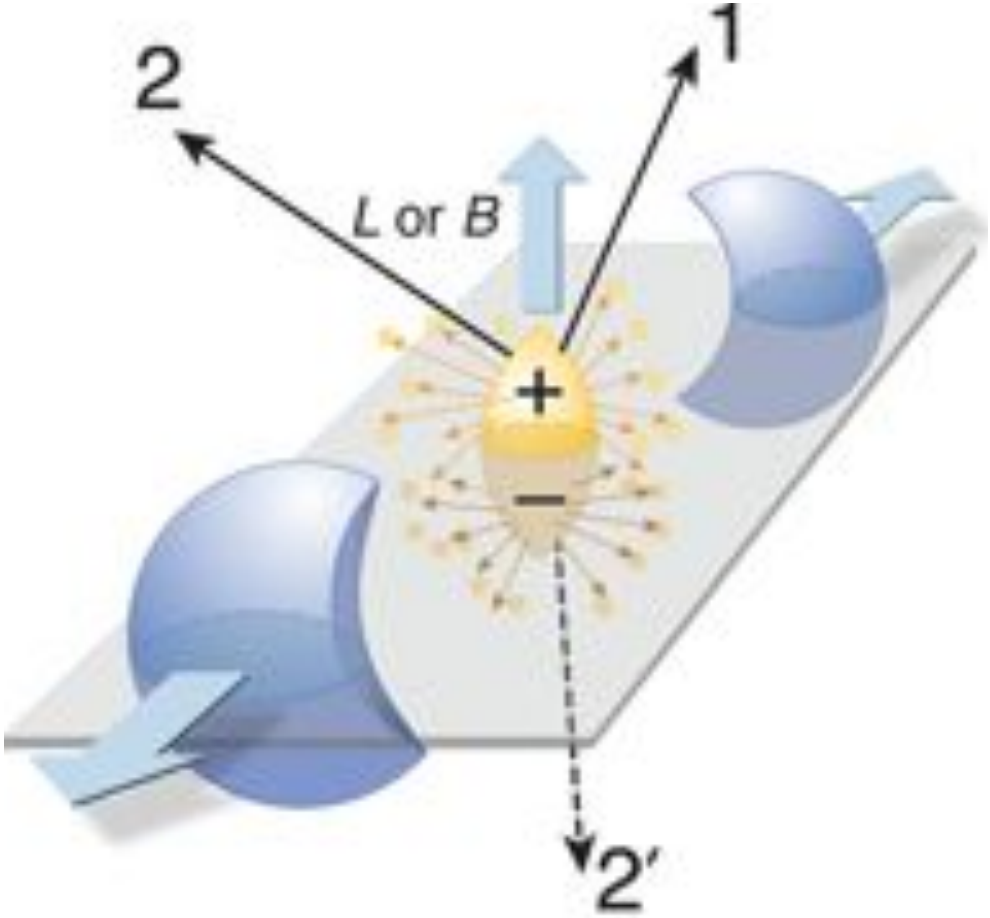
Part 5

CHIRAL MAGNETIC EFFECTS

(NONZERO DENSITY OF MATTER)

