Arizona State University

sQGP & Extreme QCD
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KITPC

Magnetism and chirality in QCD

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Role of chirality in QCD

Approximate chiral symmetry (& its breaking)

→ natural order parameter

→ dynamical origin of nucleon masses

→ spectrum of light mesons

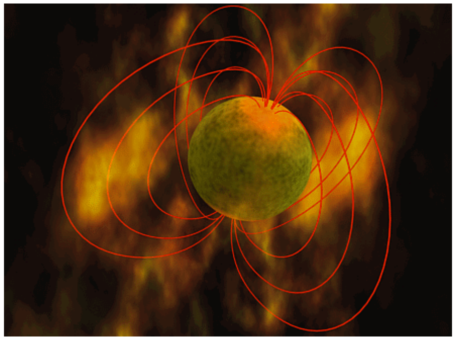
→ structure of the chiral perturbation theory

→ anomalous properties

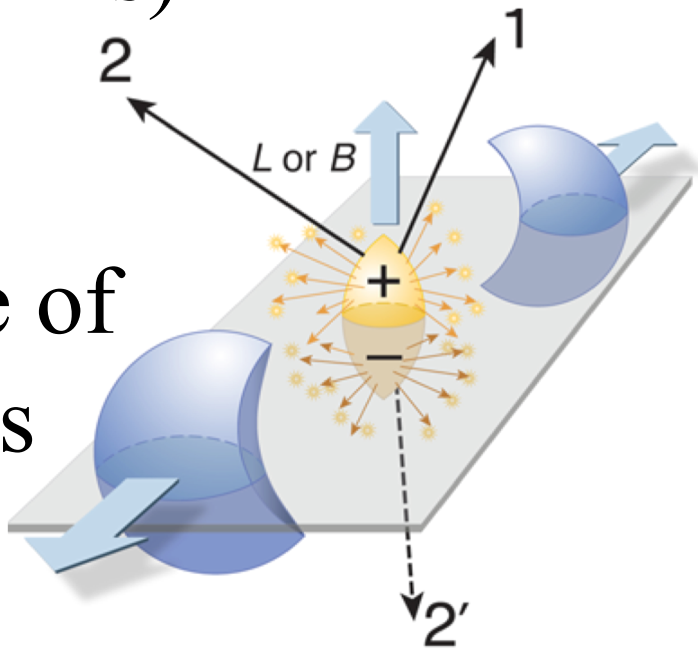
→ …

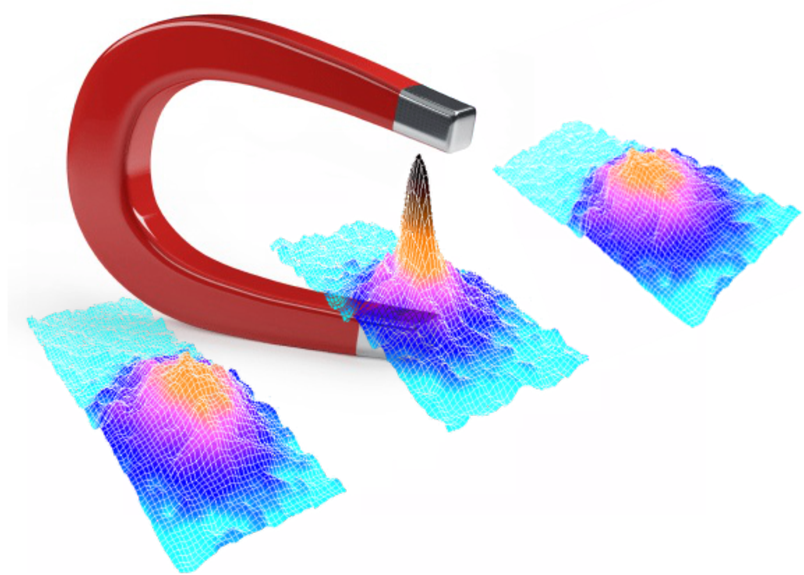
Role of magnetism in QCD

- Magnetized quark matter may exist inside compact stars
  - **10^{10} to 10^{16} G** **(10 keV to 10 MeV)**



