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Quantum Magnetic Phenomena: From QCD to Dirac semimetals

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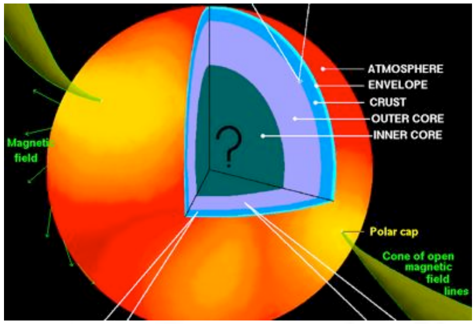
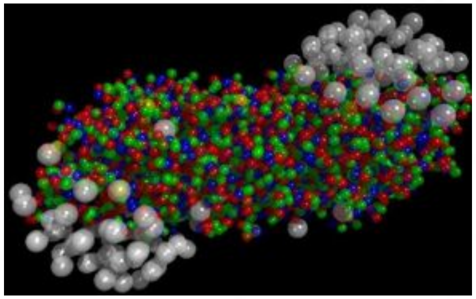
INTRODUCTION

Relativistic Matter

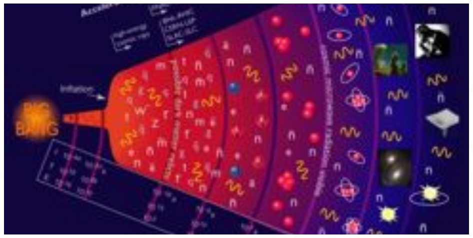
- Examples of relativistic matter
  - **Electrons, protons, quarks** inside compact stars (white dwarfs, neutron, hybrid or quark stars)



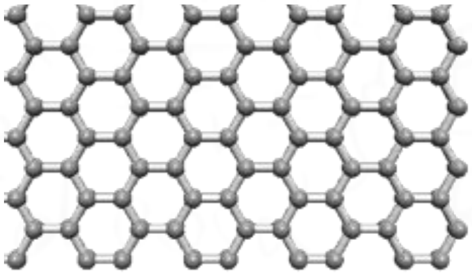
  - **Quark gluon plasma** in heavy ion collisions ($k_B T \sim 200$ MeV $\sim 10^{12}$ K)



  - **Hot matter** in the Early Universe ($k_B T \sim 100$ GeV at EW transition)



  - **Quasiparticles** in Dirac semimetals (graphene, Na_3Bi, Cd_3As_2 with zero mass Dirac fermions)



What Means "Relativistic"?

- **Relativistic matter** ($p \gg mc$)

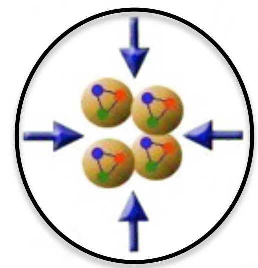
$$E = c\sqrt{p^2 + m^2c^2} \approx cp$$

compare with nonrelativistic case ($p \ll mc$)

$$E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

  – **High density** (e.g., in stars) leads to occupation of states with large momenta:

$$p \sim \hbar n^{1/3} \simeq 200 \left(\frac{n}{1\ \mathrm{fm}^3}\right)^{1/3}\ \mathrm{MeV/c}$$



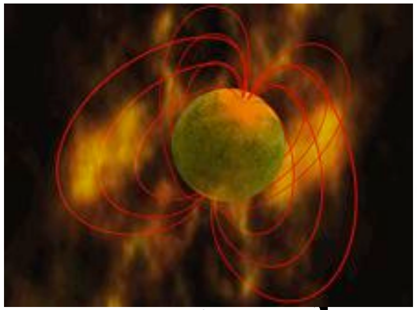
  – **High temperature** (e.g., heavy ion collisions) means energetic particles,

$$p \sim k_B T/c \simeq 200 \left(\frac{k_B T}{200\ \mathrm{MeV}}\right)\ \mathrm{MeV/c}$$

  – **Vanishing mass** (e.g., graphene) works too…

Magnetic Fields

- Strong magnetic fields exist inside *compact stars*
  - **10^{10} to 10^{15} Gauss**



- In *heavy ion collisions*, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

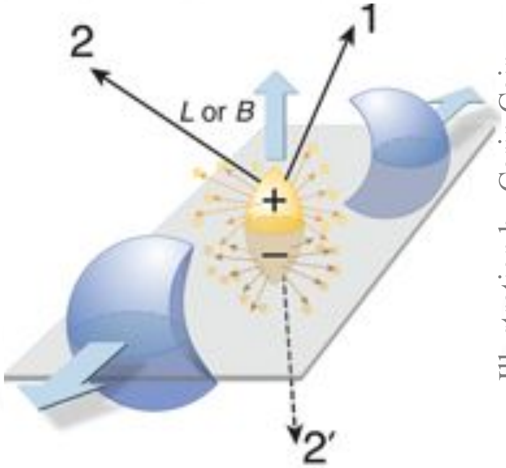
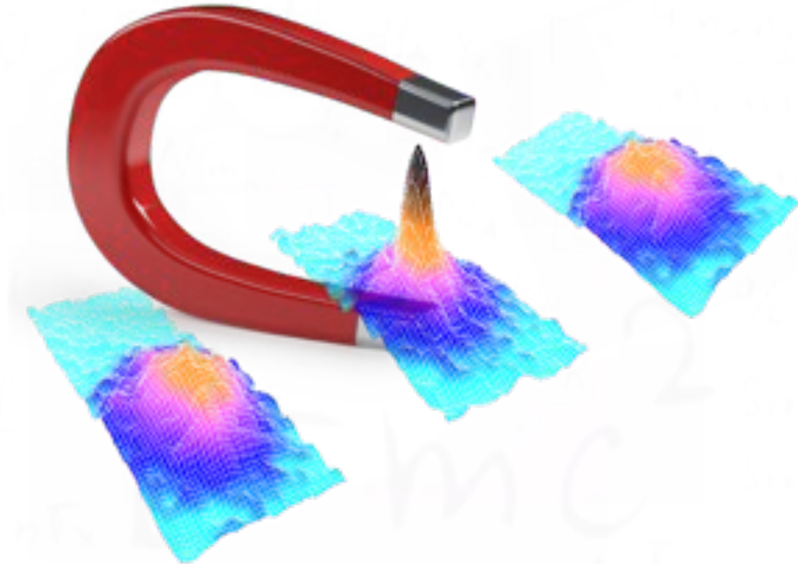


Illustration by Carin Cain

- *Early Universe*
  - **up to 10^{24} Gauss**
- *Graphene* (High Magnetic Field Laboratory)
  - **up to 5×10^5 Gauss**



MAGNETIC CATALYSIS

Review: arXiv:1207.5081

Landau levels

- Fermions in magnetic field

$$\mathcal{L} = \overline{\Psi} i \gamma^{\mu} D_{\mu} \Psi + (\text{interactions})$$

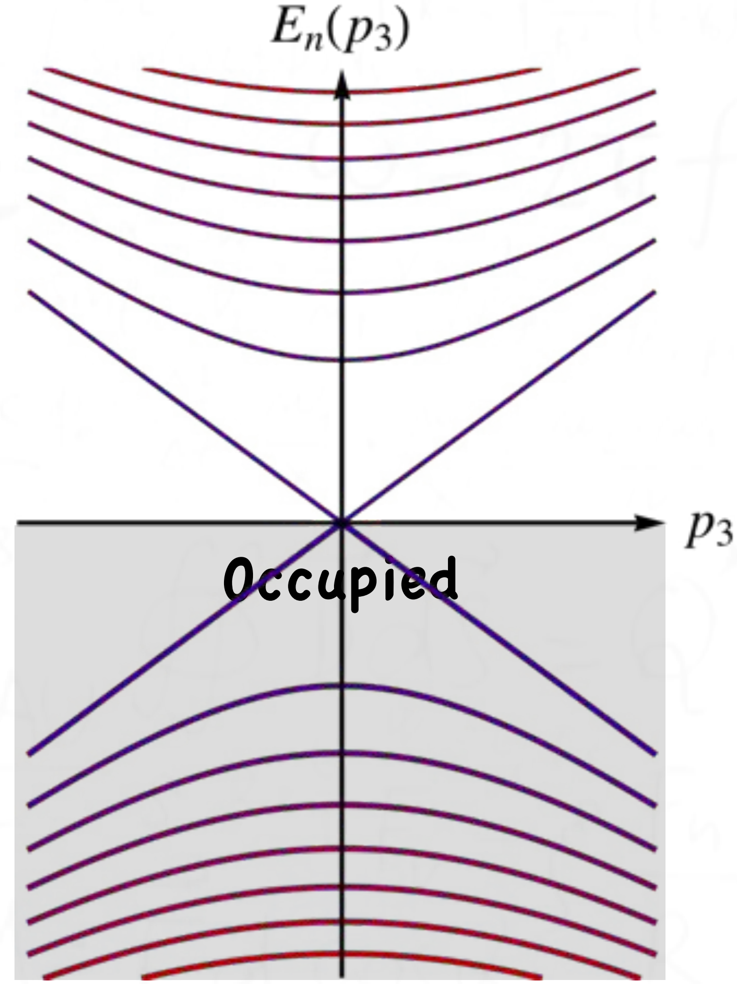
- Free energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

$s = \pm 1/2$ (spin)

where $$n = s + k + \frac{1}{2}$$

$k = 0, 1, 2, \dots$ (orbital)

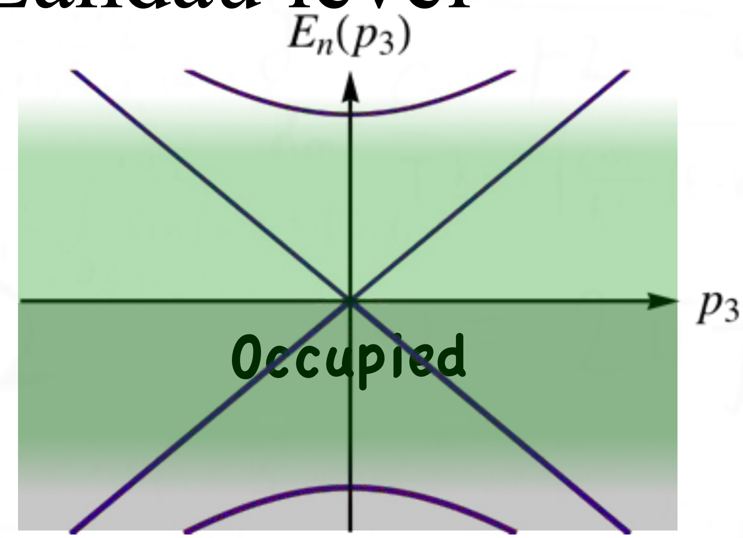


Dimensional reduction

- Low-energy is due to n=0 Landau level

$$n = 0: \qquad E_0^{(3+1)}(p_3) = \pm p_3$$

$$\left(k = 0, s = -\frac{1}{2}\right)$$



- This is (1+1)D spectrum!

