ARIZONA STATE UNIVERSITY

MAGNETIC QCD
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Chiral asymmetry:
A remarkable form of magnetization in relativistic matter

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Relativistic Matter

- **Non-relativistic**
  - Particles move much slower than the speed of light
  - Kinetic energies are much smaller than the rest energy

$$E_{\text{kin}} << E_{\text{rest}}: \quad E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

- **Relativistic**
  - Particle velocities approach the speed of light
  - Kinetic energies are comparable to, or larger than E_{rest}

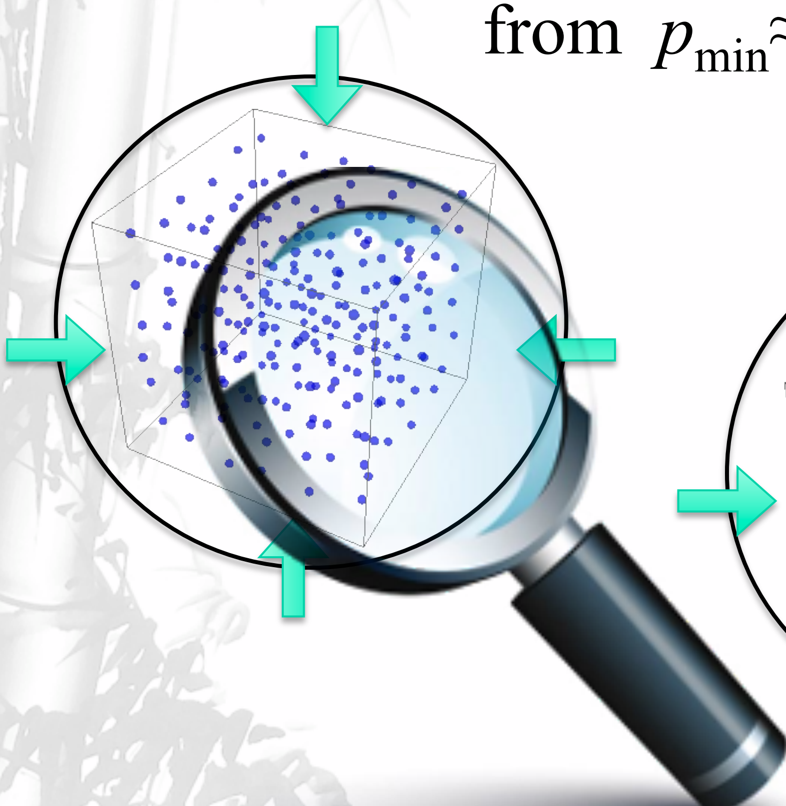
$$E_{\text{kin}} \geq E_{\text{rest}}: \quad E = c\sqrt{p^2 + m^2c^2} \approx c\,p$$

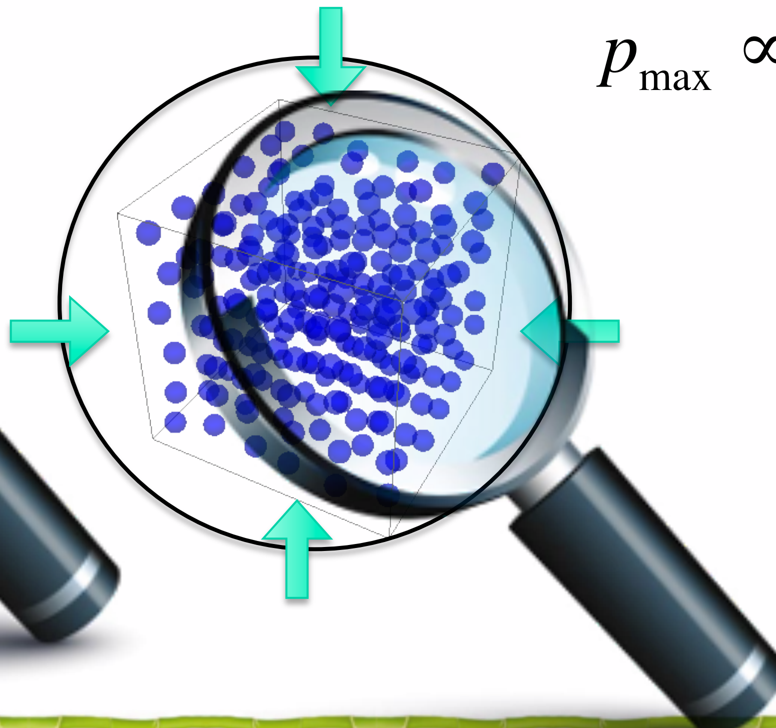
Super-dense Matter

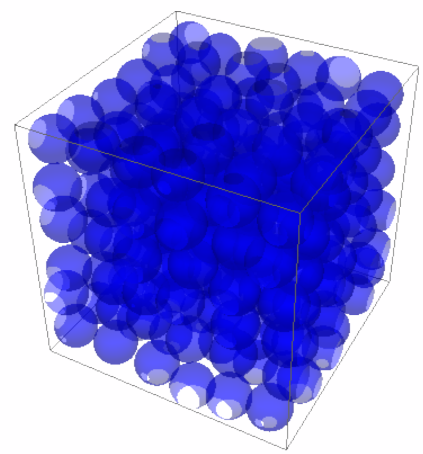
- **What happens when you squeeze matter to very high density?** (e.g., neutrons inside neutron stars)

Pauli exclusion principle: fermions cannot occupy same quantum states (they end up filling out all states from $p_{\min} \approx 0$ to $p_{\max} \propto \hbar n^{1/3}$)

$$p_{\max} \propto 200 \left(\frac{n}{1\ \mathrm{fm}^3} \right)^{1/3} \ \mathrm{MeV/c}$$

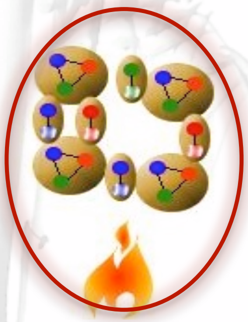


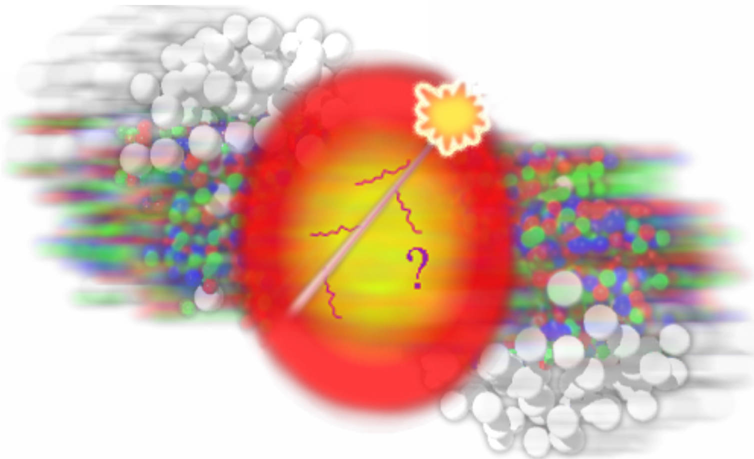


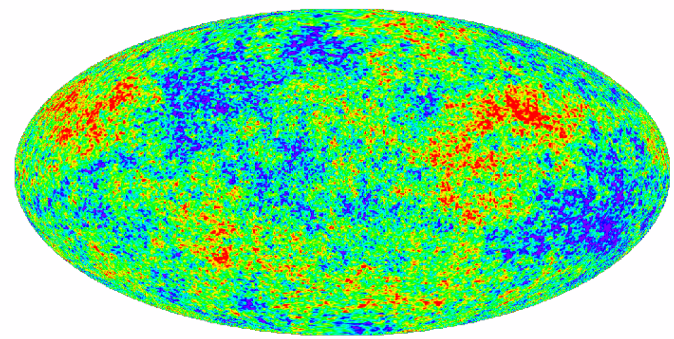


Super-hot Matter

- **What happens when you heat matter to very high temperature?** (e.g., matter in heavy ion collisions)





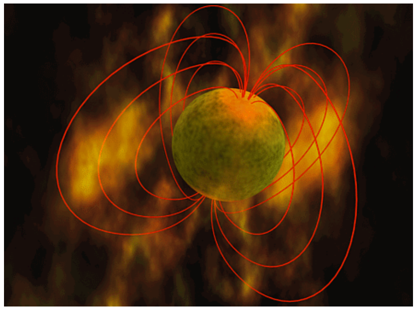


Heat is equivalent to **kinetic energy**: average kinetic energy of particles is proportional to temperature:

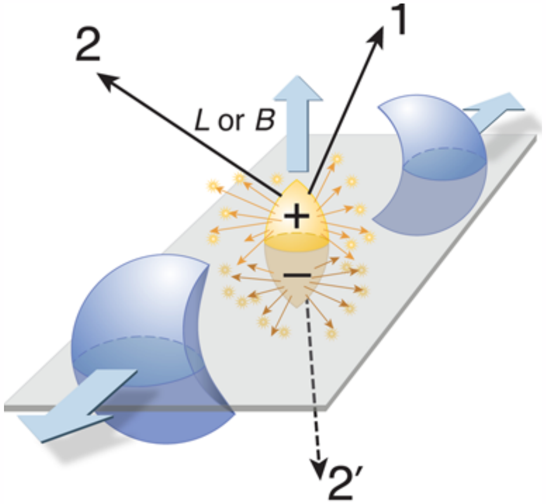
$$p \propto k_B T / c \sim 200\left(\frac{k_B T}{200\ \text{MeV}}\right) \text{MeV/c} \quad (\text{assuming } p >> mc)$$

Superstrong magnetic fields

- Strong magnetic fields are common inside compact stars
  - **10^{10} to 10^{16} G** (10 keV to 10 MeV)



- In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} G** (~ 100 MeV)

