MAGNETIC DANCE IN A QUANTUM WORLD

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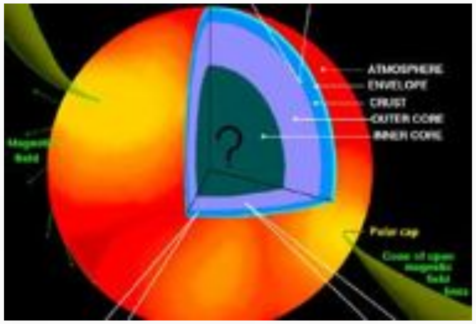
Part 1

INTRODUCTION

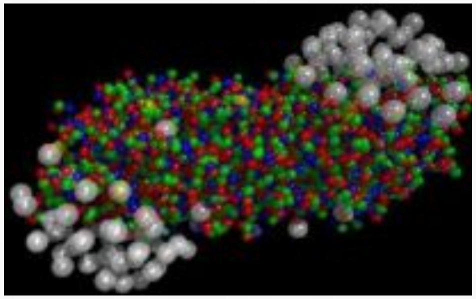
RELATIVISTIC MATTER

- Examples of relativistic matter

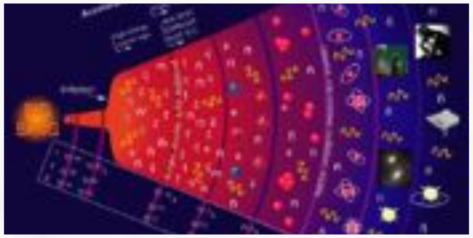
  - **Electrons, protons, quarks** inside compact stars (white dwarfs, neutron, hybrid or quark stars)



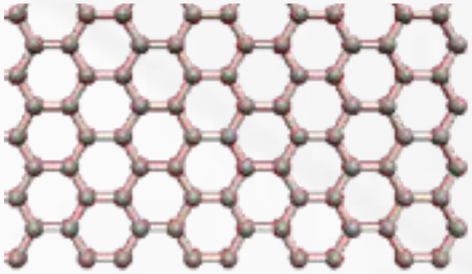
  - **Quark gluon plasma** in heavy ion collisions ($k_B T \sim 200$ MeV $\sim 10^{12}$ K)



  - **Hot matter** in the Early Universe ($k_B T \sim 100$ GeV at EW transition)



  - **Quasiparticles** in graphene (zero mass Dirac fermions)



What Means "Relativistic"?

- **Relativistic matter** $(p \gg mc)$

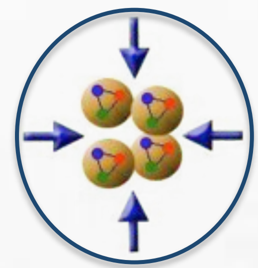
$$E = c\sqrt{p^2 + m^2c^2} \approx cp$$

compare with nonrelativistic case $(p \ll mc)$

$$E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

  – **High density** (e.g., in stars) leads to occupation of states with large momenta:

$$p \sim \hbar n^{1/3} \simeq 200 \left(\frac{n}{1\ \mathrm{fm}^3}\right)^{1/3}\ \mathrm{MeV/c}$$



  – **High temperature** (e.g., heavy ion collisions) means energetic particles,

$$p \sim k_B T/c \simeq 200 \left(\frac{k_B T}{200\ \mathrm{MeV}}\right)\ \mathrm{MeV/c}$$

  – **Vanishing mass** (e.g., graphene) works too…

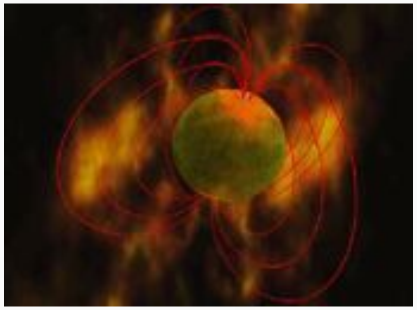
Part 2

MAGNETIC CATALYSIS

Review: arXiv:1207.5081

MAGNETIC FIELDS

- Strong magnetic fields exist inside *compact stars*
  - **10^{10} to 10^{15} Gauss**



- In *heavy ion collisions*, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

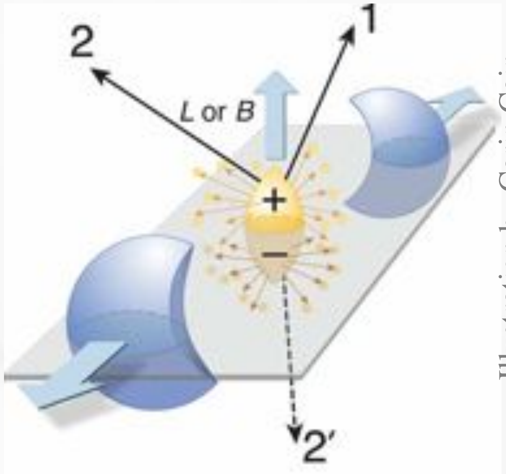


Illustration by Carin Cain

- *Early Universe*
  - **up to 10^{24} Gauss**
- *Graphene* (High Magnetic Field Laboratory)
  - **4.5×10^5 Gauss**

Landau levels

- Fermions in magnetic field

$$\mathcal{L} = \overline{\Psi} i \gamma^\mu D_\mu \Psi + (\text{interactions})$$

- Free energy spectrum

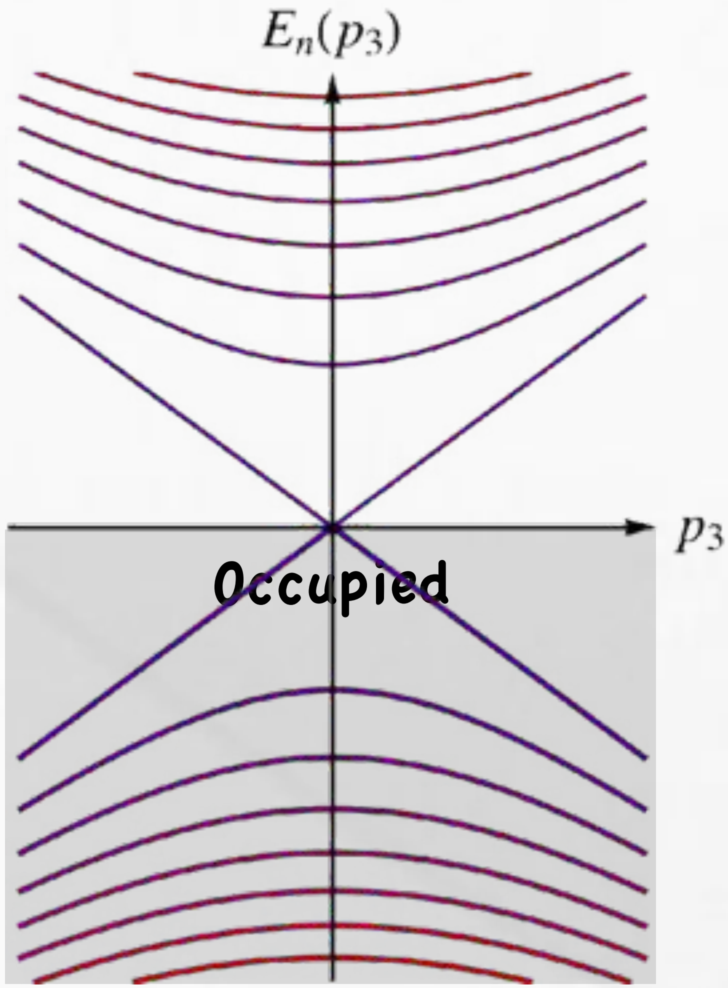
$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

$$n = s + k + \frac{1}{2}$$

$s = \pm 1/2$ (spin)

$k = 0, 1, 2, \ldots$ (orbital)

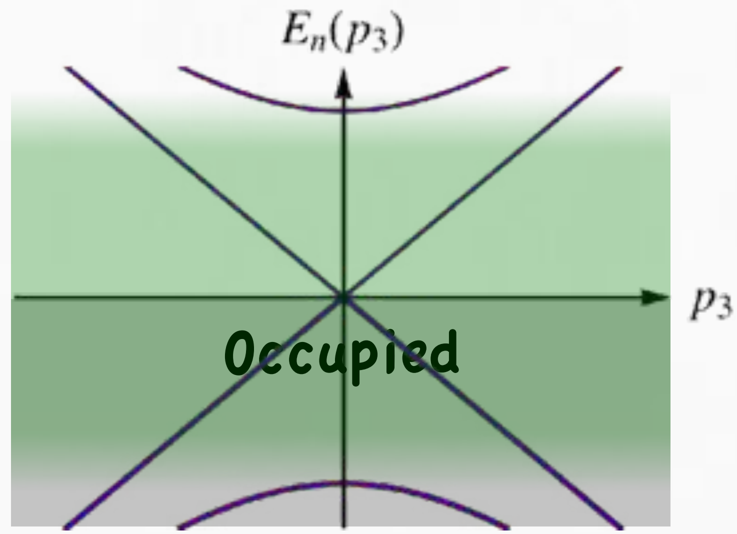


Dimensional reduction

- Low-energy is due to n=0 Landau level

$$n = 0: \qquad E_0^{(3+1)}(p_3) = \pm p_3$$

$$\left(k = 0, s = -\frac{1}{2}\right)$$



- This is (1+1)D spectrum!

- Propagator looks (1+1)D as well:

$$S(p_\parallel) \approx i\, e^{-p_\perp^2 \ell^2} \frac{\hat{p}_\parallel + m}{\hat{p}_\parallel + m} \underbrace{\left(1 - i\gamma^1\gamma^2\right)}_{s = -1/2 \text{ spin projector}}, \text{ where } \hat{p}_\parallel = p_0\gamma^0 - p_3\gamma^3$$

MAGNETIC CATALYSIS (CLUES)

- Low-energy regime is dimensionally reduced

$$D \Rightarrow D-2$$

- Density of states at $E = 0$

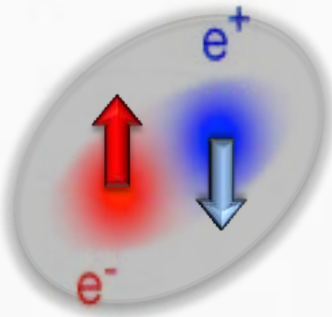
$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB|N_f}{4\pi^2}$$

(This may remind superconductivity…)

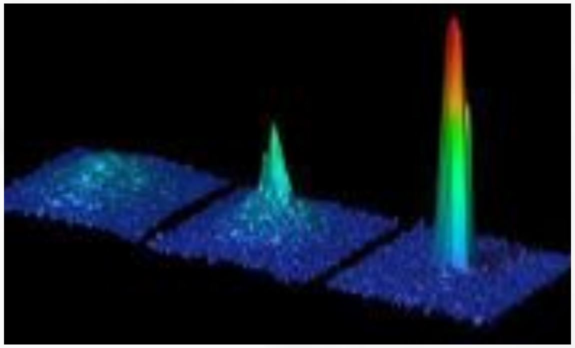
[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

MAGNETIC CATALYSIS (PHYSICS)

- n=0: particles & anti-particles
- Bound states are energetically favorable (an energy gain of E_b per pair)



- Bound states are bosons
- Bosons can (and will) occupy same zero momentum quantum state



- Bose condensate forms
- Symmetry breaking ➔ energy (mass) gap

[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

DYNAMICAL MASS

- While $m_0=0$ originally, a nonzero "dynamical" mass m_{dyn} is generated

$$m_{dyn}^{(2D)} \propto \sqrt{\alpha}\sqrt{|eB|}\ , \quad \text{and} \quad m_{dyn}^{(3D)} \propto \sqrt{|eB|}e^{-C/\alpha}$$

