

RESEARCH AT THE TIP OF A PENCIL

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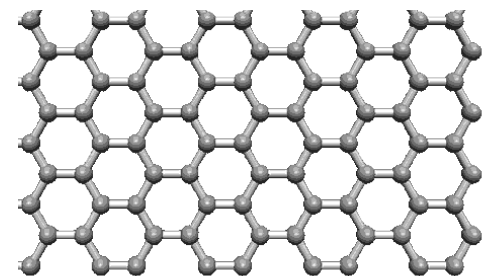
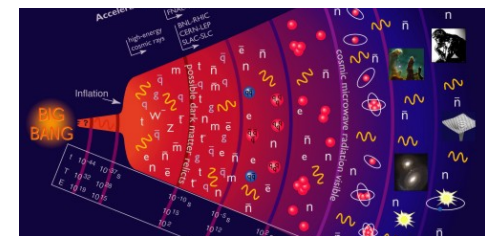
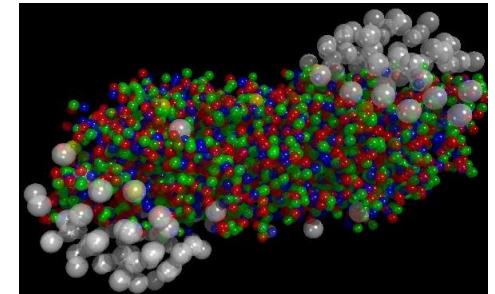
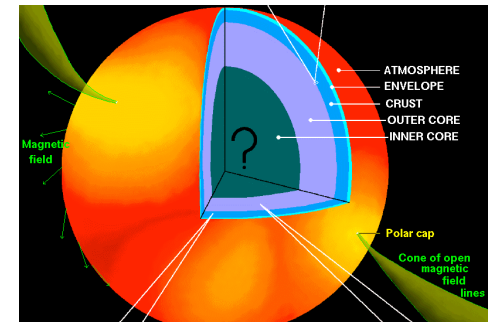
“If anybody says he can think about quantum physics without getting giddy, that only shows he has not understood the first thing about it.”

Niels Bohr

INTRODUCTION

- **Relativistic systems** (\sim speed of light)
- **Extreme densities** ($\sim 5n_0$, $n_0 \approx 3 \times 10^{17} \text{ kg/m}^3$)
- **Extreme temperatures** ($\sim 10^{12}$ Kelvin)
- **Extreme magnetic fields** ($\sim 10^{18}$ Gauss)

- Examples of relativistic matter
 - **Electrons, protons, quarks** inside compact stars (very dense matter)
 - **Quark gluon plasma** in heavy ion collisions (a very hot fireball)
 - **Hot matter** in the Early Universe (an extremely hot world)
 - **Massless particles** (e.g., quasiparticles in graphene and Dirac semimetals)





- **Non-relativistic**

- Particles move much slower than the speed of light
- Kinetic energies are much smaller than the rest energy

$$E_{\text{kin}} \ll E_{\text{rest}} : \quad E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

- **Relativistic**

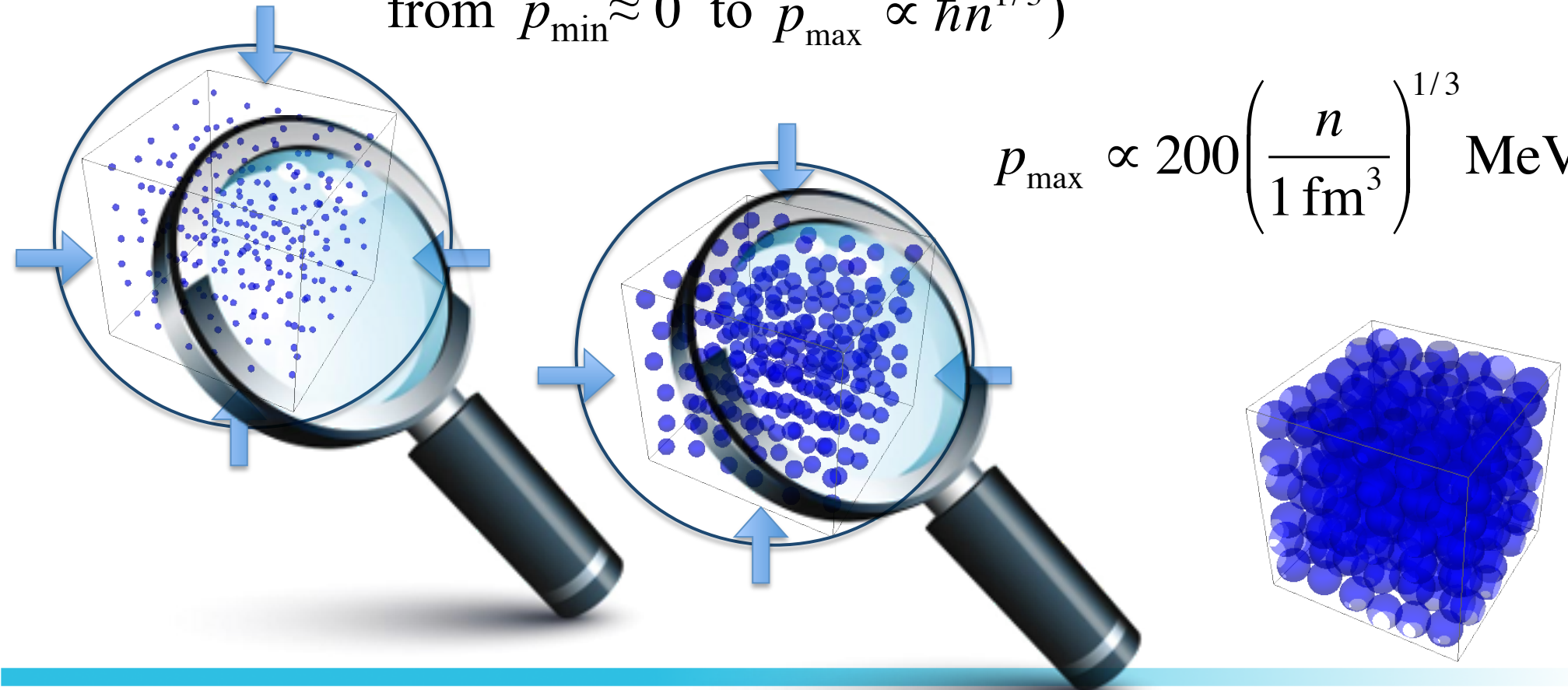
- Particle velocities approach the speed of light
- Kinetic energies are comparable to, or larger than E_{rest}

$$E_{\text{kin}} \geq E_{\text{rest}} : \quad E = c\sqrt{p^2 + m^2c^2} \approx cp$$

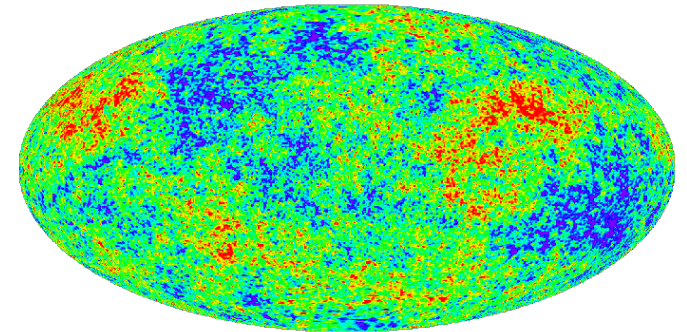
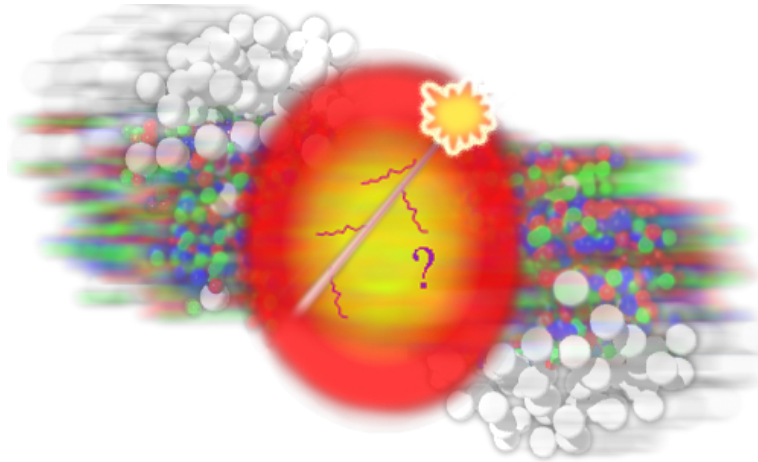
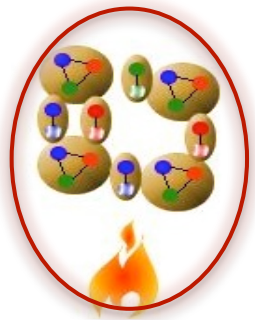
- What happens when you squeeze matter to very high density? (e.g., neutrons inside neutron stars)