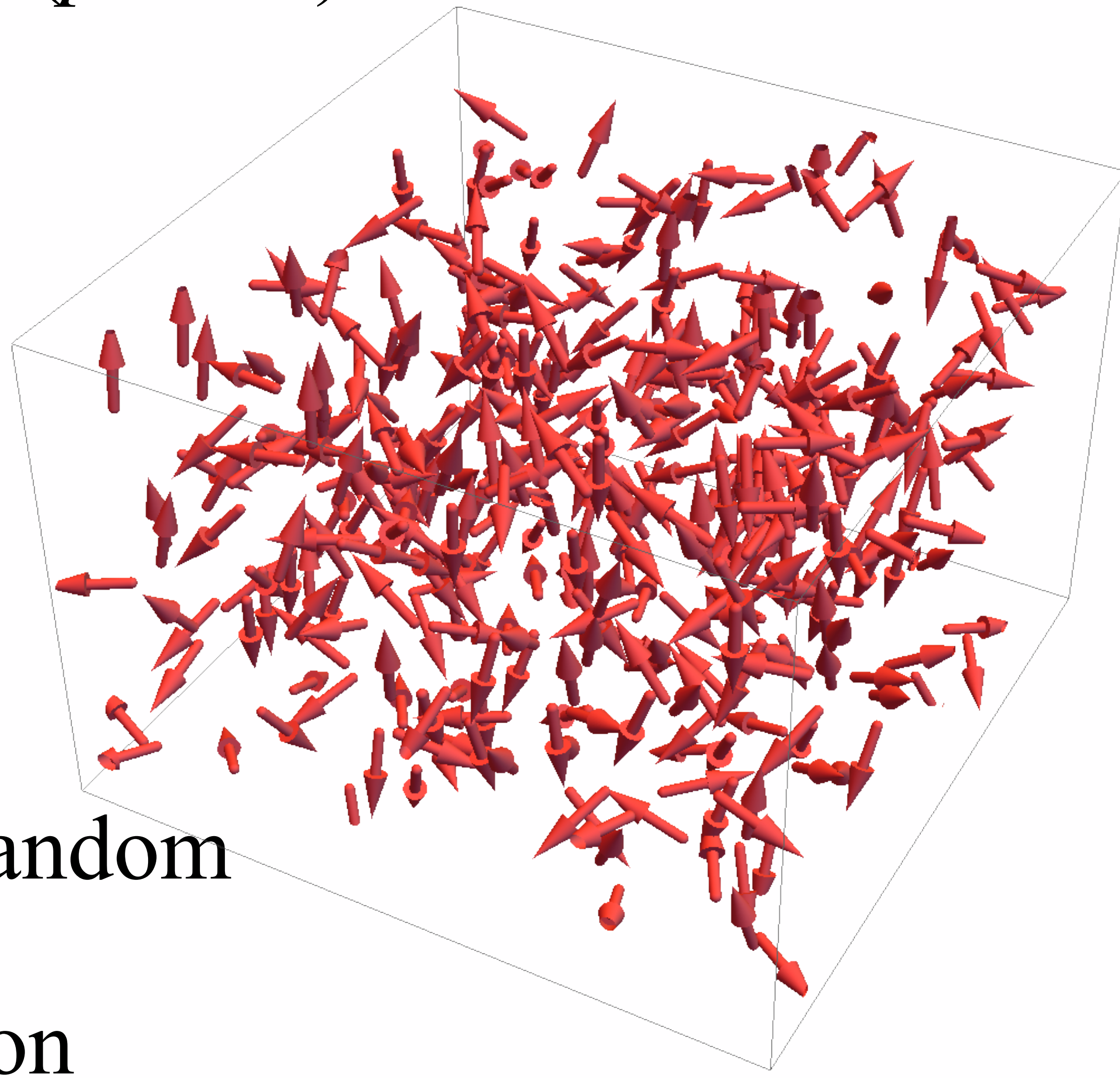


- Early Universe
  - **up to 10^{24} G** (~ 100 GeV)
- Graphene, Dirac semimetals, …
  - **10^{5} G** (~ 100 meV)

Nonrelativistic electron gas

- Nonrelativistic case ($p \ll m$)
- Electrons carry magnetic moments

$$\vec{\mu} = -\frac{g_S \mu_B}{\hbar}\vec{S}$$

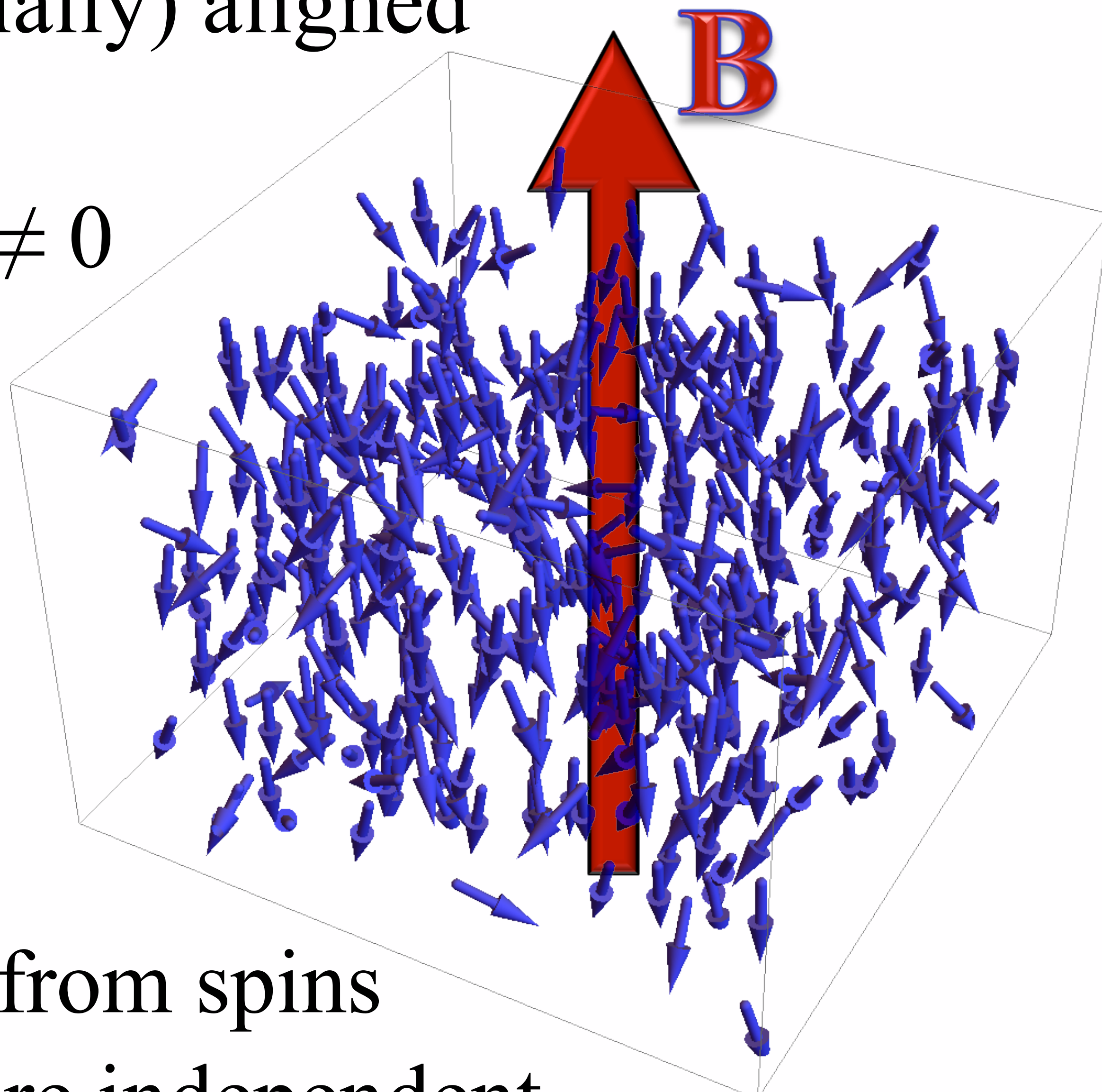


- At B=0, spins are random
- No net magnetization

Magnetization of electron gas

- B≠0: spins are (partially) aligned
- Spin magnetization ≠ 0
- Orbital motion adds some diamagnetism

$$\chi_{dia} = -\frac{1}{3}\chi_{para}$$



- Note: contributions from spins and orbital motion are independent

Relativistic electron gas

- Dirac equation with massless fermions

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla}+ie\vec{A}\right)\right]\Psi = 0$$

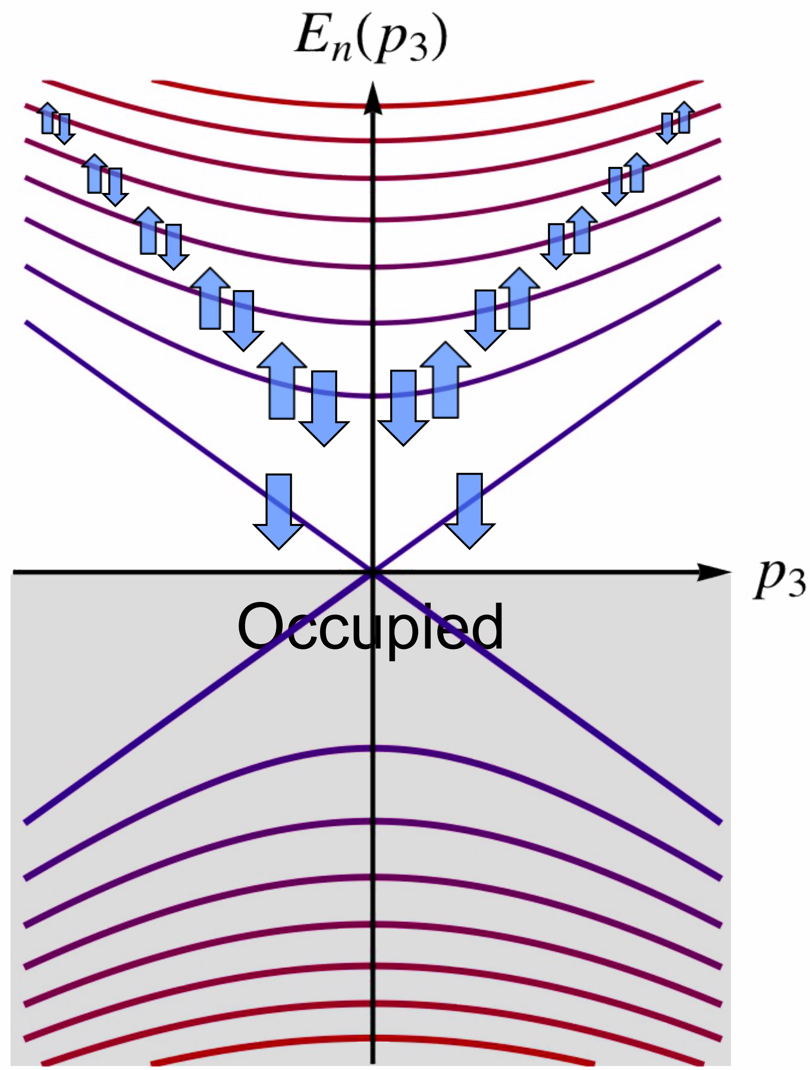
- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where $n = s + k + \frac{1}{2}$

$s = \pm 1/2$ (spin)

$k = 0,\ 1,\ 2,\ldots$ (orbital)



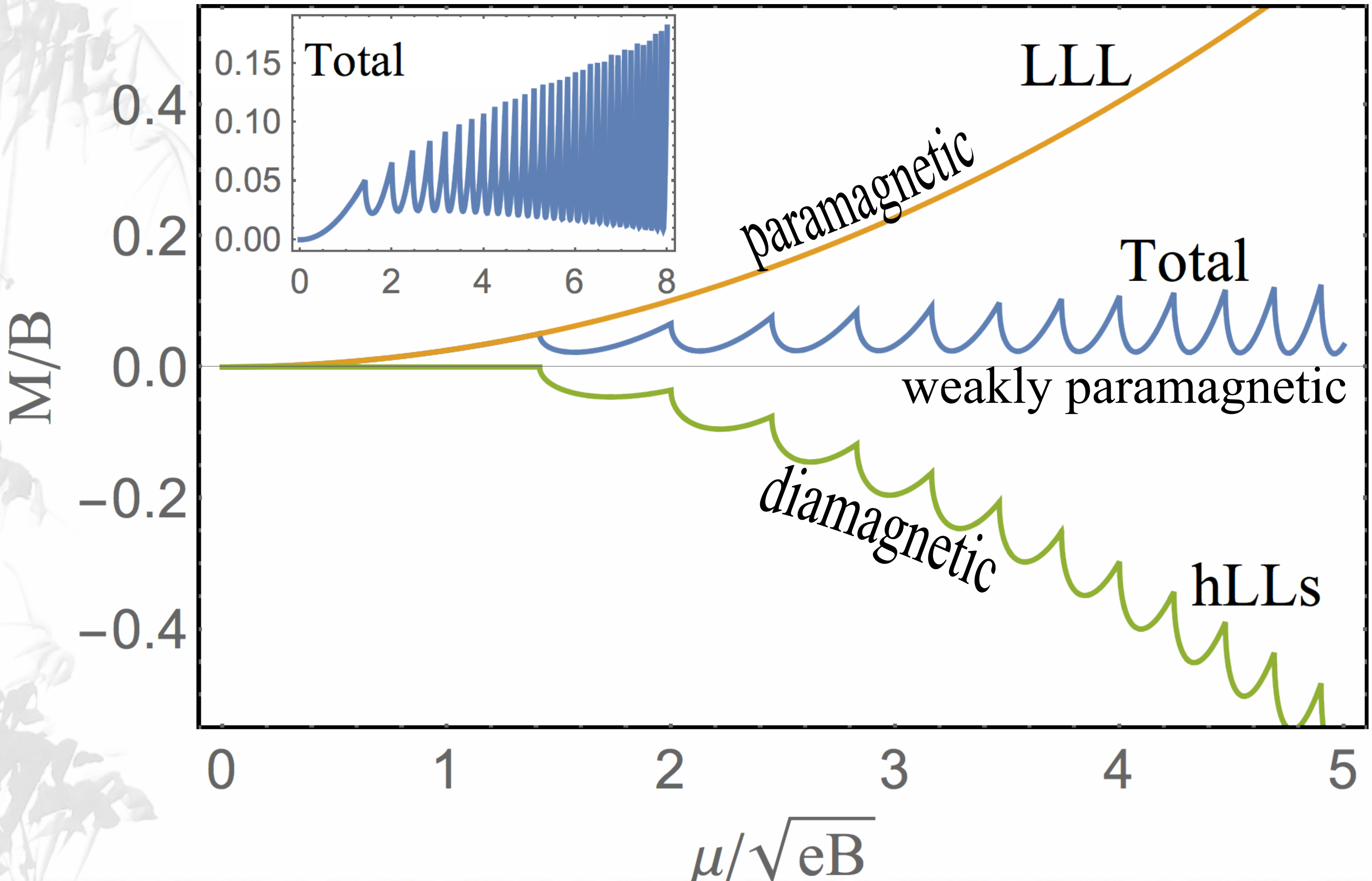
Relativistic magnetization

- Spins are aligned in LLL (n=0) − paramagnetism
- No apparent spin alignment in higher LLs (n≥1) − diamagnetism (?)
- Complication: orbital and spin contributions are inseparable in the energy
- Compare with nonrelativistic case, i.e.,

$$E_{non} \approx m + \frac{p_3^2}{2m} + \underbrace{(2k+1)\mu_B B}_{\text{orbital}} + \underbrace{2s\mu_B B}_{\text{spin}}$$

