ARIZONA STATE UNIVERSITY

QUARK COMPACT STARS
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Chiral asymmetry in magnetized stellar matter

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Chiral effects at B≠0

- Chiral magnetic effect

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Vilenkin, Phys. Rev. D 22, 3080 (1980)]
[Metlitski, Zhitnitsky, hep-ph/0505072]
[Newman, Son, hep-ph/0510049]
...

- Chiral separation effect

$$\langle \vec{j}_5 \rangle_{\text{free}} = -\frac{e\vec{B}}{2\pi^2} \mu$$

[Kharzeev, Zhitnitsky, arXiv:0706.1026]
[Kharzeev, McLerran, Warringa, arXiv:0711.0950]
[Fukushima, Kharzeev, Warringa, arXiv:0808.3382]
...

- Chiral magnetic wave

$$\delta\mu \to \delta j_5 \to \delta\mu_5 \to \delta j \to \delta\mu \to \ldots$$

[Kharzeev, Yee, arXiv:1012.6026]
[Gorbar, Miransky, Shovkovy, arXiv:1101.4954]
[Burnier, Kharzeev, Liao, Yee, arXiv:1103.1307]

- etc

Spectrum at B≠0

- Dirac equation with massless fermions

$$\left[i\gamma^0 \partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

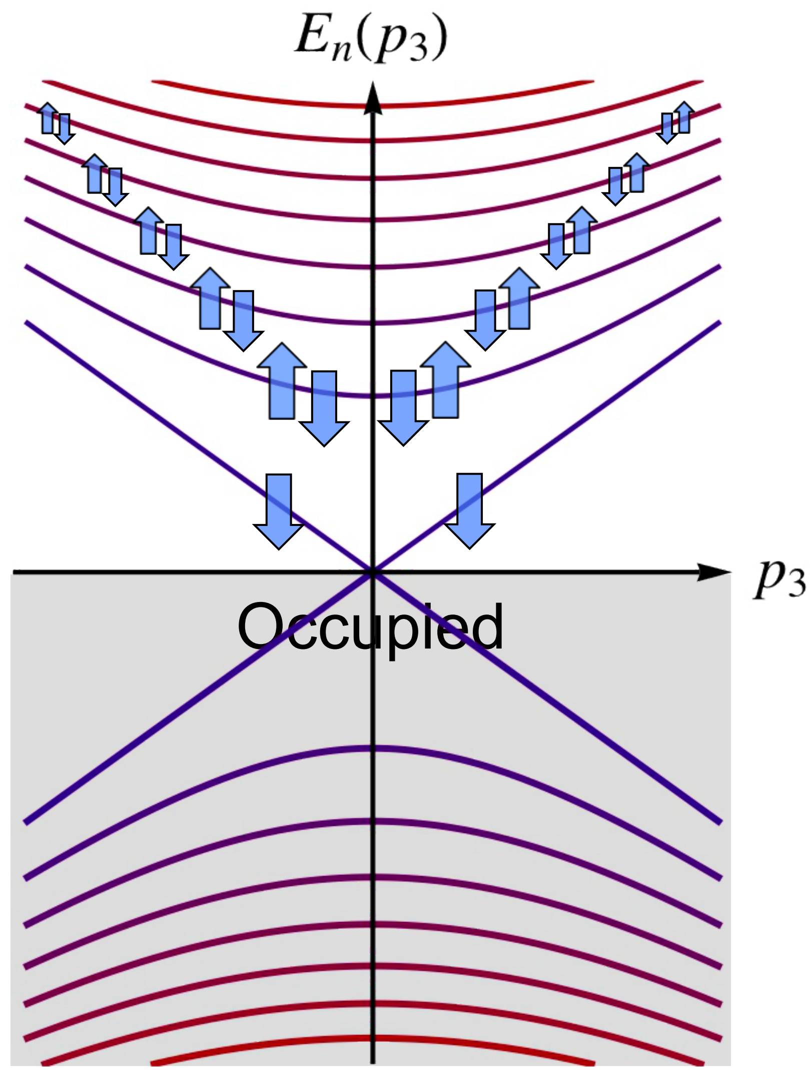
- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

$s = \pm 1/2$ (spin)