Pauli exclusion principle: fermions cannot occupy same states (they end up filling out all quantum states from $p_{\min} \approx 0$ to $p_{\max} \propto \hbar n^{1/3}$)

$$p_{\max} \propto 200 \left(\frac{n}{1 \text{ fm}^3} \right)^{1/3} \text{ MeV}/c$$



- **What happens when you heat matter to very high temperature?** (e.g., matter in heavy ion collisions)



Heat is equivalent to **kinetic energy**: average kinetic energy of particles is proportional to temperature:

$$p \propto k_B T / c \sim 200 \left(\frac{k_B T}{200 \text{ MeV}} \right) \text{ MeV}/c$$

- **Can matter be made of massless particles?**

Yes! Electron quasiparticles masquerade as massless particles in some materials (no rest mass energy)

- **Examples:**

- Graphene



2D (planar) materials

- $\text{Bi}_{1-x}\text{Sb}_x$ alloy with $x \approx 0.03$

- cadmium arsenide Cd_3As_2

3D materials

- potassium bismuthide Na_3Bi

- Strong magnetic fields are common inside compact stars
 - 10^{10} to 10^{15} Gauss
- In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
 - 10^{18} to 10^{19} Gauss
- Early Universe
 - up to 10^{24} Gauss
- High Magnetic Field Laboratory
 - 4.5×10^5 Gauss

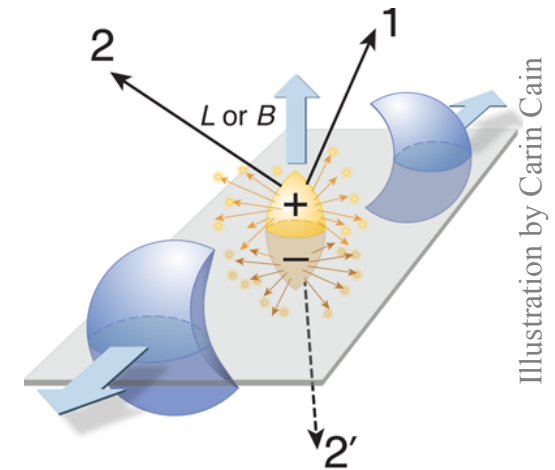
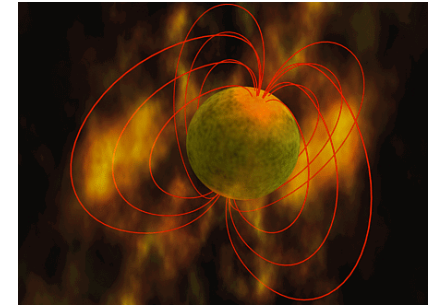


Illustration by Carin Cain

- Dynamical origin of mass
- Interplay of symmetry and dynamics
- Exotic phases of matter at extreme densities/
temperatures
- Effect of superstrong magnetic fields on
vacuum/matter

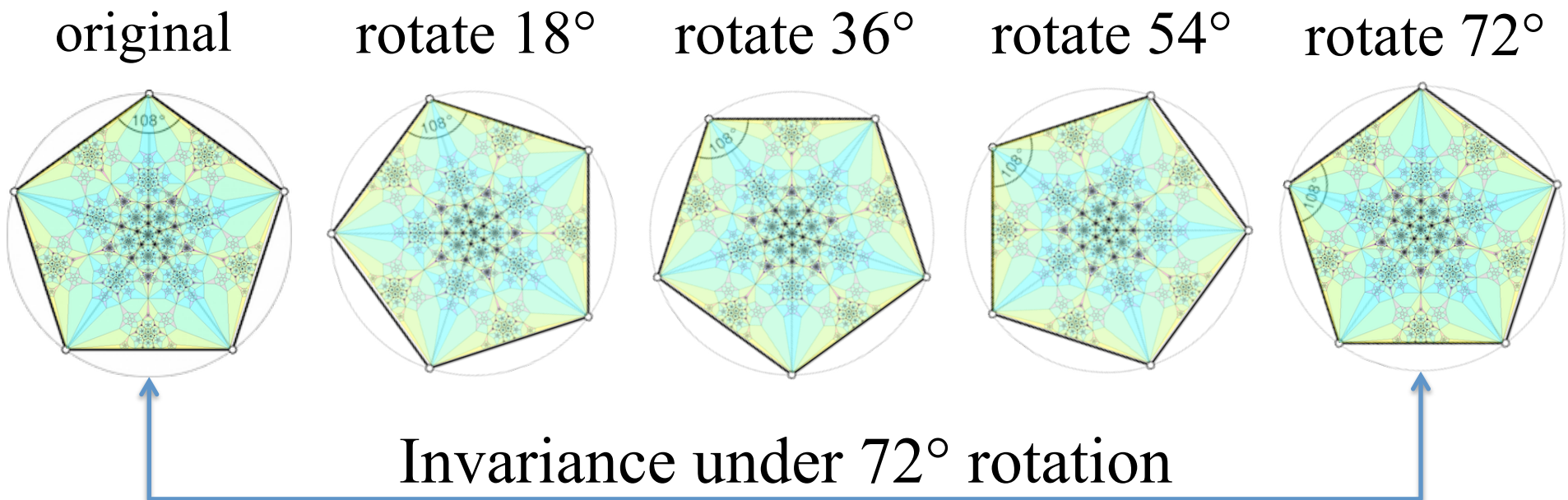
“It is only slightly overstating the case to say that physics is the study of symmetry”