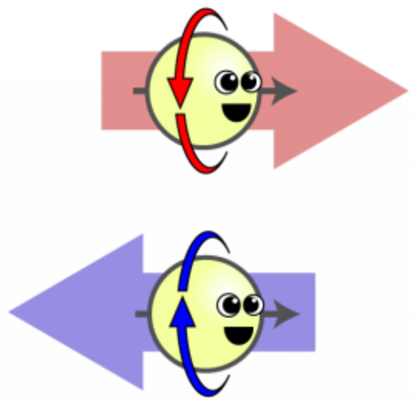
“Weak” paramagnetism

$$M = \frac{e\mu^2}{4\pi^2} + \frac{e}{2\pi^2}\sum_{n=1}^{n_{\max}}\left(\mu\sqrt{\mu^2 - 2neB} - 4n\,|eB|\ln\frac{\mu - \sqrt{\mu^2 - 2neB}}{\sqrt{2neB}}\right)$$



Spin vs. orbital motion

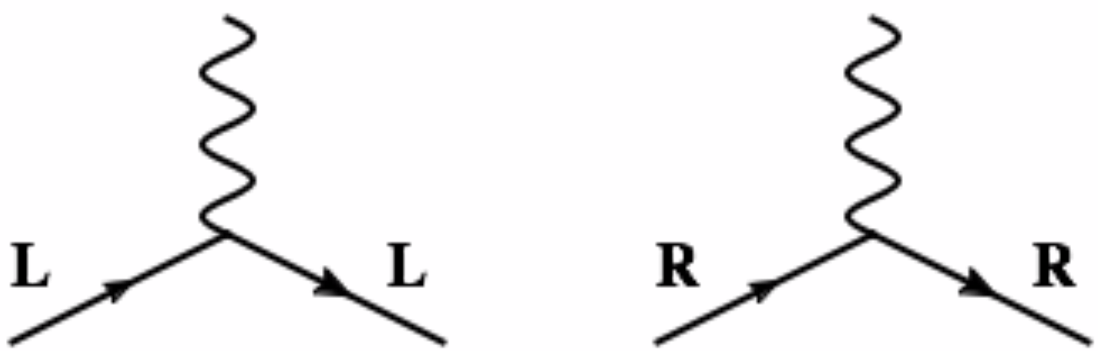
- Helicity/chirality of massless (ultrarelativistic) fermions is (≈) conserved



L-handed

R-handed

- Chirality does not change in elementary QED interactions



Asymmetry: LLL ➡ hLLs

- LLL is spin polarized and chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are L-handed
  - states with $p_3>0$ (and $s=\downarrow$) are R-handed

- When scattering off LLL, particles in higher LLs must "flip" both spin & momentum

- B-field $\oplus$ interactions = chiral asymmetry

L-handed prefer $s\downarrow$ and $p_3<0\ (\downarrow)$

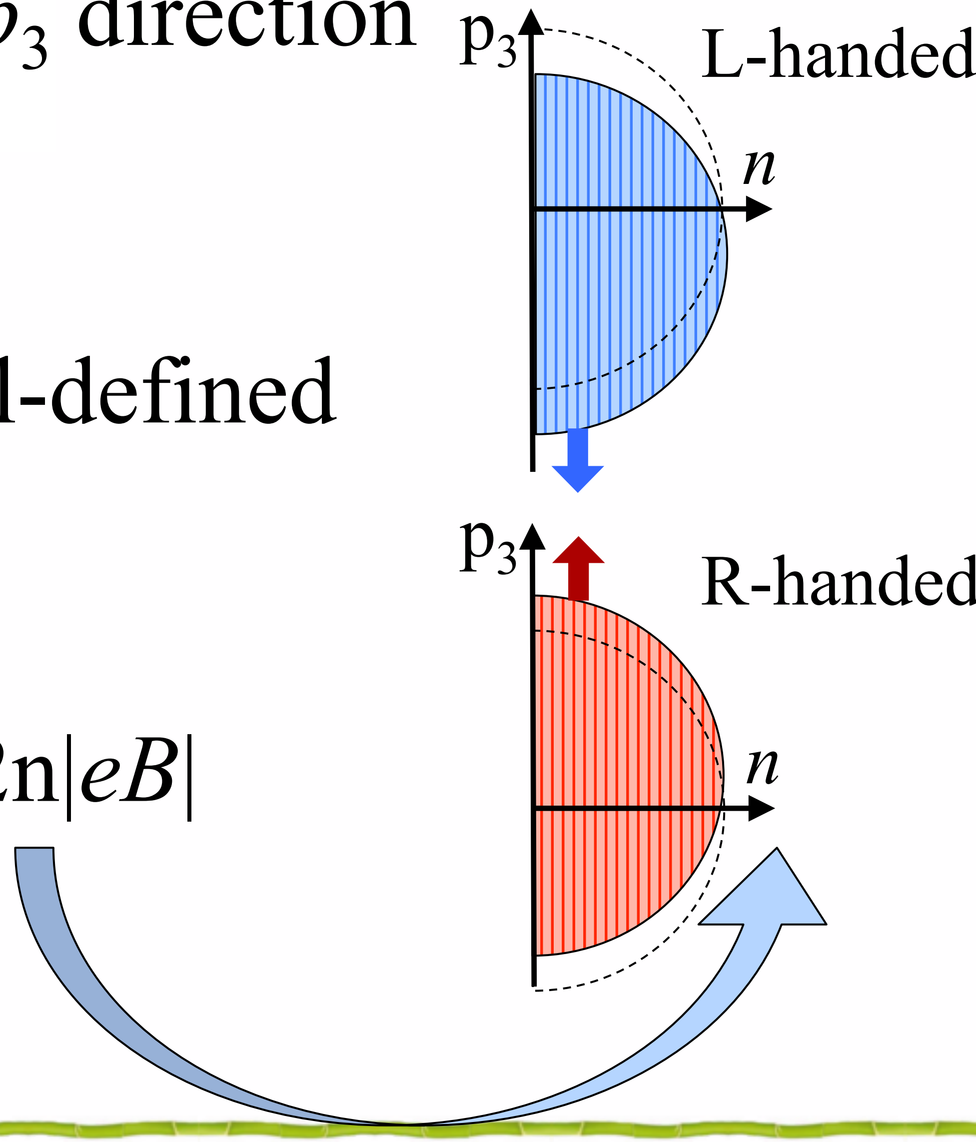
R-handed prefer $s\downarrow$ and $p_3>0\ (\uparrow)$

Chiral asymmetry

- Anticipated outcome: L- & R-handed Fermi surfaces shift in p_3 direction

Note: $\mathbf{p}_\perp$ is not well-defined

$p_\perp^2$ is replaced by $2n|eB|$



Seed chiral asymmetry

- LLL gives a seed chiral asymmetry, which is quantified by the induced axial current (CSE)

$$\left\langle \vec{j}_5 \right\rangle_{\text{free}} = -\frac{e\vec{B}}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Is this an exact result connected directly with the chiral anomaly relation?
- Not really, see [Gorbar, Miransky, Shovkovy, Wang, PRD **88** (2013) 025025]

Chiral shift at low energies

- Ground state expectation value of the axial current (CSE)

$$\left\langle \bar{\psi}\gamma^3\gamma^5\psi \right\rangle = -\frac{eB}{2\pi^2}\mu \quad \text{with} \quad \vec{B} = (0,0,B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

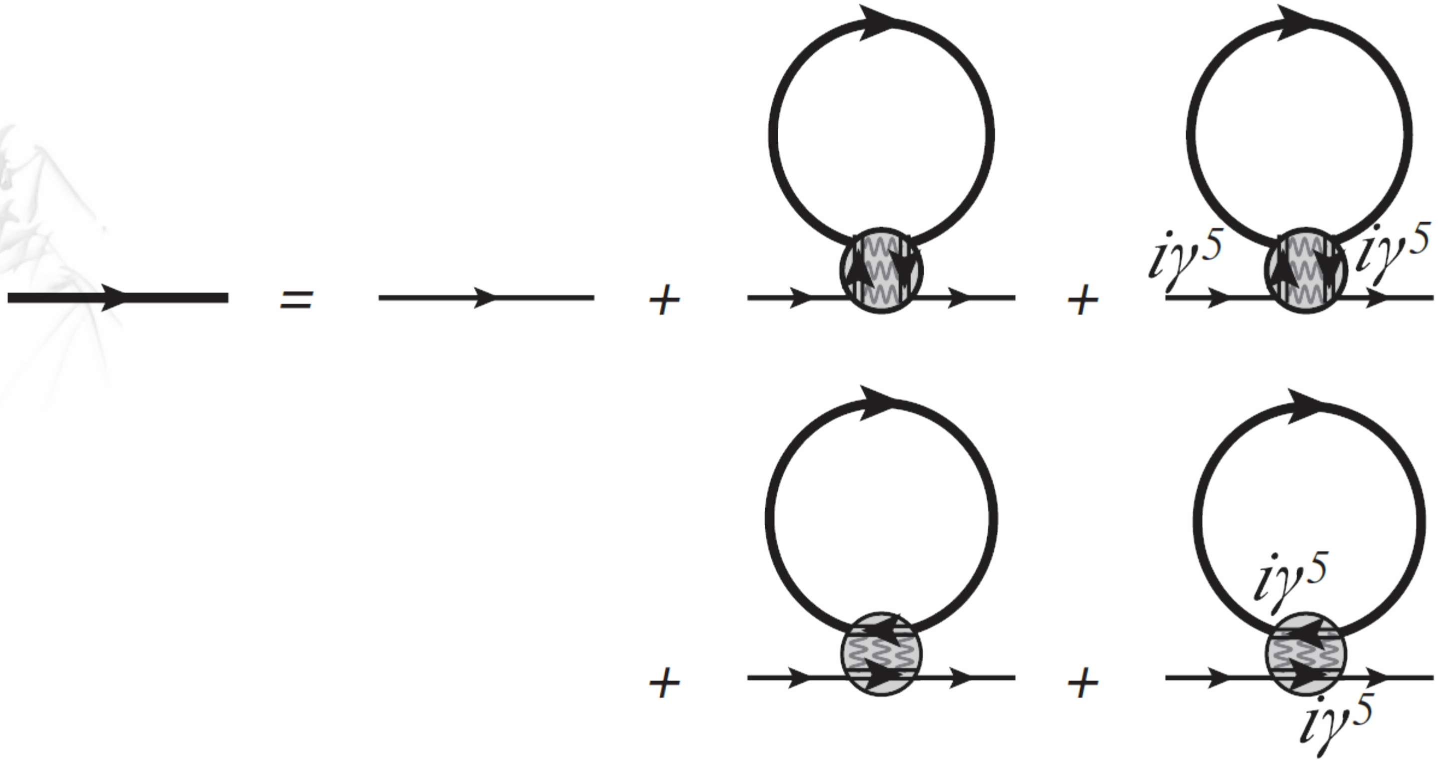
should induce a dynamical (chiral shift) parameter Δ associated with the condensate,

$$\delta L = \Delta \bar{\psi}\gamma^3\gamma^5\psi$$

- Note: Δ=0 is not protected by any symmetry

NJL model: quick check

- NJL model (local interaction)