- Propagator also looks like in (1+1)D:

$$S(p_\parallel) \approx i\, e^{-p_\perp^2 \ell^2} \frac{\hat{p}_\parallel + m}{\hat{p}_\parallel + m} \underbrace{\left(1 - i\gamma^1\gamma^2\right)}_{s=-1/2 \text{ spin projector}}, \text{ where } \hat{p}_\parallel = p_0\gamma^0 - p_3\gamma^3$$

Magnetic Catalysis (clues)

- Low-energy regime is dimensionally reduced

$$D \Rightarrow D-2$$

- Density of states at $E = 0$

$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB|N_f}{4\pi^2}$$

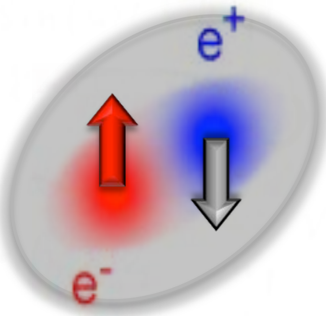
- Attractive interaction → gap in the spectrum

(This may remind superconductivity…)

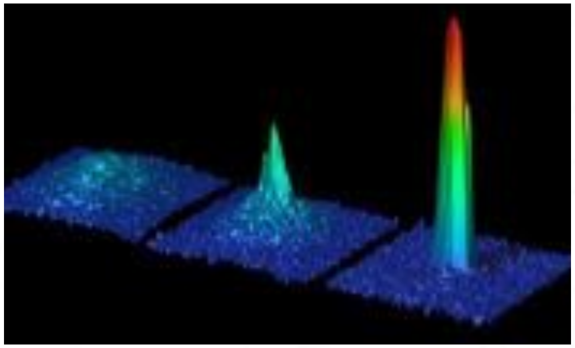
[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

Magnetic Catalysis (physics)

- n=0: particles & anti-particles
- Bound states are energetically favorable (an energy gain of E_b per pair)



- Bound states are bosons
- Bosons can (and will) occupy same zero momentum quantum state



- Bose condensate forms
- Symmetry breaking → energy (mass) gap

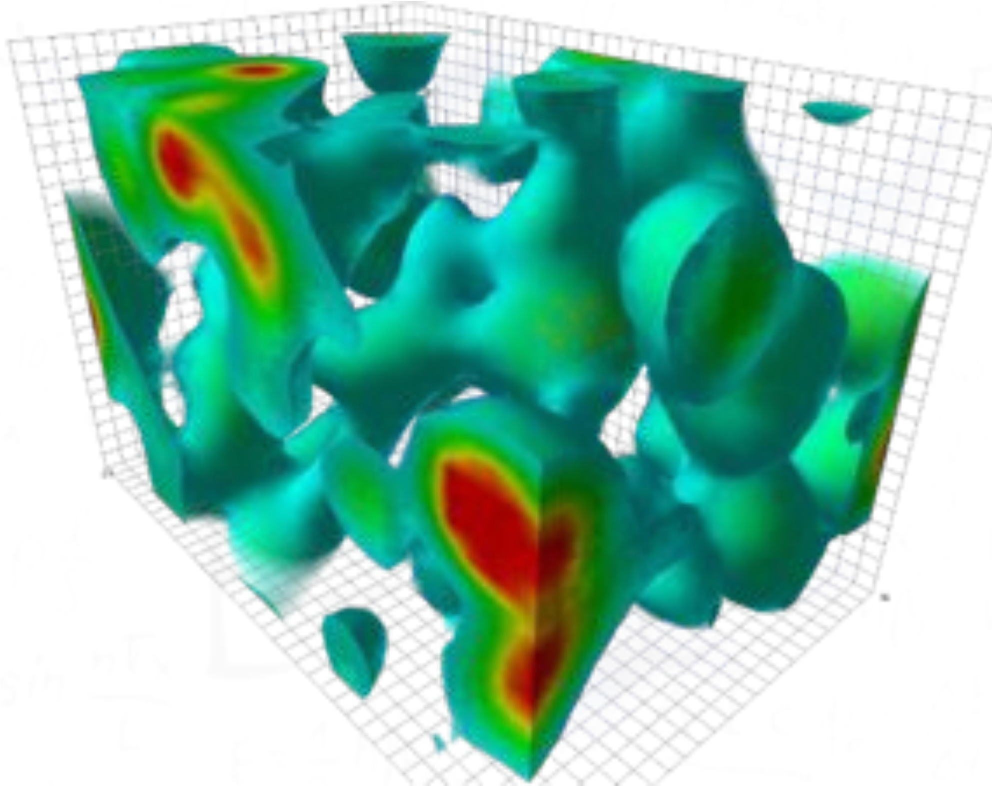
[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

Dynamical Mass

- While $m_0=0$ originally, a nonzero "dynamical" mass m_{dyn} is generated

$$m_{dyn}^{(2D)} \propto \sqrt{\alpha}\sqrt{|eB|}\,, \quad \text{and} \quad m_{dyn}^{(3D)} \propto \sqrt{|eB|}e^{-C/\alpha}$$

- This happens even at the *weakest* interaction ("catalysis")
- Dimensional *reduction* and *finite* density of states at E=0 play the key role
- The phenomenon is largely *insensitive* to model details



Credit: Centre for the Subatomic Structure of Matter, University of Adelaide

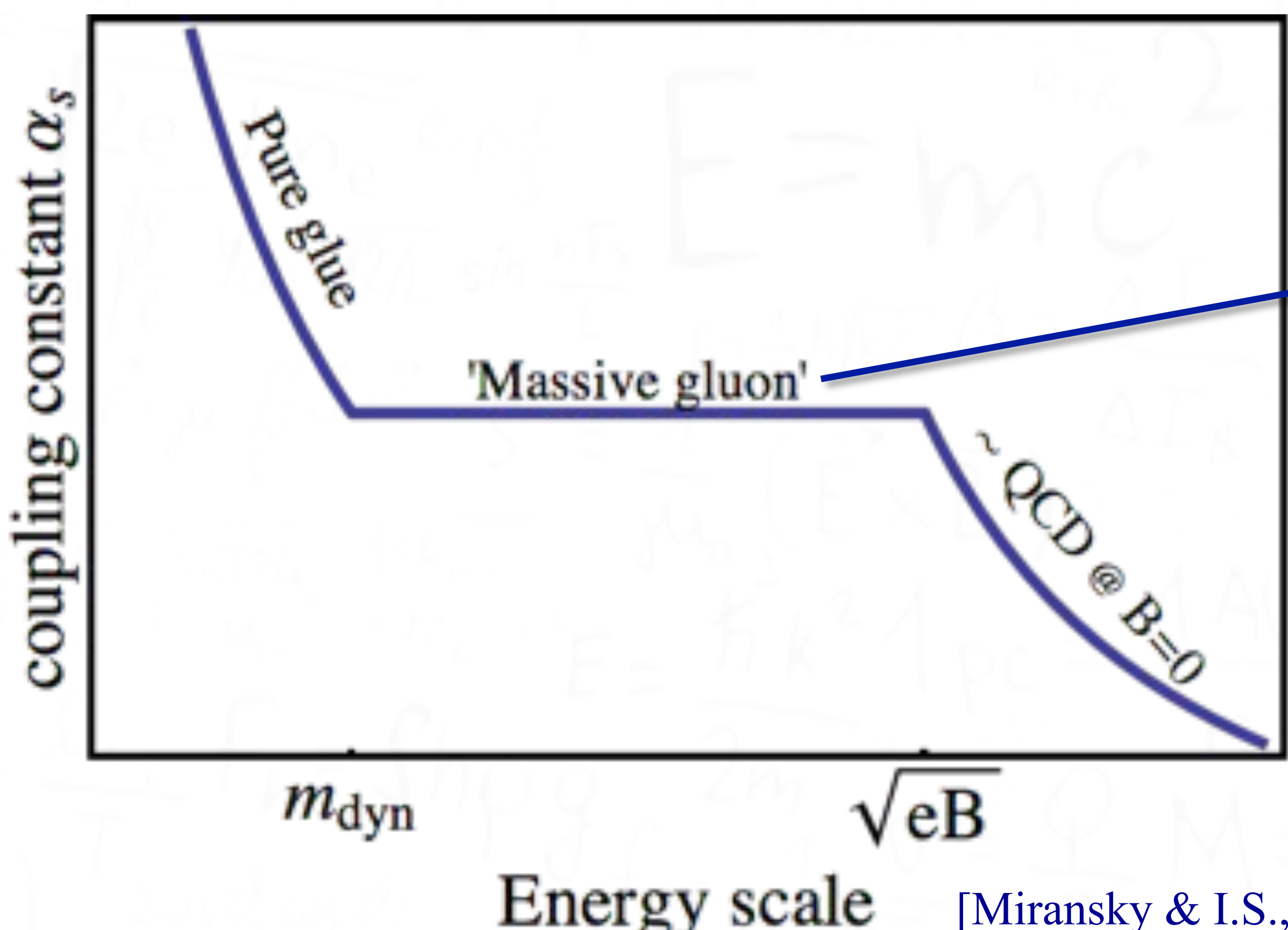
MAGNETIC CATALYSIS IN QCD

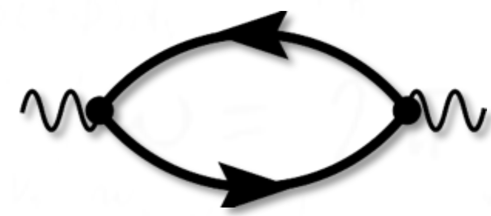
QCD in a magnetic field

- Weak coupling, (semi-)perturbative regime:

$$\sqrt{|eB|} >> \Lambda_{\mathrm{QCD}}$$

- Running QCD coupling





$$m_f^2 << \left|k_{\parallel}^2\right| << \sqrt{|eB|}$$

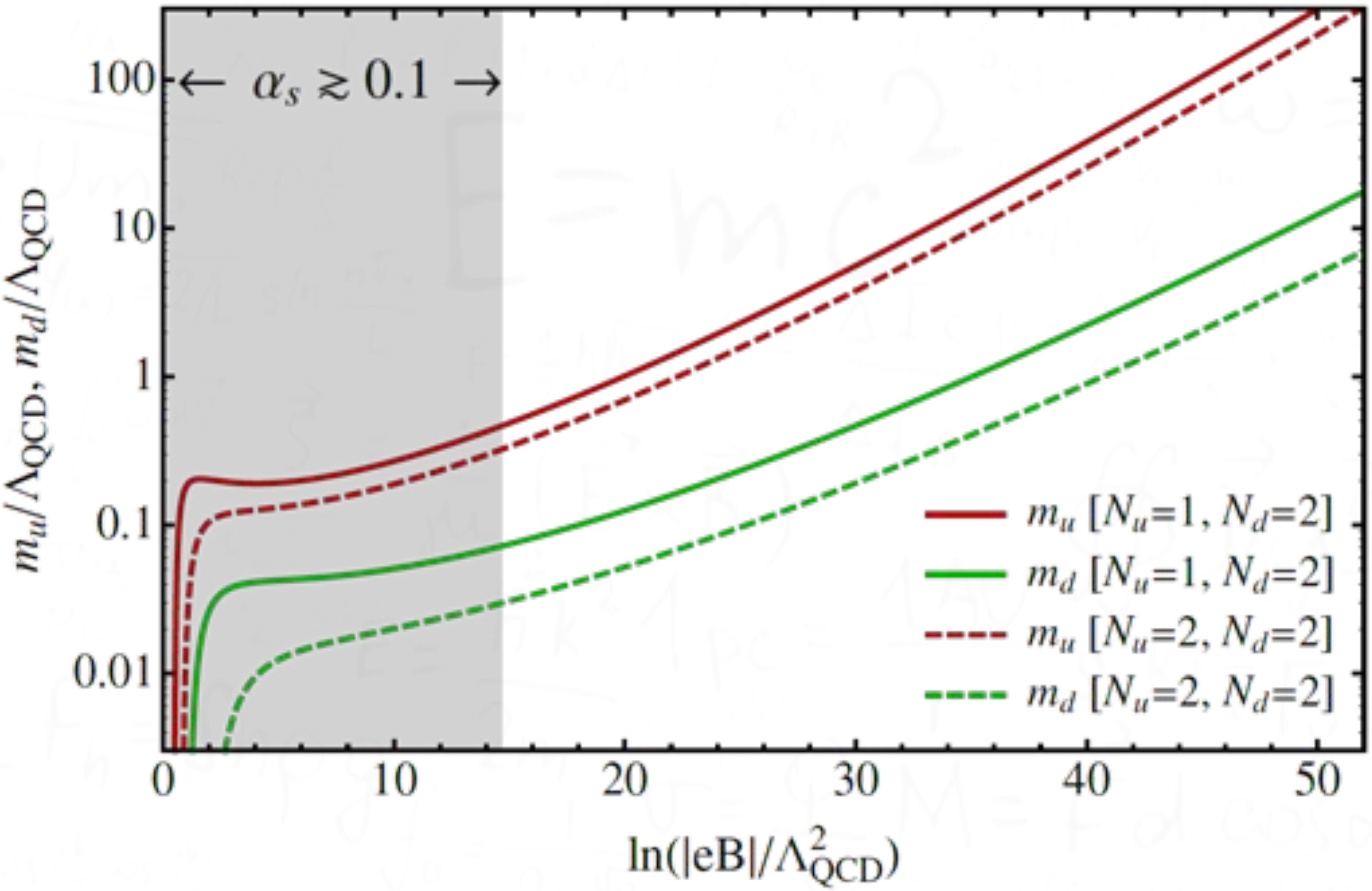
$$M_g^2 \approx \frac{\alpha_s}{\pi} \sum_f \left|e_f B\right|$$