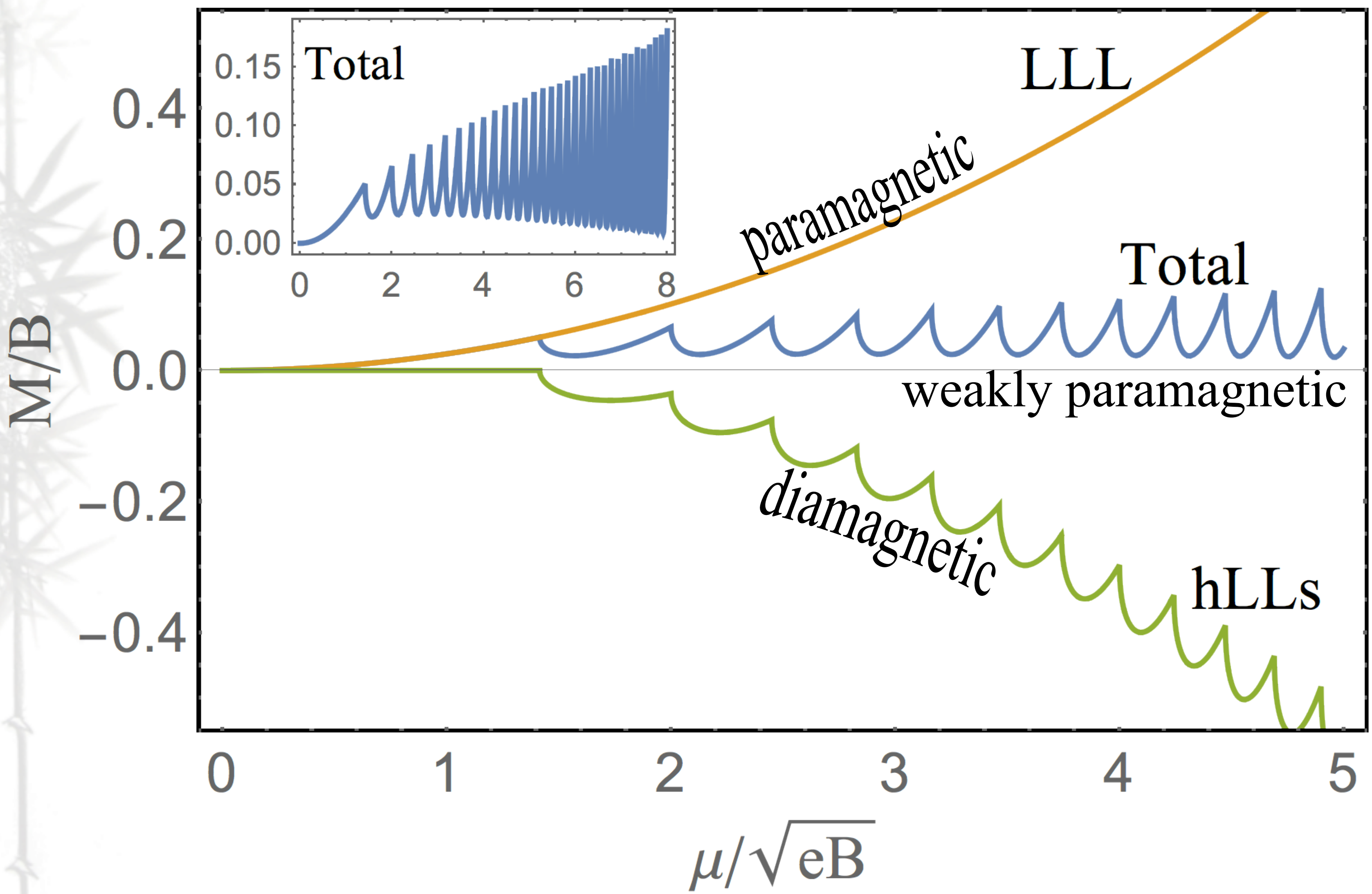
where $n = s + k + \frac{1}{2}$

$k = 0, 1, 2, \ldots$ (orbital)



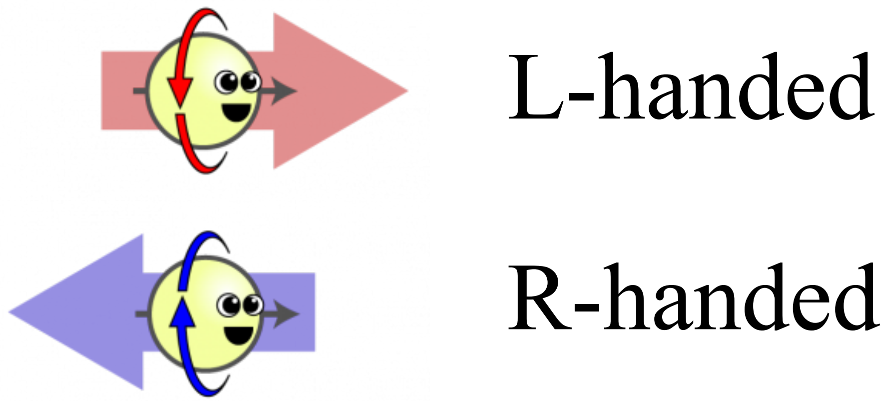
Relativistic magnetization

$$M = \frac{e\mu^2}{4\pi^2} + \frac{e}{2\pi^2}\sum_{n=1}^{n_{\max}}\left(\mu\sqrt{\mu^2 - 2neB} - 4n|eB|\ln\frac{\mu - \sqrt{\mu^2 - 2neB}}{\sqrt{2neB}}\right)$$