- This happens even at the *weakest* interaction ("catalysis")
- The phenomenon is *universal* (model details are irrelevant)
- Dimensional *reduction* is the key ingredient (massless bound states form = symmetry breaking)

UNIVERSALITY OF MC

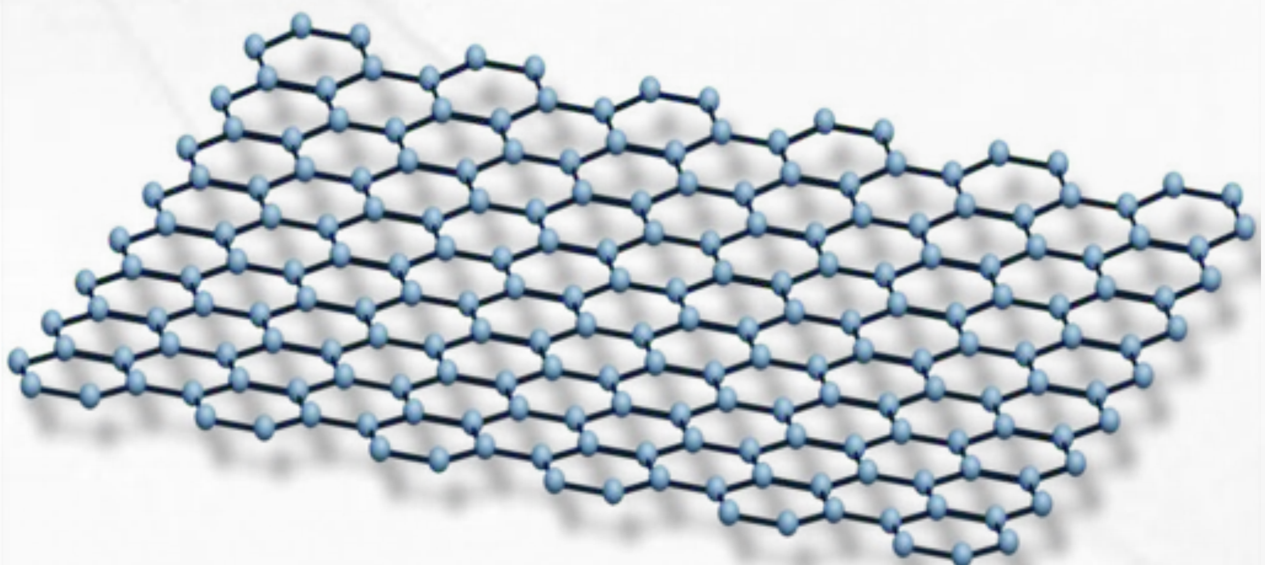
- **Input**
  - Spin-1/2 charged particles and B≠0
  - Attractive particle-antiparticle interaction
- **Output**
  - Dimensional reduction D->D-2 (low energies)
  - Bound states form and condense
  - Symmetry breaks down
  - Dynamical mass is generated

Part 3

GRAPHENE

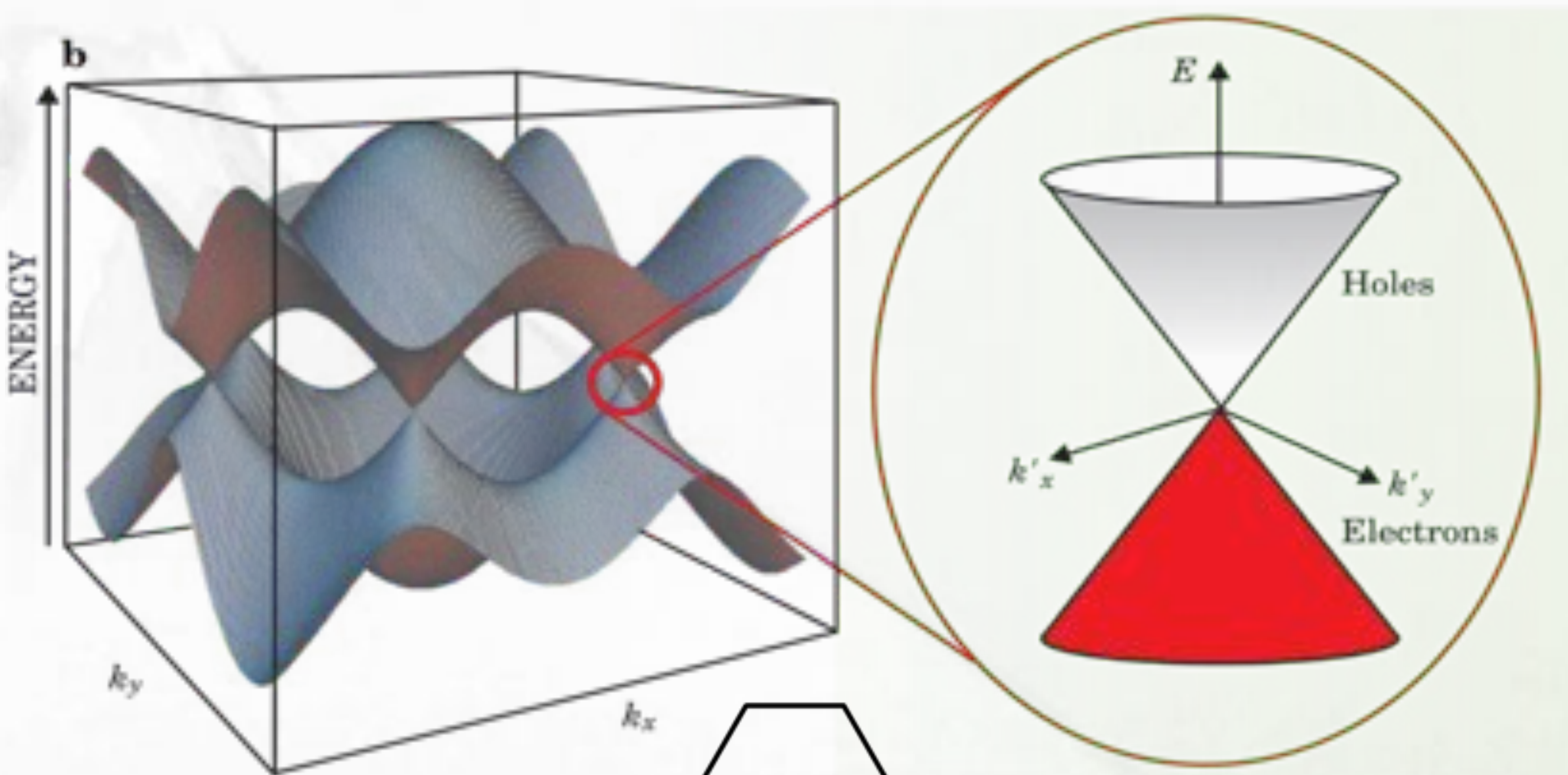
GRAPHENE

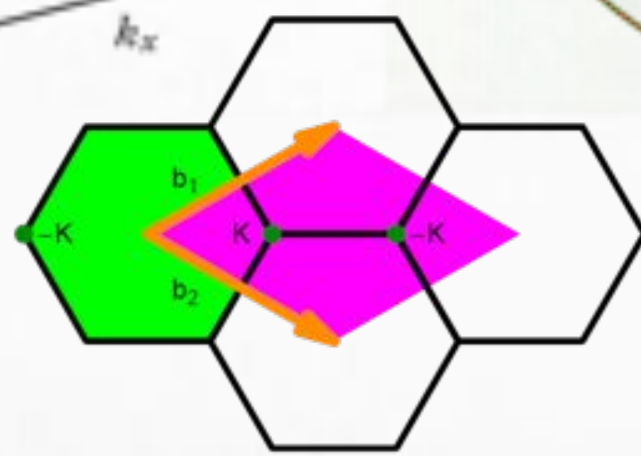
- It is a single atomic layer of graphite
  [Novoselov et al., Science **306**, 666 (2004)]
- 2D crystal with hexagonal lattice of carbon atoms



- Interesting basic physics
- Great promise for applied physics

DISPERSION RELATION





DIRAC FERMIONS IN GRAPHENE

- Low energy quasiparticles are **massless** Dirac fermions $(\upsilon_F = c/300)$

- Spinor:

$$\Psi_s = \begin{pmatrix} \psi_{KAs} \\ \psi_{KBs} \\ \psi_{K'Bs} \\ \psi_{K'As} \end{pmatrix}$$

- Low-energy model with U(4) global symmetry:

$$H_0 = \upsilon_F \int d^2 r \, \overline{\Psi}_s \left(\gamma^1 \pi_x + \gamma^2 \pi_y \right) \Psi_s$$

[Wallace, Phys. Rev. **71,** 622 (1947)]

[Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)]

MAGNETIC CATALYSIS IN GRAPHENE

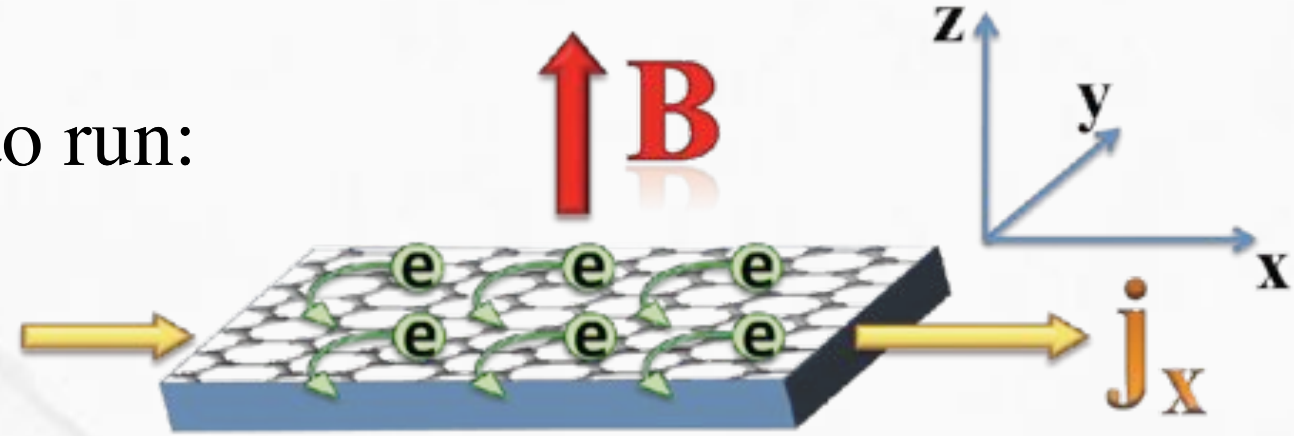
- Charge carriers are spin-½ fermions with m=0
- $m_{dyn} \neq 0$ is expected in a strong magnetic field

[Khveshchenko, PRL **87**, 206401 (2001)]
[Gorbar, Gusynin, Miransky, Shovkovy, PRB **66** (2002) 045108]

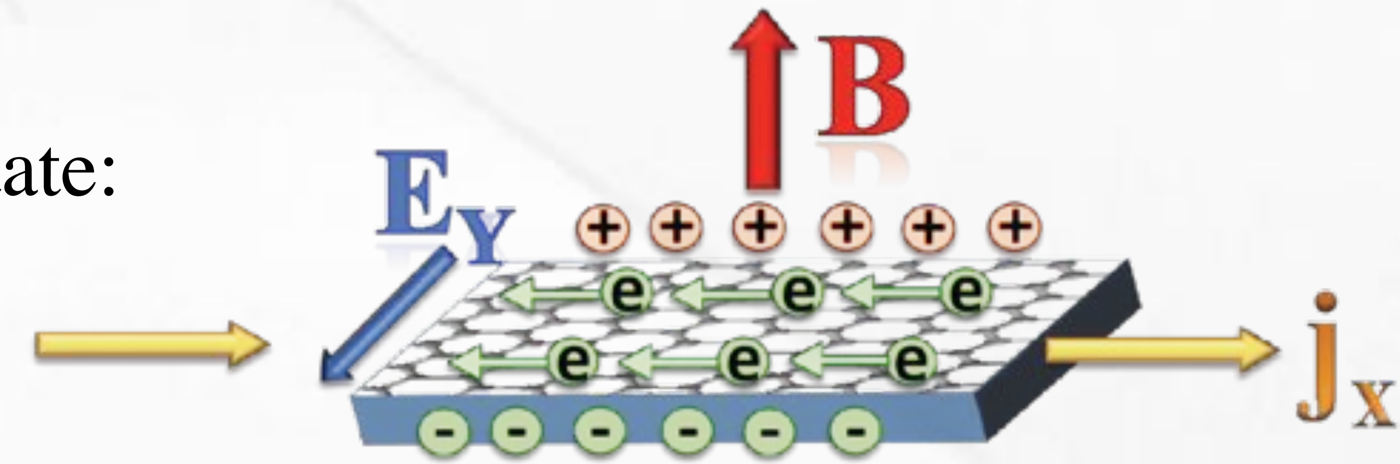
- Possible complications:
  - many types of "Dirac" masses in **2D**
  - competition with quantum Hall ferromagnetism
  - nonzero electron/hole density
  - impurities, lattice defects, ripples, etc.
- How to test this experimentally?