[Miransky & I.S., Phys. Rev. D 66 (2002) 045006]

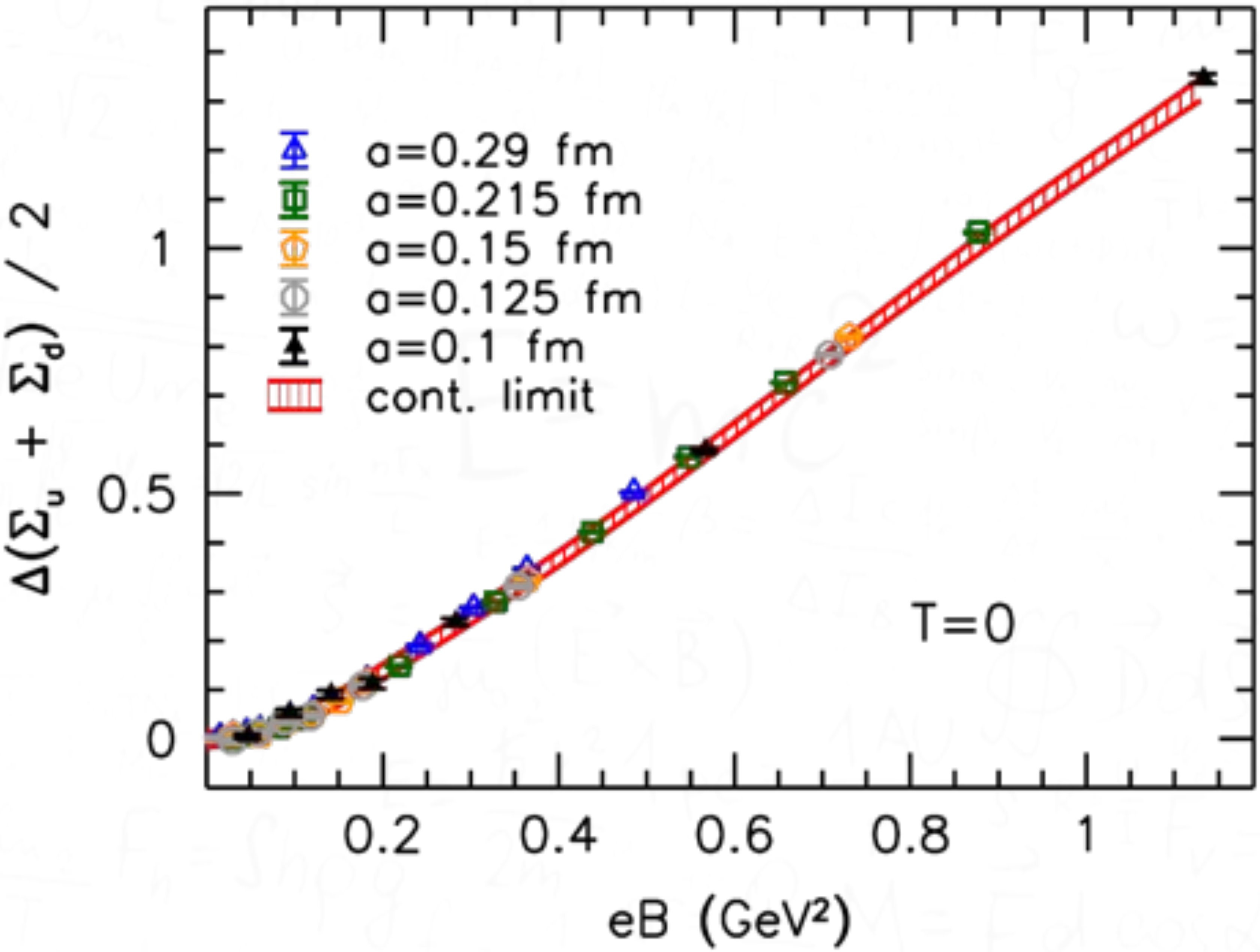
Dynamical quark masses

- Approximate dynamical mass:

$$m_f^2 \propto |e_f B| \alpha_s^{3/2} \exp\left(-\frac{4\pi N_c}{\alpha_s (N_c^2 - 1)\ln\left(C/\alpha_s\right)}\right)$$

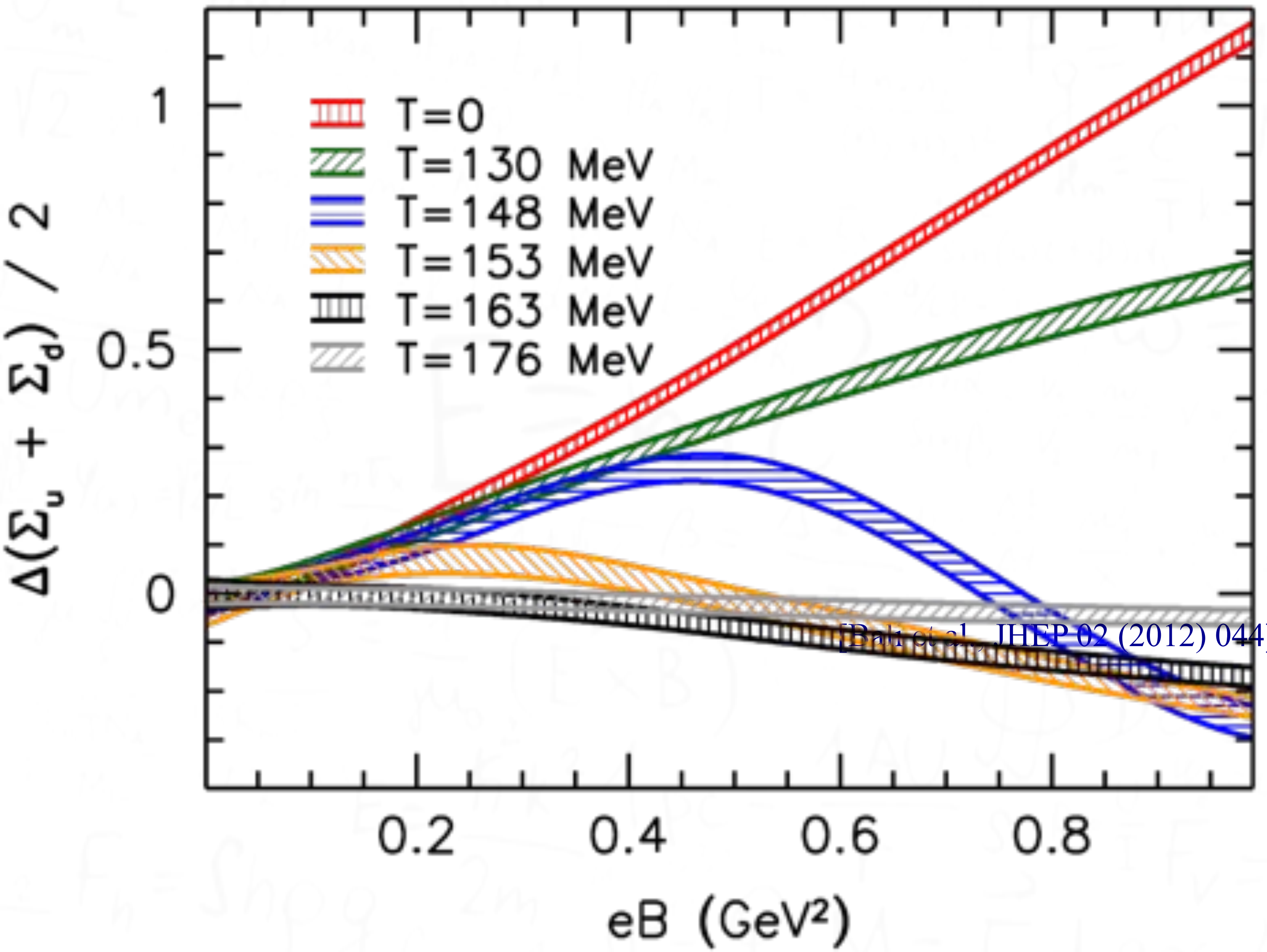


Catalysis at T=0 (lattice)



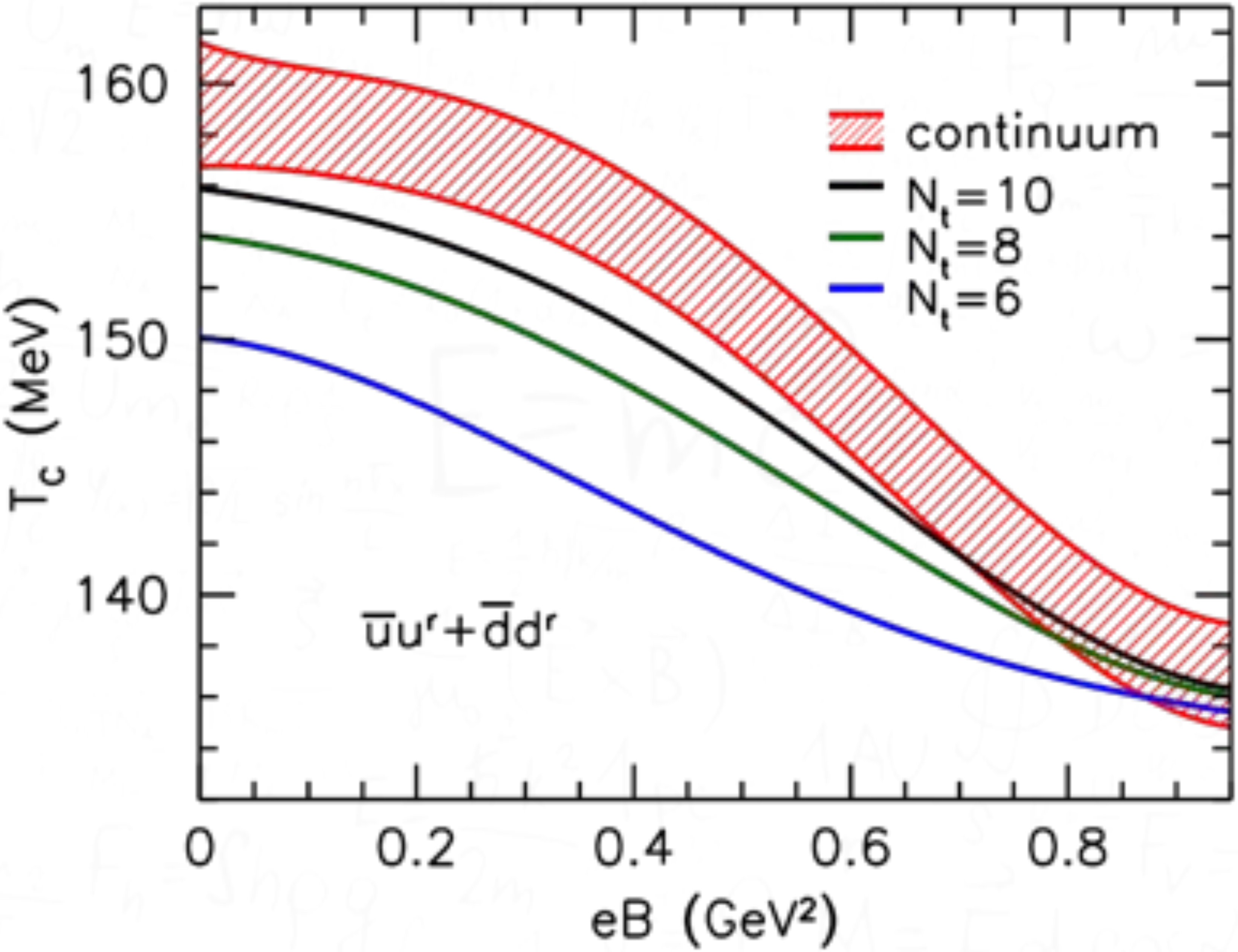
[Bali et al., Phys. Rev. D86, 071502 (2012)]