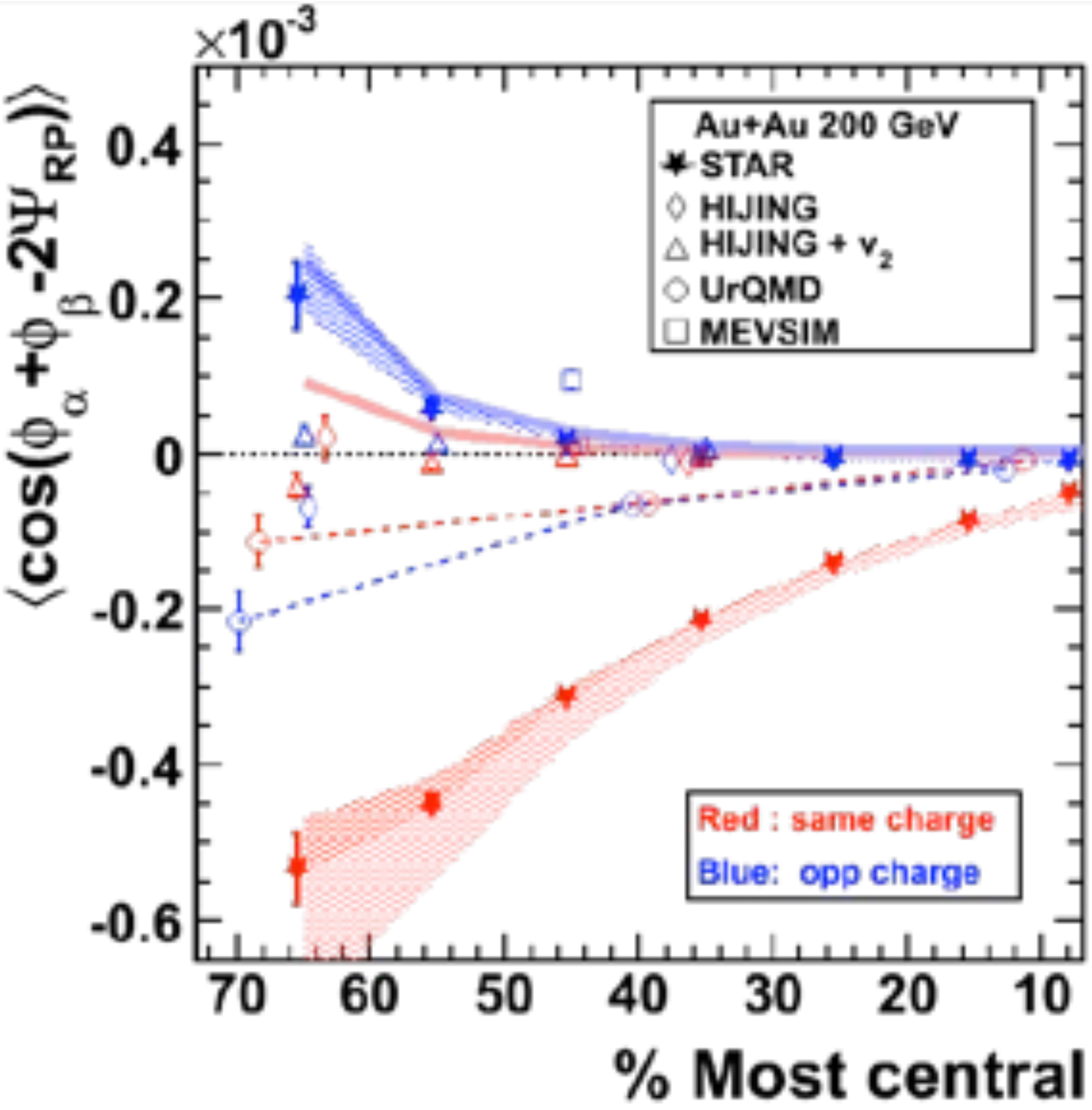
CHIRAL MAGNETIC EFFECT

- A specific spatial pattern of electric currents (or charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

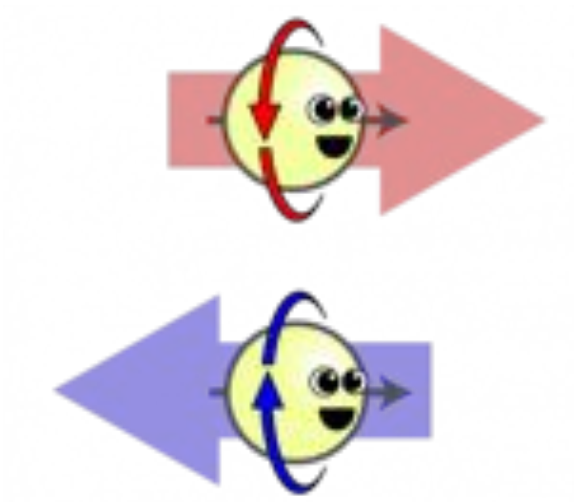
EXPERIMENTAL EVIDENCE



[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]
[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]

HELICITY/CHIRALITY

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved



Righ-handed

Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)
- At quantum level, however, such symmetry is anomalous

"CONTINUITY" EQUATION

- Continuity equation for the chiral charge

$$\frac{\partial \rho_5}{\partial t} - \vec{\nabla} \vec{j}_5 = -\frac{e^2}{4\pi^2}(\vec{E} \cdot \vec{B})$$