Quantum Hall Effect

- General setup
  - Current starts to run:



  - Steady state:



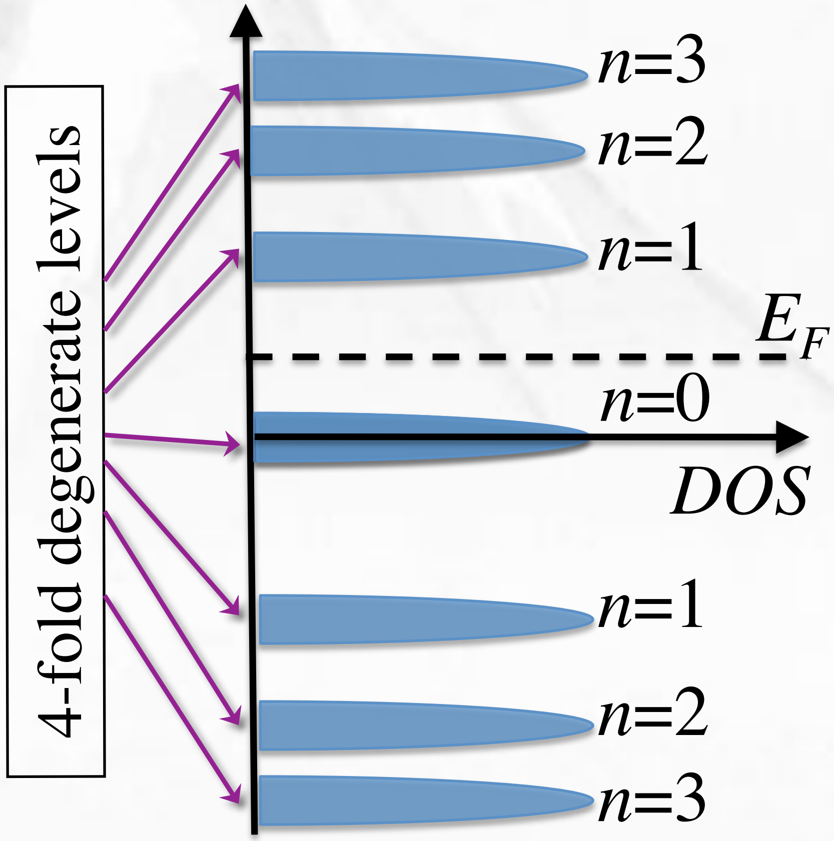
  - Hall conductivity:

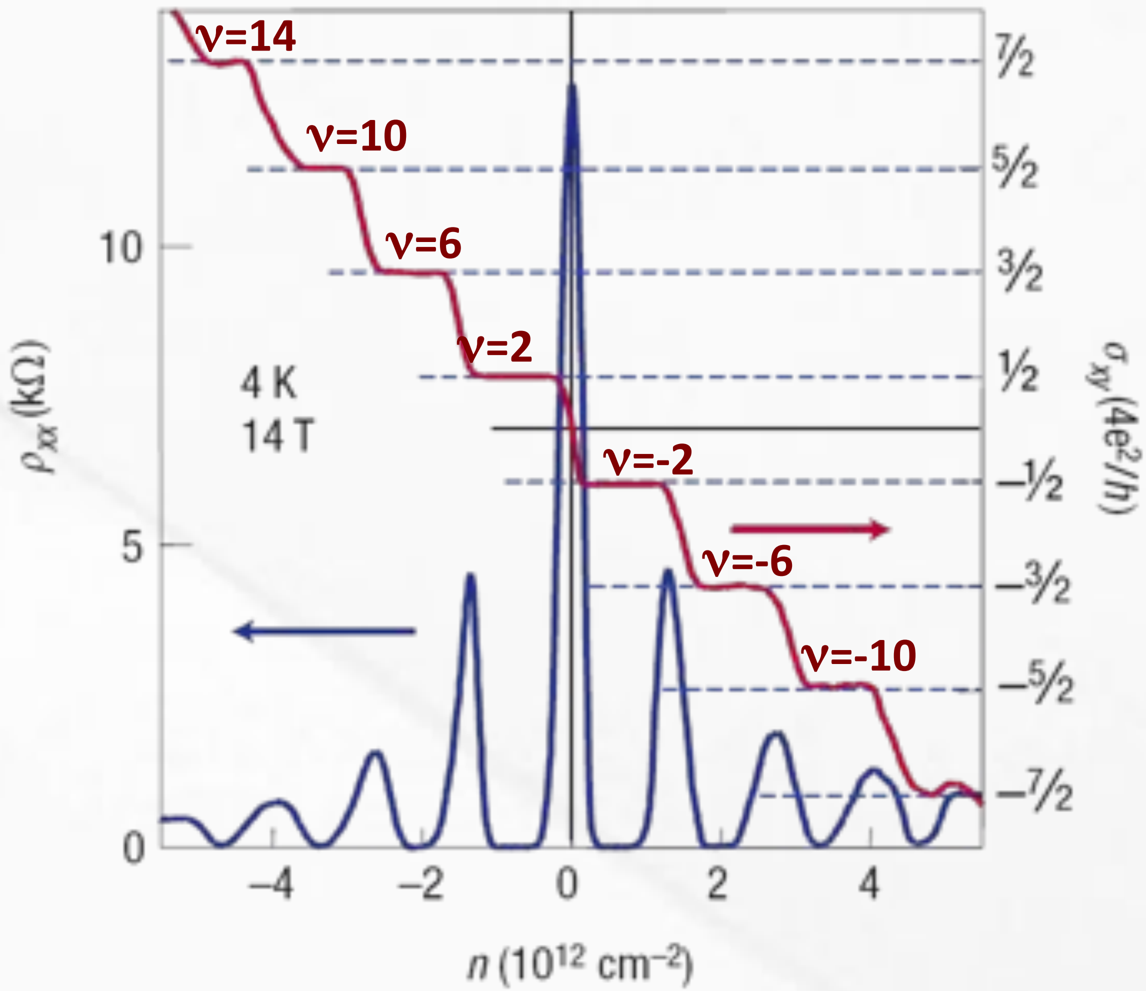
$$j_x = \sigma_{xy} E_y$$

QHE in Graphene

$$E_n = \pm\sqrt{2\hbar v_F^2 n|eB|}$$

$$\sigma_{xy} = \nu\frac{e^2}{h} = 4\left(n+\frac{1}{2}\right)\frac{e^2}{h}$$



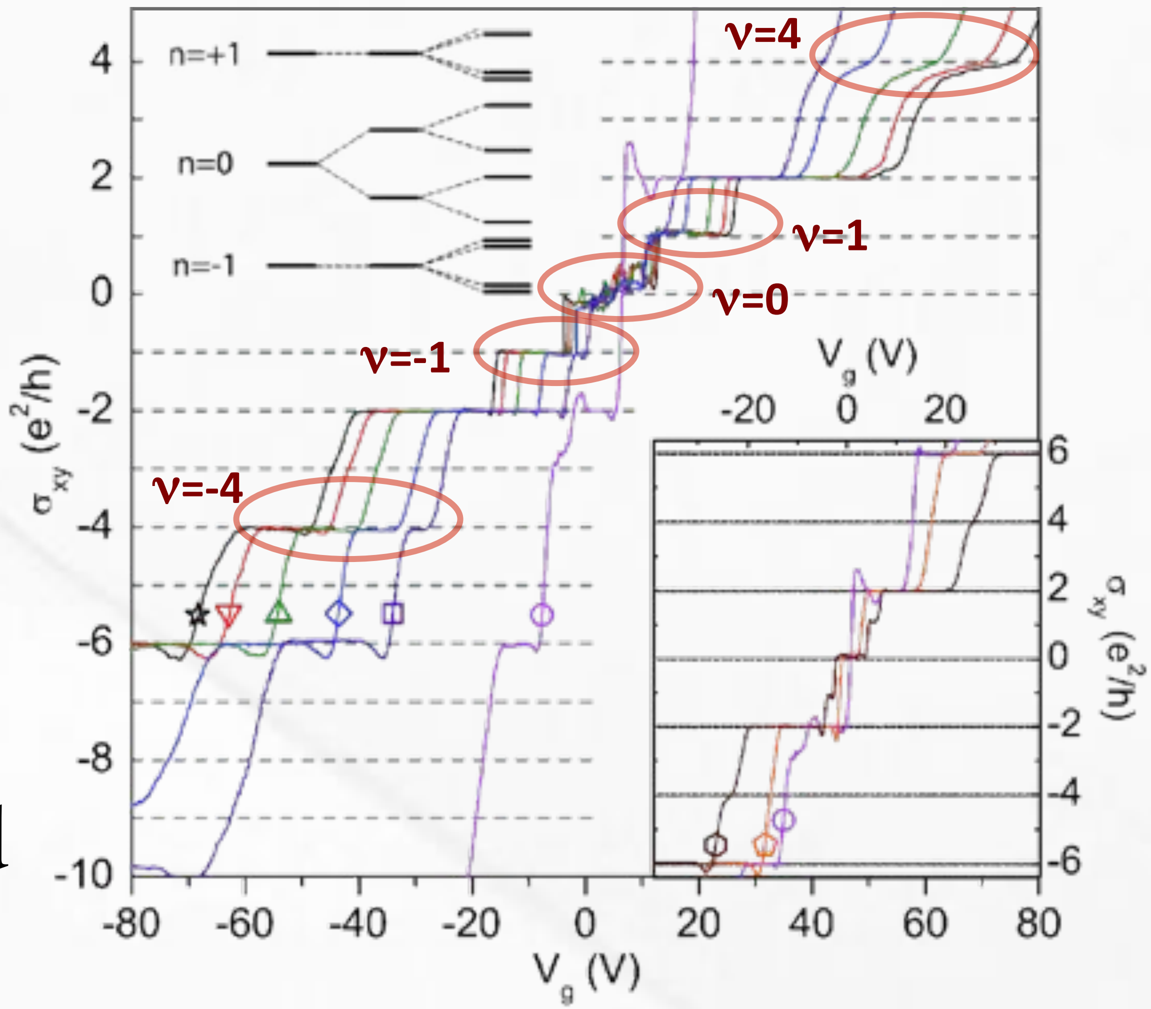


[Gusynin, Sharapov, Phys. Rev. Lett. **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, Phys. Rev. B **73**, 125411 (2006)]
[Novoselov et al., Nature **438**, 197 (2005)], [Zhang et al., Nature **438**, 201 (2005)]