- The equation for the chiral shift

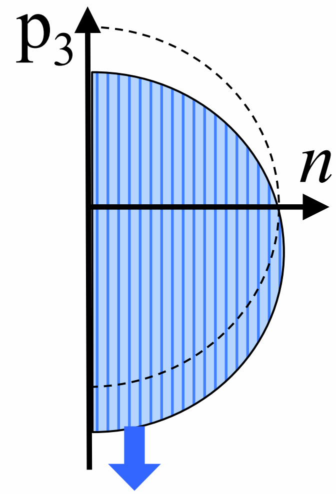
$$\Delta = -\frac{1}{2} G_{\text{int}} \left\langle \bar{\psi} \gamma^3 \gamma^5 \psi \right\rangle \approx \frac{G_{\text{int}} eB}{4\pi^2} \mu$$

Chiral shift @ Fermi surface

- Chirality is $\approx$ well-defined at Fermi surface ($|p_3| \gg m$)
- L-handed Fermi surface:

$$n>0: \quad p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

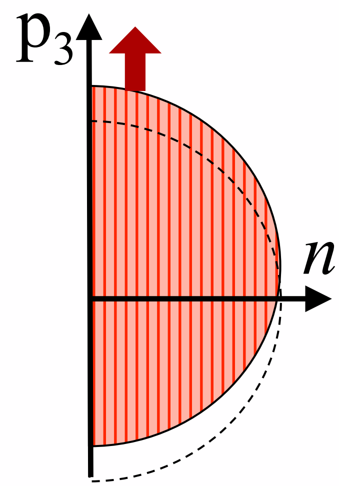
$$p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



- R-handed Fermi surface:

$$n>0: \quad p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

Pause to think

- NJL model supports the conceptual idea,

  but it is a poor toy model because

  – it is nonrenormalizable

  – it is unable to properly deal with anomalies

  – results depend strongly on the cutoff

  – induced chiral shift is same for all LLs

Magnetized QED plasma

- The Lagrangian density

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\gamma^{\nu}D_{\nu} + \mu\gamma^{0} - m\right)\psi$$

- Self-energy in coordinate space

$$\Sigma(x,y) = -4i\,\pi\,\alpha\,\gamma^{\mu}\,S(x,y)\gamma^{\nu}D_{\mu\nu}(x-y)$$

where

$$S(x,y) = e^{i\Phi(x,y)}\bar{S}(x-y)$$

$$\Sigma(x,y) = e^{i\Phi(x,y)}\bar{\Sigma}(x-y)$$

and Φ(x,y) is the Schwinger phase

Weak field expansion

By making use of the expansion in small B,

$$\bar{S}^{(0)}(k) = i\frac{(k_0+\mu)\gamma^0 - \mathbf{k}\cdot\gamma + m}{\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2 - \mathbf{k}^2 - m^2}$$

$$\bar{S}^{(1)}(k) = -\gamma^1\gamma^2 eB\frac{(k_0+\mu)\gamma^0 - k_3\gamma^3 + m}{\left[\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2 - \mathbf{k}^2 - m^2\right]^2}$$

Then, to leading (linear) order in B, we derive

$$\bar{\Sigma}^{(1)}(p) = -4i\pi\alpha\int\frac{d^4k}{(2\pi)^4}\gamma^\mu\,\bar{S}^{(1)}(k)\gamma^\nu D_{\mu\nu}(k-p)$$

Leading order result (B→0)

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta(p) + \gamma^0\gamma^5\mu_5(p)$$

Near Fermi surface ($p_0 \to 0$, $|\mathbf{p}| \to p_F$)

$$\Delta(p) \approx \frac{\alpha eB\mu}{\pi m^2}\left(\ln\frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1\right)$$

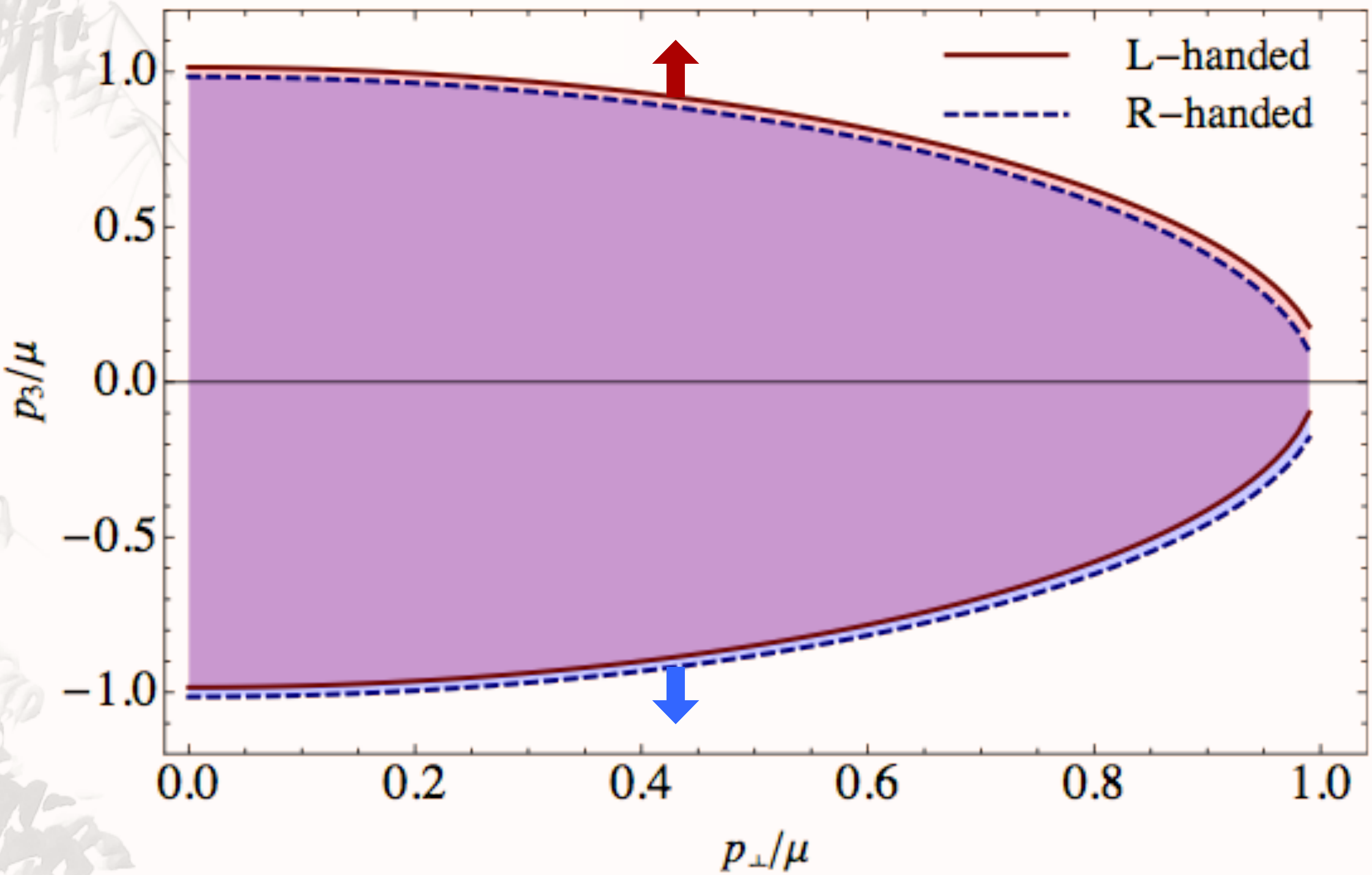
$$\mu_5(p) \approx -\frac{\alpha eB\mu}{\pi m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1\right)$$

[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Dispersion relations in QED

- Let us use the condition (for a small B)

$$\mathrm{Det}\left[i\bar{S}^{-1}(p)+\bar{\Sigma}^{(1)}(p)\right]=0$$



[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

QED in strong field

Self-energy in the Landau-level representation:

$$\bar{\Sigma}(p) = 2e^{-p_\perp^2 l^2} \sum_{n=0}^{\infty} (-1)^n \left(-\gamma^0 \delta\mu_n - \gamma^3\gamma^5 \Delta_n - \gamma^0\gamma^5 \mu_{5,n} + m_n + \ldots\right)\left[P_- L_n - P_+ L_{n-1}\right] - \ldots$$

where $\delta\mu_n$, Δ_n, $\mu_{5,n}$, … are "projections" of the self-energy on the *n*th Landau level,

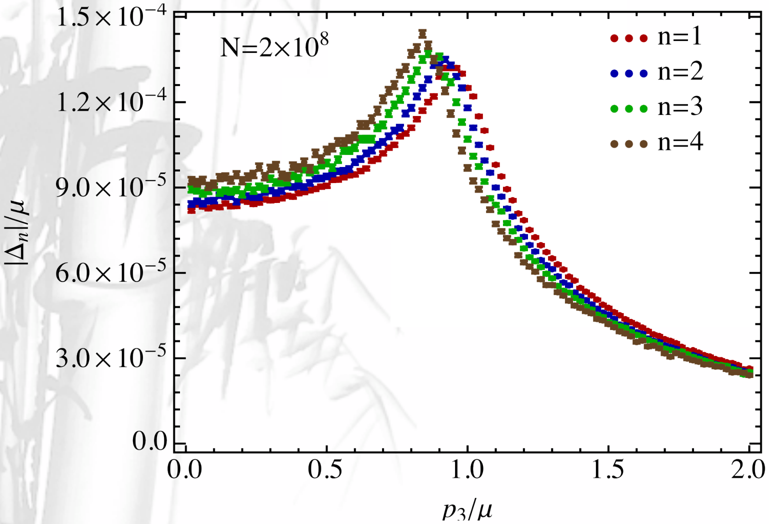
$$\Delta_n(p_0, p_3) = \frac{(-1)^n l^2}{8\pi} \int d^2 p_\perp e^{-p_\perp^2 l^2} \left[L_n + L_{n+1}\right] Tr\left[\gamma^0 \bar{\Sigma}(p)\right]$$

where

$$\bar{\Sigma}(p) = -4i\pi\alpha \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \bar{S}(k) \gamma^\nu D_{\mu\nu}(k-p)$$