Philip W. Anderson

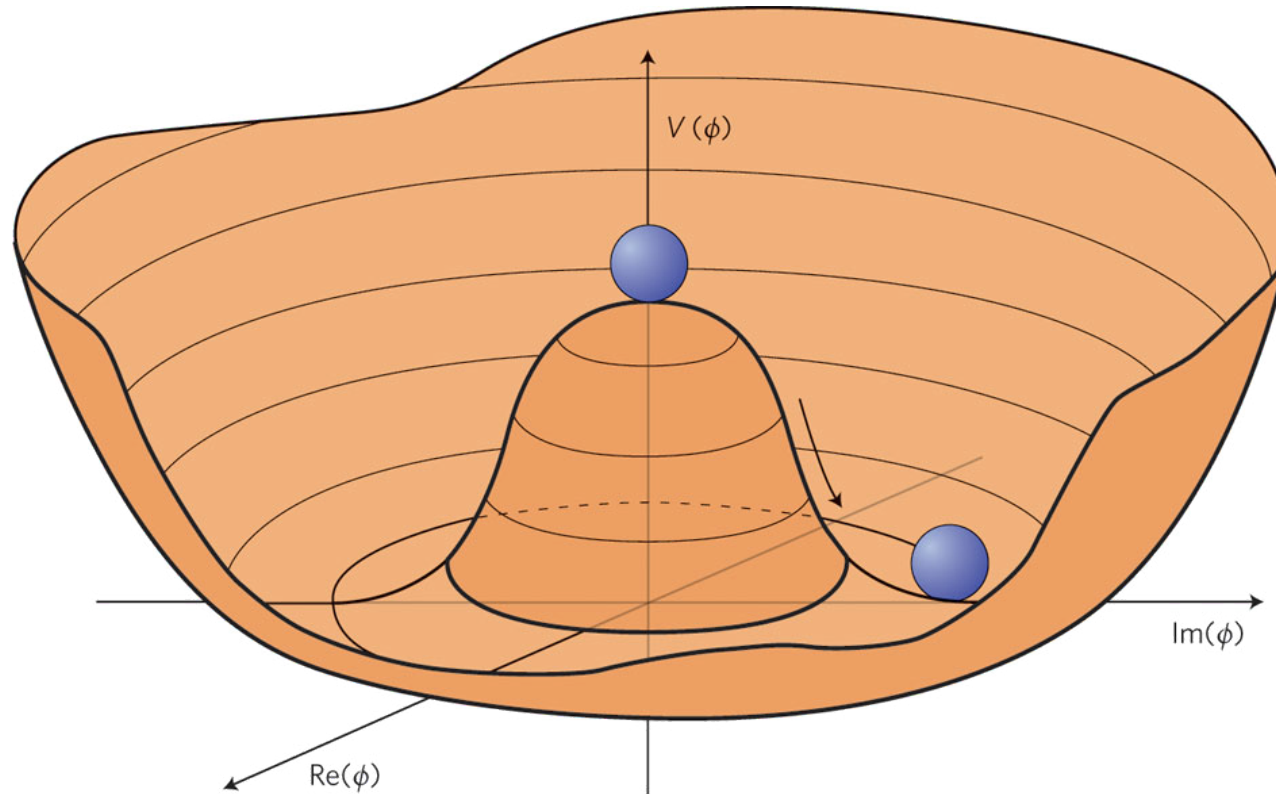
SYMMETRY & ORIGIN OF MASS

WHAT IS SYMMETRY?

- Wikipedia:
 - sense of harmonious and beautiful proportion and balance
 - invariance under certain transformation
- Example:



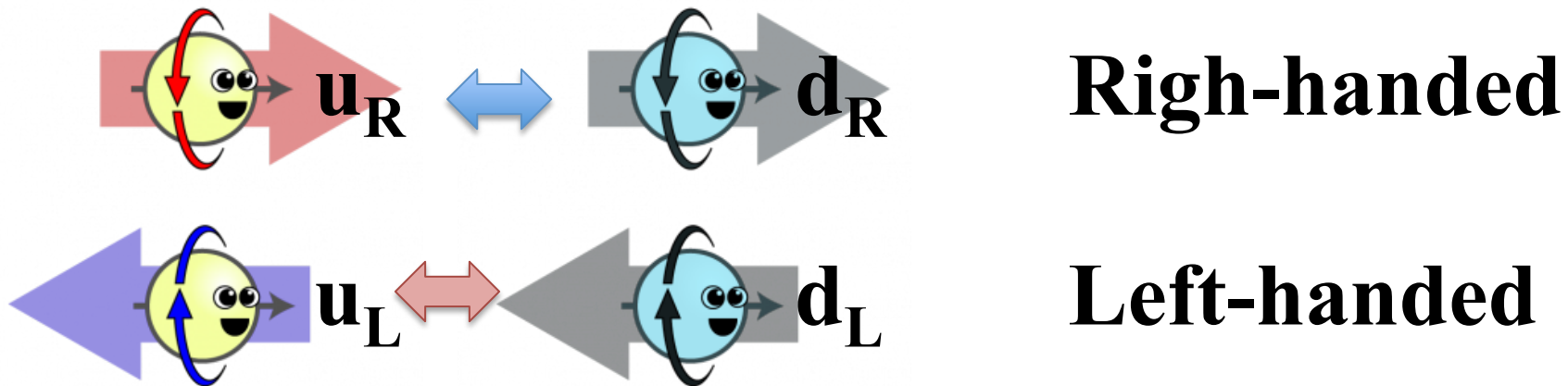
- Underlying laws are symmetric, but the system/ground state changes under a symmetry transformation



- Symmetry may refer to “internal” symmetries (e.g. rotations in color/flavor spaces, rescaling, etc.)

MASS VS. SYMMETRY

- **Massless** ($m=0$) fermions enjoy chiral symmetry (“rotation” of left-handed and right-handed particles in flavor space)

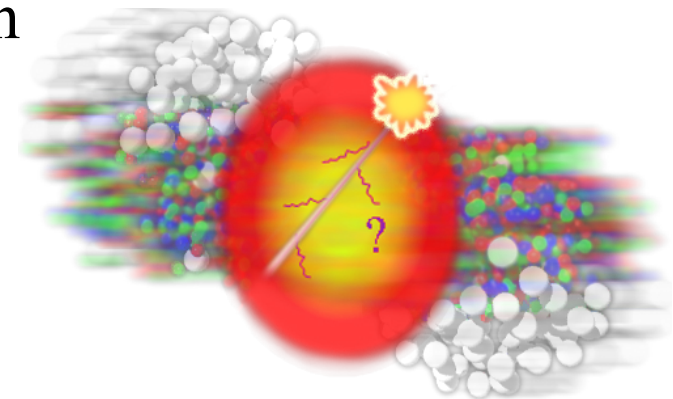
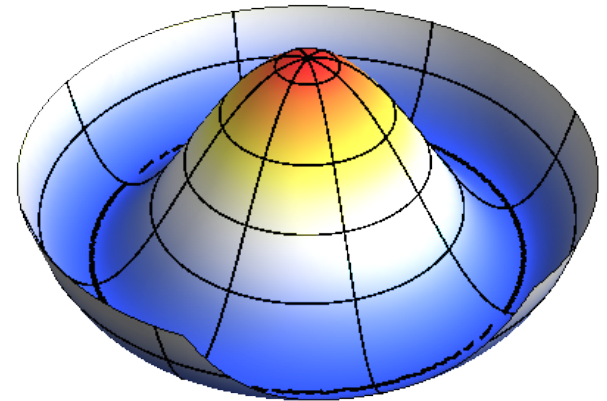


- Introduction of a **nonzero mass** for fermions breaks chiral symmetry

$$|m\rangle \propto C_1|L\rangle + C_2|R\rangle \quad \approx \quad \langle L | \text{Image} | R \rangle$$

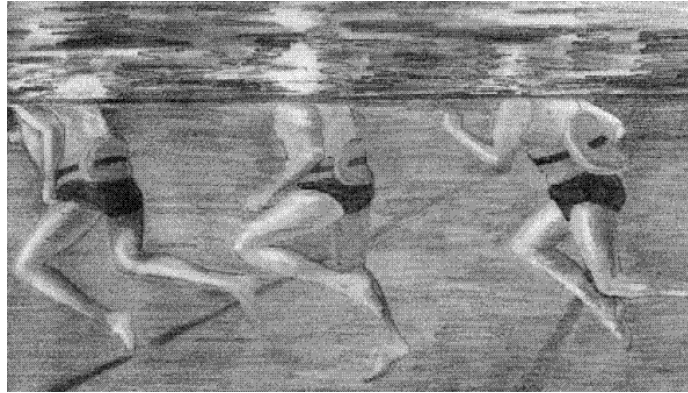


- Hadron properties strongly suggest an approximate chiral symmetry that is broken “spontaneously”
 - Mass hierarchy of pions and nucleons ($m_\pi \ll m_N$)
 - Energy dependence of pion-nucleon interaction
 - Discovery of multiplets of hadrons with similar properties
 - Relationships between various interaction strengths
 - Properties of quark-gluon plasma in heavy-ion collision

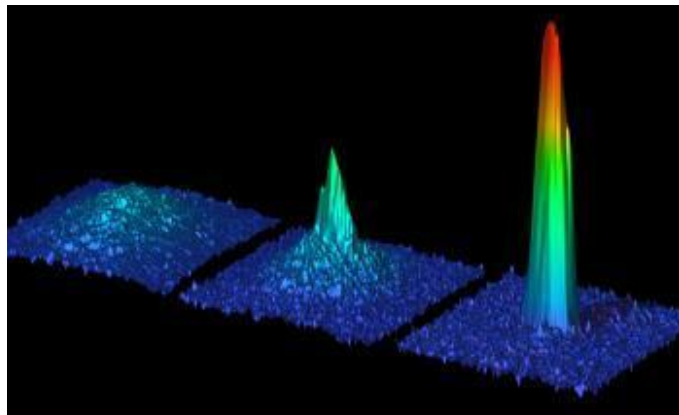


HOW SYMMETRY BREAKS?