ANOMALOUS QHE

- New plateaus at
  - ν=0
  - ν=±1
  - ν=±3
  - ν=±4
- Some Landau level degeneracy is lifted

Zhang et al., PRL **96,** 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]
[Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)]
[Checkelsky et al., Phys. Rev. Lett. **100**, 206801 (2008)]
[Xu Du et al., Nature **462**, 192 (2009)]

Magnetic Catalysis in Graphene

- Charge carriers are massless Dirac fermions
- Spectrum in magnetic field:

$$E_n = \pm\sqrt{2\hbar \upsilon_F^2 n |eB|}$$

- Degenerate E=0 level with particles & holes
- Electron-hole (excitonic) pairing occurs
- $m_{dyn} \neq 0$ is generated
- In qualitative agreement with experiment

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **66** (2002) 045108]

Order parameters

- Many order parameters may be generated (pairing from different valleys/sublattices)

- Dirac masses [triplet under $U(2)_s$]

$$\tilde{\Delta}_s: \quad \overline{\Psi} P_s \Psi = \psi^{+}_{KAs}\psi_{KAs} - \psi^{+}_{KBs}\psi_{KBs} + \psi^{+}_{K'As}\psi_{K'As} - \psi^{+}_{K'Bs}\psi_{K'Bs}$$

(charge-density wave)

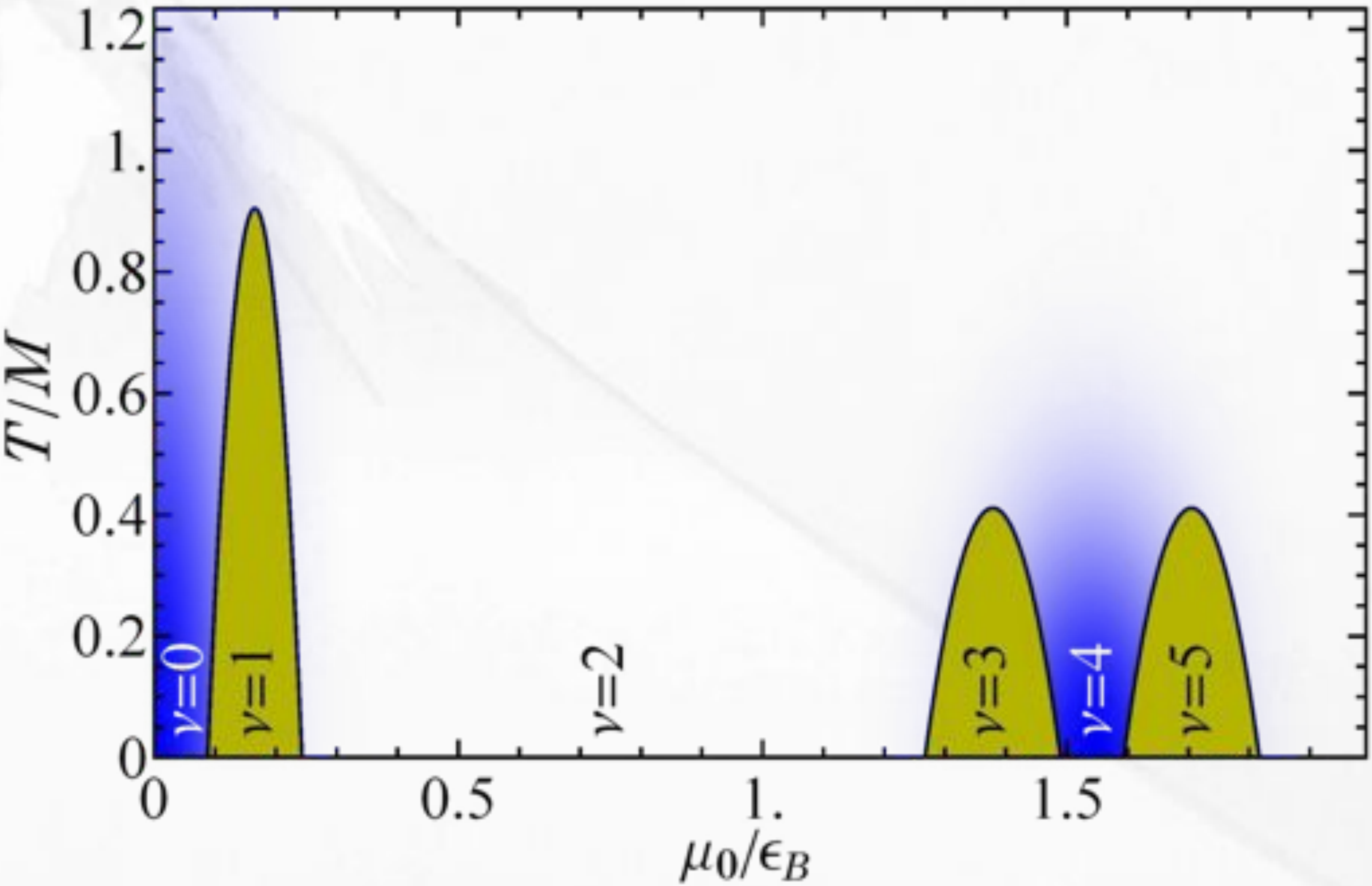
- Haldane masses [singlet under $U(2)_s$]

$$\Delta_s: \quad \overline{\Psi}\gamma^3\gamma^5 P_s \Psi = \psi^{+}_{KAs}\psi_{KAs} - \psi^{+}_{KBs}\psi_{KBs} - (\psi^{+}_{K'As}\psi_{K'As} - \psi^{+}_{K'Bs}\psi_{K'Bs})$$

- + spin & pseudo-spin densities

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **78** (2008) 085437]
[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scr. T **146** (2012) 014018]

PHASE DIAGRAM



THEORETICAL COMPLICATIONS

- Competition between Dirac & Haldane masses is subtle
- Symmetry breaking lattice effects
- Dynamical screening effects
- Competition with quantum Hall ferromagnetism
- Nonzero electron/hole density (ν>0)
- impurities, lattice defects, ripples, etc.

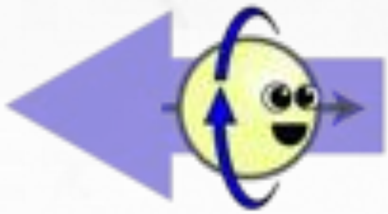
Part 4

CHIRAL EFFECTS

HELICITY/CHIRALITY

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved

Righ-handed



Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)

- The symmetry is anomalous at quantum level

Chiral Magnetic Effect

- Chiral charge is produced by topological QCD configurations

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2}\int d^3x\, F_a^{\mu\nu}\tilde{F}^a_{\mu\nu}$$

- Random fluctuations with nonzero chirality in each event

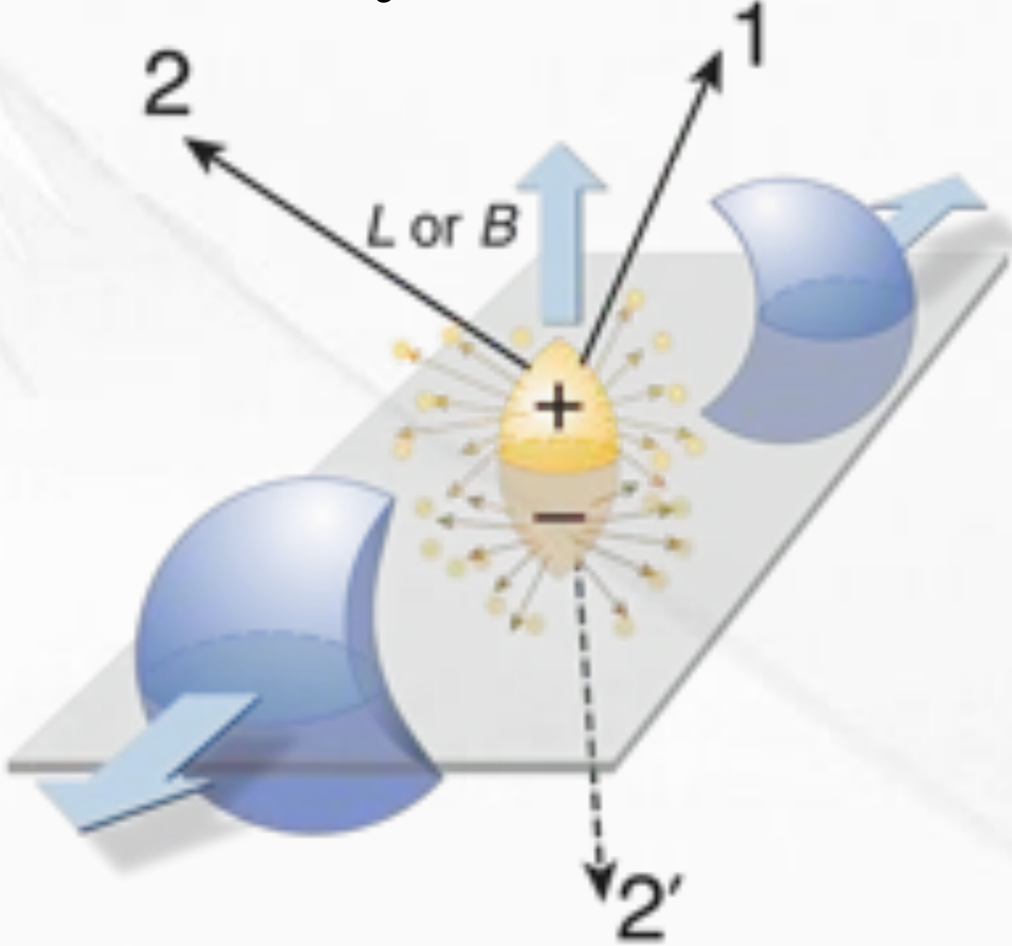
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- Driving electric current