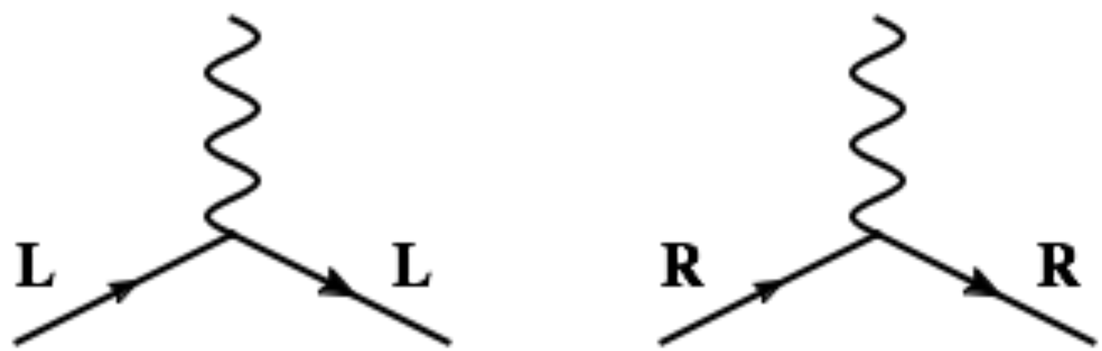


Spin vs. orbital motion

- Helicity/chirality of massless (ultrarelativistic) fermions is (≈) conserved



- Chirality does not change in elementary QED interactions



Asymmetry: LLL ➟ hLLs

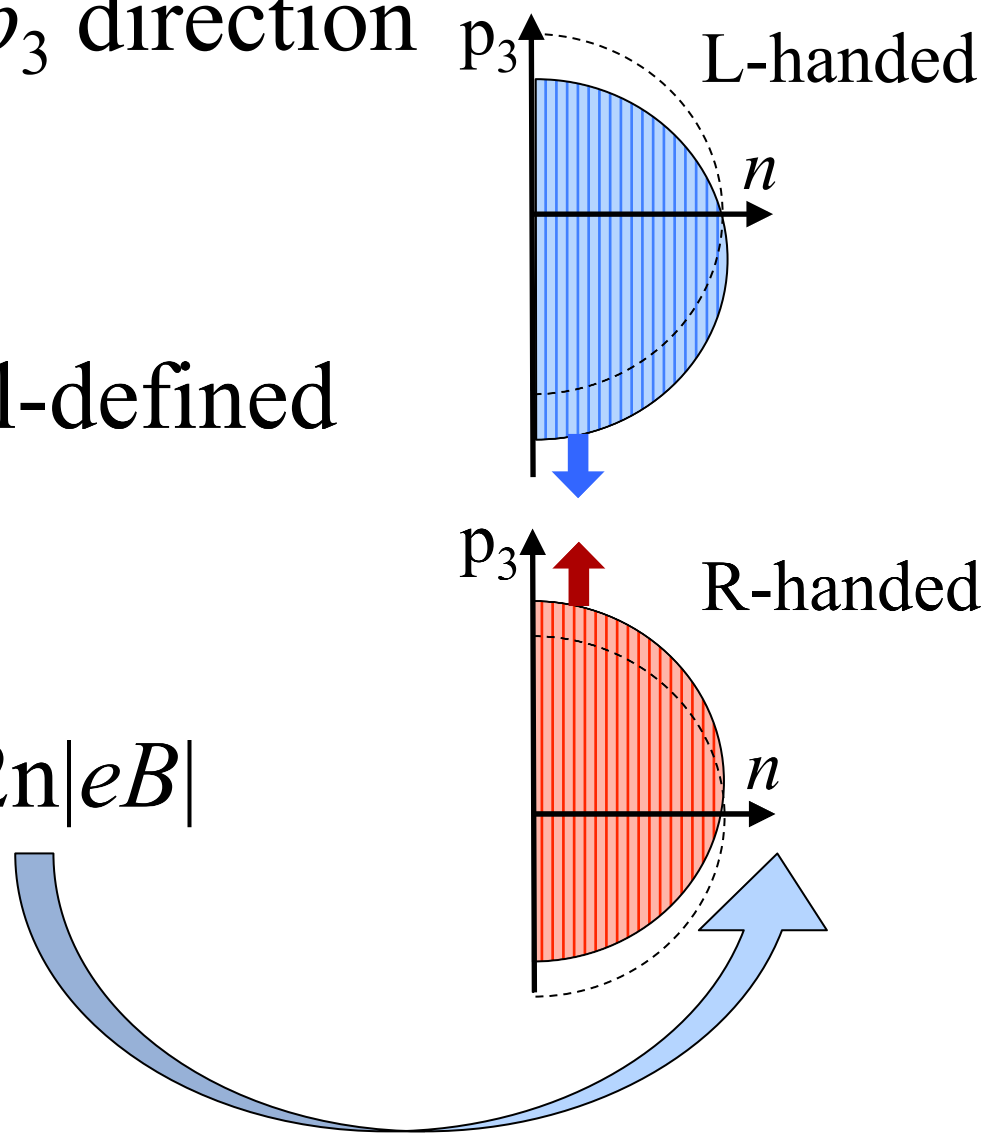
- LLL is spin polarized and chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are L-handed
  - states with $p_3>0$ (and $s=\downarrow$) are R-handed
- When scattering off LLL, particles in higher LLs must “flip” both spin & momentum
- B-field $\oplus$ interactions = chiral asymmetry

L-handed prefer $s=\downarrow$ and, thus, $p_3<0$

R-handed prefer $s=\downarrow$ and, thus, $p_3>0$

Chiral asymmetry

- Anticipated outcome: L- & R-handed Fermi surfaces shift in p_3 direction



Note: $\mathbf{p}_\perp$ is not well-defined

$p_\perp^2$ is replaced by $2n|eB|$

Chiral shift at low energies

- Ground state expectation value of the axial current (CSE)

$$\left\langle \bar{\psi}\gamma^3\gamma^5\psi \right\rangle = -\frac{eB}{2\pi^2}\mu \quad \text{with} \quad \vec{B} = (0,0,B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