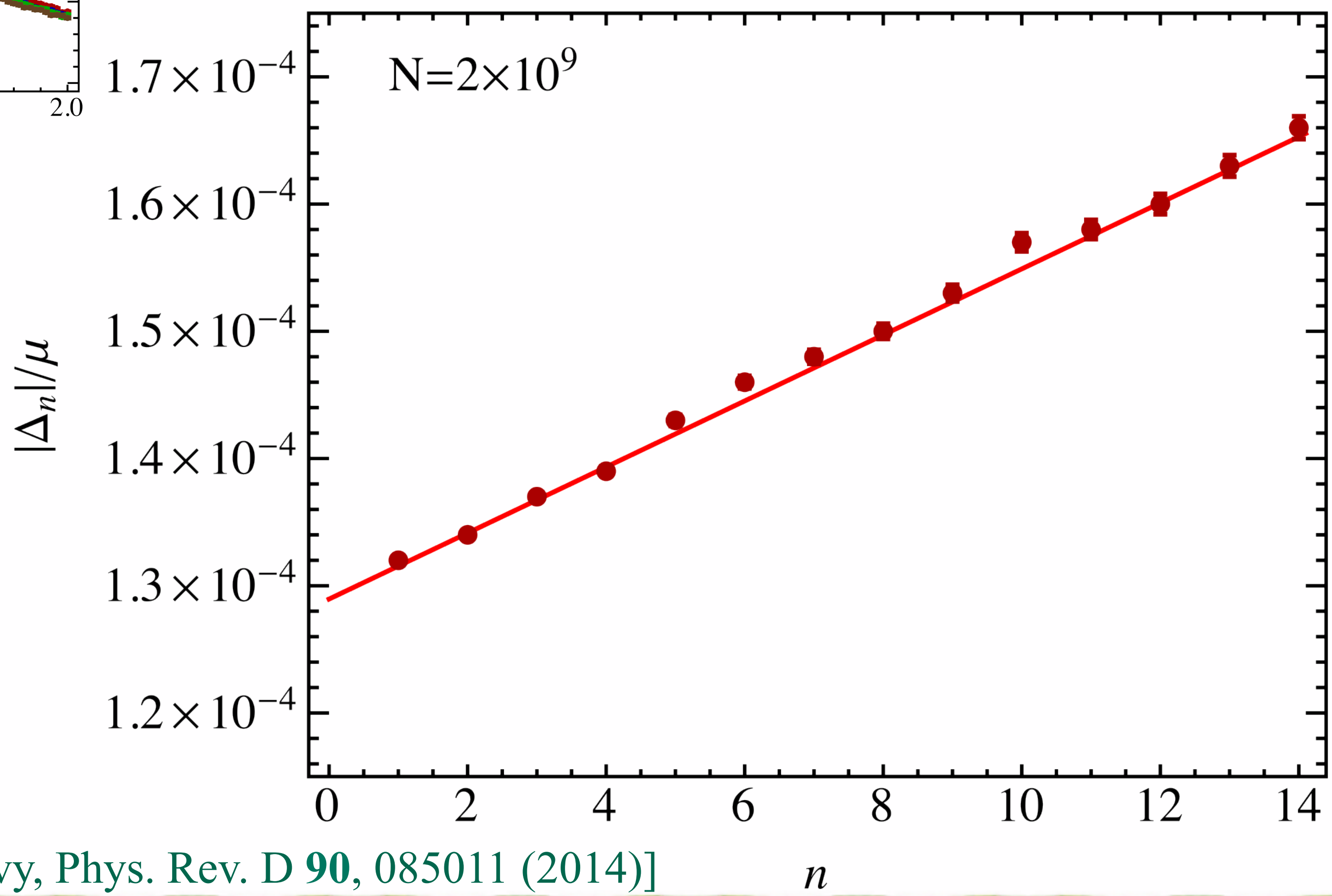
[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025025 (2013)]

QED in strong field: results (Δ)



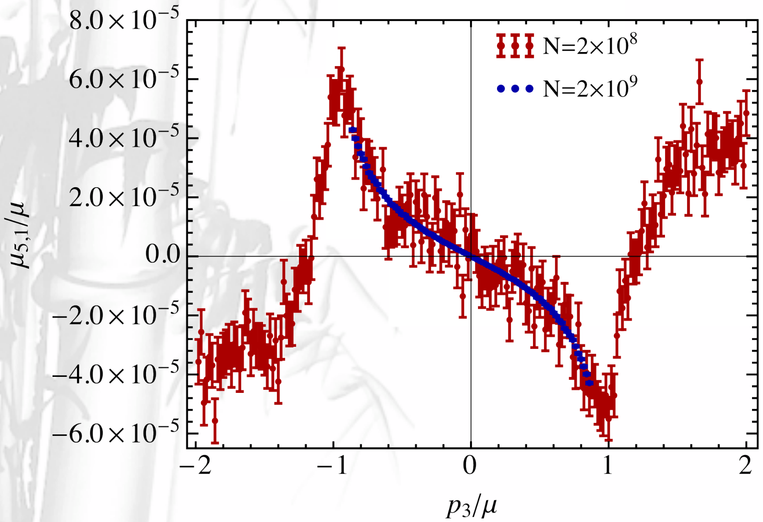
Model fit:

$$\Delta_n = -\frac{\alpha|eB|}{\mu}\left(0.53 + 0.32\frac{|eB|n}{\mu^2}\right)$$



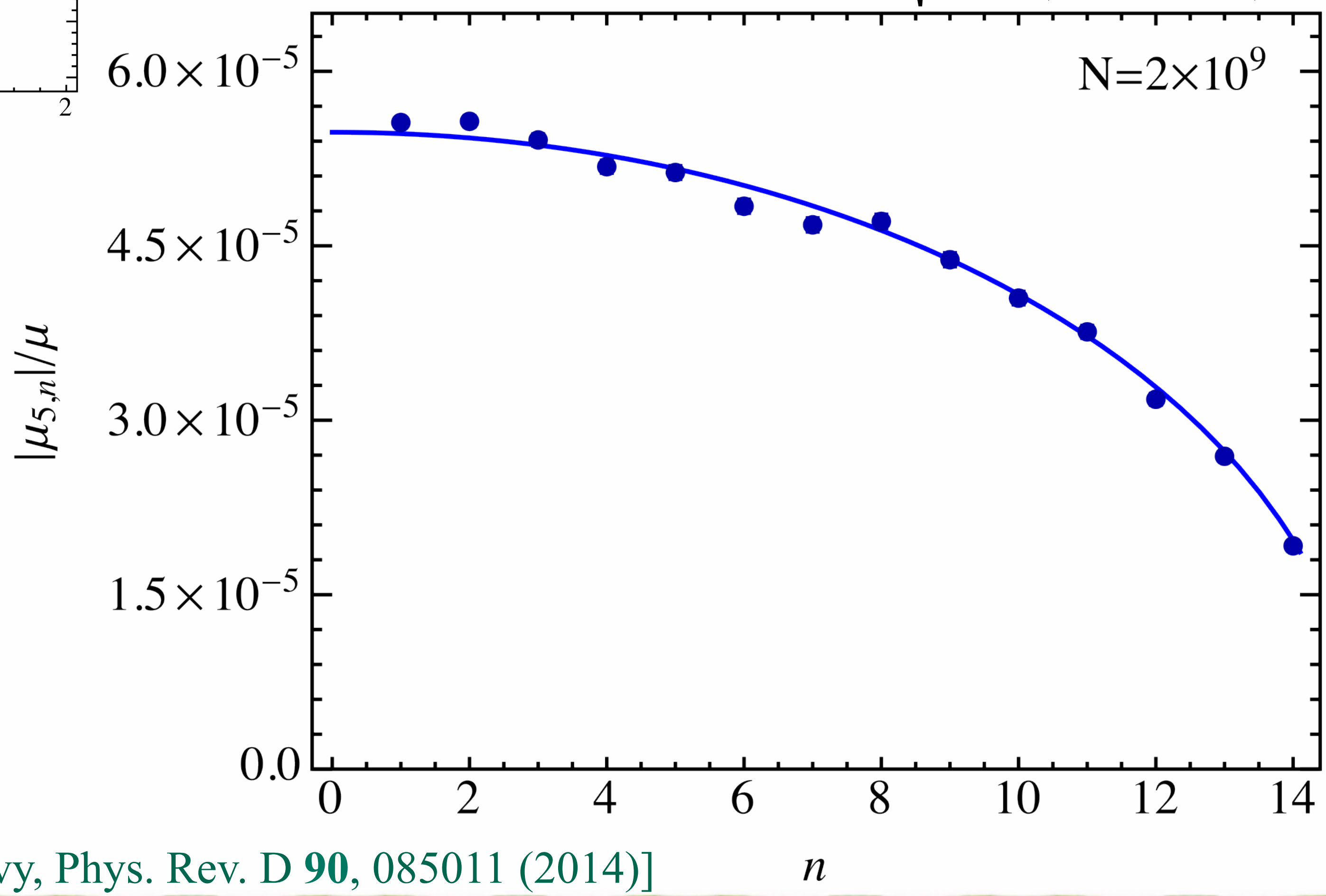
[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: results (μ_5)



Model fit:

$$\mu_{5,n} = -0.225\frac{\alpha|eB|}{\mu}\sqrt{1-\left(\frac{2n|eB|}{\mu^2}\right)^2}$$

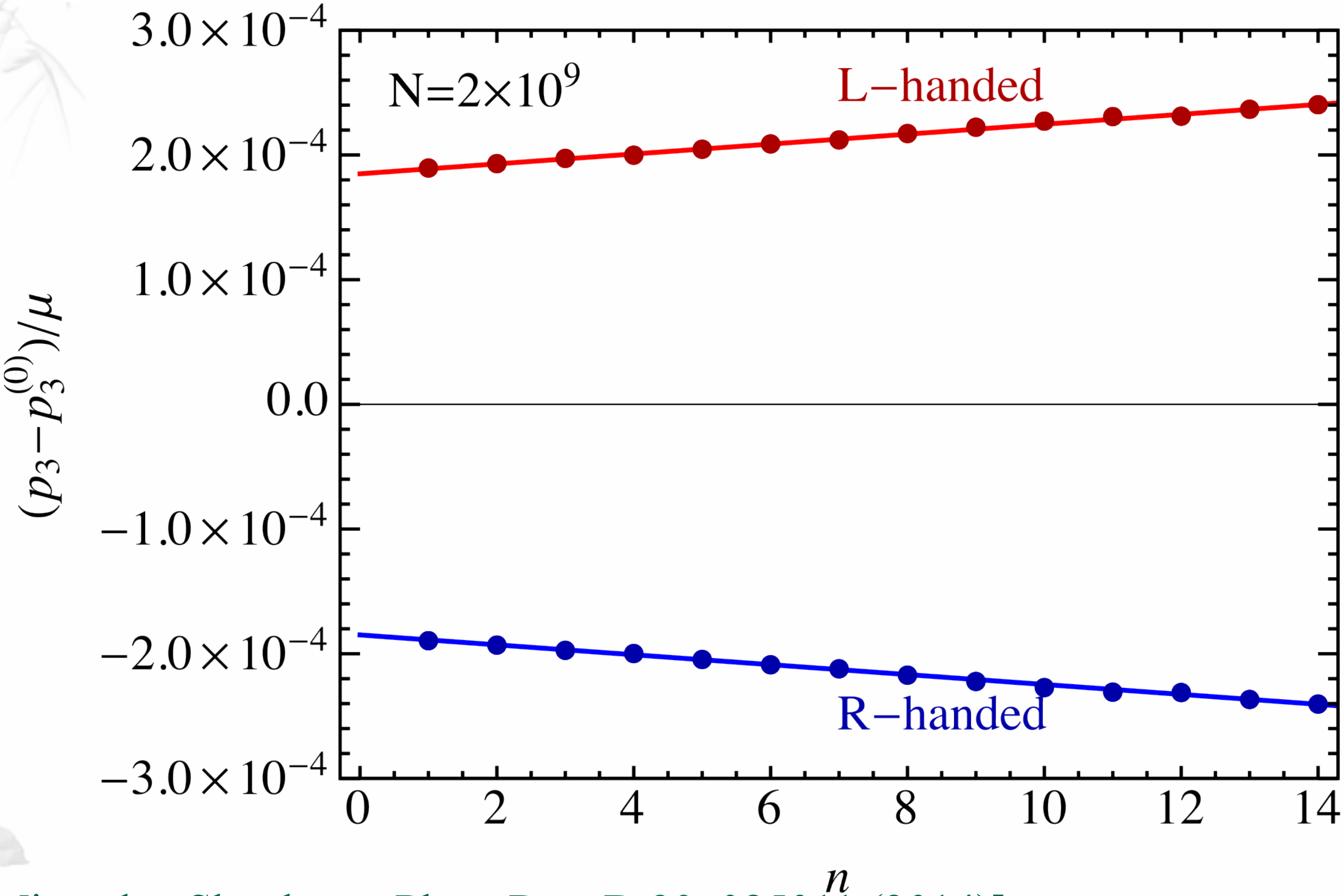


[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: results (δp_3)

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu}\left(0.76 + 0.49\frac{|eB| n}{\mu^2}\right)$$



[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

How large is the asymmetry?