$$\langle \vec{j} \rangle = -\frac{e^2\vec{B}}{2\pi^2}\mu_5$$

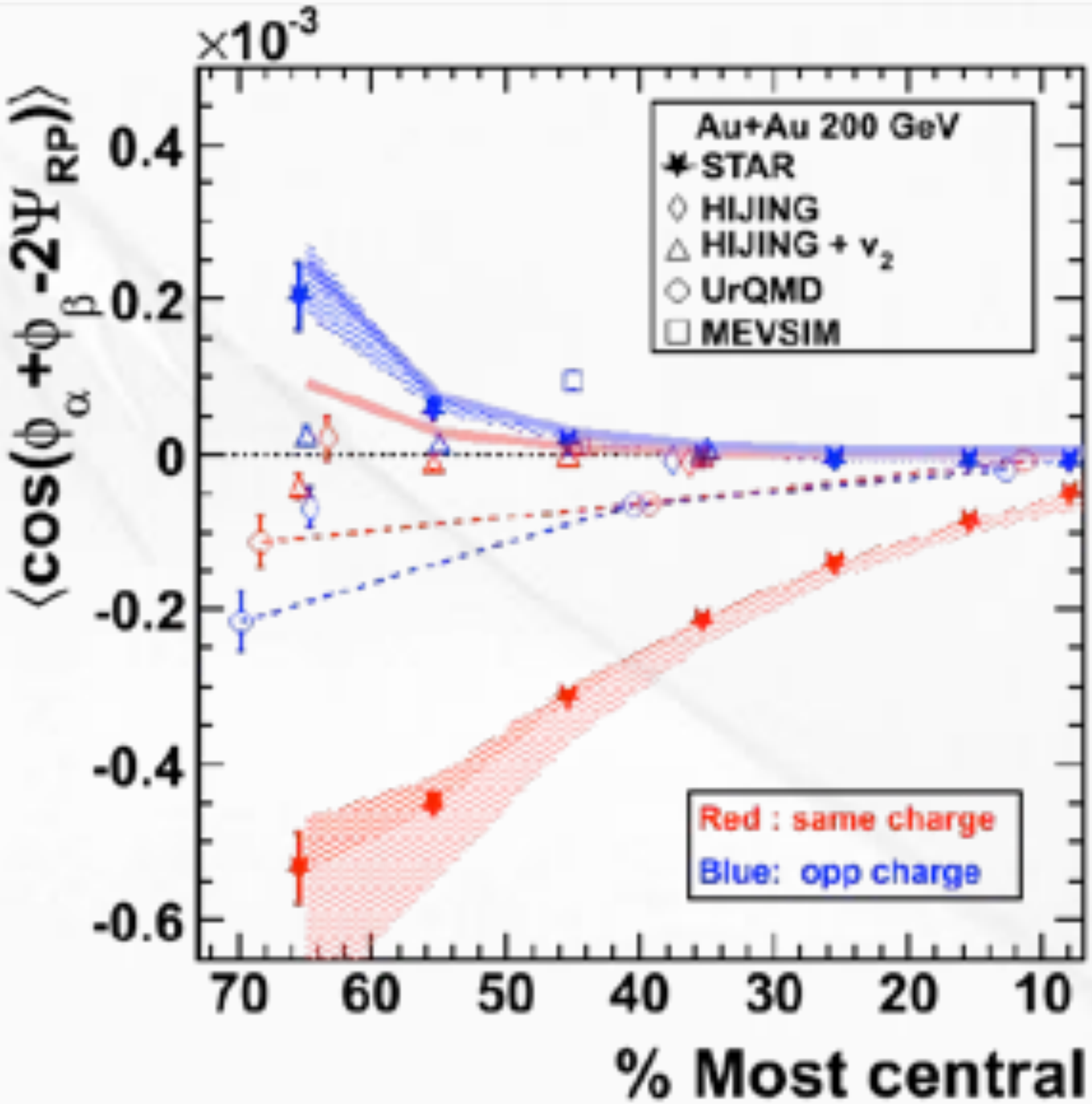
Dipole CME

- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

EXPERIMENTAL EVIDENCE



[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]
[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]

Chiral separation effect

- Axial current induced by fermion chemical potential

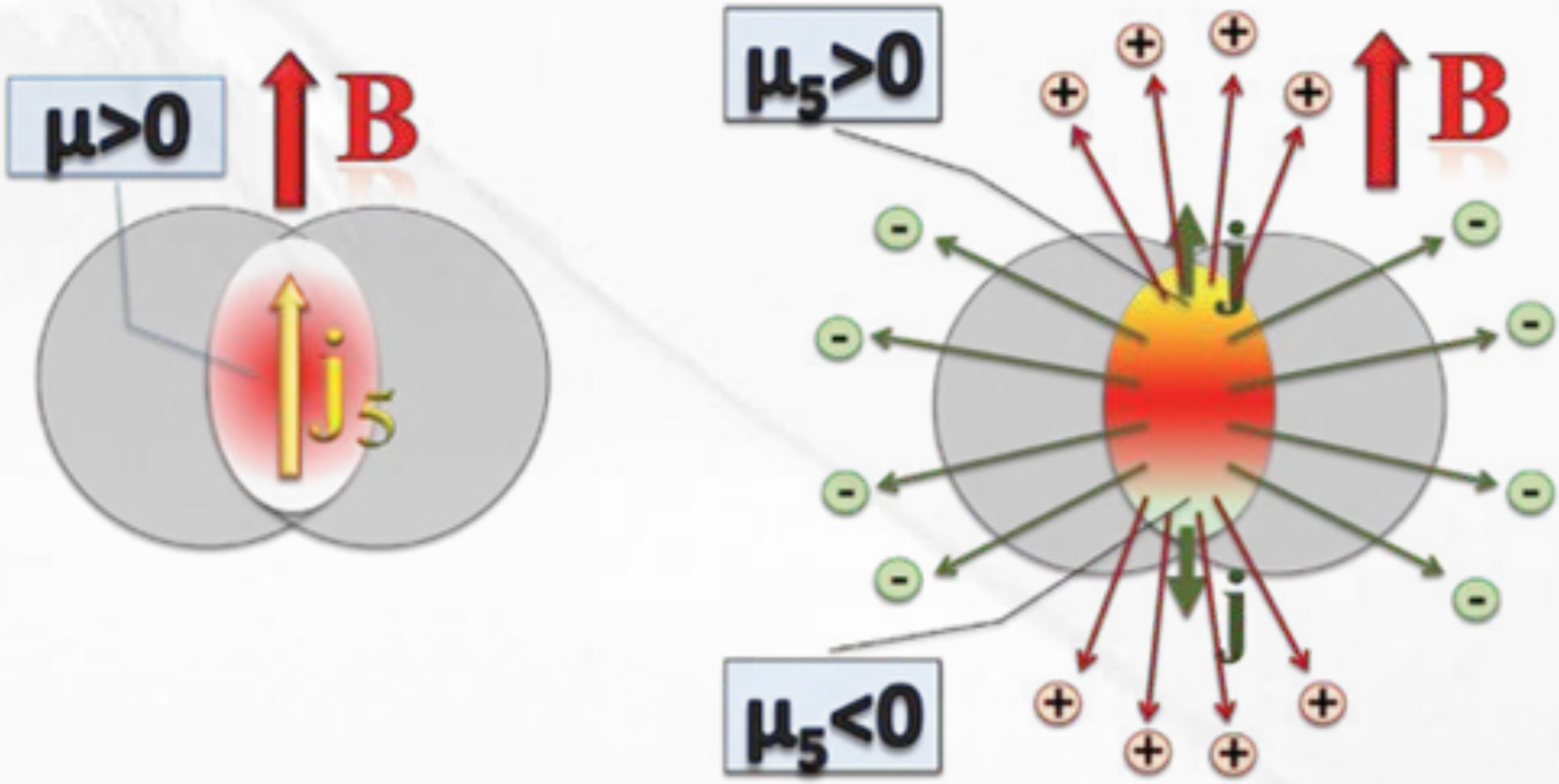
$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Exact result (is it?), which follows from chiral anomaly relation
- No radiative corrections expected…

QUADRUPOLE CME

- Start from a small baryon density and B≠0



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

BEYOND CSE/CME

- Any radiative corrections to CSE?

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu + \ldots \qquad \text{(yes)}$$

- Any dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \Delta \overline{\psi} \gamma^3 \gamma^5 \psi \qquad \text{(yes)}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83 (2011) 085003]

- Note: Δ=0 is not protected by any symmetry

Chiral shift & Fermi surface

- Chirality is $\approx$ well defined at Fermi surface $\left(|k_3| >> m\right)$
- L-handed Fermi surface:

$$n = 0: \qquad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \qquad k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

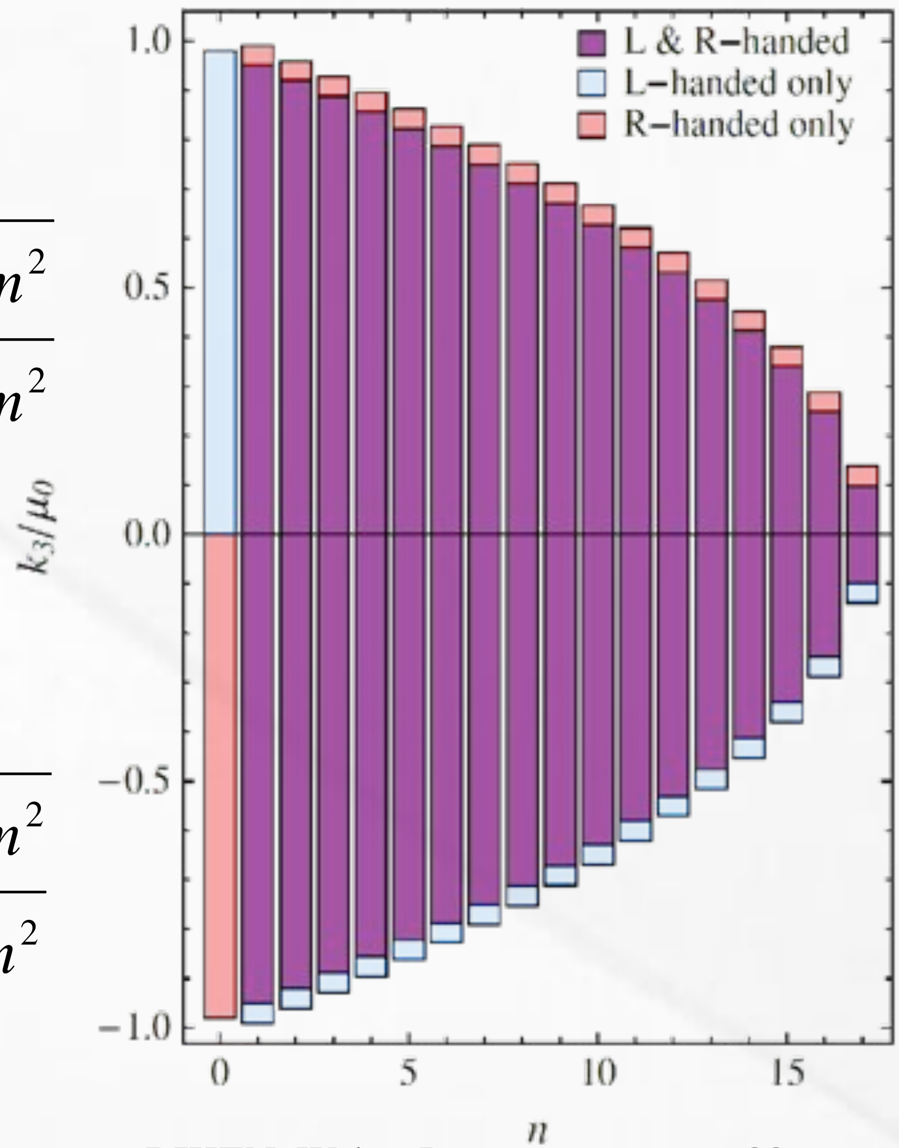
$$k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \qquad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \qquad k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

$$k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$



QED: YES

$$\overline{\Sigma}^{(1)}(p) = -4i\pi \int \frac{d^4k}{(2\pi)^4} \gamma^{\mu} \,\overline{S}^{(1)}(k)\gamma^{\nu} D_{\mu\nu}(k-p)$$