- Because of strong interactions, almost massless quarks ($m \sim 5 \text{ Mev}/c^2$) become heavy ($m \sim 300 \text{ Mev}/c^2$)

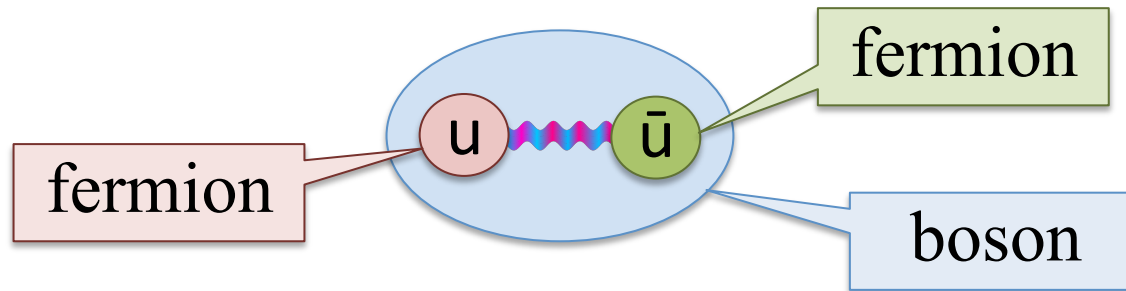


- The analogue of water is a certain (quark-antiquark) Bose condensate in the physical vacuum



SOME KEY POINTS

- Interaction must be sufficiently strong

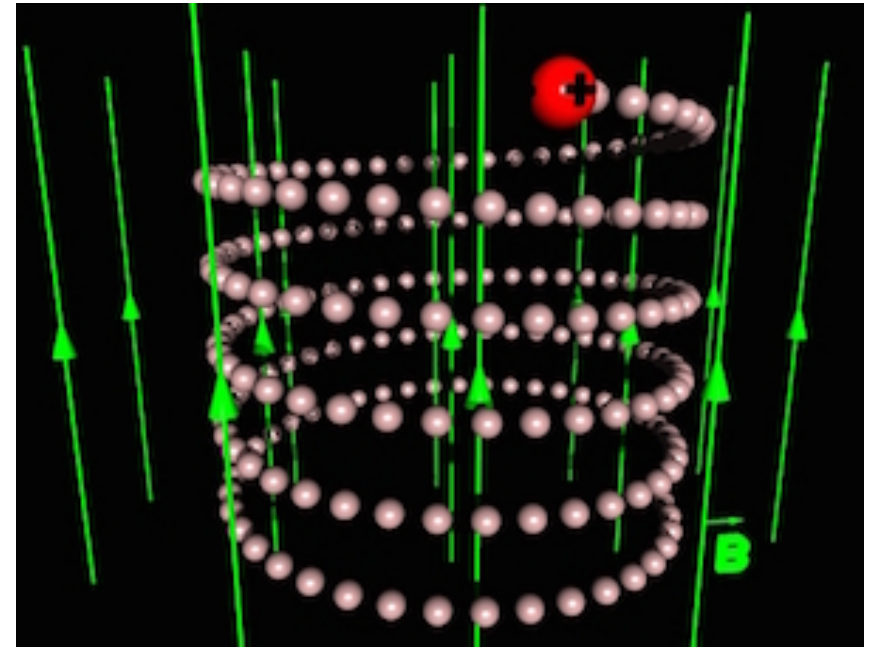


- Bose condensate fills/defines the vacuum
- Fermions (e.g., quarks, protons, etc.) become massive
- Light (massless) pions are predicted
- Mass can be “melted” away by high temperature/density

MAGNETIC CATALYSIS

Review: Lect. Notes Phys. 871 (2013) 13-49

- Magnetic field *constrains* perpendicular motion of charged particles
- Even *arbitrarily* weak attractive interaction is sufficient to form bound states
- Condensate forms & symmetry breaks down
- Fermions become massive!



This is the essence of **magnetic catalysis**

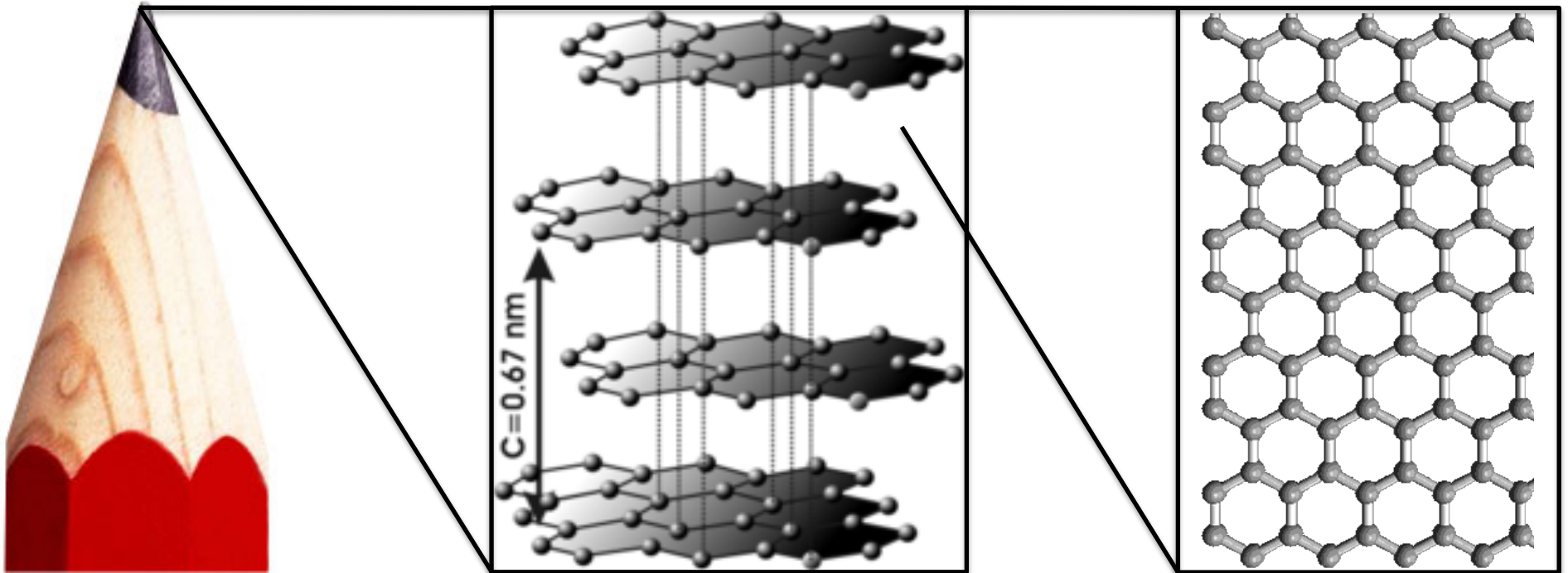
- A nonzero “dynamical” mass m_{dyn} is generated

$$m_{\text{dyn}}^{(2D)} \propto \sqrt{\alpha} \sqrt{|eB|}, \quad \text{and} \quad m_{\text{dyn}}^{(3D)} \propto \sqrt{|eB|} e^{-C/\alpha}$$

- This happens even at the *weakest* interaction (“catalysis”)
- The phenomenon is *universal* (most model details are irrelevant)
- Dimensional *reduction* is the key ingredient (it makes interaction more efficient)