In QED:

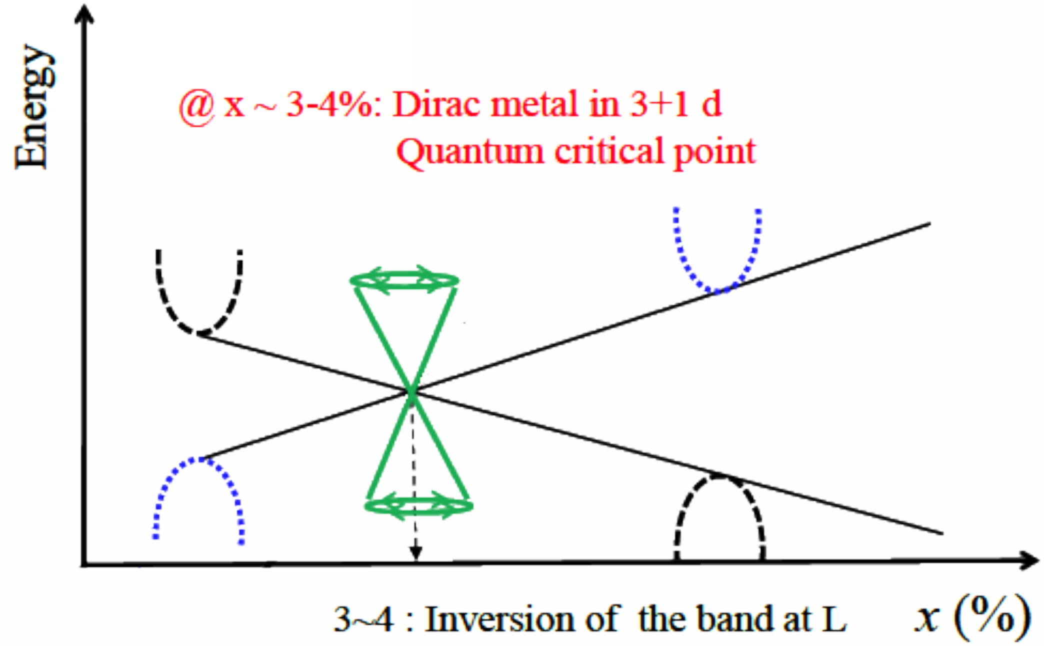
$$\frac{\alpha|eB|}{\mu} \approx 40\left(\frac{B}{10^{17}\,\mathrm{G}}\right)\left(\frac{100\,\mathrm{MeV}}{\mu}\right)\mathrm{keV}$$

In QCD:

$$\frac{\alpha_s|eB|}{\mu} \approx 1\left(\frac{B}{10^{17}\,\mathrm{G}}\right)\left(\frac{400\,\mathrm{MeV}}{\mu}\right)\mathrm{MeV}$$

Dirac semimetals

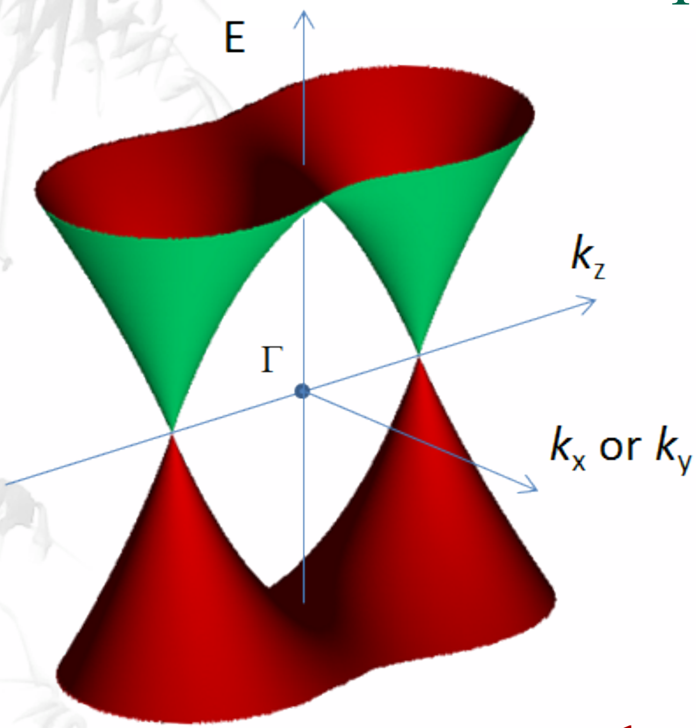
- Solid state materials with Dirac quasiparticles:
  - $Bi_{1-x}Sb_x$ alloy



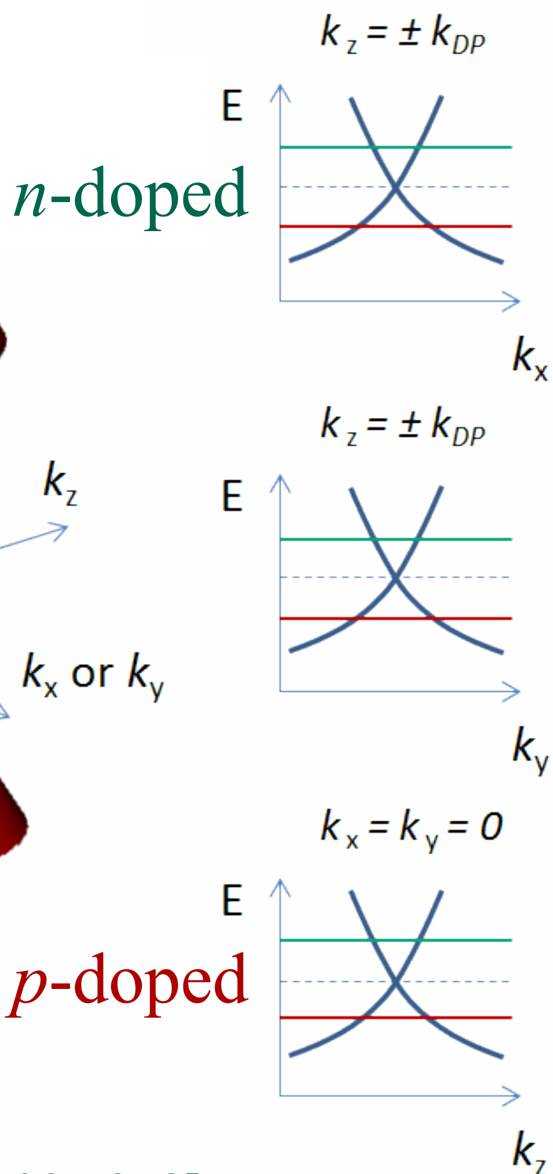
- "New" 3D Dirac materials (ARPES):
  - Na_3Bi [Z. K. Liu et al., arXiv:1310.0391]
  - Cd_3As_2 [M. Neupane et al., arXiv:1309.7892] [S. Borisenko et al., arXiv:1309.7978]

Cadmium arsenide

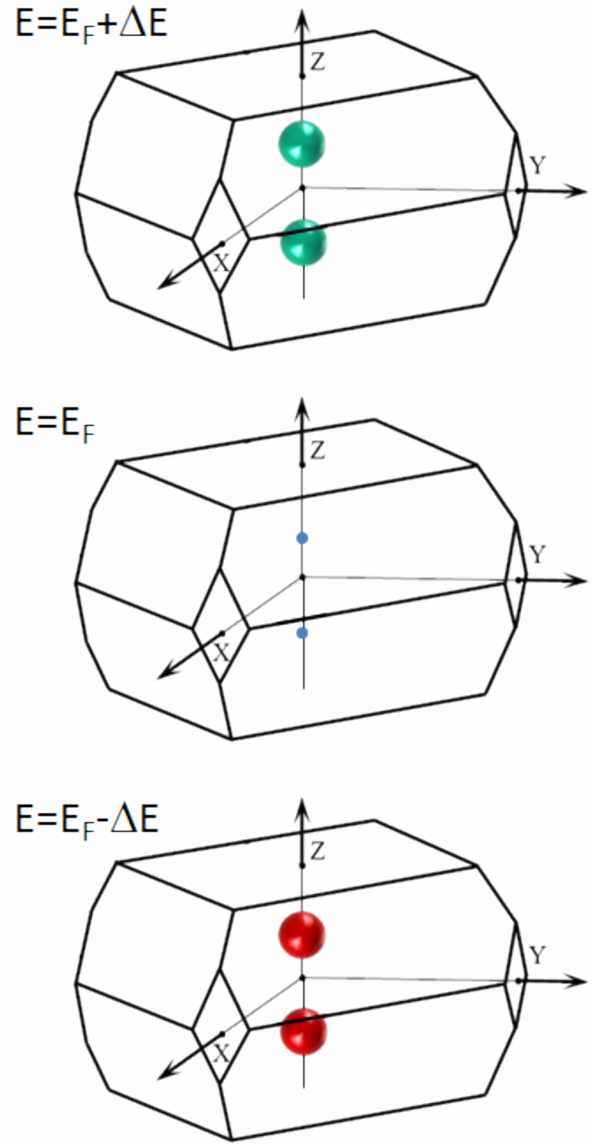
3D Dirac semimetal Cd_3As_2



Dispersion

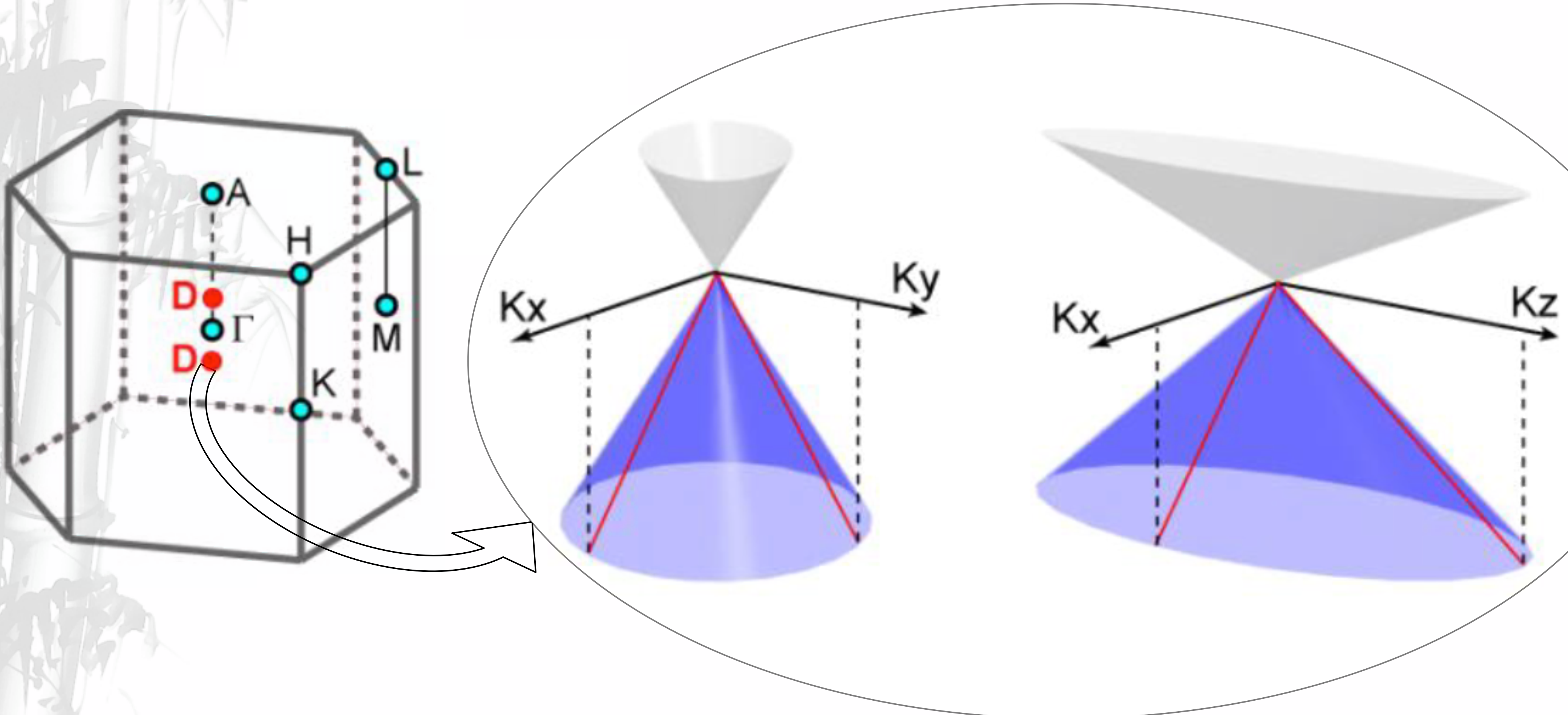


Fermi surface



[S. Borisenko et al., arXiv:1309.7978]