(Inverse) Catalysis at T≠0



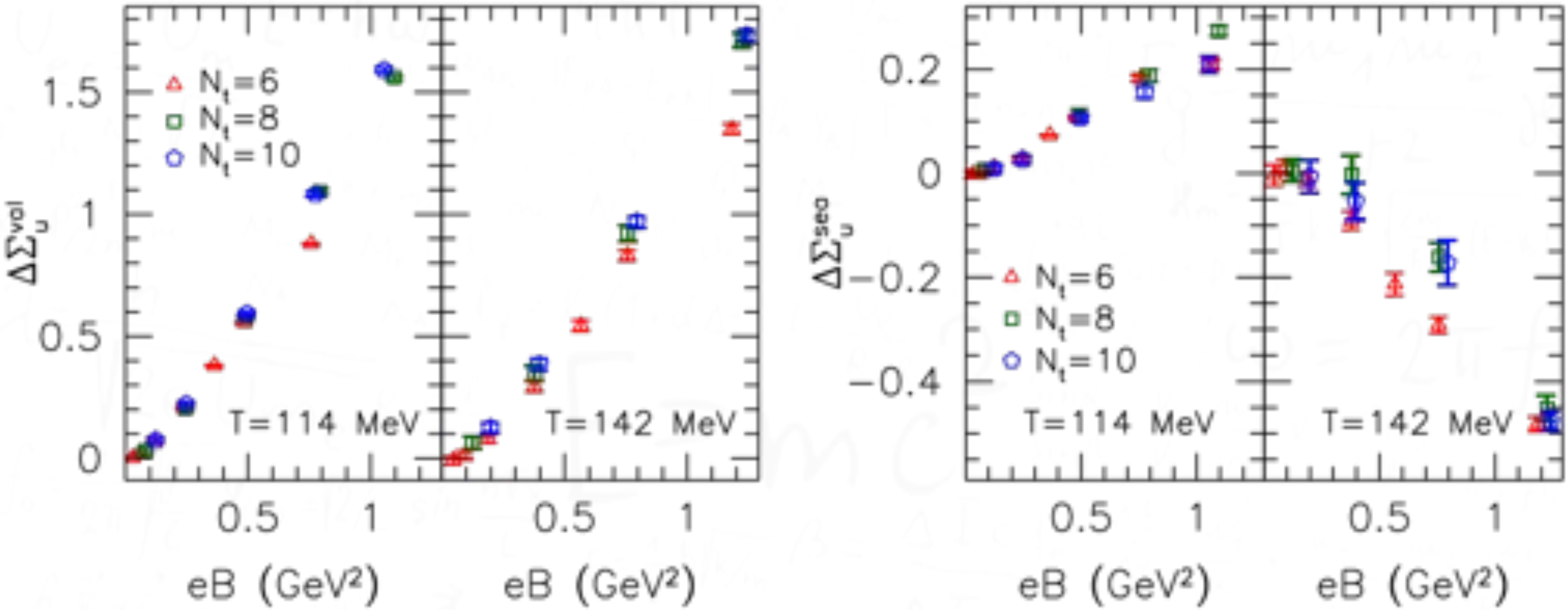
[Bali et al., Phys. Rev. D86, 071502 (2012)]

T_c vs. B (inverse catalysis)



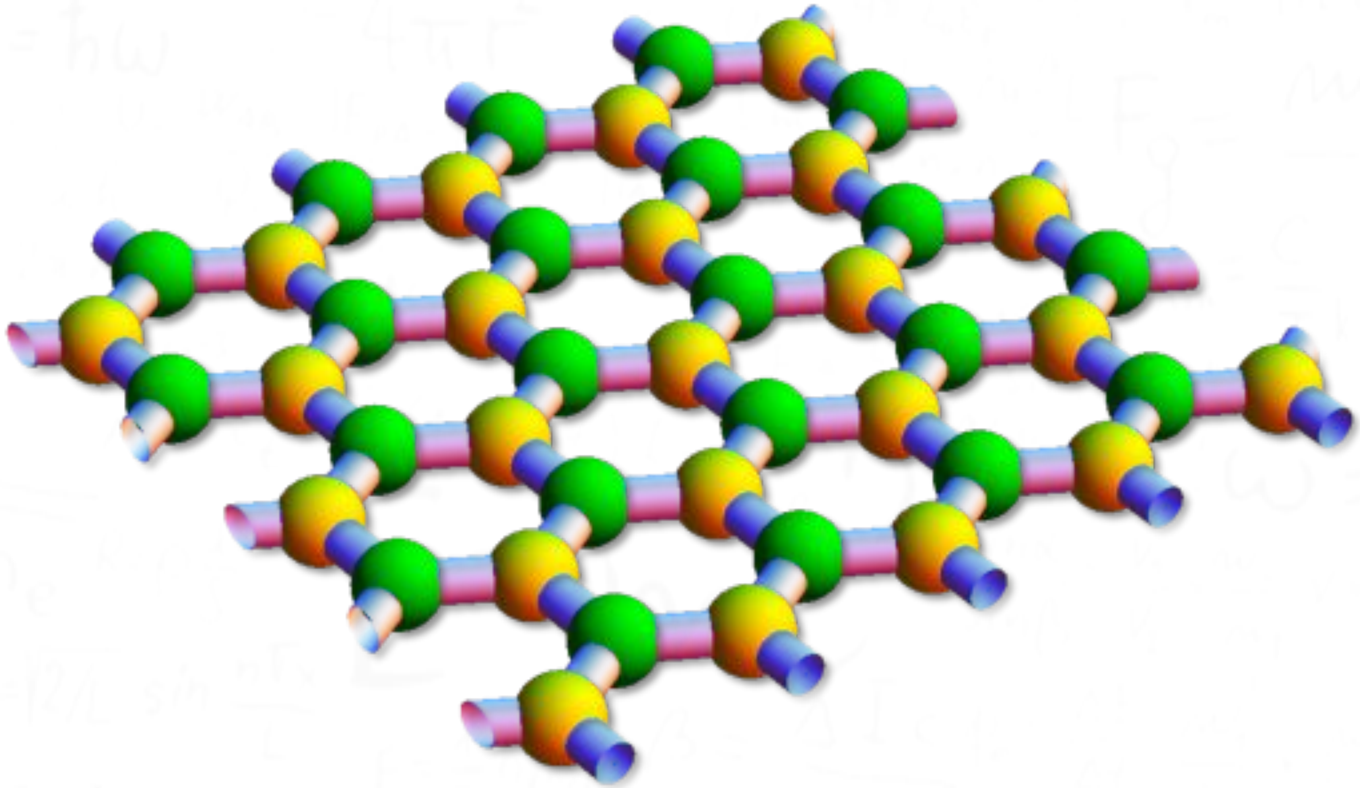
[Bali et al., JHEP 02, 044 (29012)]

Valence vs. sea



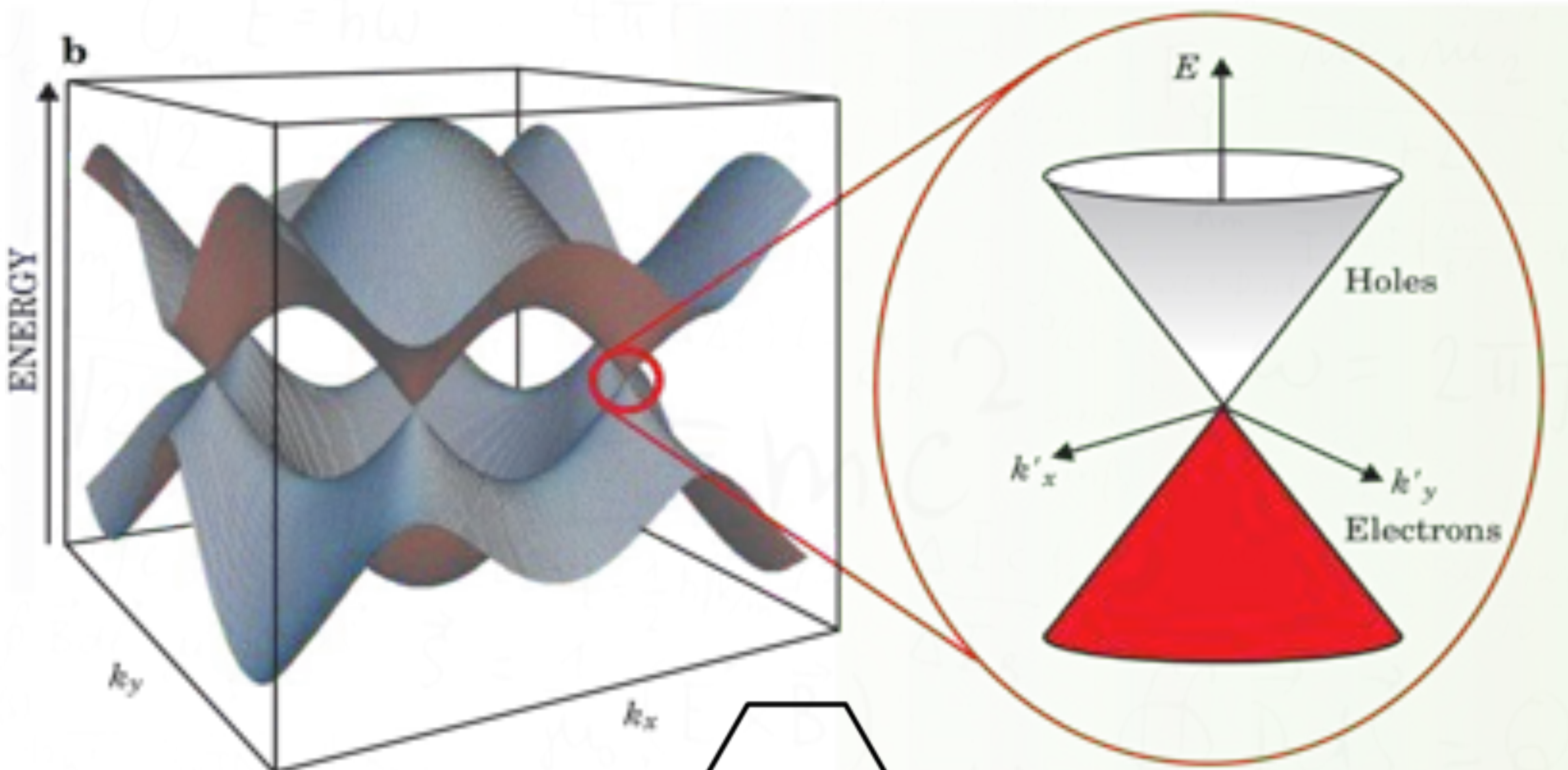
[Bruckmann, G. Endrodi, T. G. Kovacs, arXiv:1303.3972]