which is of topological nature and exact

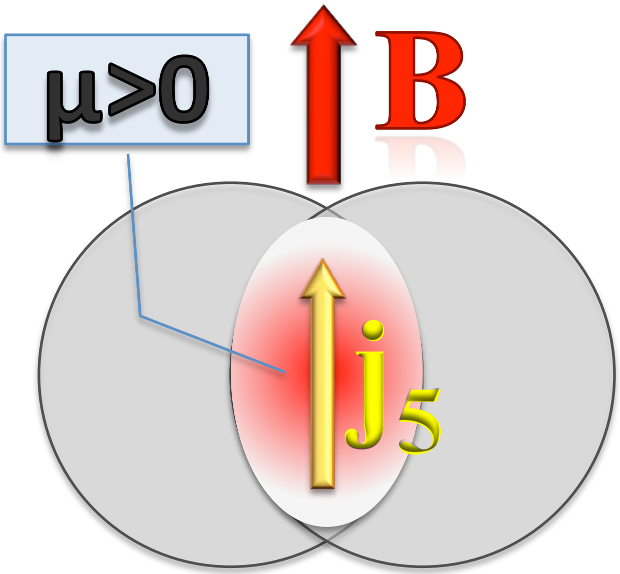
- Among its consequences are the relations:

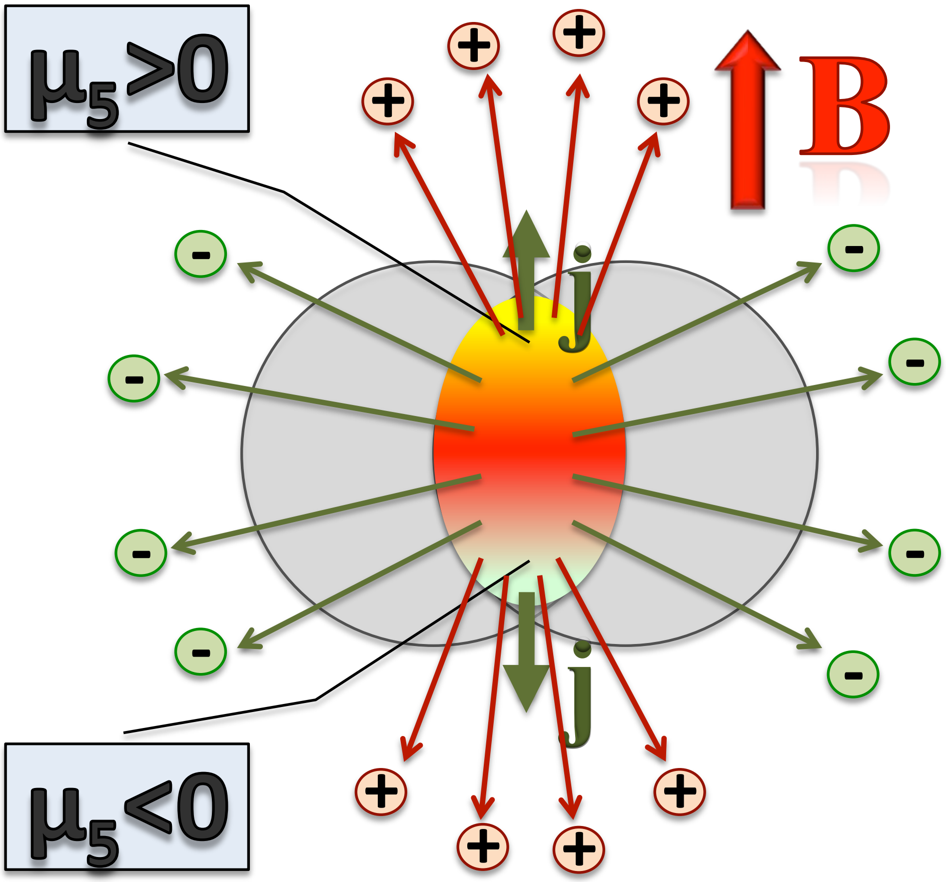
$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu \qquad \left\langle \vec{j} \right\rangle = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

- These relations are the key relations leading to the *chiral magnetic effect*

CME: CHARGE CORRELATIONS

- Start from a small baryon density and B≠0





- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

BEYOND CME

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2}\mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

- Any new physics when interaction is included?

- Yes! Chiral shift is dynamically generated (resembling $\mathrm{m}_{\mathrm{dyn}}$ in the magnetic catalysis)