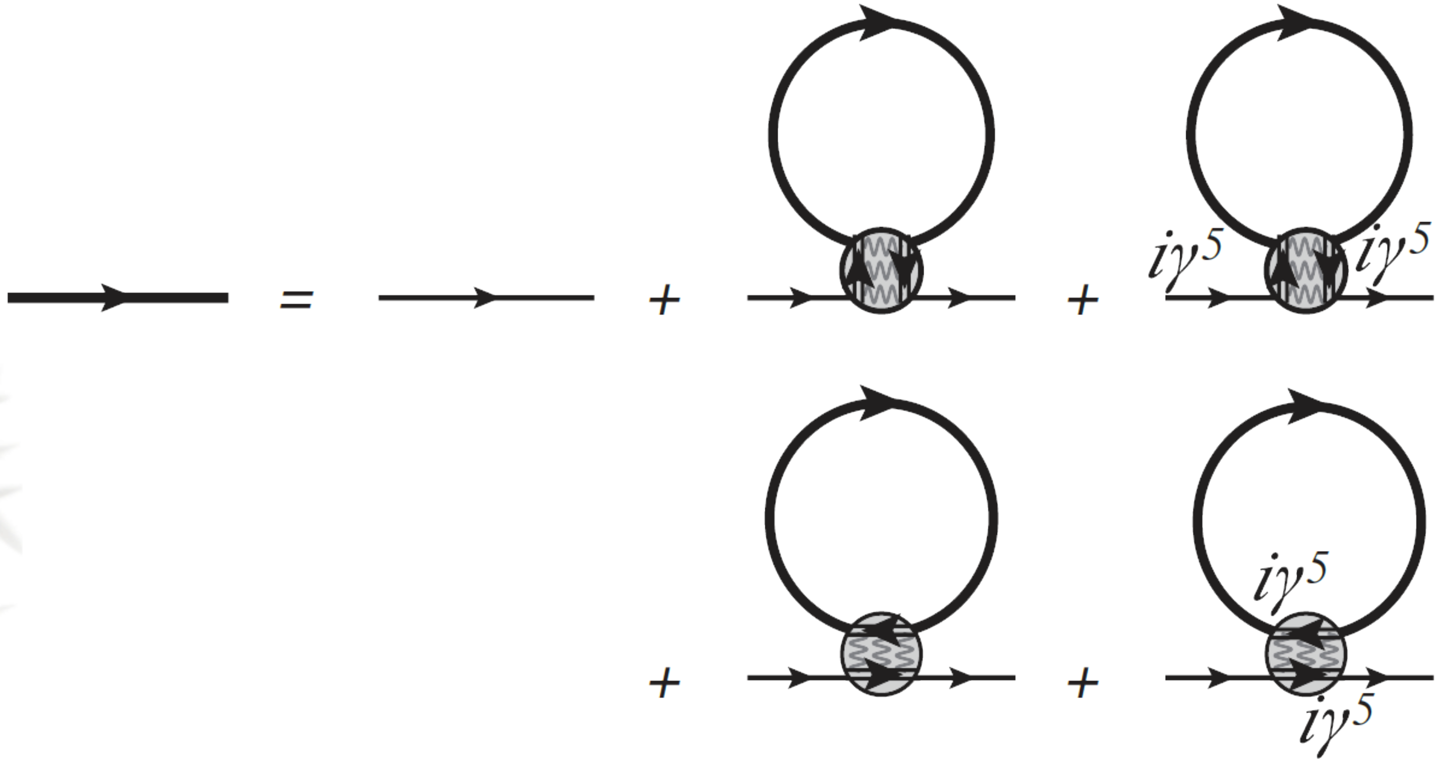
should induce a dynamical (chiral shift) parameter Δ associated with the condensate,

$$\delta L = \Delta\, \bar{\psi}\gamma^3\gamma^5\psi$$

(Δ=0 is not protected by any symmetry)

NJL model: quick check

- NJL model (local interaction)



- The equation for the chiral shift

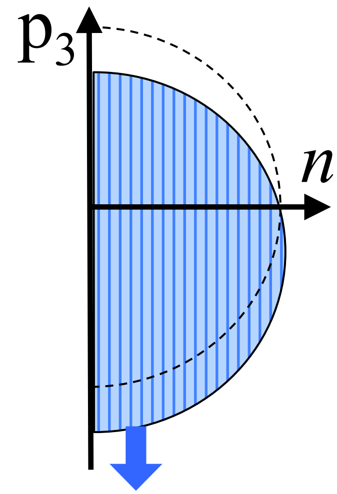
$$\Delta = -\frac{1}{2} G_{\text{int}} \left\langle \bar{\psi} \gamma^3 \gamma^5 \psi \right\rangle \approx \frac{G_{\text{int}} eB}{4\pi^2} \mu$$

Chiral shift @ Fermi surface

- Chirality is $\approx$ well-defined at Fermi surface ($|p_3| \gg m$)
- L-handed Fermi surface:

$$n>0: \quad p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

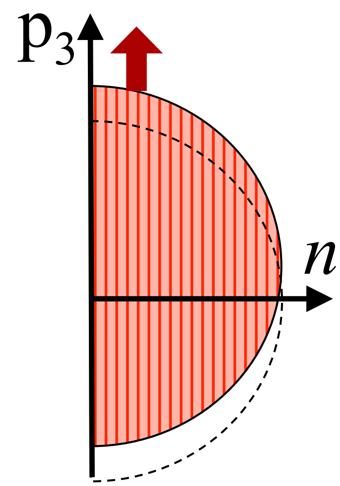
$$p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



- R-handed Fermi surface:

$$n>0: \quad p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

QED in weak field (B→0)

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta(p) + \gamma^0\gamma^5\mu_5(p)$$

Near Fermi surface ($p_0 \to 0$, $|\mathbf{p}| \to p_F$)

$$\Delta(p) \approx \frac{\alpha\, eB\,\mu}{\pi\, m^2}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