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta + \gamma^0\gamma^5\mu_5(p) + \cdots$$

where, in the small B limit,

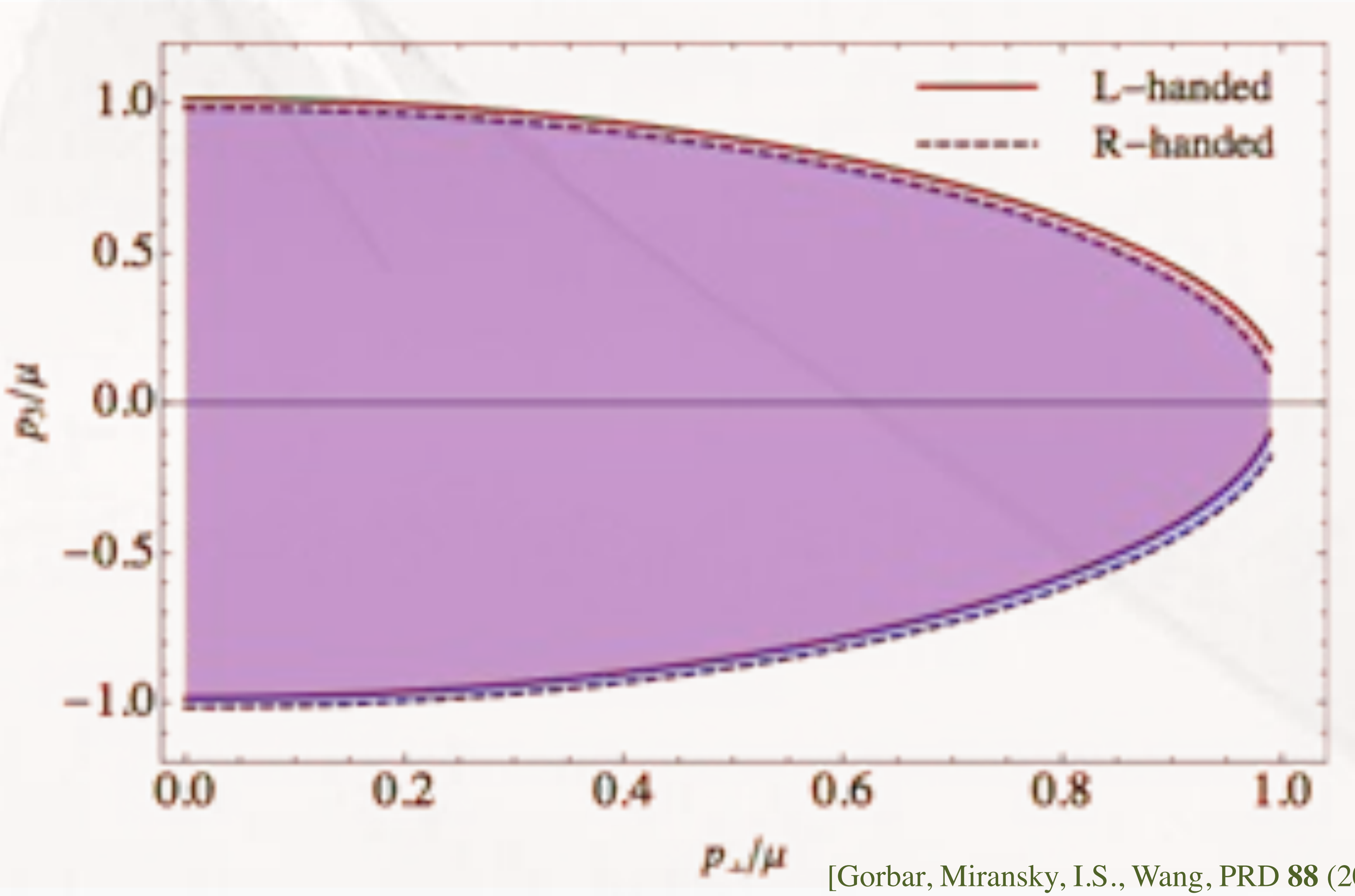
$$\Delta \approx \frac{\alpha\, eB\,\mu}{\pi\, m^2}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

$$\mu_5(p) \approx -\frac{\alpha\, eB\,\mu}{\pi\, m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

Dispersion relations in QED

- Let us use the condition (for a small B)

$$\mathrm{Det}\left[i\,\bar{S}^{-1}(p) + \bar{\Sigma}^{(1)}(p) \right] = 0$$



[Gorbar, Miransky, I.S., Wang, PRD **88** (2013) 025043]

Axial current in QED

- Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}\left(i\gamma^{\mu}D_{\mu} + \mu\gamma^{0} - m\right)\psi + \text{(counterterms)}$$

- Axial current

$$\left\langle j_5^3 \right\rangle = -Z_2\,\mathrm{tr}\left[\gamma^3\gamma^5 G(x,x)\right]$$

- To leading order in coupling $\alpha = e^2/(4\pi)$

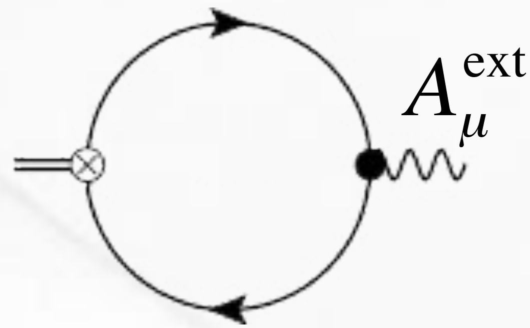
$$G(x,y) = S(x,y) + i\int d^4u\, d^4v\, S(x,u)\Sigma(u,v)S(v,y)$$

[Gorbar, Miransky, Shovkovy, Wang, PRD **88** (2013) 025025]

Expansion in external field

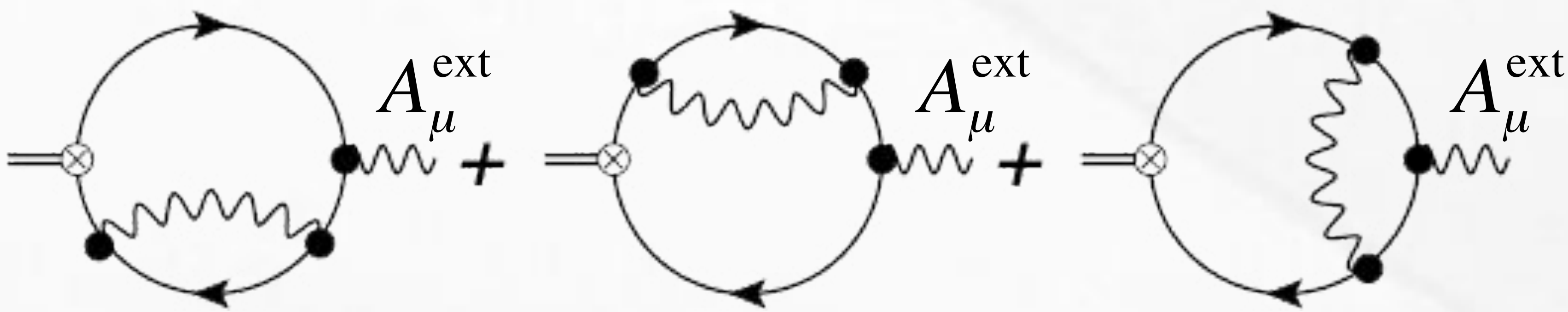
- Expand $S(x,y)$ in powers of gauge field A_μ^{ext}
- To leading order in coupling,

$$\left\langle j_5^3 \right\rangle_0 =$$



- Next-order radiative corrections are

$$\left\langle j_5^3 \right\rangle_\alpha =$$



RESULT (m<<μ)

- Final result (loops+counterterms)

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m}+\ln\frac{m_\gamma^2}{m^2}+\frac{4}{3}\right)-\frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma}-\frac{11}{12}\right)$$

- Unphysical dependence on photon mass because infrared physics with

$$m_\gamma \le |k_0|, |k_3| \le \sqrt{|eB|}$$

not captured properly

- Note: similar problem exists in calculation of Lamb shift

[Gorbar, Miransky, Shovkovy, Wang, PRD **88** (2013) 025025]

BEYOND PERTURBATIVE EXPANSION

- Perpendicular momenta cannot be defined with accuracy better than

$$\left|\Delta \mathbf{k}_{\perp}\right|_{\min} \sim \sqrt{|eB|}$$

(In contrast to the tacit assumption in using expansion in powers of B-field)

- Screening effects provide a natural infrared regulator

$$m_{\gamma} \Rightarrow \sqrt{\alpha}\,\mu$$

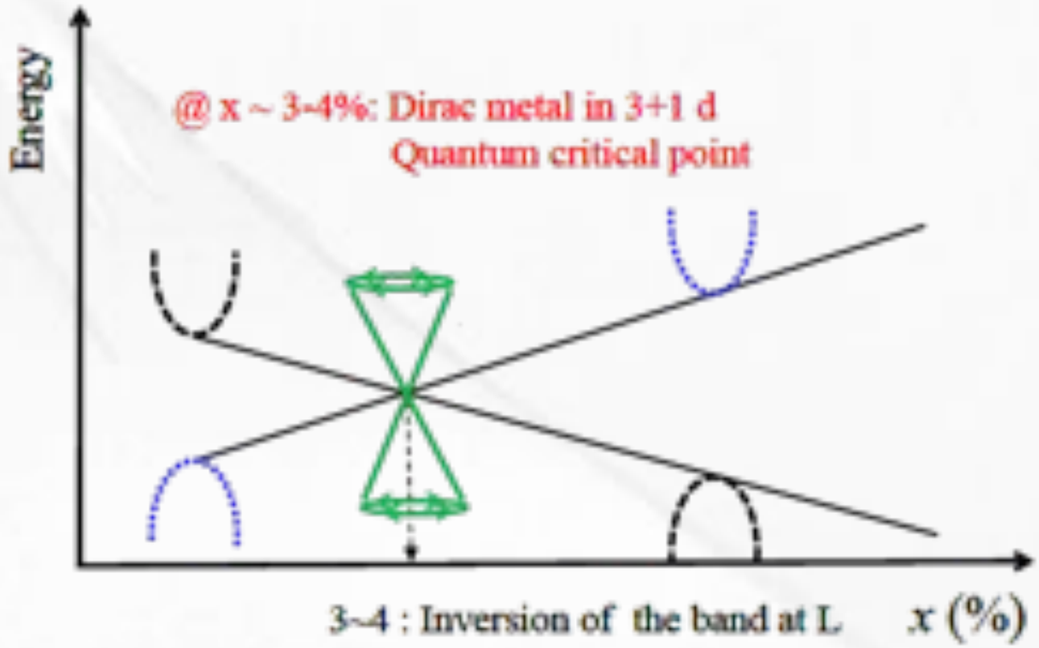
(Formally, this goes beyond the leading order in coupling)