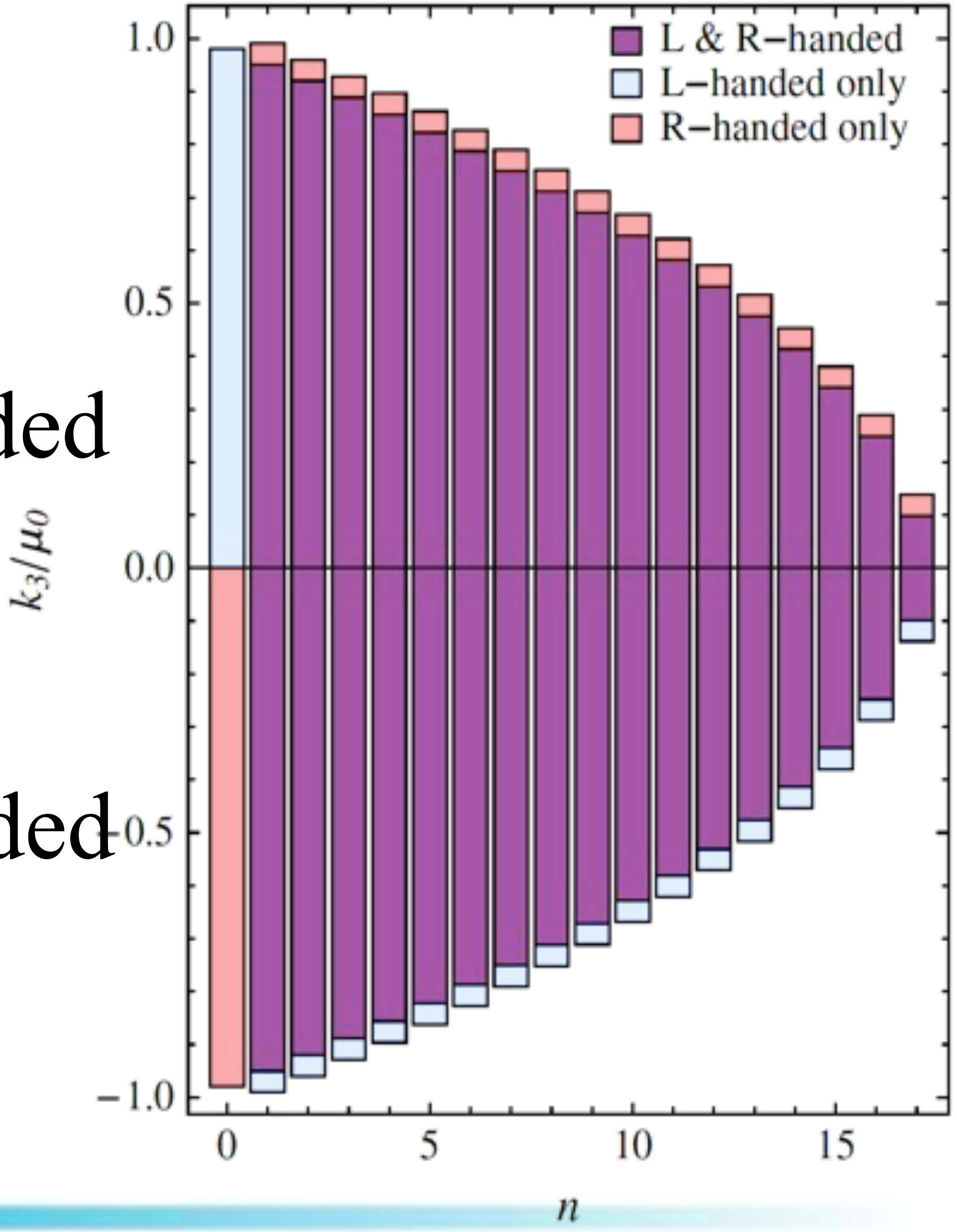
CHIRAL SHIFT AND FERMI SURFACE

- Chirality is a “good” concept at large density

$$|k_3| >> m$$

- Fermi surface of L-handed fermions is shifted in negative *z*-direction

- Fermi surface of R-handed fermions is shifted in positive *z*-direction



PHYSICS DUE TO CHIRAL SHIFT

- Chiral shift induces a chiral asymmetry at the Fermi surface

- Chiral shift can be induced in Dirac semimetals (affects magnetoresistance)

- Potential applications:
  - Pulsar kicks
  - Facilitation of supernova explosions
  - modified Chiral magnetic effect
  - Making Weyl semimetals from Na_3Bi & Cd_3As_2

SUMMARY

- Studies of relativistic matter in magnetic field are relevant for many branches of physics
- The underlying physics is conceptually rich
- Recent developments include
  - Magnetic catalysis […]
  - Chiral magnetic effect [Fukushima, Kharzeev, Warringa, PRD **78**, 074033 (2008)]
  - Chiral shift [Gorbar, Miransky, Shovkovy, PRC **80**, 032801 (R) (2009)]
  - Chiral magnetic spiral [Basar, Dunne, Kharzeev, PRL 104, 232301 (2010)]
  - Magnetic properties of Dirac semimetals [arXiv:1312.0027]
  - Quantum Hall Effect in graphene […]
  - and many others […]