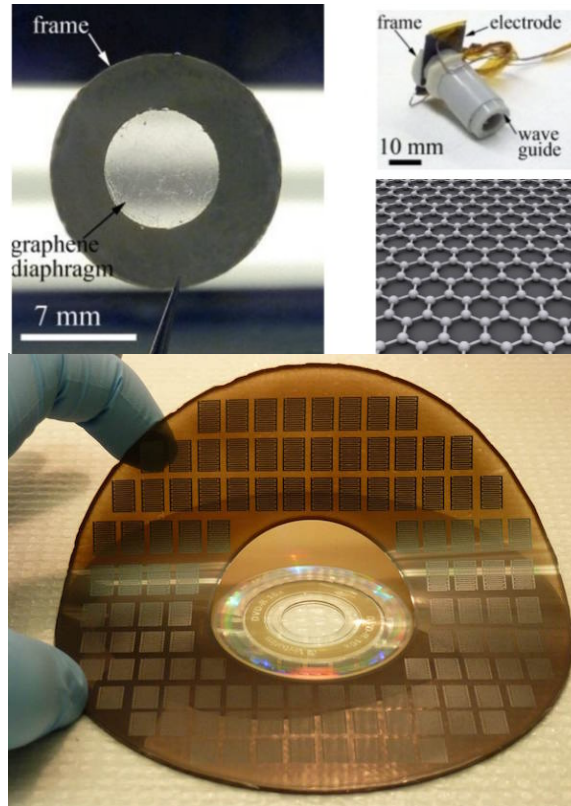
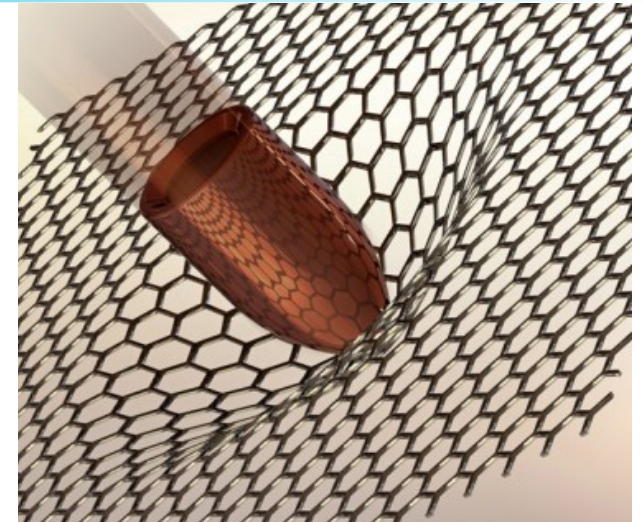
APPLICATIONS

- It is a single atomic layer of graphite
[Novoselov et al., *Science* **306**, 666 (2004)]
- 2D crystal with hexagonal lattice of carbon atoms



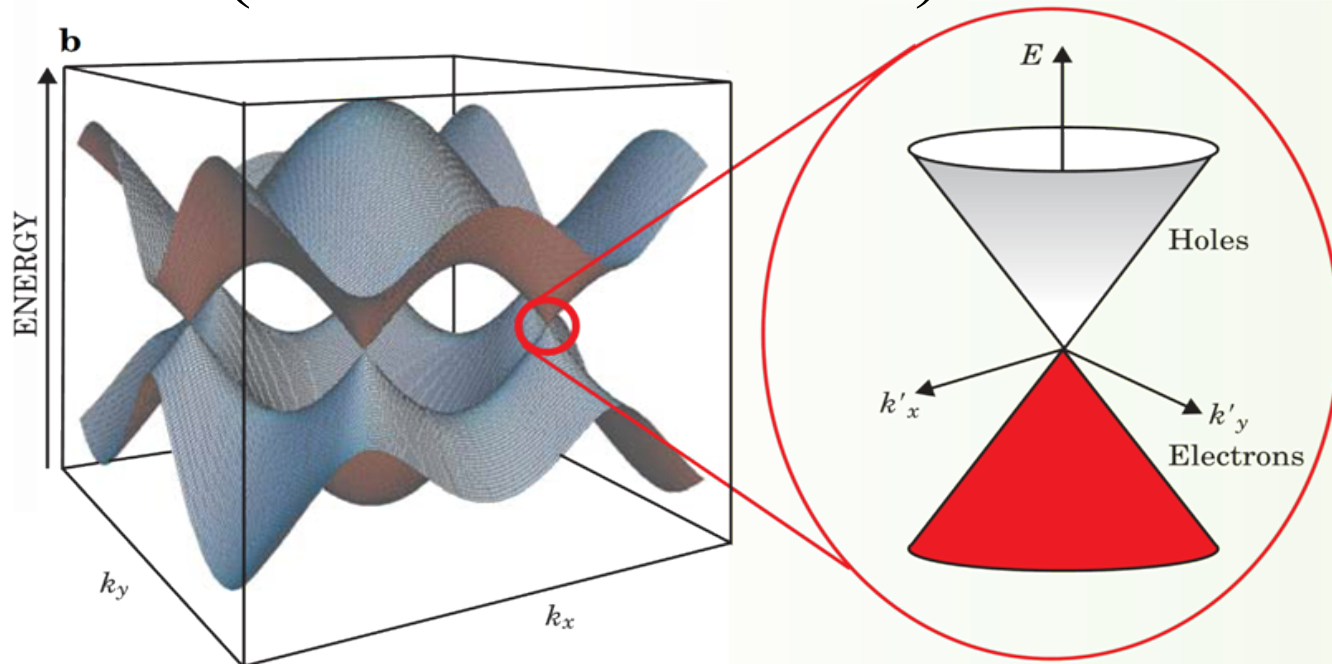
- Interesting basic physics
- Great promise for applied physics ...

- 100 times stronger than steel
- great heat conductor
- great conductor of electricity
- nearly transparent



Galapad Settler retails for US\$399. Photo: Galapad

- Charge carriers (\sim electrons)
 - spin- $1/2$ fermions
 - have large speed, $v \approx c/300$
 - behave like massless particles
- In essence, they behave like relativistic particles (“Dirac fermions”) in vacuum



- **Prediction:**

A nonzero dynamical mass should be generated in a strong magnetic field

[Khveshchenko, PRL **87**, 206401 (2001)]

[Gorbar, Gusynin, Miransky, Shovkovy, PRB **66** (2002) 045108]

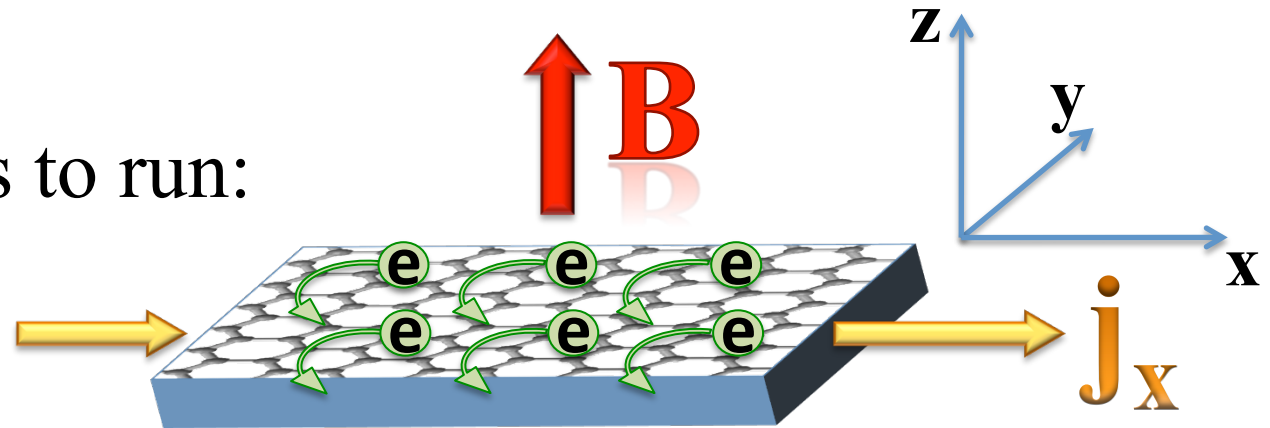
- **Possible complications:**

- many types of “Dirac” masses in **2D**
- competition with quantum Hall ferromagnetism
- nonzero electron/hole density
- impurities, lattice defects, ripples, etc.