Part 5

DIRAC SEMIMETAL

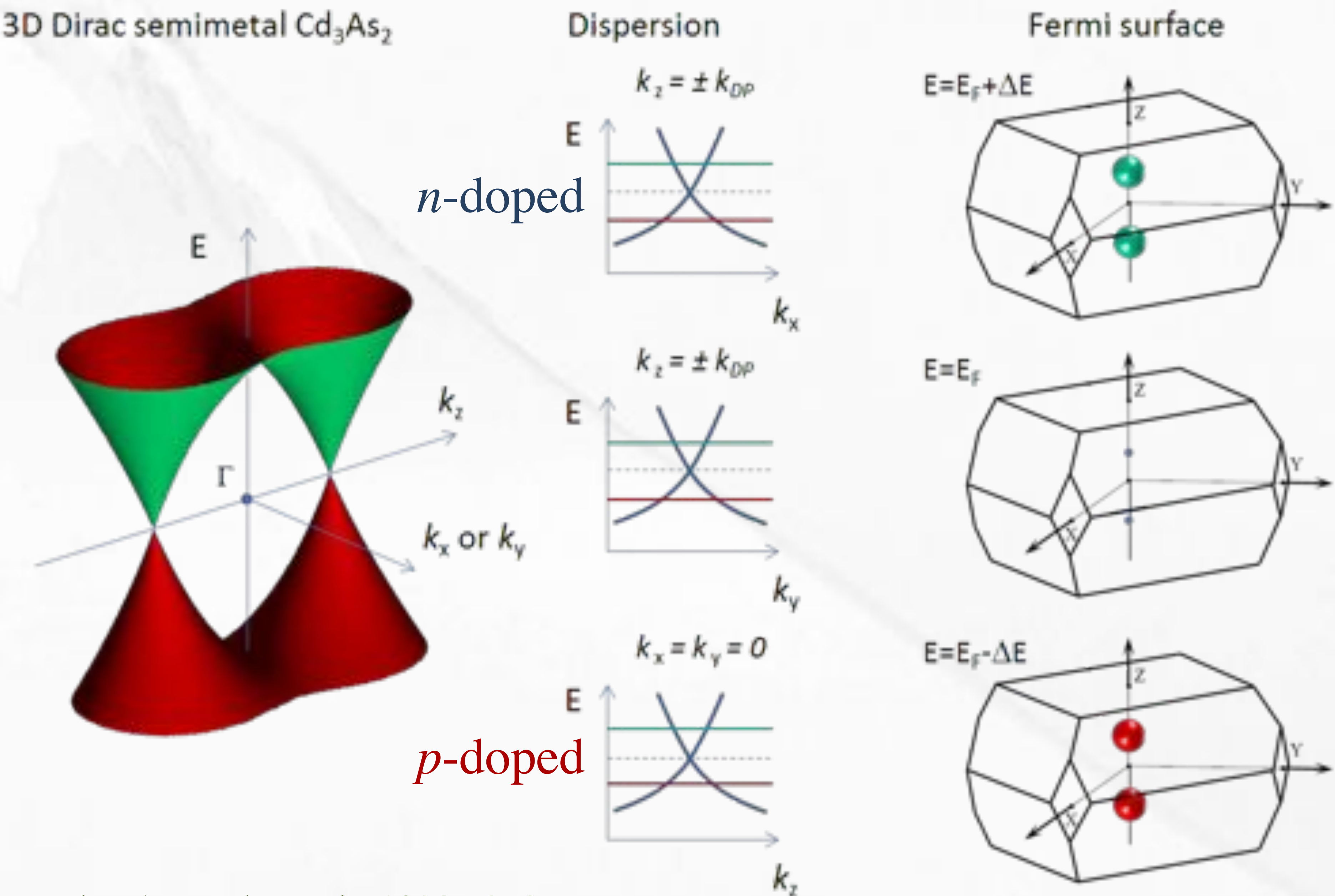
DIRAC SEMIMETALS

- Solid state materials with Dirac quasiparticles:
  – $Bi_{1-x}Sb_x$ alloy



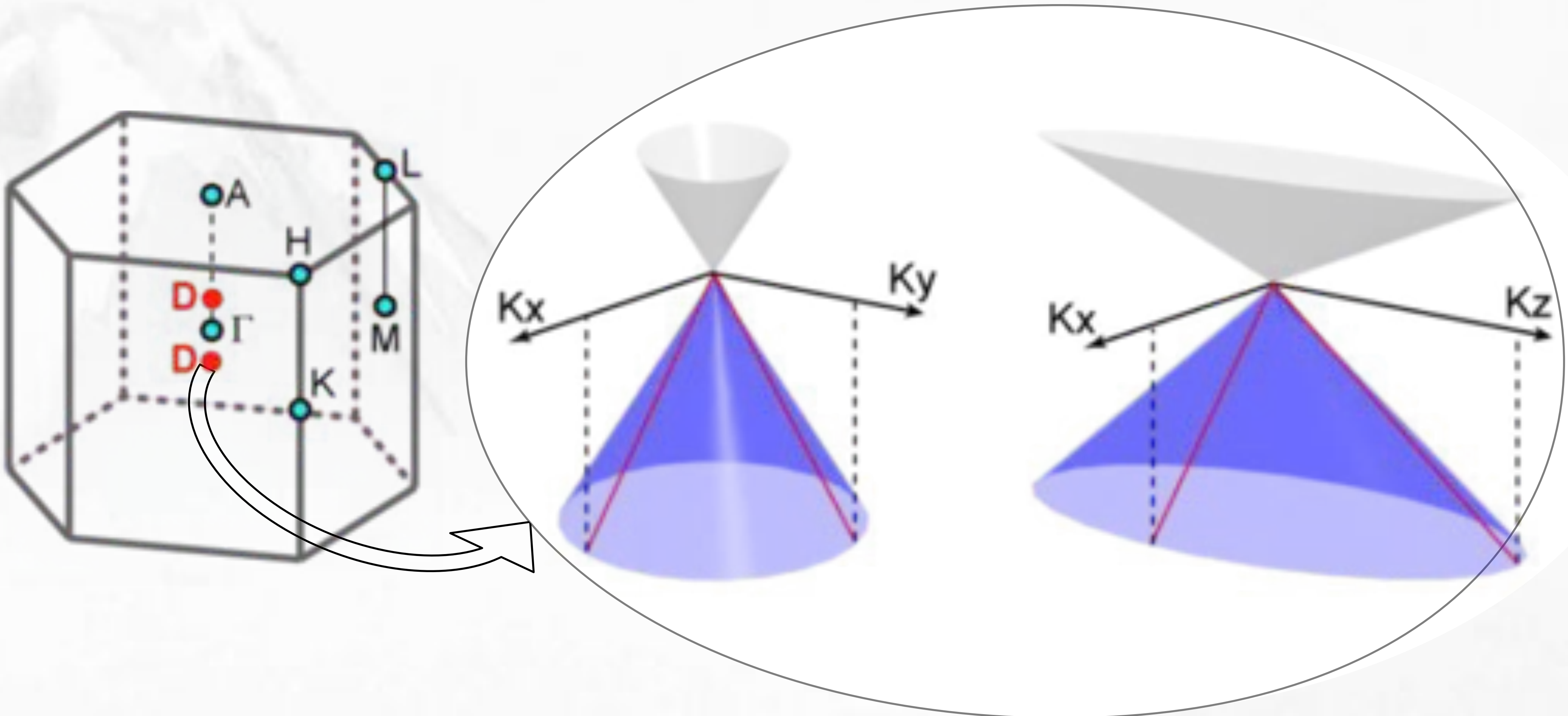
- "New" 3D Dirac materials (ARPES):
  – Na_3Bi [Z. K. Liu et al., arXiv:1310.0391]
  – Cd_3As_2 [M. Neupane et al., arXiv:1309.7892] [S. Borisenko et al., arXiv:13

Cadmium arsenide



[S. Borisenko et al., arXiv:1309.7978]

POTASSIUM BISMUTHIDE



In the vicinity of 3D Dirac points:

$$E = v_x k_x + v_y k_y + v_z k_z$$

[Z. K. Liu et al., arXiv:1310.0391]

DIRAC INTO WEYL SEMIMETAL

- Hamiltonian of a Dirac semimetal

$$H^{(D)} = \int d^3 r \overline{\psi} \left[-i v_F \left(\vec{\gamma} \cdot \vec{\nabla} \right) - \mu_0 \gamma^0 \right] \psi + H_{\text{int}}$$

cf. Weyl semimetal

"chiral shift"

$$H^{(W)} = \int d^3 r \overline{\psi} \left[-i v_F \left(\vec{\gamma} \cdot \vec{\nabla} \right) - \left(\vec{b} \cdot \vec{\gamma} \right) \gamma^5 - \mu_0 \gamma^0 \right] \psi + H_{\text{int}}$$

- In a Dirac semimetal, a nonzero chiral shift $\vec{b}$ will be induced when $B \neq 0$, i.e.,

$$\vec{b} \propto -\frac{g}{v_F^2 c} \mu_0 e \vec{B}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B **88**, 165105 (2013)]

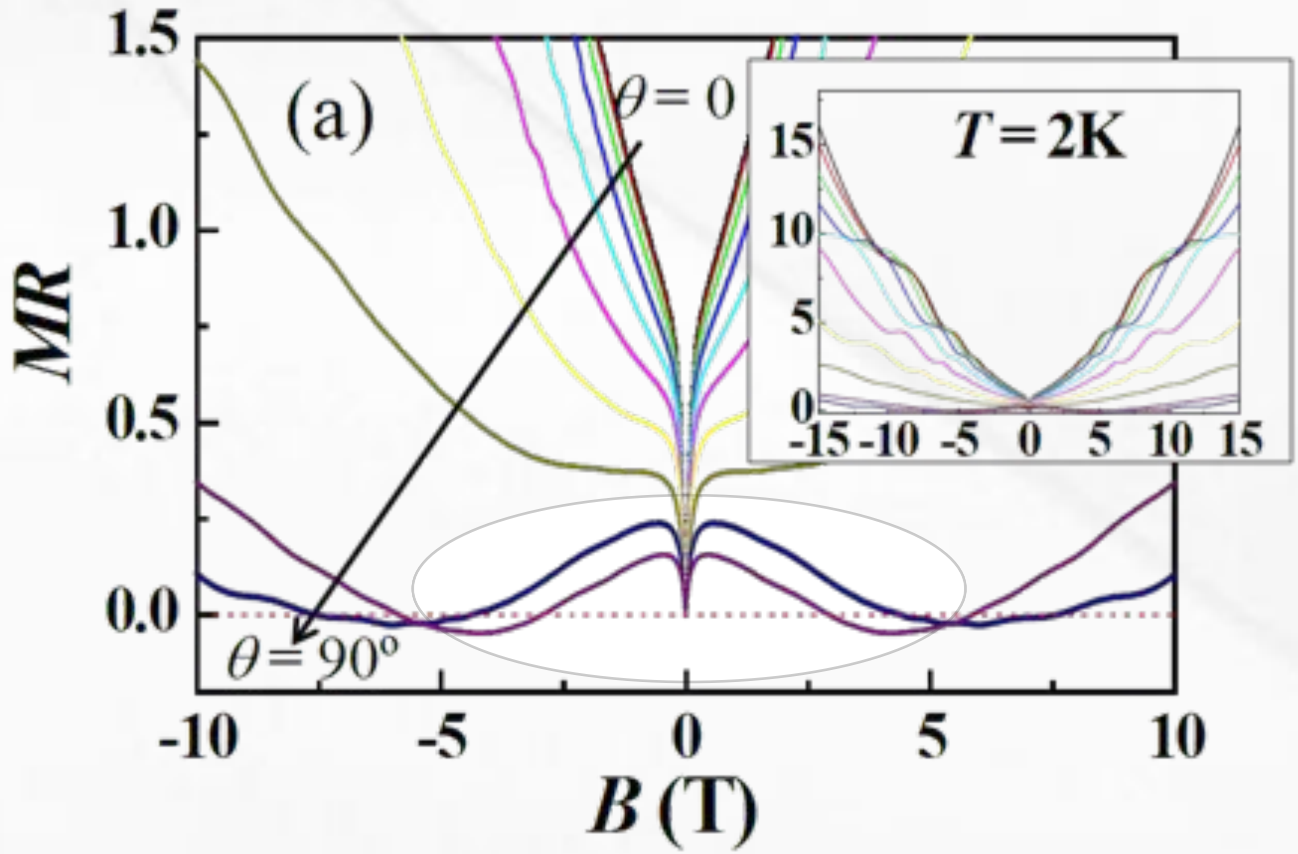
NEGATIVE MAGNETORESISTANCE

- ρ_{33} is expected to decrease with B because

  $\sigma_{33} \propto B^2$ (weak B) [Son & Spivak, Phys. Rev. B 88, 104412 (2013)]