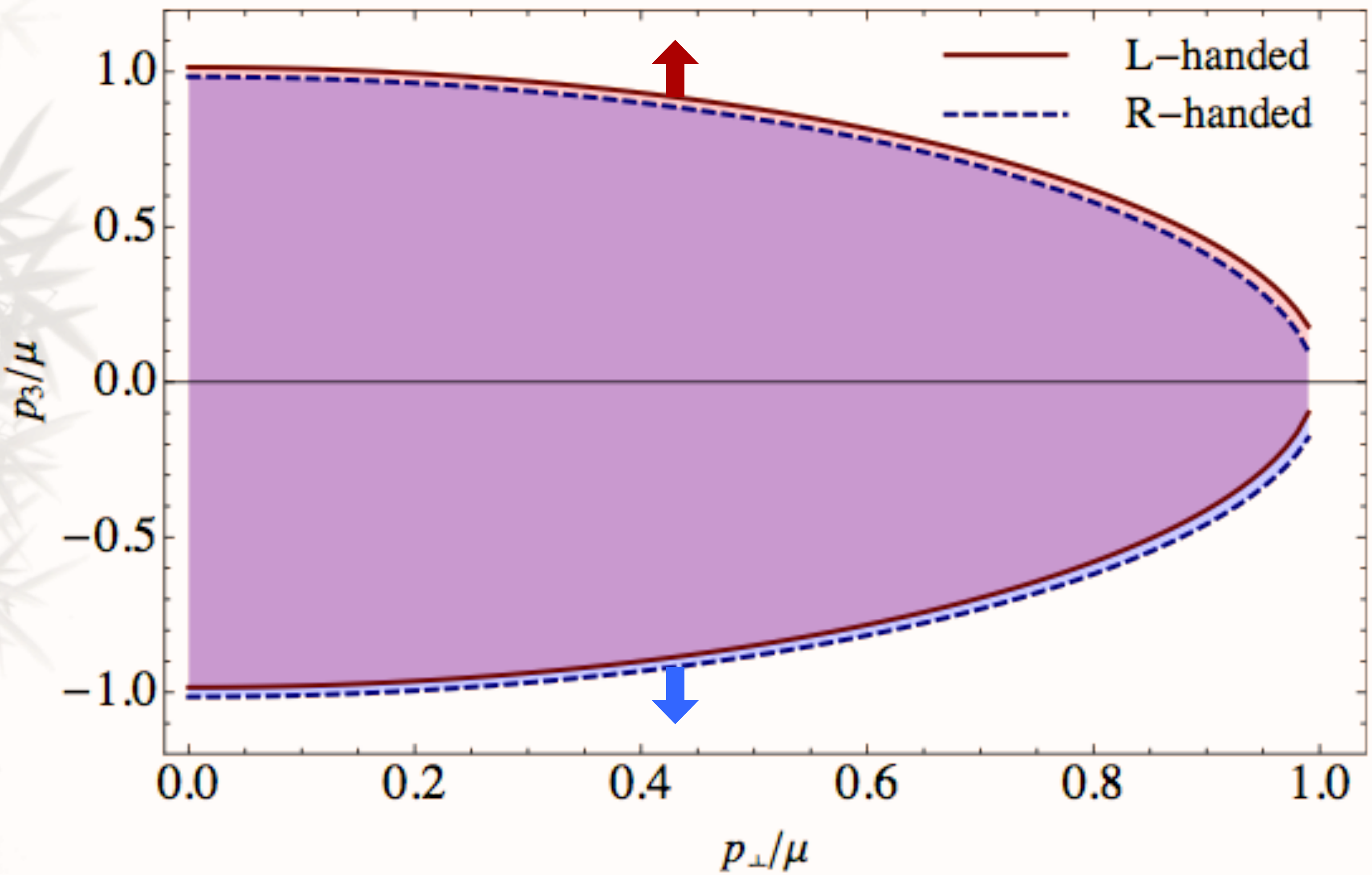
$$\mu_5(p) \approx -\frac{\alpha\, eB\,\mu}{\pi\, m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Dispersion relations in QED

- Let us use the condition (for a small B)

$$\mathrm{Det}\left[i\bar{S}^{-1}(p)+\bar{\Sigma}^{(1)}(p)\right]=0$$



[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

QED in strong field

Self-energy in the Landau-level representation:

$$\bar{\Sigma}(p) = 2e^{-p_\perp^2 l^2} \sum_{n=0}^{\infty} (-1)^n \left(-\gamma^0 \delta\mu_n - \gamma^3\gamma^5 \Delta_n - \gamma^0\gamma^5 \mu_{5,n} + m_n + \ldots\right)\left[P_- L_n - P_+ L_{n-1}\right] - \ldots$$

where $\delta\mu_n$, Δ_n, $\mu_{5,n}$, … are "projections" of the self-energy on the *n*th Landau level,

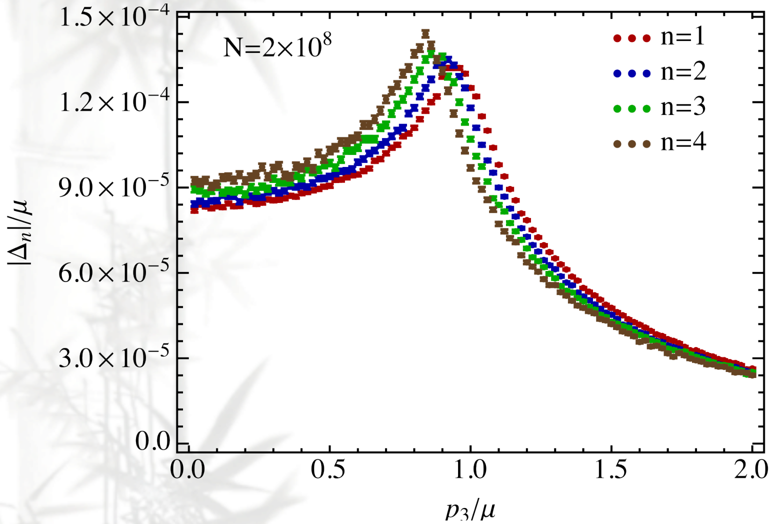
$$\Delta_n(p_0, p_3) = \frac{(-1)^n l^2}{8\pi} \int d^2 p_\perp e^{-p_\perp^2 l^2} \left[L_n + L_{n+1}\right] Tr\left[\gamma^0 \bar{\Sigma}(p)\right]$$

where

$$\bar{\Sigma}(p) = -4i\pi\alpha \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \bar{S}(k) \gamma^\nu D_{\mu\nu}(k-p)$$