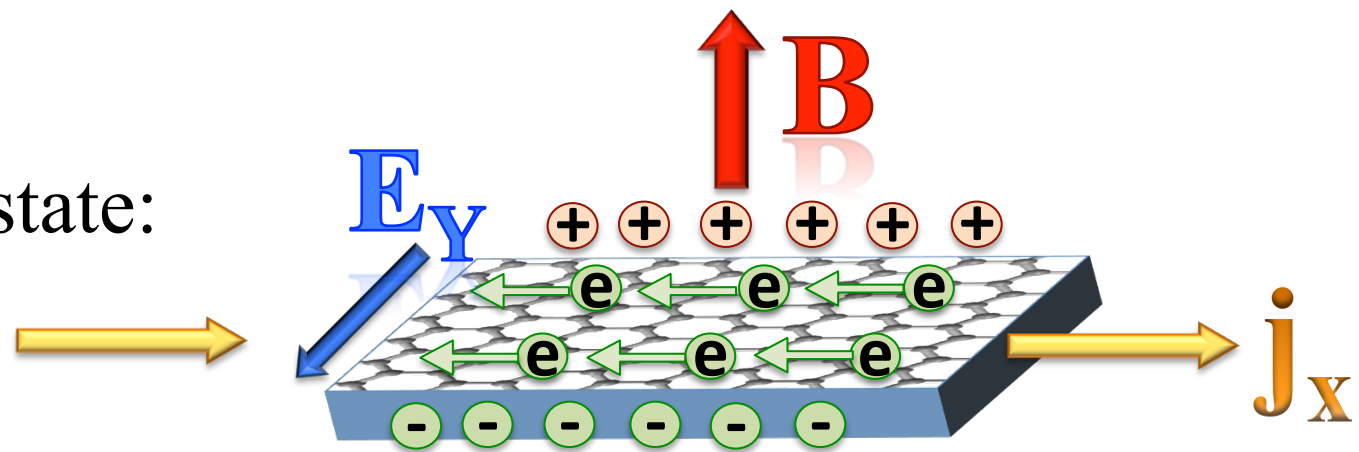
- **How to test this experimentally?**

- General setup

- Current starts to run:

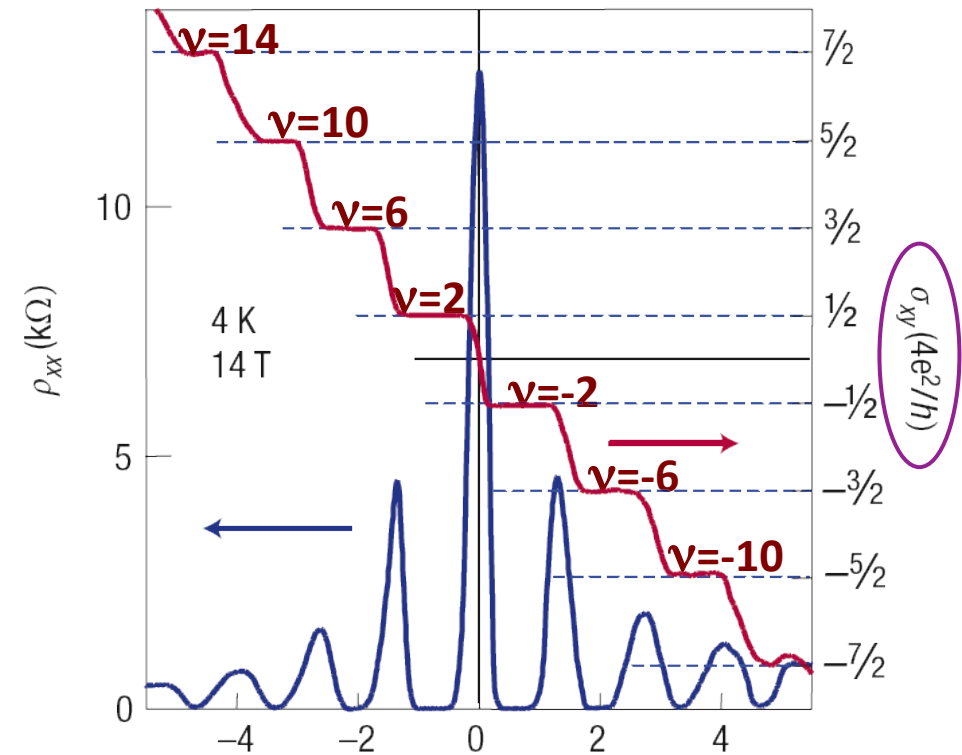
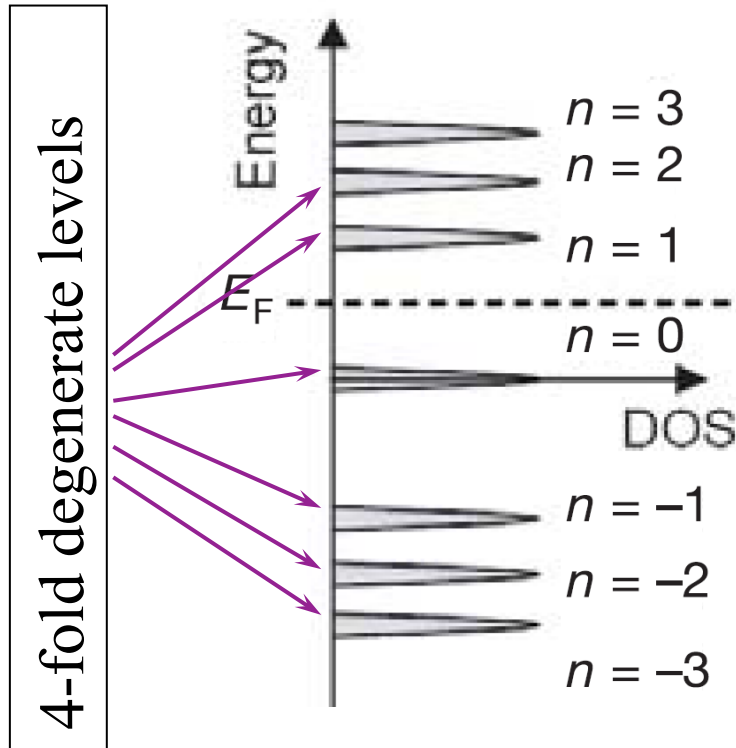


- Steady state:



- Hall conductivity σ_{xy} :

$$\mathbf{j}_x = \sigma_{xy} \mathbf{E}_y$$



$$E_n = \text{sgn}(n) \sqrt{2e\hbar v_F^2 |n| B}$$

$$\sigma_{xy} = \frac{ve^2}{h} = \frac{4e^2}{h} \left(n + \frac{1}{2} \right)$$

- [Gusynin, Sharapov, PRL **95**, 146801 (2005)]
- [Peres, Guinea, Castro Neto, PRB **73**, 125411 (2006)]
- [Novoselov et al., Nature **438**, 197 (2005)]
- [Zhang et al., Nature **438**, 201 (2005)]