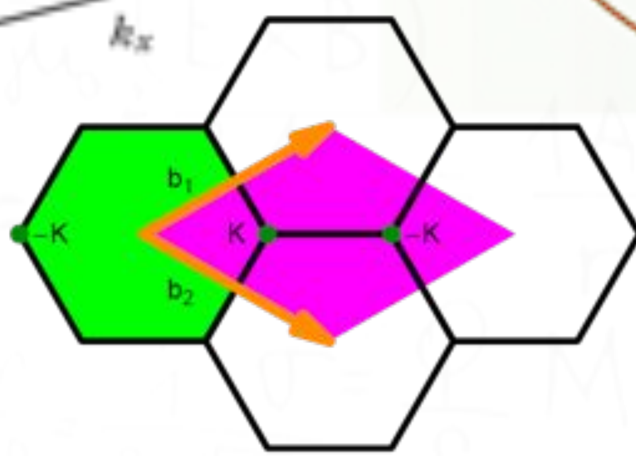
- Hints of gluon screening (?)
- See also [Ilgenfritz et al. Phys. Rev. D 89, 054512 (2014)]



MAGNETIC CATALYSIS IN GRAPHENE

Dirac Fermions in graphene





Low-energy theory

- Low energy quasiparticles are **massless** Dirac fermions $\left(v_F = c/300\right)$

- Spinor:

$$\Psi_s = \begin{pmatrix} \psi_{KAs} \\ \psi_{KBs} \\ \psi_{K'Bs} \\ \psi_{K'As} \end{pmatrix}$$

- Low-energy model with U(4) global symmetry:

$$H_0 = v_F \int d^2 r \, \overline{\Psi}_s \left(\gamma^1 \pi_x + \gamma^2 \pi_y \right) \Psi_s$$

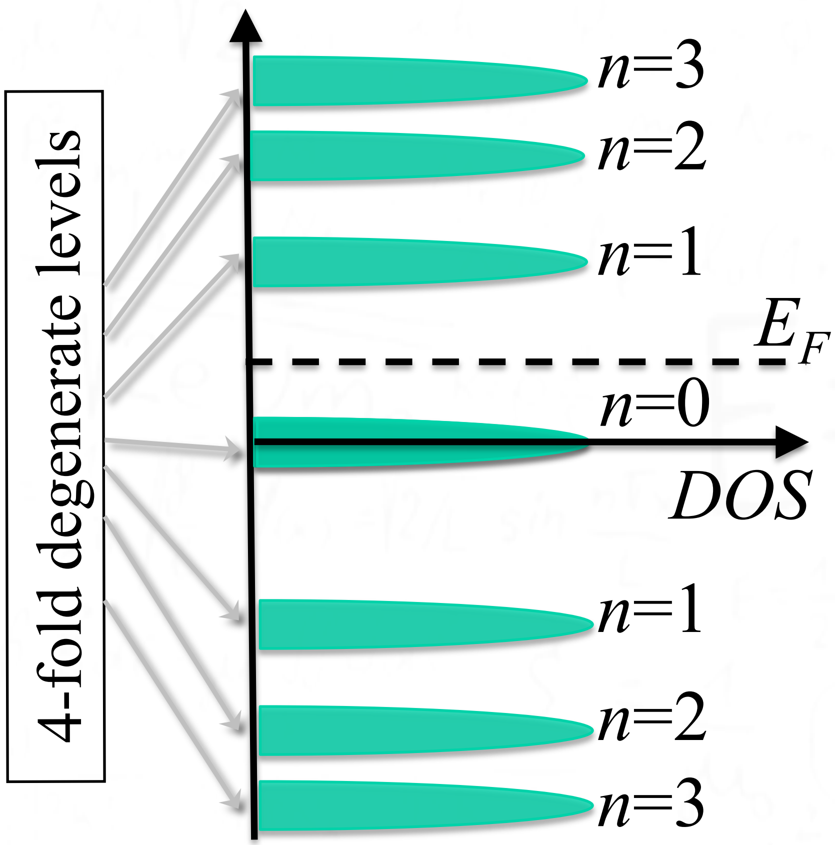
[Wallace, Phys. Rev. **71,** 622 (1947)]

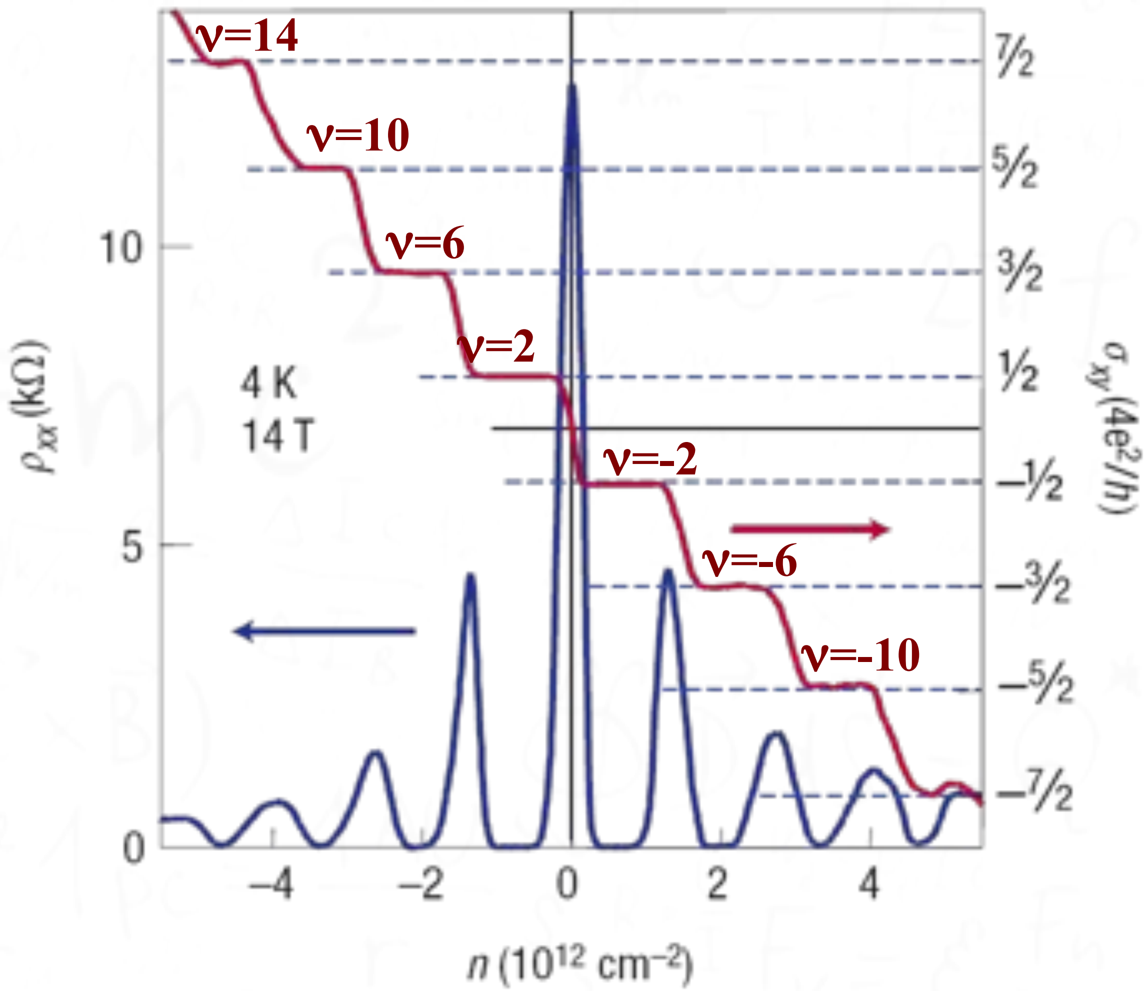
[Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)]

Quantum Hall Effect

$$E_n = \pm\sqrt{2\hbar v_F^2 n|eB|}$$

$$\sigma_{xy} = \nu \frac{e^2}{h} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$$





[Gusynin, Sharapov, Phys. Rev. Lett. **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, Phys. Rev. B **73**, 125411 (2006)]
[Novoselov et al., Nature **438**, 197 (2005)], [Zhang et al., Nature **438**, 201 (2005)]

Anomalous QHE

- New plateaus at

  ν=0

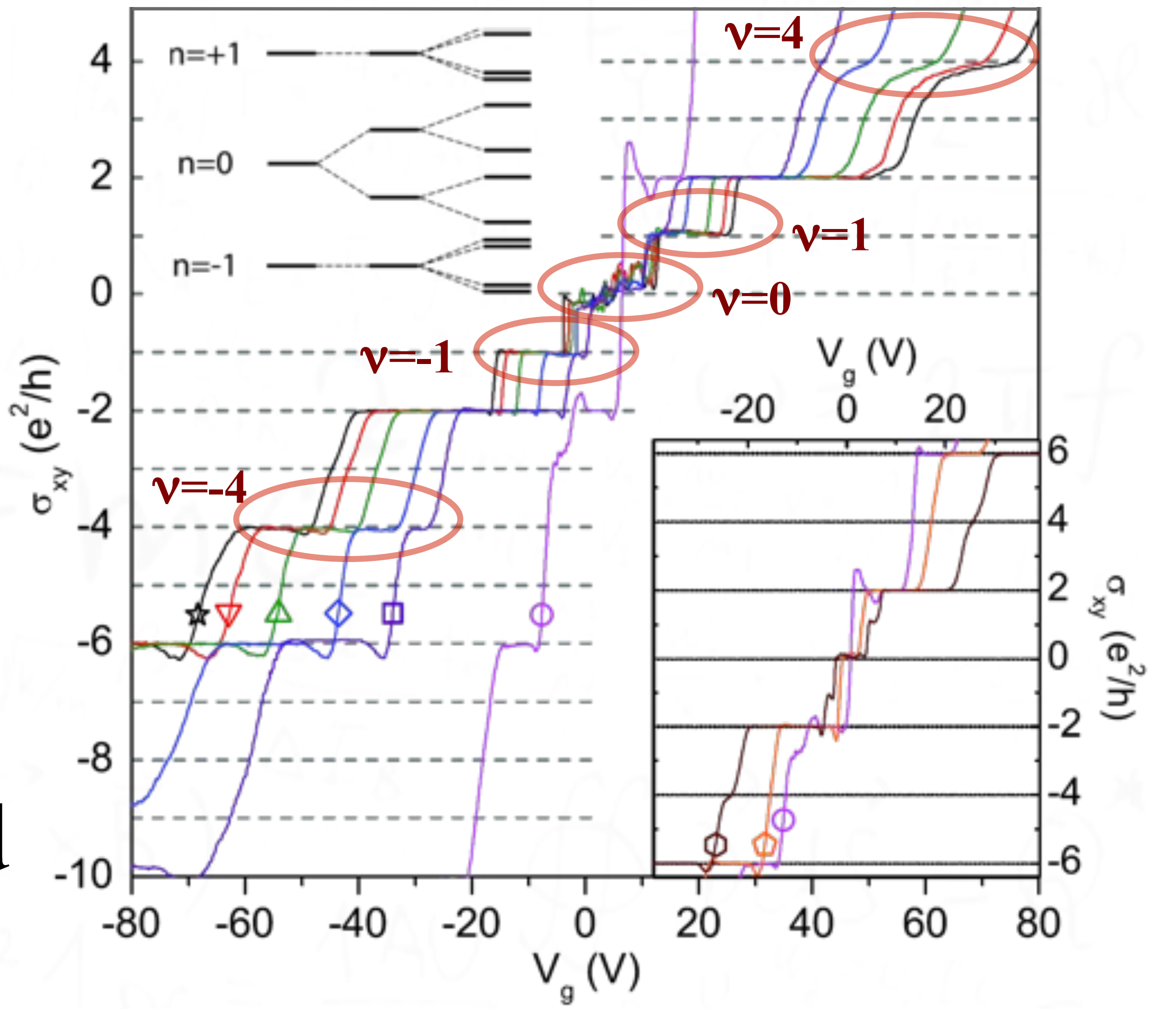
  ν=±1

  ν=±3

  ν=±4

- Some Landau level degeneracy is lifted

Zhang et al., PRL **96,** 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]
[Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)]
[Checkelsky et al., Phys. Rev. Lett. **100**, 206801 (2008)]
[Xu Du et al., Nature **462**, 192 (2009)]