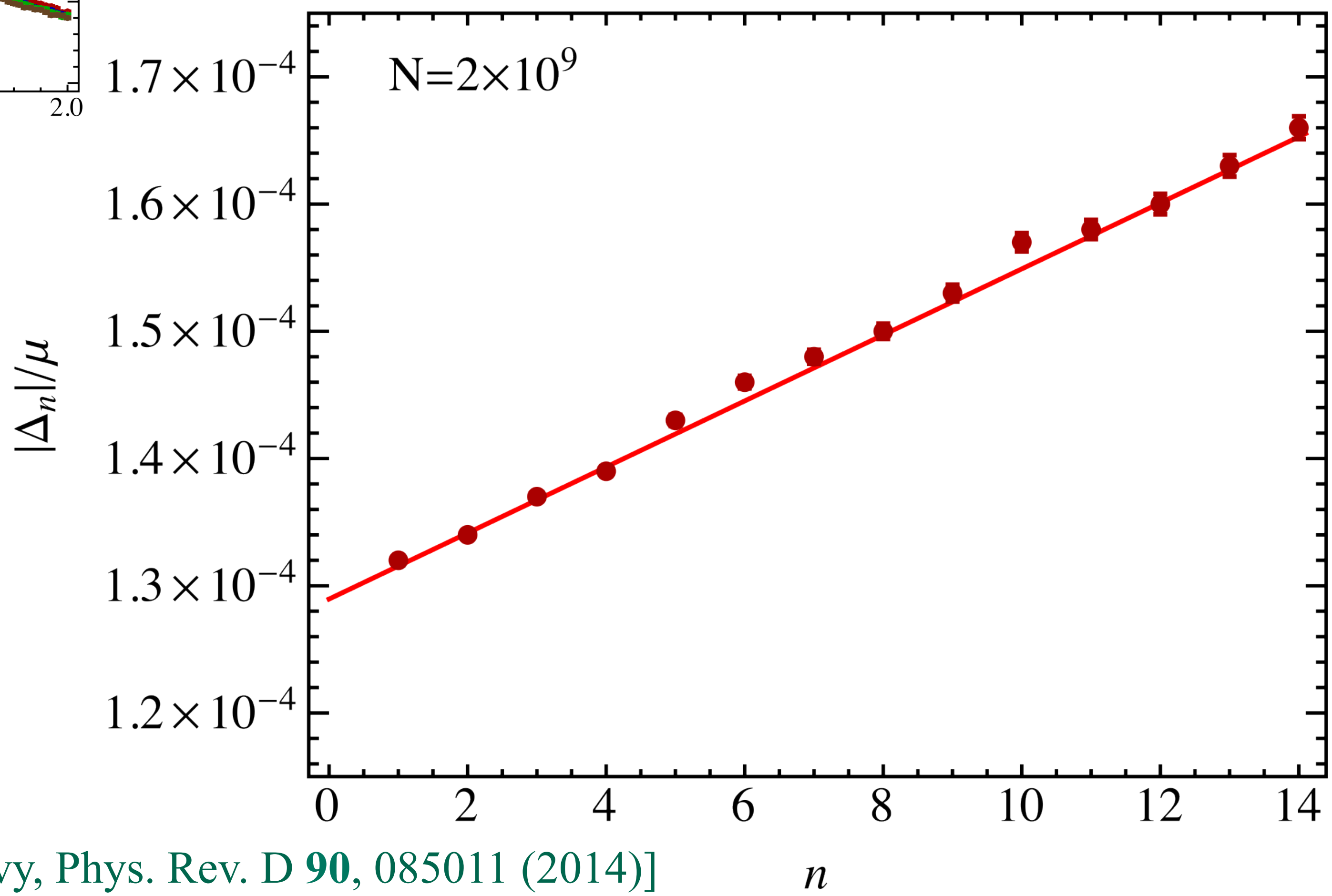
[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025025 (2013)]

QED in strong field: Δ_n



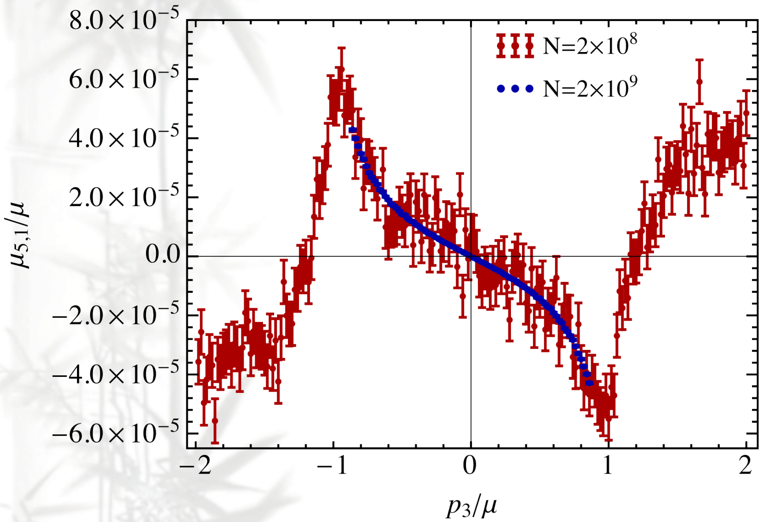
Model fit:

$$\Delta_n = -\frac{\alpha|eB|}{\mu}\left(0.53 + 0.32\frac{|eB|n}{\mu^2}\right)$$