Potassium bismuthide



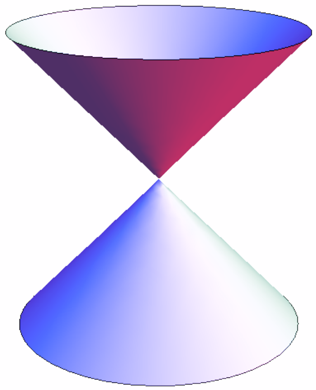
In the vicinity of 3D Dirac points:

$$E = v_x k_x + v_y k_y + v_z k_z$$

[Z. K. Liu et al., arXiv:1310.0391]

μ=0: Semimetal → Insulator

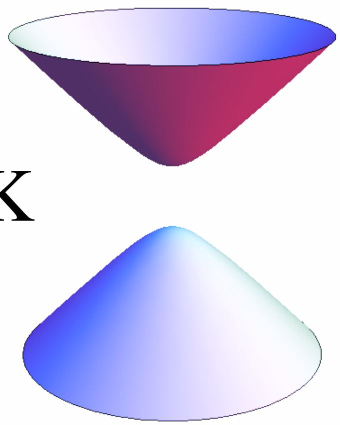
- Doping → neutrality point (μ=0)



Magnetic catalysis

[Shovkovy, Lect. Notes Phys. **871** (2013) pp. 13-49]

- Magnetic field B and small temperature: mass gap generation



$$m_{\rm dyn} \sim 10^{-3}\sqrt{|eB|} \approx 8\times 10^{-3}\sqrt{B[T]}\ \mathrm{eV} \approx 90\sqrt{B[T]}\ \mathrm{K}$$

(assuming that coupling constant α ≈ 1)

- Experimental signatures are expected in transport measurements

μ≠0: Dirac → Weyl semimetal

- Hamiltonian of a Dirac semimetal

$$H^{(D)} = \int d^3r\,\overline{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\nabla}\right) - \mu_0\gamma^0\right]\psi + H_{\text{int}}$$

cf. Weyl semimetal

"chiral shift"

$$H^{(W)} = \int d^3r\,\overline{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\nabla}\right) - \left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5 - \mu_0\gamma^0\right]\psi + H_{\text{int}}$$

- In a Dirac semimetal, a nonzero chiral shift $\vec{b}$ will be induced when $B\neq 0$, i.e.,

$$\vec{b} \propto -\frac{g}{v_F^2 c}\mu_0 e\vec{B}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B **88**, 165105 (2013)]

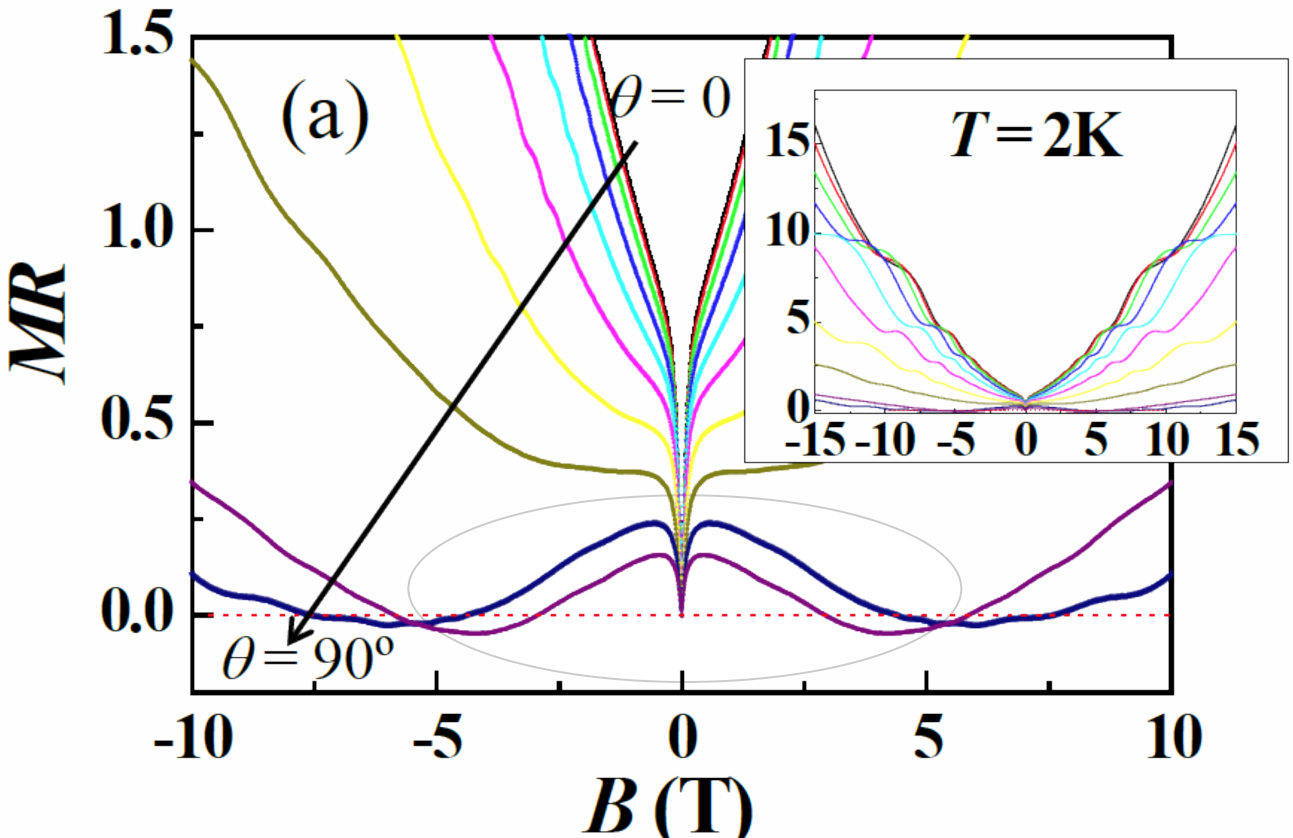
Negative magnetoresistance

- ρ_{33} is expected to decrease with B because

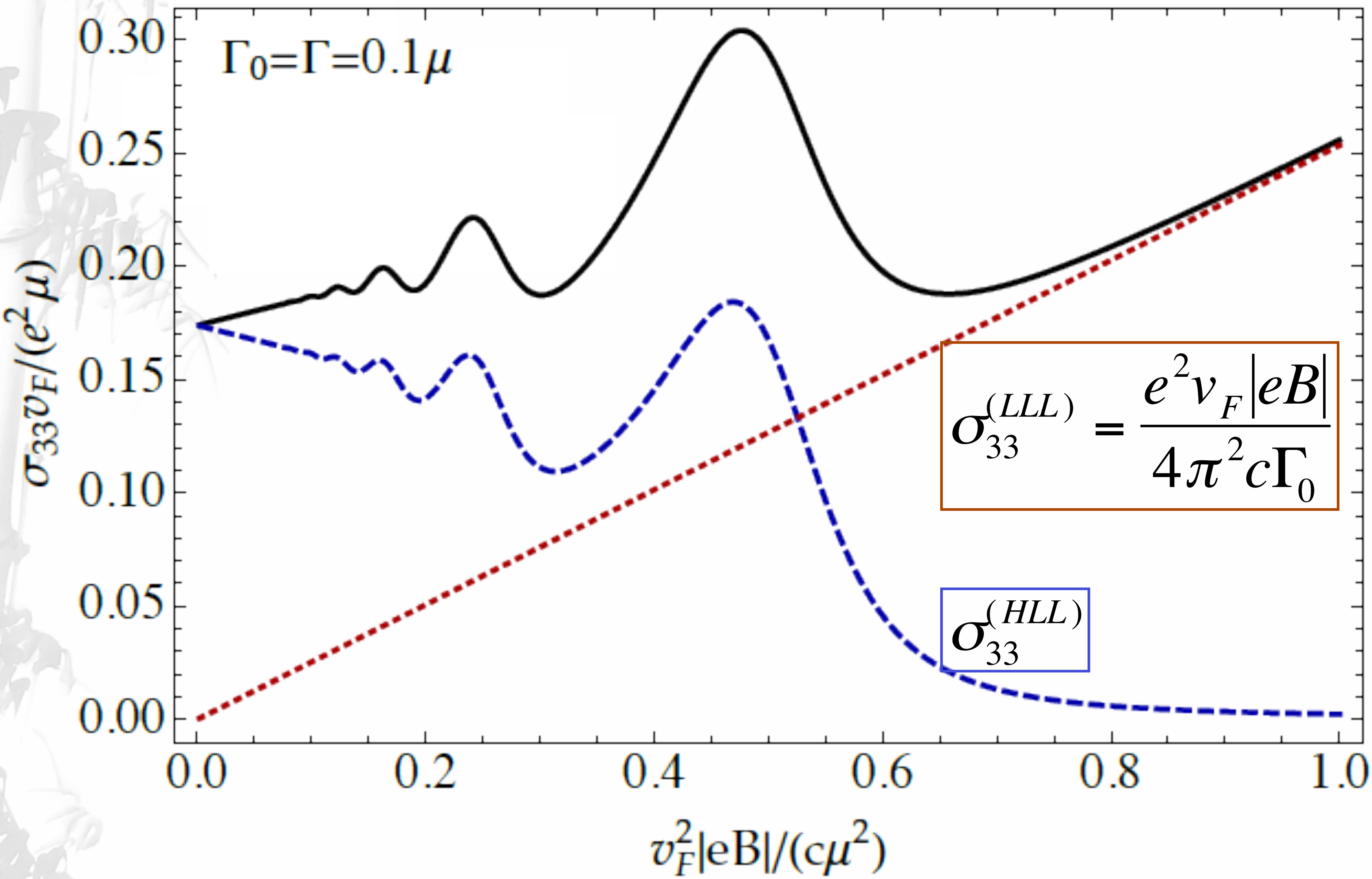
  $\sigma_{33} \propto B^2$ (weak B) [Son & Spivak, Phys. Rev. B 88, 104412 (2013)]

  $\sigma_{33} \propto B$ (strong B) [Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]

- Experimental confirmation (?) [Kim, et al., arXiv:1307.6990]