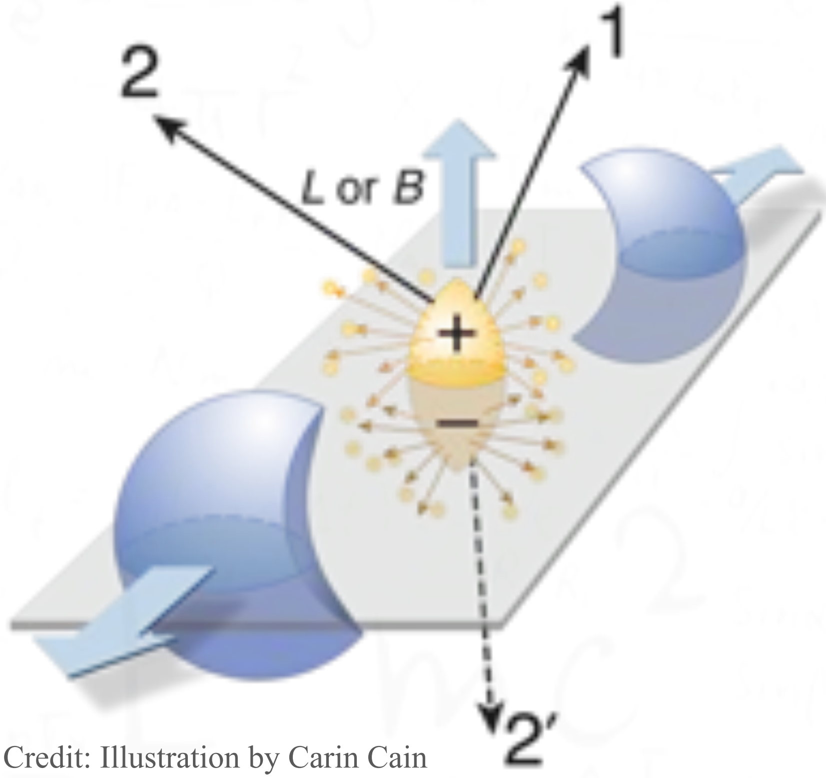
Magnetic Catalysis in Graphene

- Charge carriers are massless Dirac fermions
- Spectrum in magnetic field:

$$E_n = \pm\sqrt{2\hbar v_F^2 n|eB|}$$

- Degenerate E=0 level with particles & holes
- Electron-hole (excitonic) pairing occurs
- $m_{dyn} \neq 0$ is generated
- In qualitative agreement with experiment

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **66** (2002) 045108]

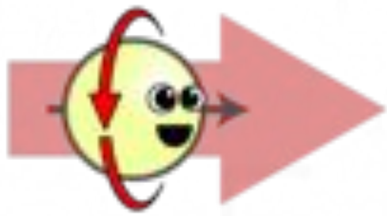


Credit: Illustration by Carin Cain

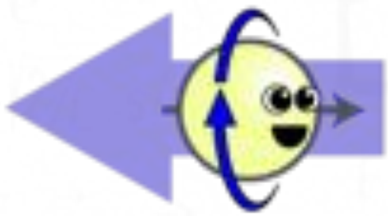
NONZERO DENSITY: CHIRAL SHIFT

Helicity/Chirality

- Helicities of massless (or ultra-relativistic) particles are (approximately) conserved



Righ-handed



Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)

- The symmetry is anomalous at quantum level

"Continuity" equation

- Continuity equation for the chiral charge

$$\frac{\partial \rho_5}{\partial t} - \vec{\nabla} \cdot \vec{j}_5 = -\frac{e}{2\pi^2} \left(\vec{B} \cdot \vec{E} \right)$$

- Among its consequences are the relations:

$$\left\langle \vec{j}_5 \right\rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu \qquad \left\langle \vec{j} \right\rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

- These are key relations of the *chiral magnetic effect*

[Kharzeev, McLerran, Warringa, Nucl. Phys. A 803, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]

Beyond CSE/CME

- Any radiative corrections to CSE?

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu + \ldots$$

[Metlitski & Zhitnitsky, Phys Rev D 72, 045011 (2005)]
[Newman & Son, Phys. Rev. D 73 (2006) 045006]

- Is there a dynamical parameter Δ (“chiral shift”) associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \Delta \overline{\psi}\gamma^3\gamma^5\psi$$

[Δ=0 is not protected by any symmetry]

- Yes! Chiral shift is dynamically generated

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83 (2011) 085003]

Self-energy at B≠0

$$\overline{\Sigma}^{(1)}(p) = -4i\pi \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \, \overline{S}^{(1)}(k) \gamma^\nu \, D_{\mu\nu}(k-p)$$

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta + \gamma^0\gamma^5\mu_5(p)$$

where

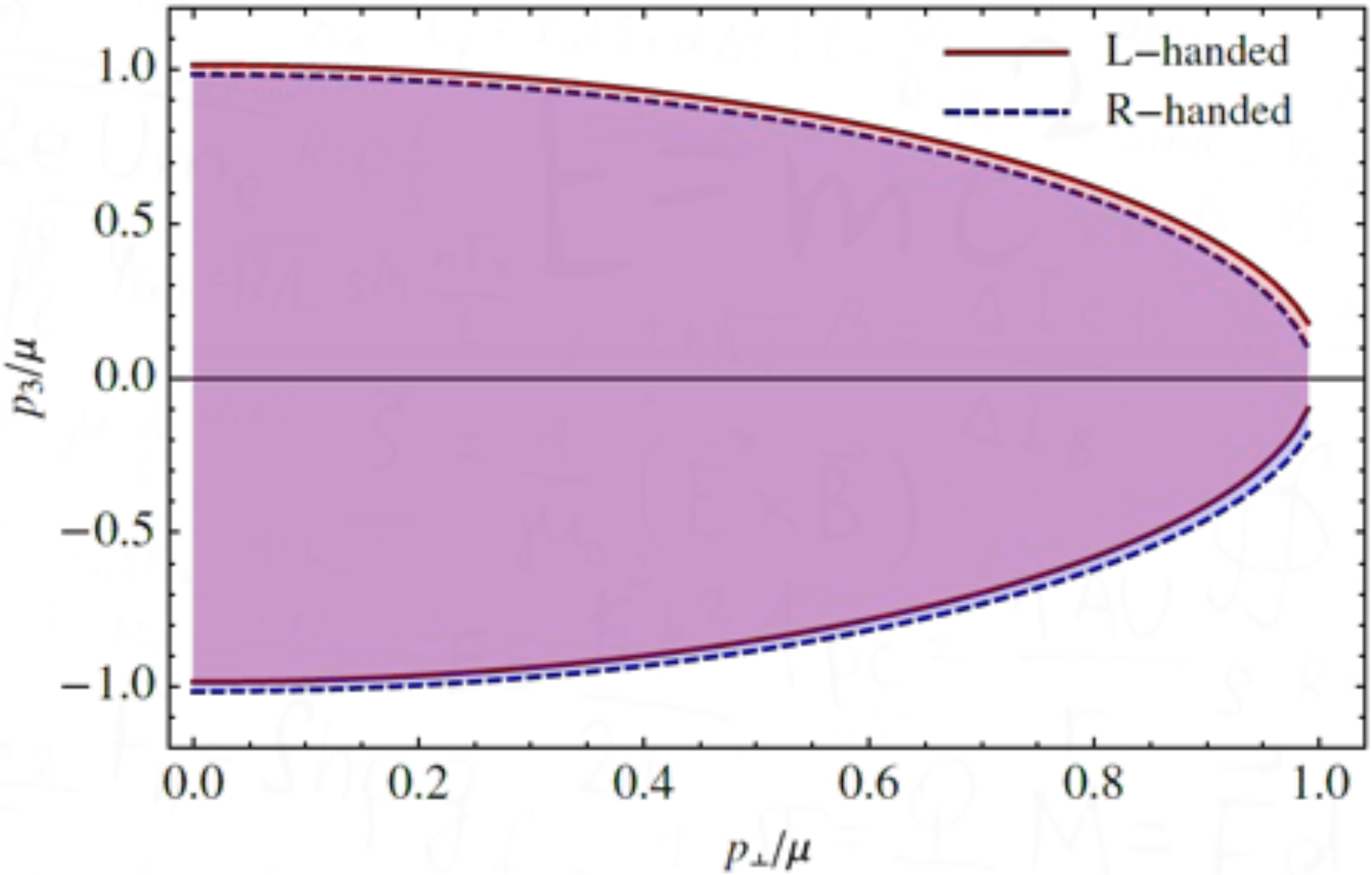
$$\Delta \approx \frac{\alpha\, eB\, \mu}{\pi\, m^2}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

$$\mu_5(p) \approx -\frac{\alpha\, eB\, \mu}{\pi\, m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

Chiral shift and Fermi surface

- QED: Fermi surface of L-handed (R-handed fermions is shifted in negative (positive) z-direction

$$\mathrm{Det}\left[i\,\overline{S}^{-1}(p)+\overline{\Sigma}^{(1)}(p)\right]=0$$



[Gorbar, Miransky, Shovkovy, Wang, PRD **88** (2013) 025043]

Corrections to axial current

- Final result (loops+counterterms)

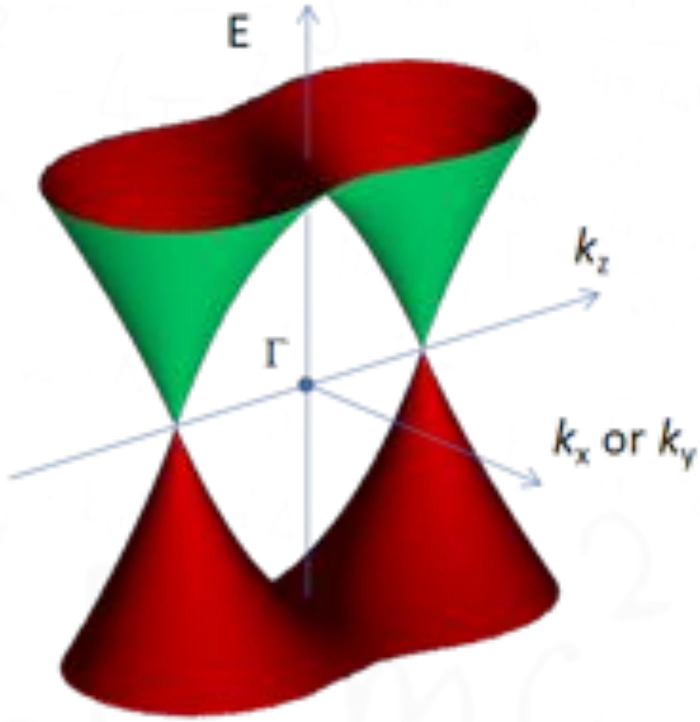
$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

- Unphysical dependence on photon mass because infrared physics with

$$m_\gamma \leq |k_0|, |k_3| \leq \sqrt{|eB|}$$

not captured properly

- Note: similar problem exists in calculation of Lamb shift [Gorbar, Miransky, Shovkovy, Wang, PRD 88 (2013) 025025]