} **Theory**

} **Experiment**

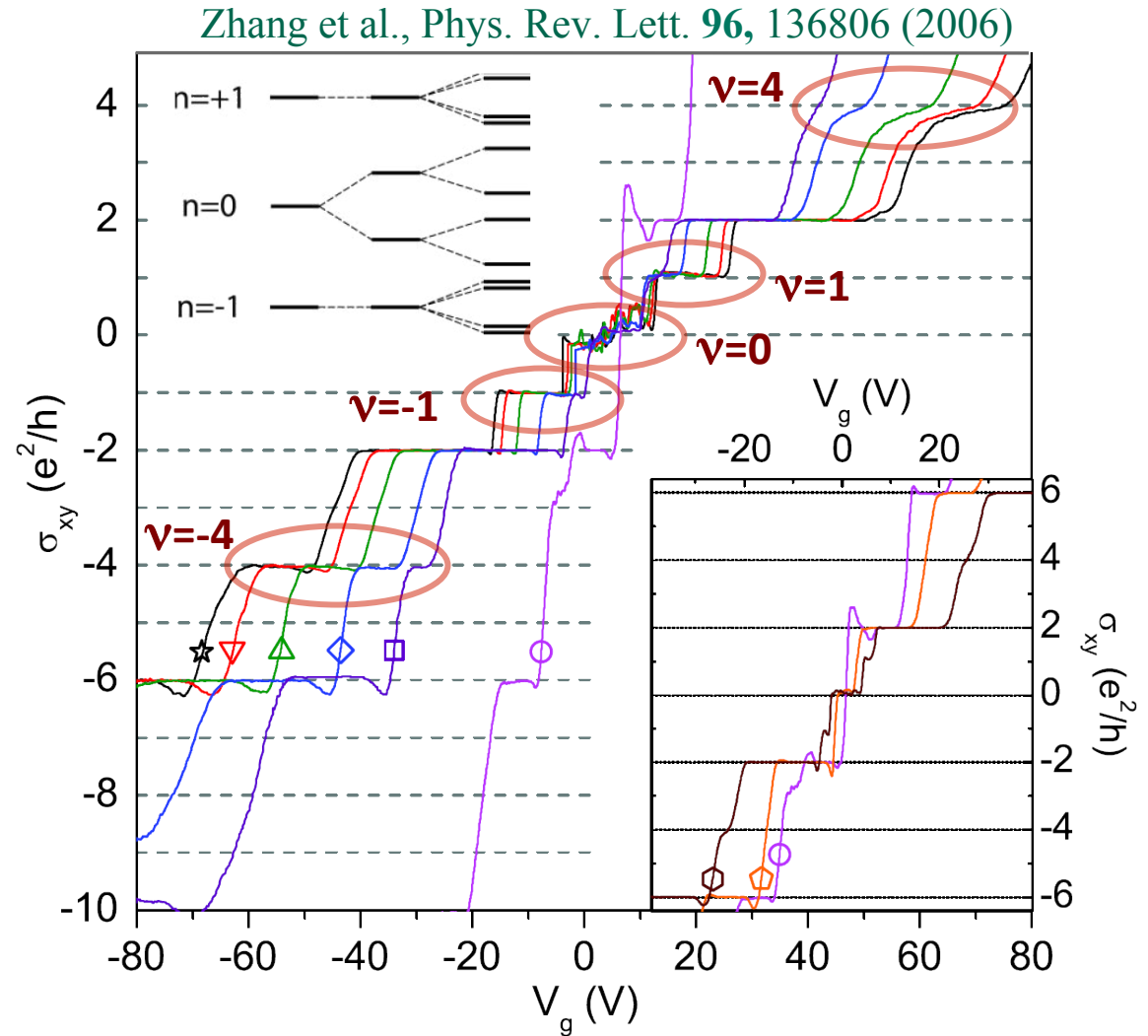
- New plateaus are observed at

$$\nu=0$$

$$\nu=\pm 1$$

$$\nu=\pm 3$$

$$\nu=\pm 4$$



[Novoselov et al., Science **315**, 1379 (2007)]

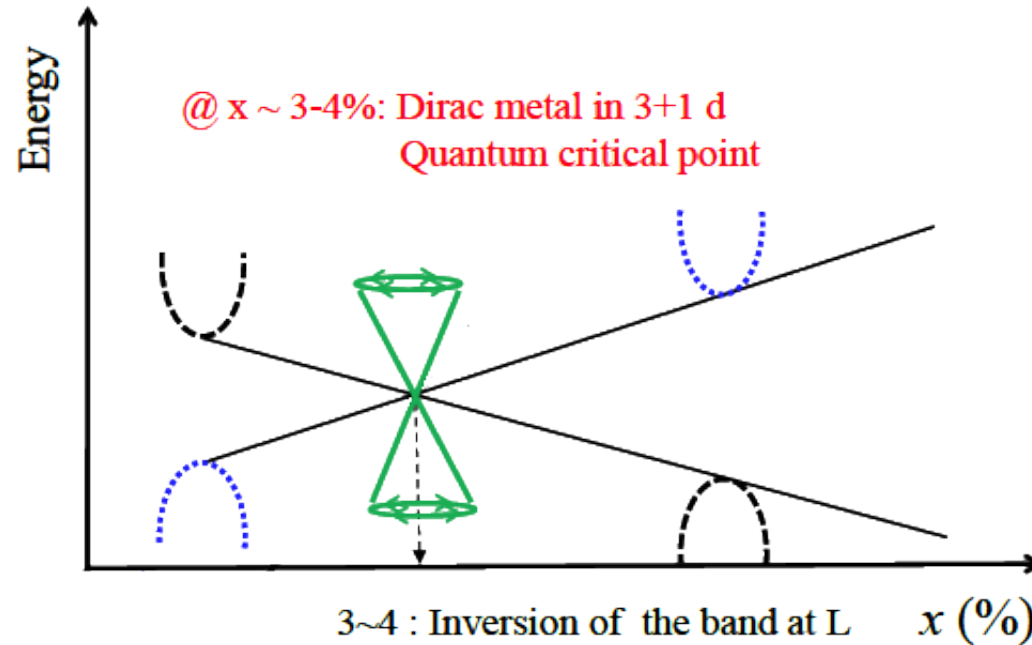
[Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)]

[Checkelsky et al., Phys. Rev. Lett. **100**, 206801 (2008)]

[Xu Du et al., Nature **462**, 192 (2009)]

IS “3D GRAPHENE” POSSIBLE?

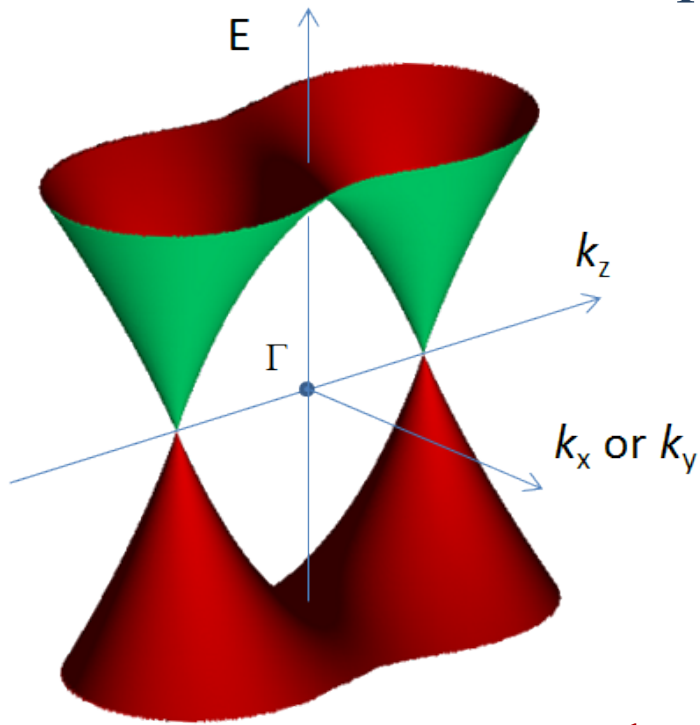
- “Old” 3D materials with Dirac quasiparticles:
 - $\text{Bi}_{1-x}\text{Sb}_x$ alloy