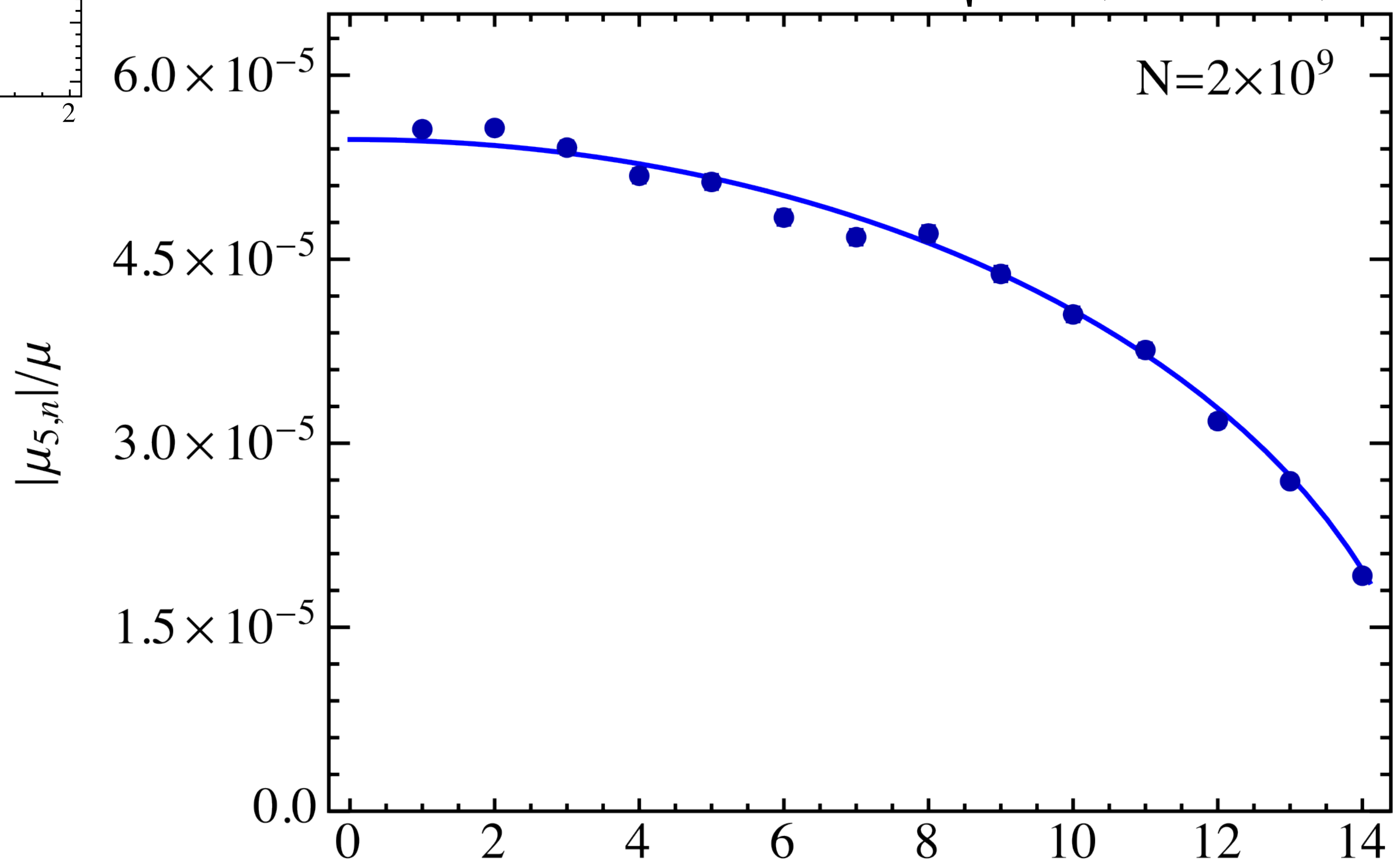
[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: $\mu_{5,n}$



Model fit:

$$\mu_{5,n} = -0.225\frac{\alpha|eB|}{\mu}\sqrt{1-\left(\frac{2n|eB|}{\mu^2}\right)^2}$$