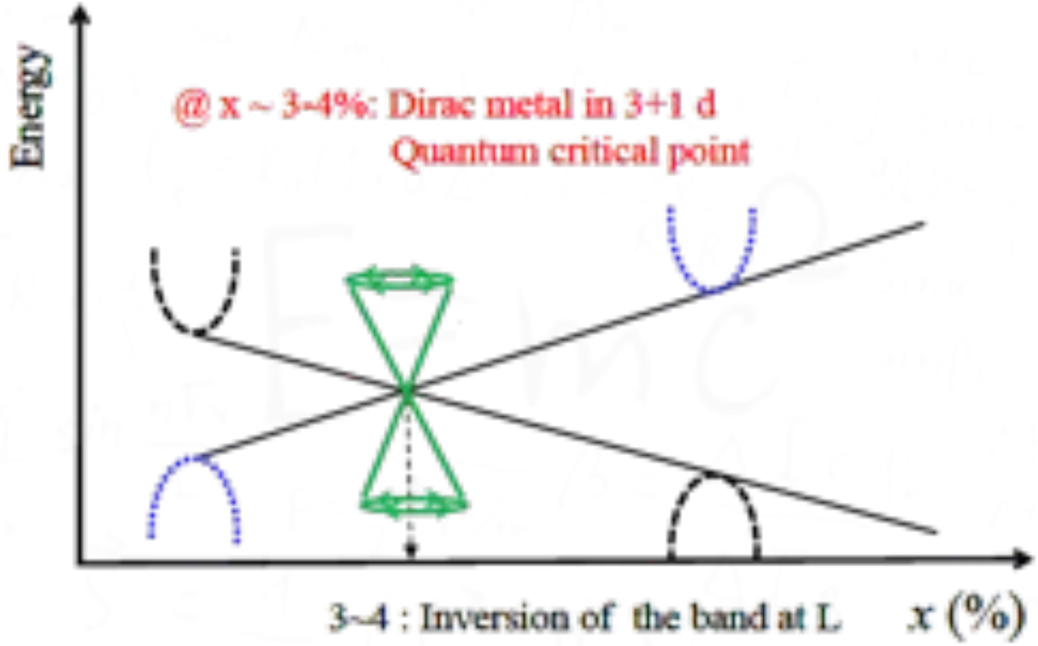


Credits: Borisenko et al., arXiv:1309.7978

DIRAC SEMIMETALS

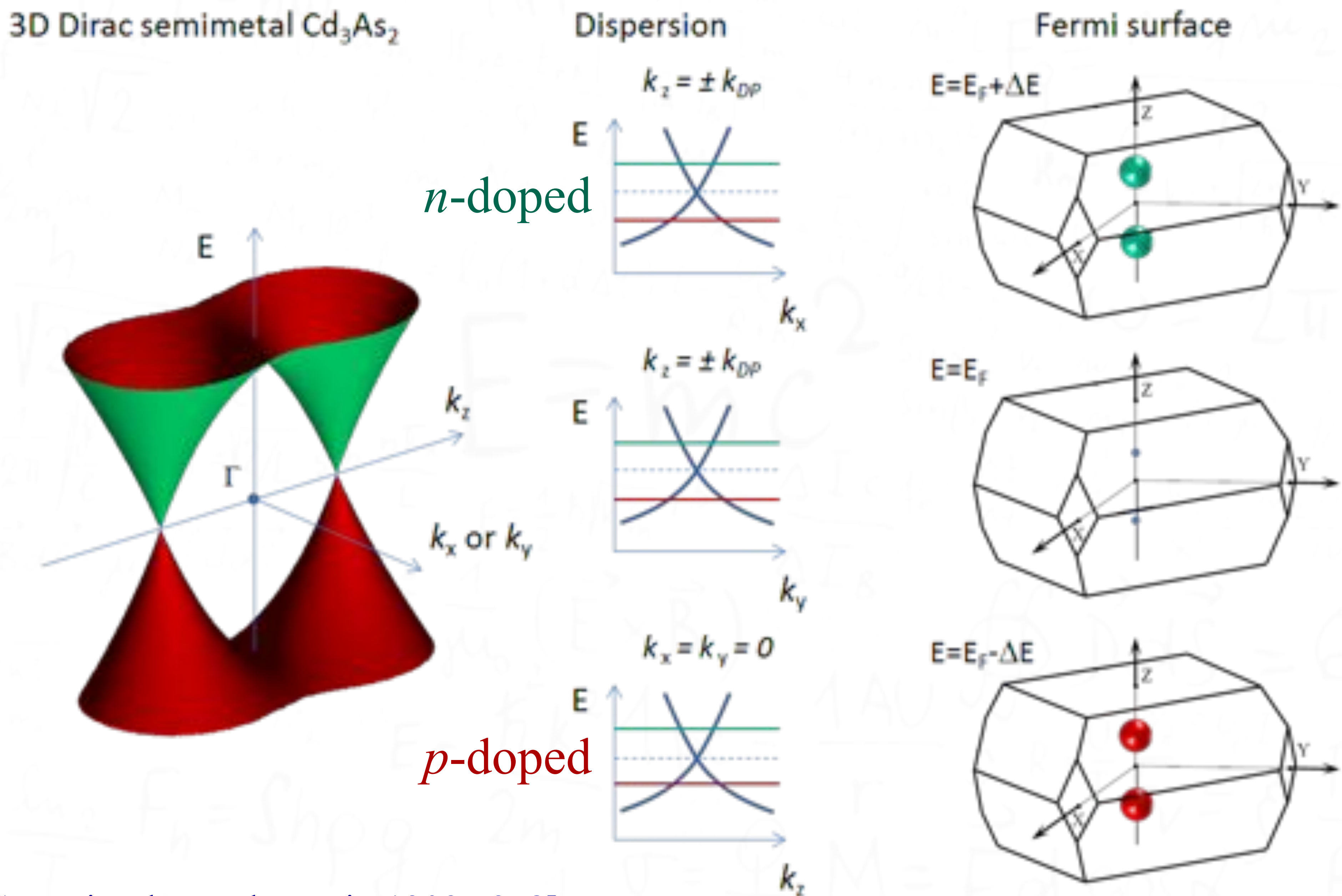
Dirac semimetals

- Solid state materials with Dirac quasiparticles:
  - $Bi_{1-x}Sb_x$ alloy



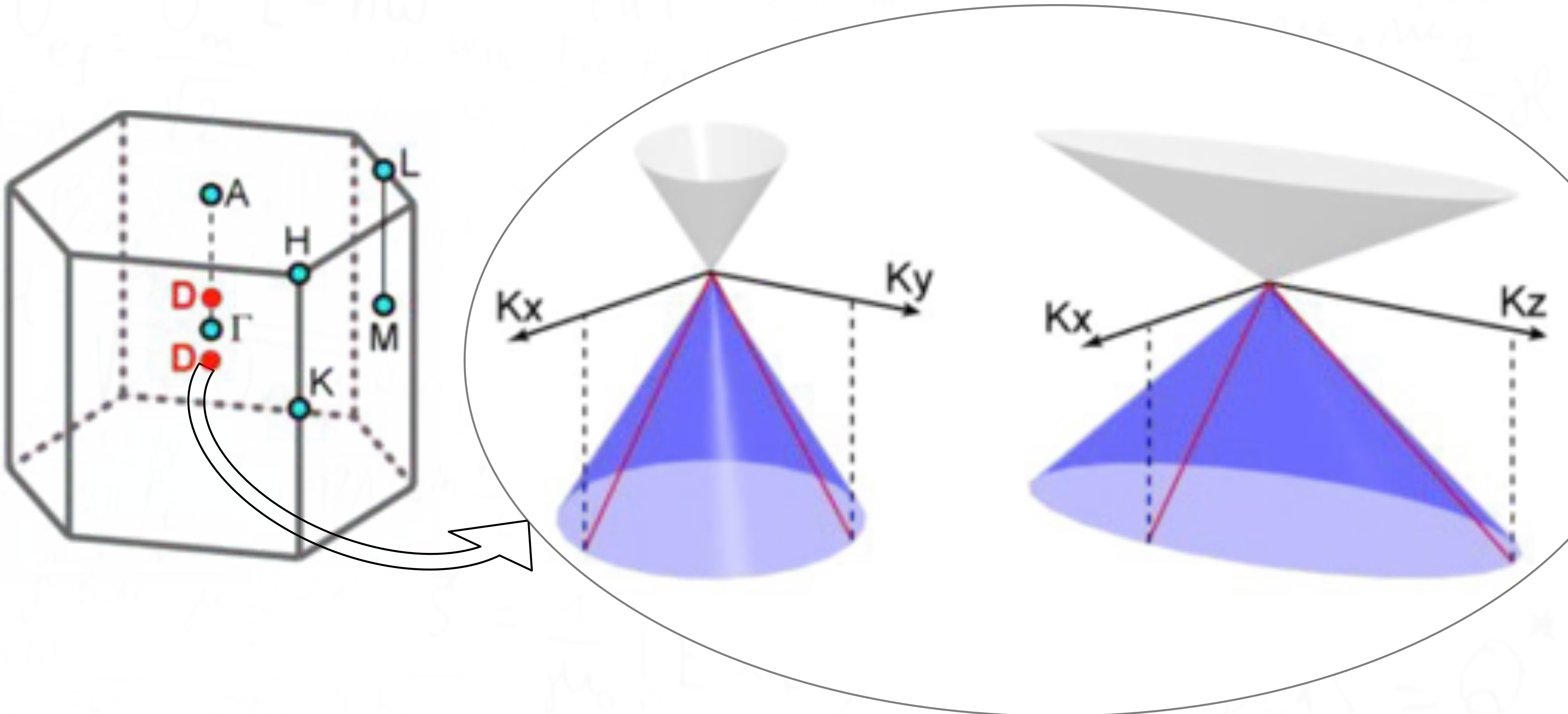
- "New" 3D Dirac materials (ARPES):
  - Na_3Bi [Z. K. Liu et al., arXiv:1310.0391]
  - Cd_3As_2 [M. Neupane et al., arXiv:1309.7892] [S. Borisenko et al., arXiv:1309.7978]

Cadmium arsenide



[S. Borisenko et al., arXiv:1309.7978]

Potassium bismuthide



In the vicinity of 3D Dirac points:

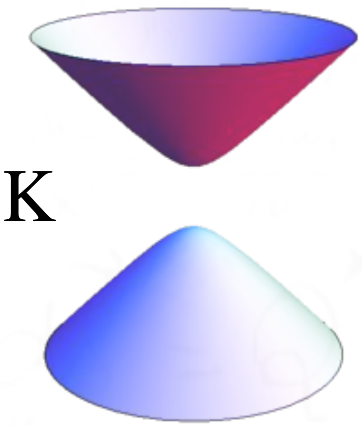
$$E = v_x k_x + v_y k_y + v_z k_z$$

[Z. K. Liu et al., arXiv:1310.0391]

μ=0: Semimetal → Insulator

- Doping → neutrality point (μ=0)

- Magnetic field B and small temperature: mass gap generation



$$m_{\text{dyn}} \sim 10^{-3}\sqrt{|eB|} \approx 8\times 10^{-3}\sqrt{B[T]}\ \text{eV} \approx 90\sqrt{B[T]}\ \text{K}$$

(assuming that coupling constant $\alpha \approx 1$)

- Experimental signatures are expected in transport measurements

$\mu \neq 0$: Dirac → Weyl metal

- Hamiltonian of a Dirac semimetal

$$H^{(D)} = \int d^3 r \bar{\psi} \left[-i v_F \left(\vec{\gamma} \cdot \vec{\nabla} \right) - \mu_0 \gamma^0 \right] \psi + H_{\text{int}}$$

cf. Weyl semimetal

“chiral shift”

$$H^{(W)} = \int d^3 r \bar{\psi} \left[-i v_F \left(\vec{\gamma} \cdot \vec{\nabla} \right) - \left(\vec{b} \cdot \vec{\gamma} \right) \gamma^5 - \mu_0 \gamma^0 \right] \psi + H_{\text{int}}$$

- In a Dirac semimetal, a nonzero chiral shift $\vec{b}$ will be induced when $B \neq 0$, i.e.,

$$\vec{b} \propto -\frac{g}{v_F^2 c} \mu_0 e \vec{B}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B **88**, 165105 (2013)]