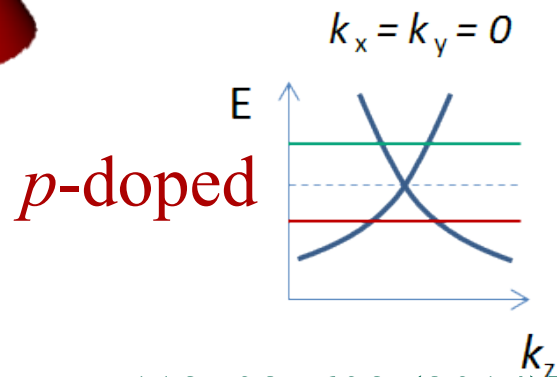
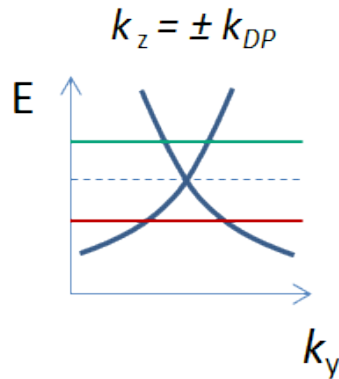
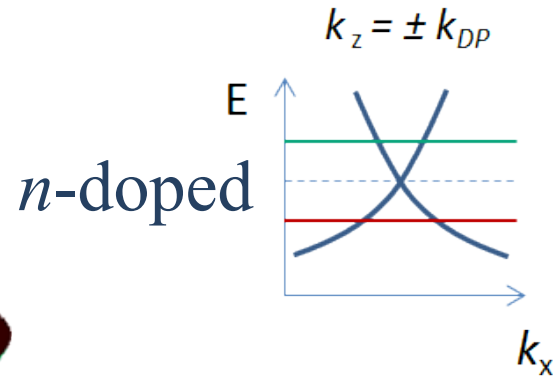
- “New” 3D Dirac materials:
 - Na_3Bi [Liu et al., Science 343, 864 (2014)]
 - Cd_3As_2 [Neupane et al., Nat. Commun. 5, 3786 (2014)]
[Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)]

CADMIUM ARSENIDE

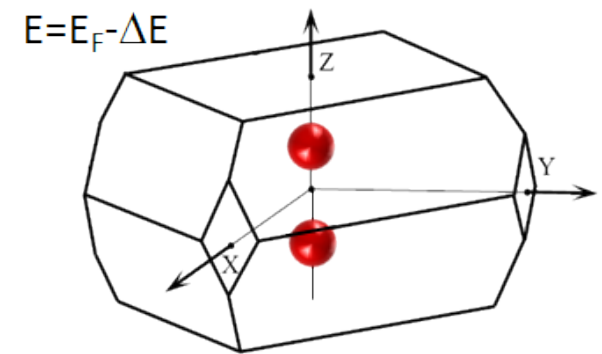
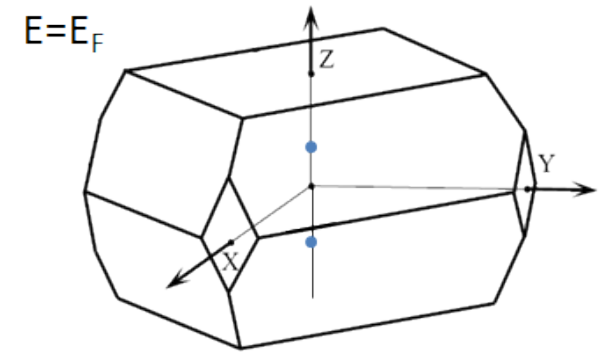
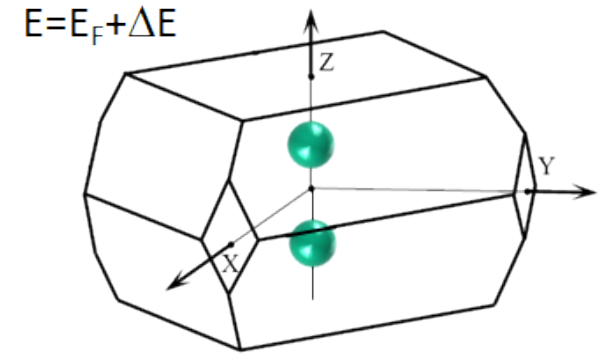
3D Dirac semimetal Cd_3As_2



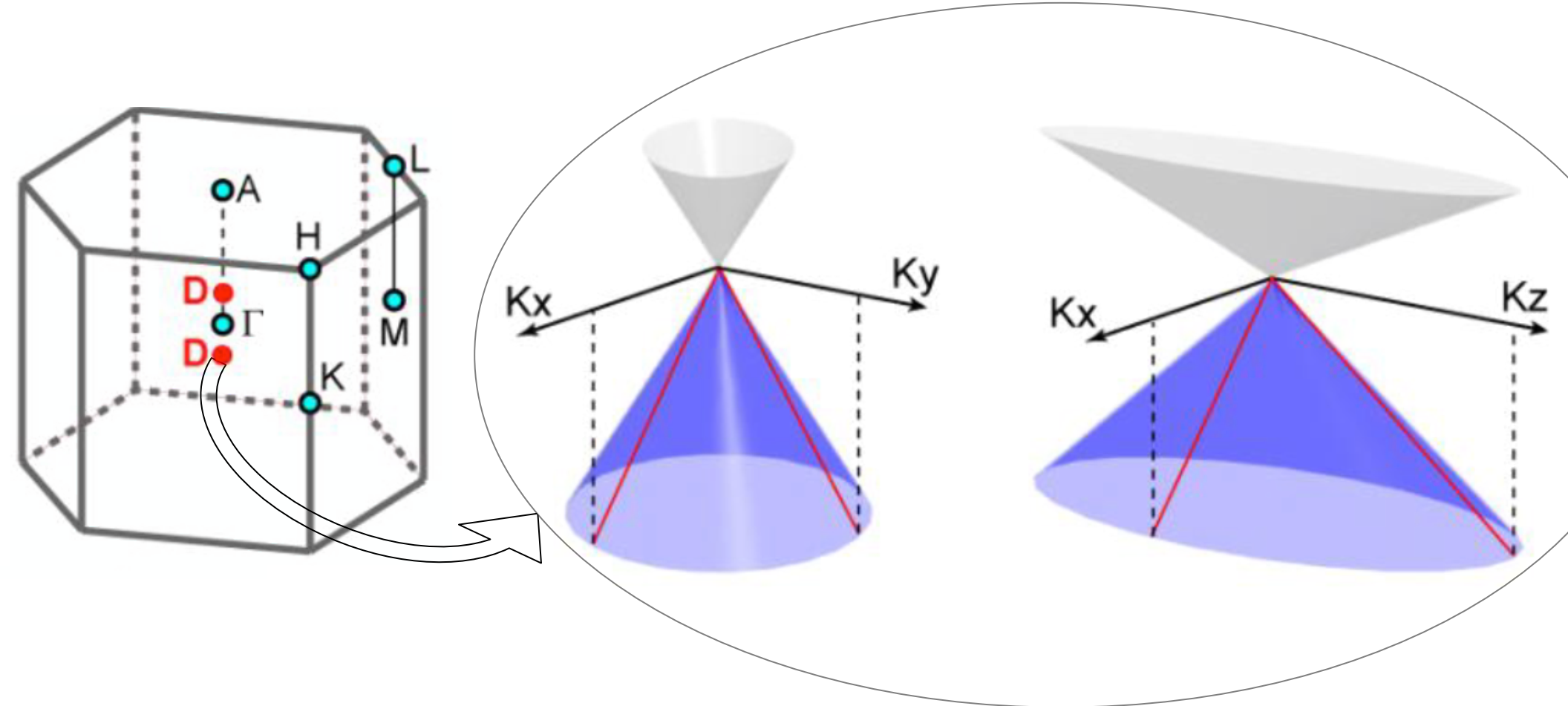
Dispersion



Fermi surface



[Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)]



In the vicinity of 3D Dirac points:

$$E = v_x k_x + v_y k_y + v_z k_z$$

[Liu et al., Science 343, 864 (2014)]

- Relativistic matter in magnetic field is relevant for many branches of physics
- The underlying physics is very rich
- A partial list of phenomena
 - Magnetic catalysis
 - Chiral magnetic effect
 - Chiral shift
 - Chiral magnetic spiral
 - Magnetic properties of Dirac semimetals
 - Quantum Hall Effect in graphene
 - ...



Physics Reports

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In Press, Corrected Proof — Note to users



Quantum field theory in a magnetic field: From quantum chromodynamics to graphene and Dirac semimetals

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