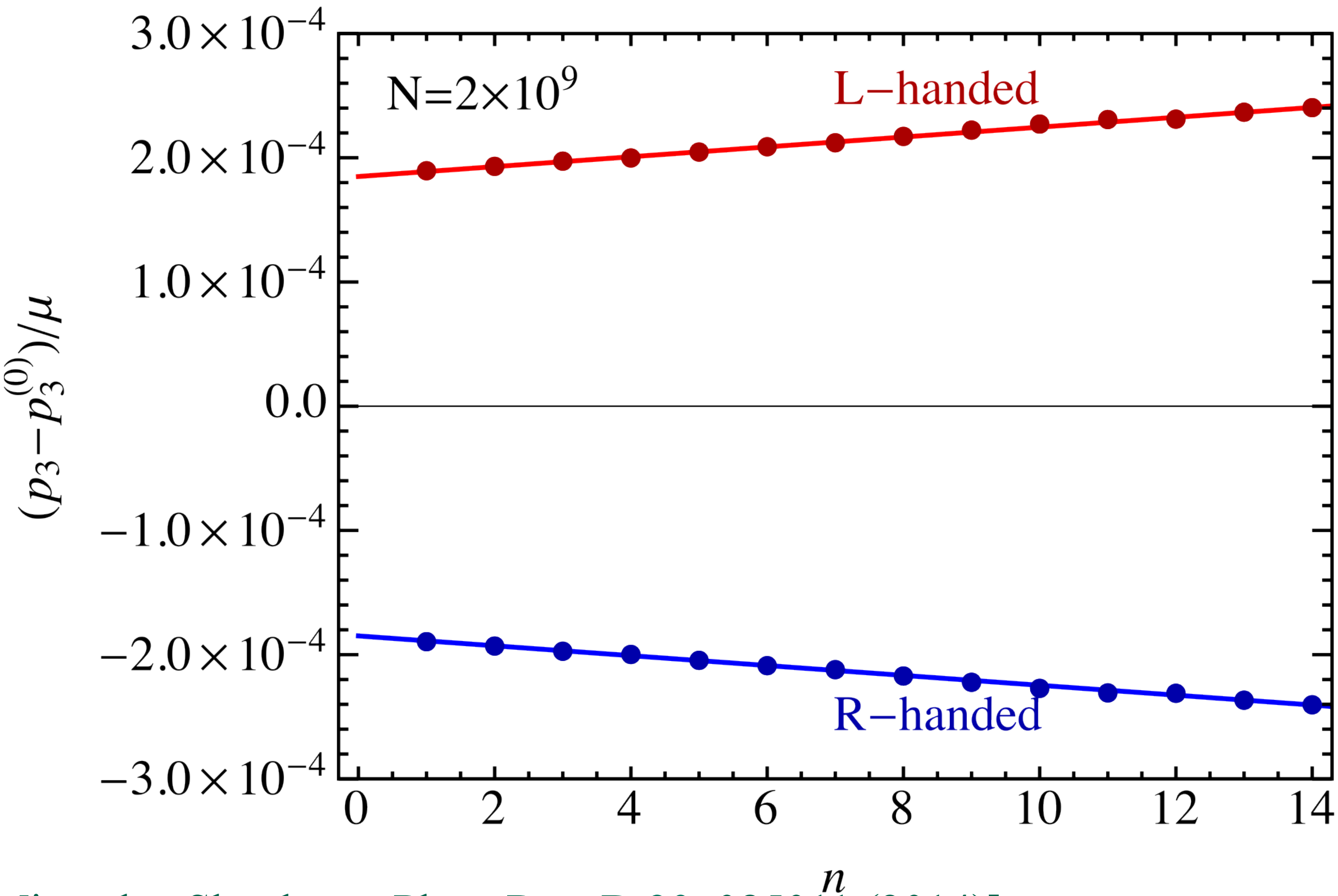


[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: δp_3

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu} \left(0.76 + 0.49 \frac{|eB| n}{\mu^2} \right)$$



[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

How large is the asymmetry?

In QED:

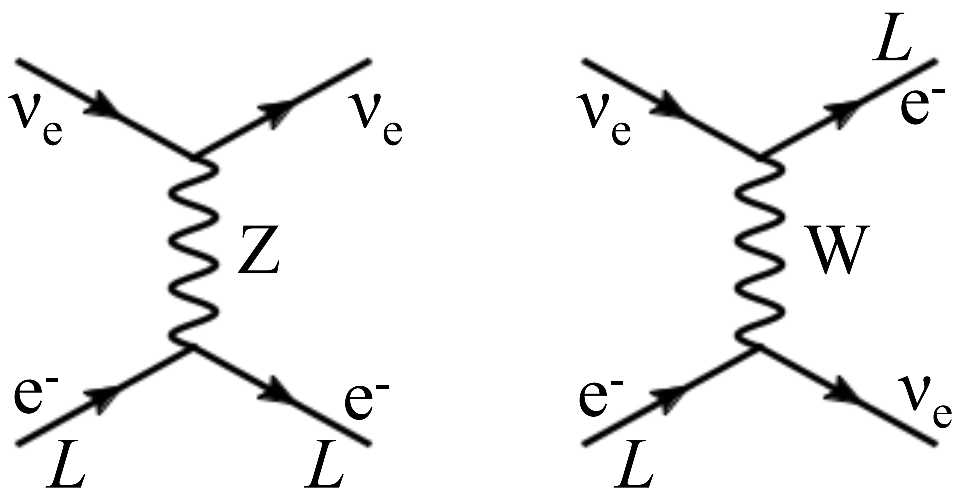
$$\frac{\alpha|eB|}{\mu} \approx 0.4\left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{100\,\mathrm{MeV}}{\mu}\right)\mathrm{MeV}$$

In QCD:

$$\frac{\alpha_s|eB|}{\mu} \approx 10\left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{400\,\mathrm{MeV}}{\mu}\right)\mathrm{MeV}$$

Neutrino asymmetry

- Neutrinos scatter off L-handed fermions only



- An asymmetric L-handed Fermi surface with

$$\delta p_3 \sim \alpha\,|eB|/\mu$$

should scatter ν_e's more preferably in the direction opposite to **B**

Total momentum estimate

- Total momentum carrier by L-handed fermions
  - QED (B=10^{18} G and μ=100 MeV):

$$P \sim N\,\delta p \sim 10^{55}\,\frac{\alpha\left|eB\right|}{\mu} \sim (1\,\mathrm{km/s})M_{Sun}$$

  - QCD (B=10^{18} G and μ=400 MeV):

$$P \sim N\,\delta p \sim 10^{55}\,\frac{\alpha_s\left|eB\right|}{\mu} \sim (30\,\mathrm{km/s})M_{Sun}$$

- Pulsar kicks? Possible, but questions remain…

Summary

- LLL chiral asymmetry plus **interactions** generate chiral shift/asymmetry in higher LLs
- Chiral asymmetry shifts the L-handed and R-handed **Fermi surfaces** along **B**-field direction
- Chiral asymmetry can produce asymmetric **neutrino emission** and generate **pulsar kicks**
- The mechanism is more promising for quark stars, but may even affect all compact stars