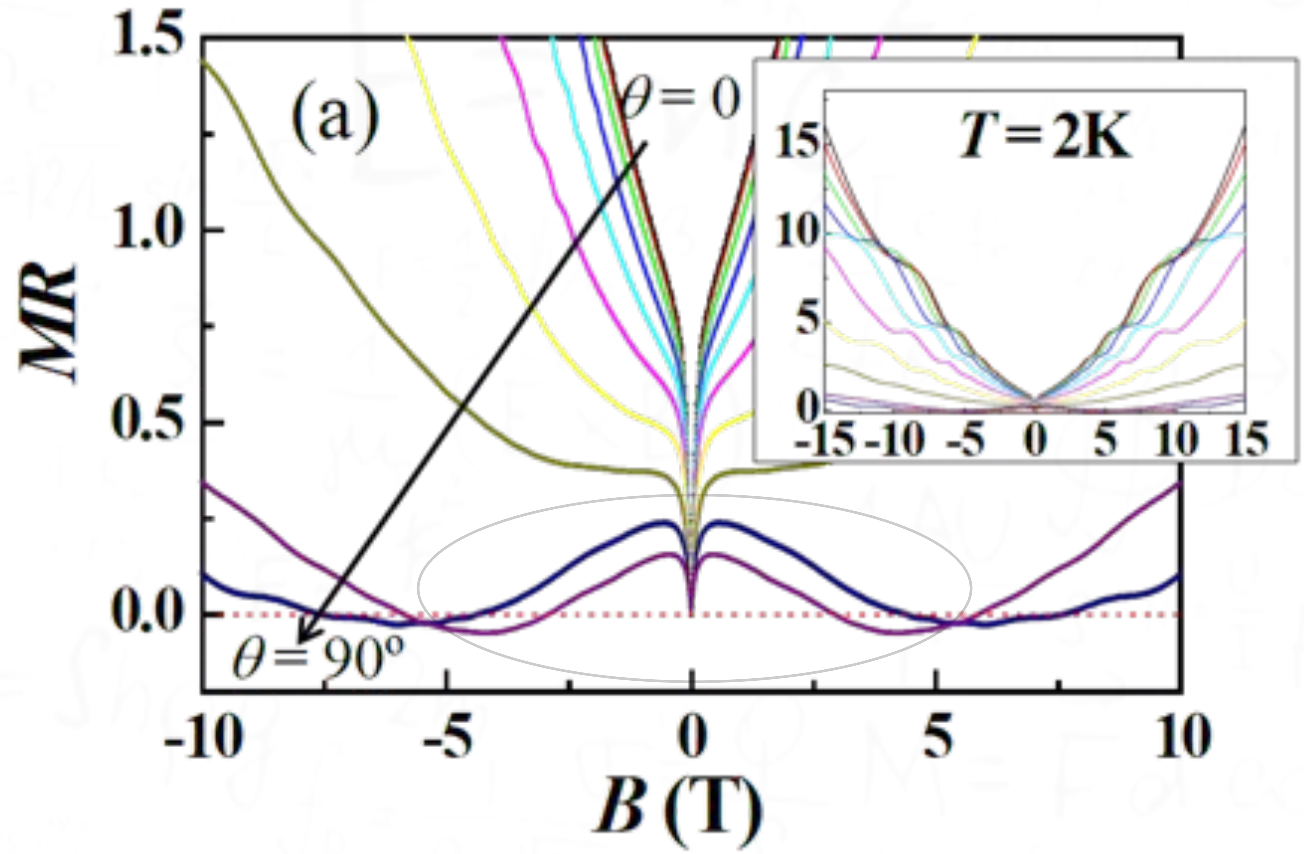
Negative magnetoresistance

- ρ_{33} is expected to decrease with B because

  $\sigma_{33} \propto B^2$ (weak B) [Son & Spivak, Phys. Rev. B 88, 104412 (2013)]

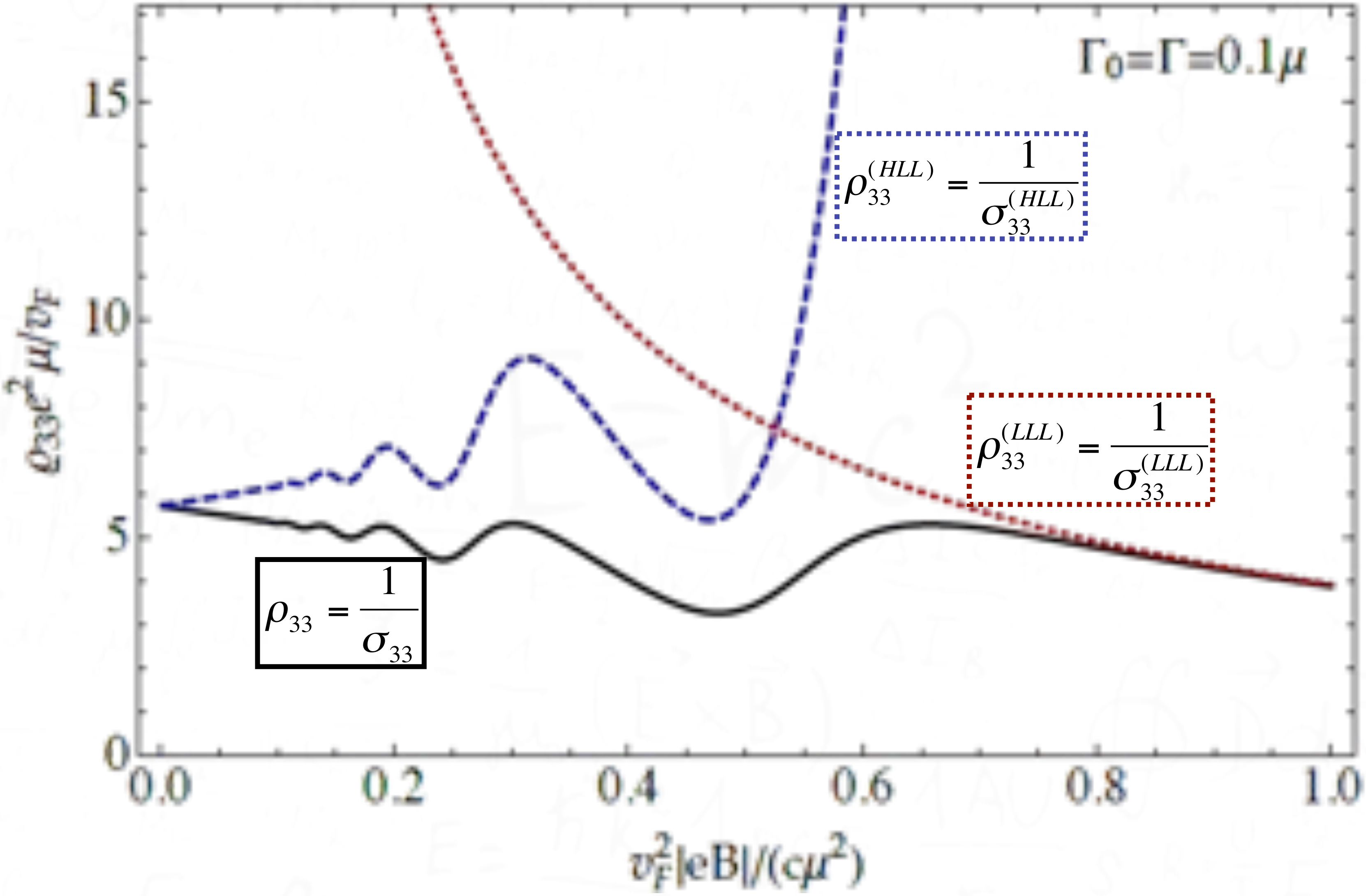
  $\sigma_{33} \propto B$ (strong B) [Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]

- Experimental confirmation? [Kim, et al., PRL 111, 246603 (2013)]



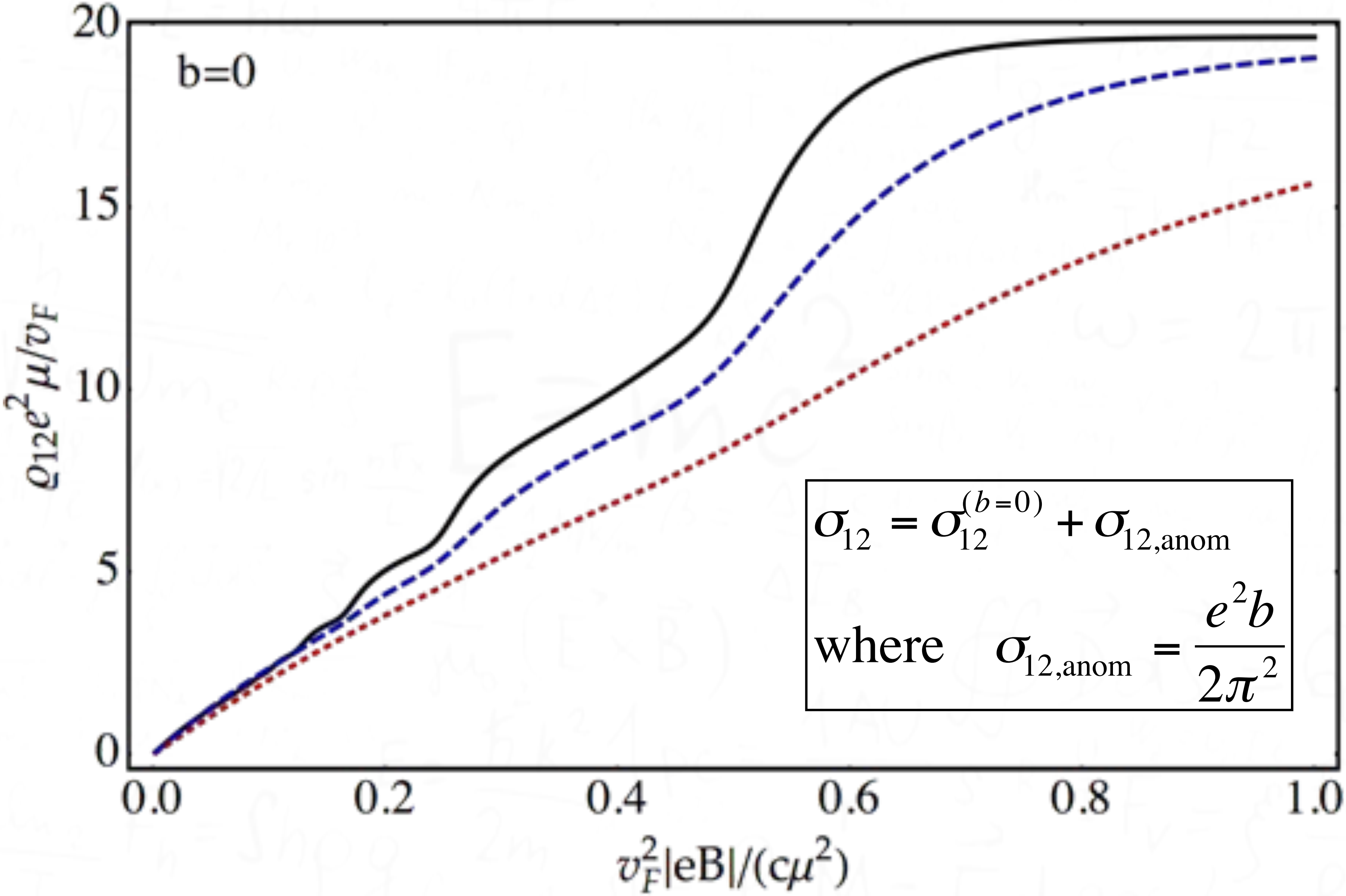
Longitudinal resistivity

[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89 (2014) 085126]



- Note: $\sigma_{33}=\sigma_{33}^{(LLL)}+\sigma_{33}^{(HLL)}$, where $\sigma_{33}^{(LLL)}=\frac{e^2 v_F|eB|}{4\pi^2 c\Gamma_0}$

Transverse off-diagonal ρ_{12}



$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\mathrm{anom}}$$

where $\sigma_{12,\mathrm{anom}} = \dfrac{e^2 b}{2\pi^2}$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89 (2014) 085126]

Summary

- Relativistic matter in magnetic fields is relevant for many branches of physics
- The underlying physics is conceptually rich
- A short list of recent developments
  - Magnetic catalysis (QCD, graphene, Dirac metals)
  - Chiral magnetic/separation effect (QCD, Dirac metals)
  - Chiral shift (QED/QCD plasma, Dirac/Weyl metals)
  - Chiral magnetic spiral …
  - and many others …