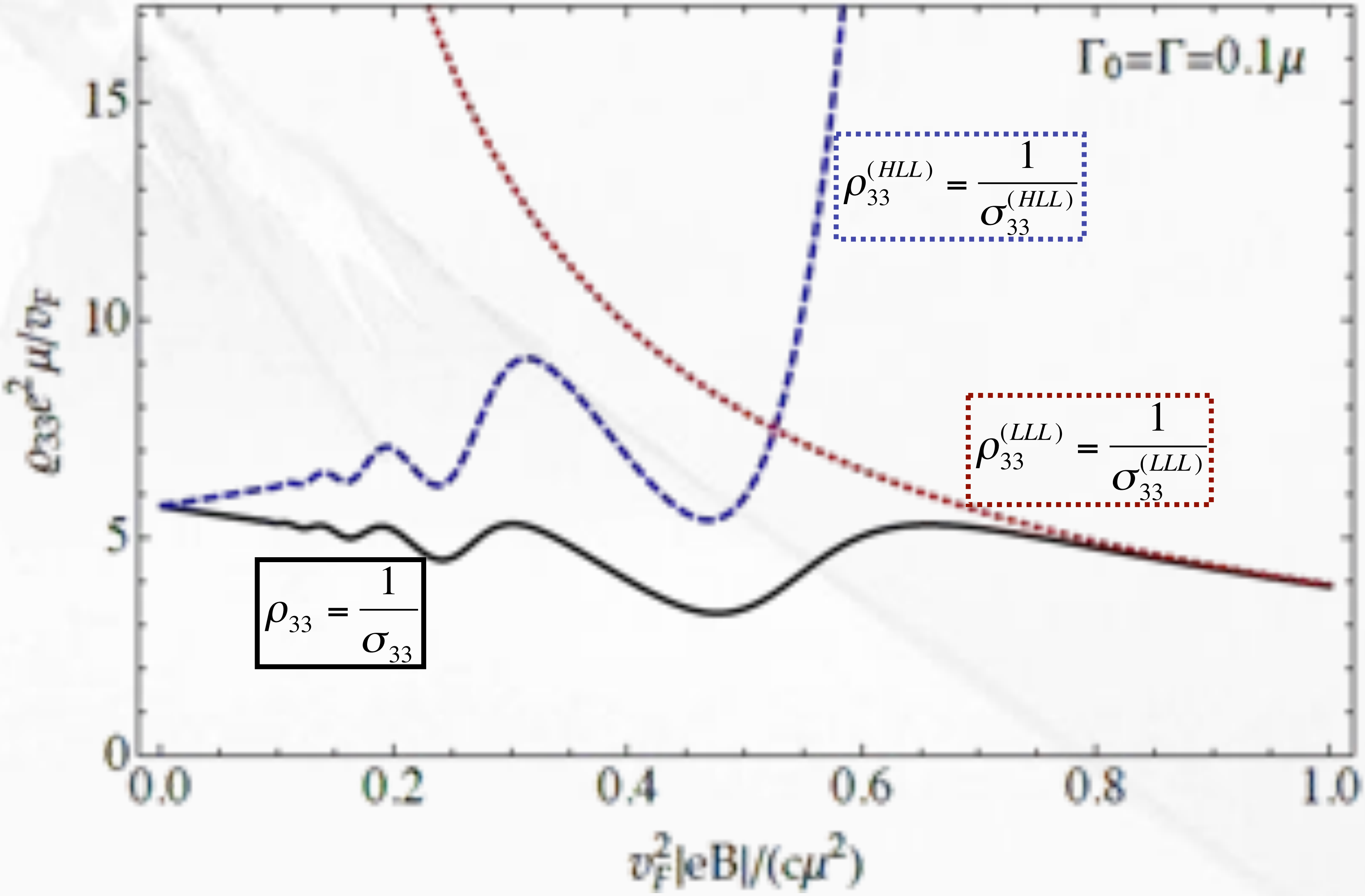
  $\sigma_{33} \propto B$ (strong B) [Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]

- Experimental confirmation? [Kim, et al., PRL 111, 246603 (2014)]



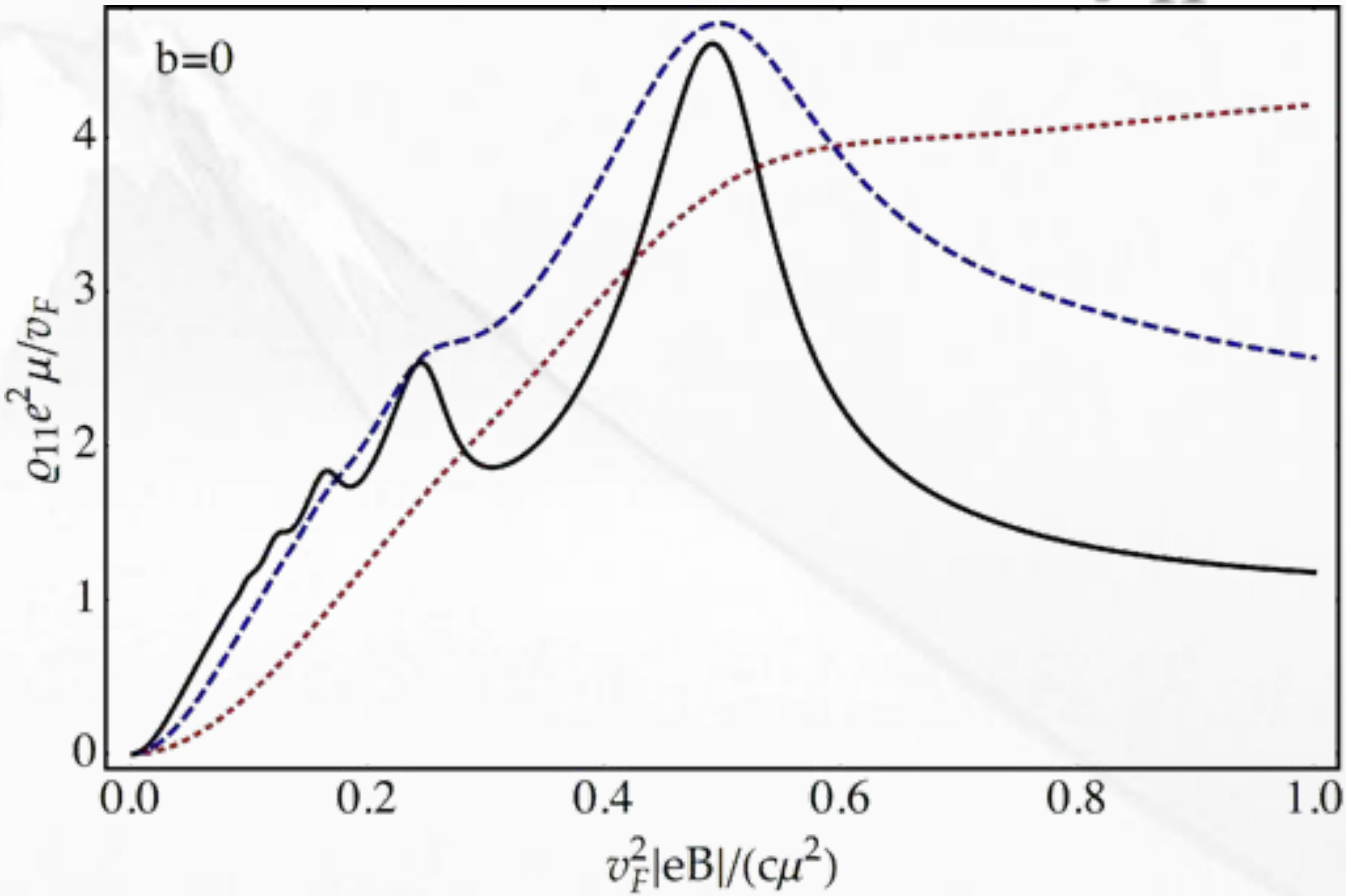
LONGITUDINAL RESISTIVITY

[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89 (2014) 085126]



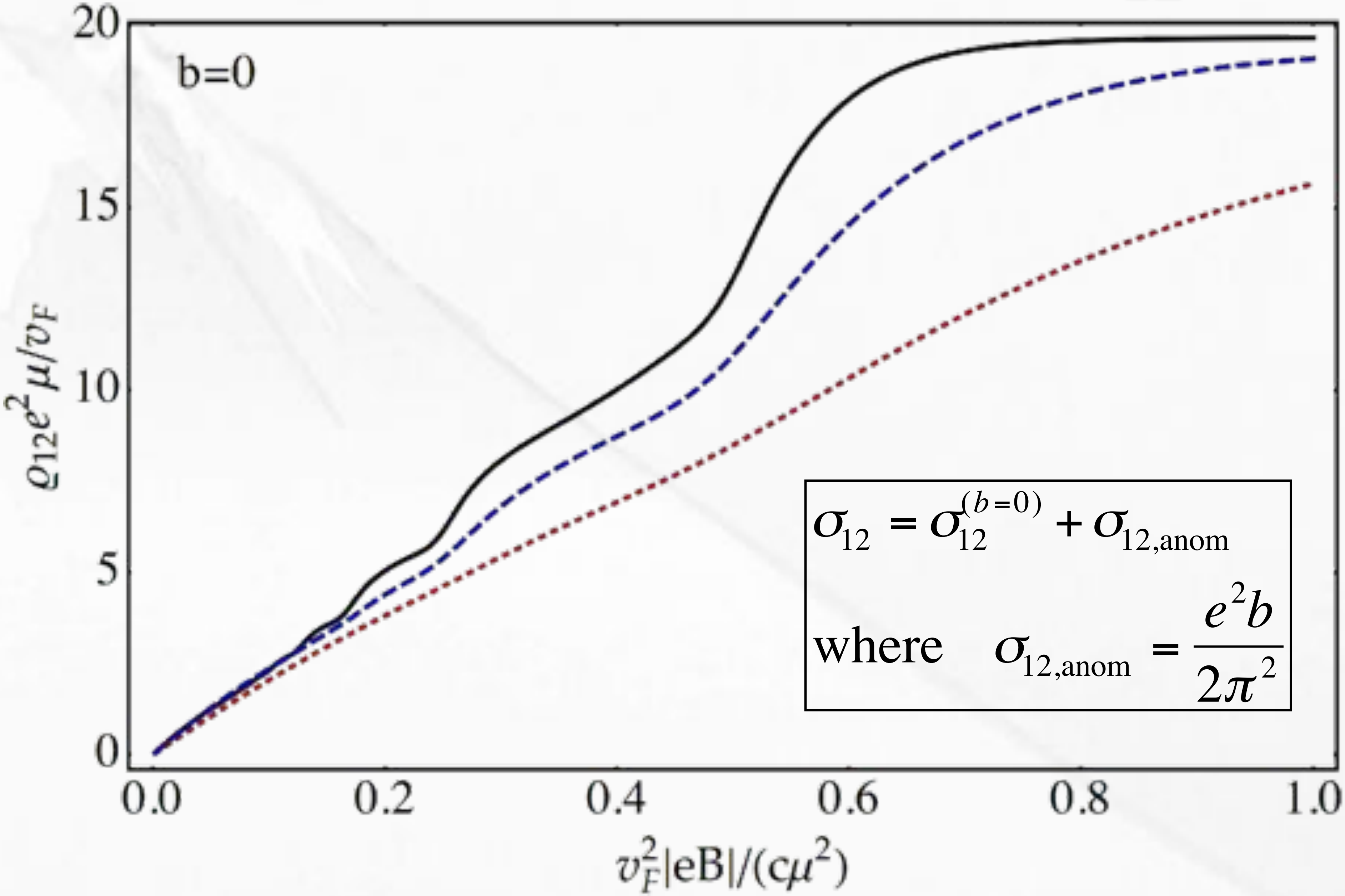
- Note: $\sigma_{33} = \sigma_{33}^{(LLL)} + \sigma_{33}^{(HLL)}$, where $\sigma_{33}^{(LLL)} = \frac{e^2 v_F |eB|}{4\pi^2 c \Gamma_0}$

Transverse resistivity ρ_{11}



[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89 (2014) 085126]

TRANSVERSE OFF-DIAGONAL ρ_{12}



$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\text{anom}}$$

where $\sigma_{12,\text{anom}} = \frac{e^2 b}{2\pi^2}$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89 (2014) 085126]

SUMMARY

- Studies of relativistic matter in magnetic field are relevant for many branches of physics
- The underlying physics is conceptually rich
- Recent developments include
  - Magnetic catalysis
  - Chiral magnetic/separation effect
  - Chiral shift
  - Chiral magnetic spiral
  - and many others