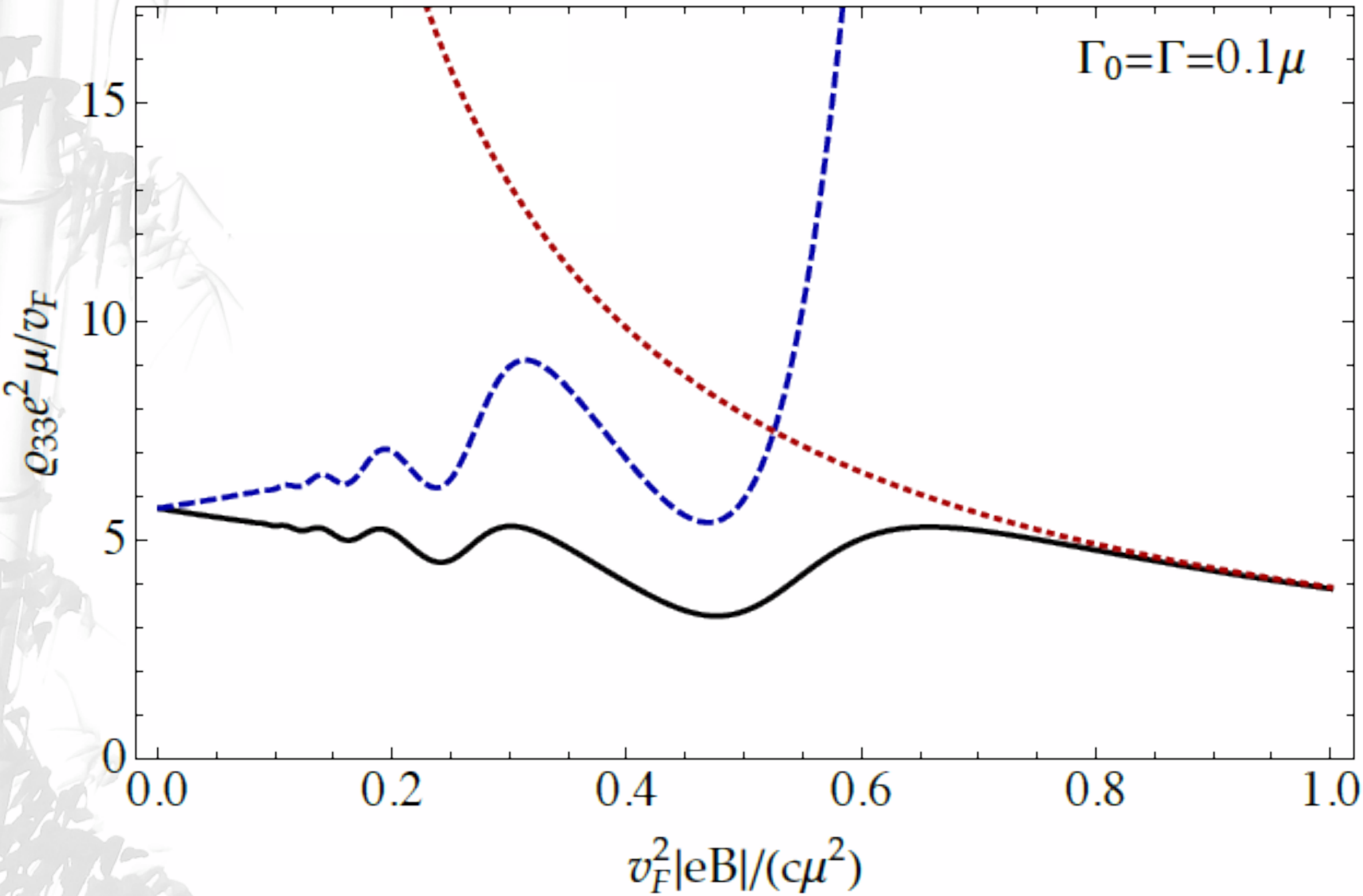
Longitudinal conductivity



- σ_{33} grows with *B* even in Dirac semimetals (*b*=0)

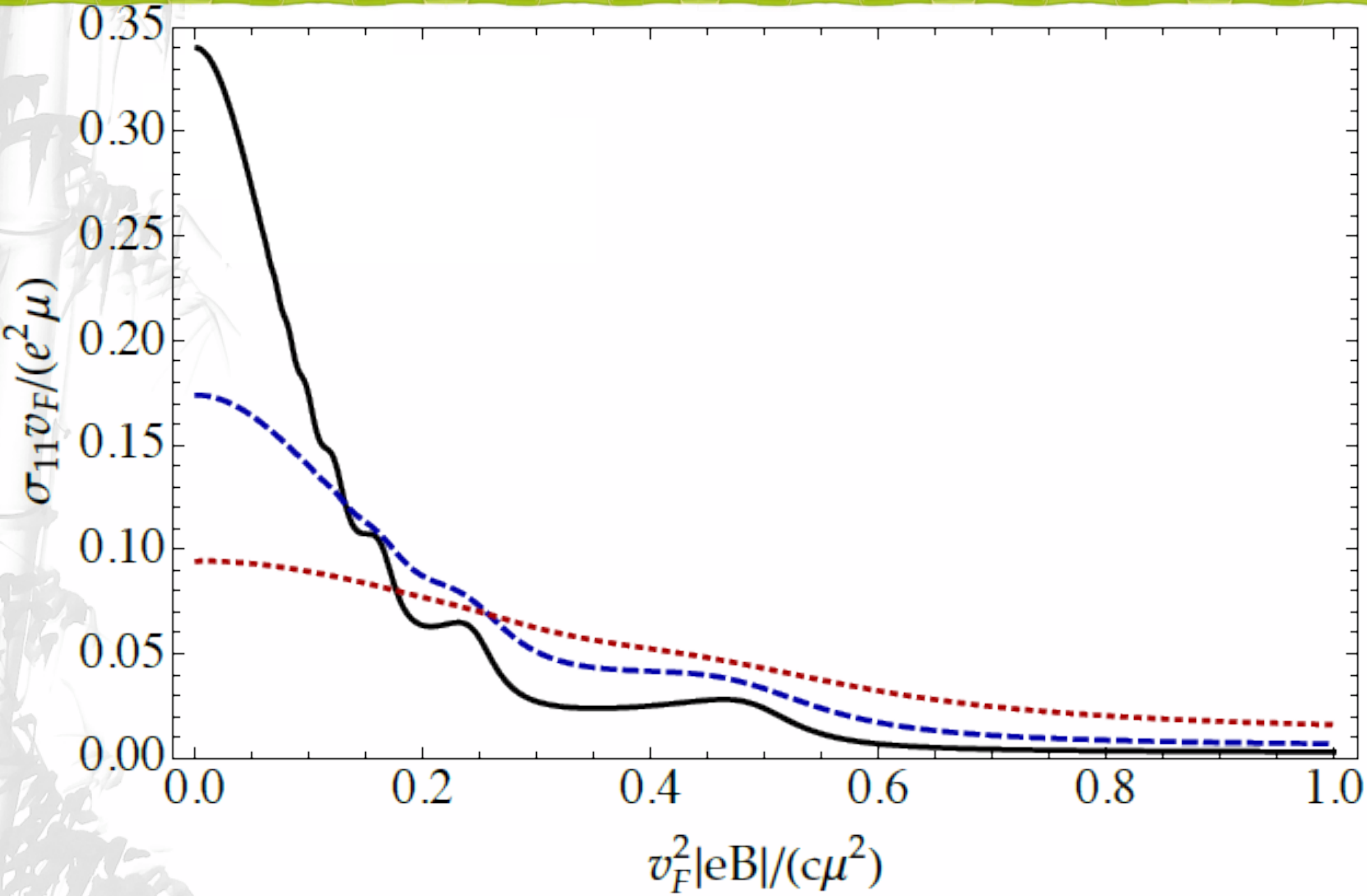
[Gorbar, Miransky, Shovkovy, Phys. Rev. B **89**, 085126 (2014)]

Longitudinal resistivity



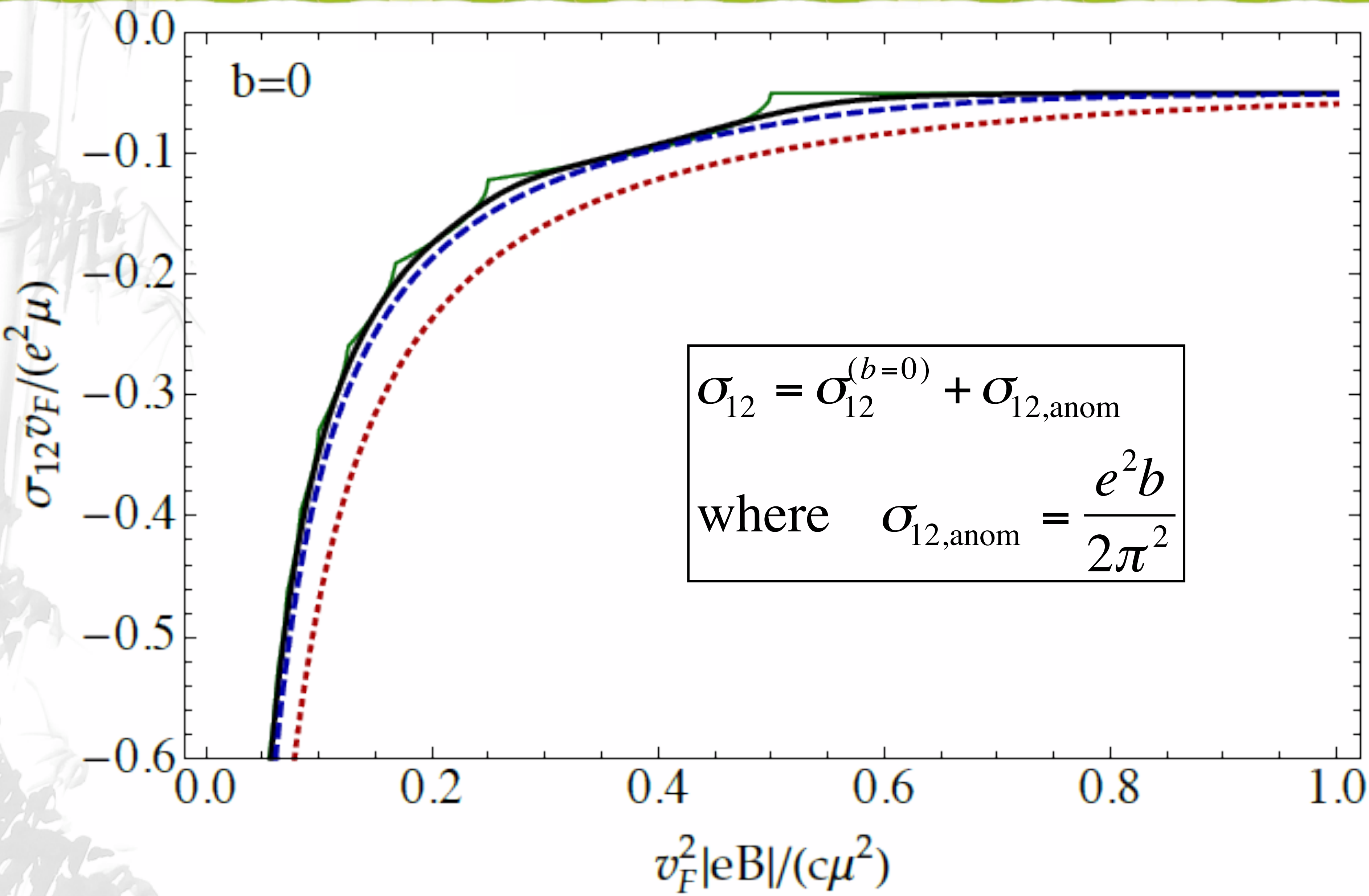
[Gorbar, Miransky, Shovkovy, Phys. Rev. B **89**, 085126 (2014)]

Transverse diagonal σ_{11}



[Gorbar, Miransky, Shovkovy, Phys. Rev. B **89**, 085126 (2014)]

Transverse off-diagonal σ_{12}



$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\text{anom}}$$

where $\sigma_{12,\text{anom}} = \dfrac{e^2 b}{2\pi^2}$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B **89**, 085126 (2014)]

Summary

- Chiral asymmetry is **intrinsic** property of LLL
- **Interactions** promote asymmetry from LLL to higher LLs
- Chiral **shift** is the measure of the asymmetry
- Chiral asymmetry shifts the L-handed and R-handed **Fermi surfaces** along **B**-field direction
- Nonzero **B** turns Dirac → Weyl matter
- In condensed matter: Dirac → Weyl metals