- Magnetized quark-gluon plasma is produced in heavy ion collisions ($\Delta t \approx 10^{-24}$ s)
  - **10^{18} to 10^{19} G** **(~ 100 MeV)**
- Magnetic field is a useful probe of nonperturbative QCD properties
  - **10^{21} G** **(~ 1 GeV)**





MAGNETIZED QCD VACUUM

[Miransky & Shovkovy, Physics Reports 576 (2015) pp. 1-209]

Landau spectrum at B≠0

- Dirac equation with massless fermions

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla}+ie\vec{A}\right)\right]\Psi = 0$$

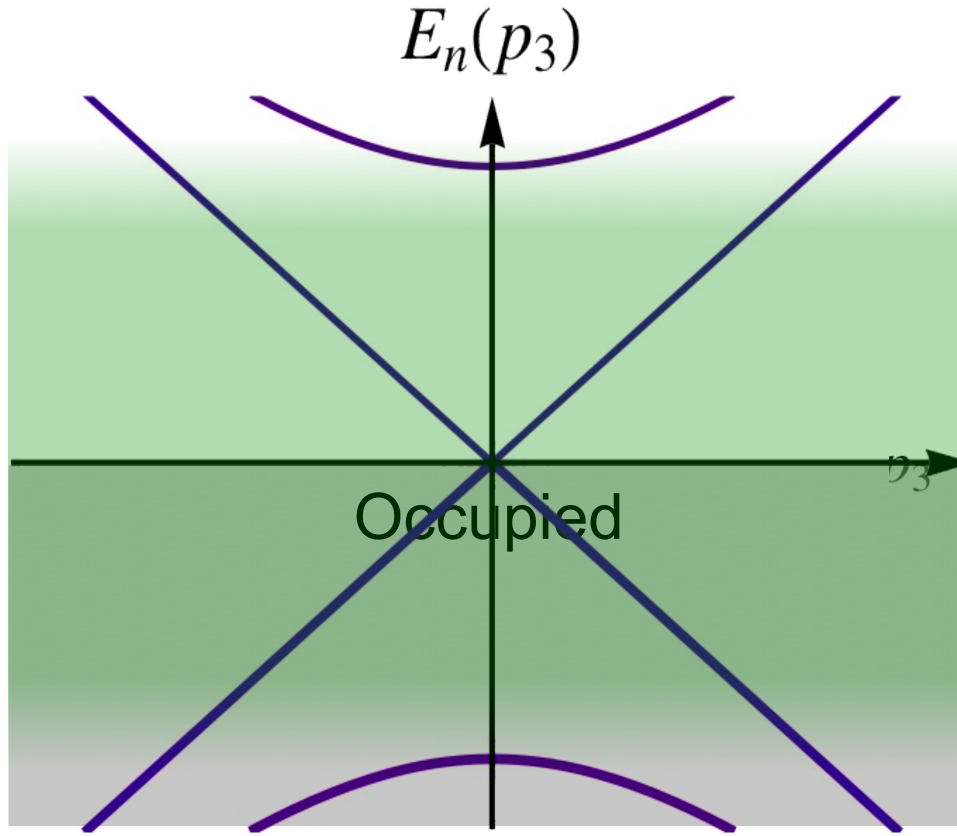
- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where $n = s + k + \frac{1}{2}$

$s = \pm 1/2$ (spin)

$k = 0, 1, 2, \ldots$ (orbital)



Magnetic catalysis: idea

- Low-energy fermion dynamics is dimensionally reduced

$$D \to D - 2$$

- Nonzero density of states at $E = 0$

$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB|N_f}{4\pi^2}$$

- Attractive interaction → symmetry breaking

  (reminiscent of superconductivity…)

[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73** (1994) 3499]

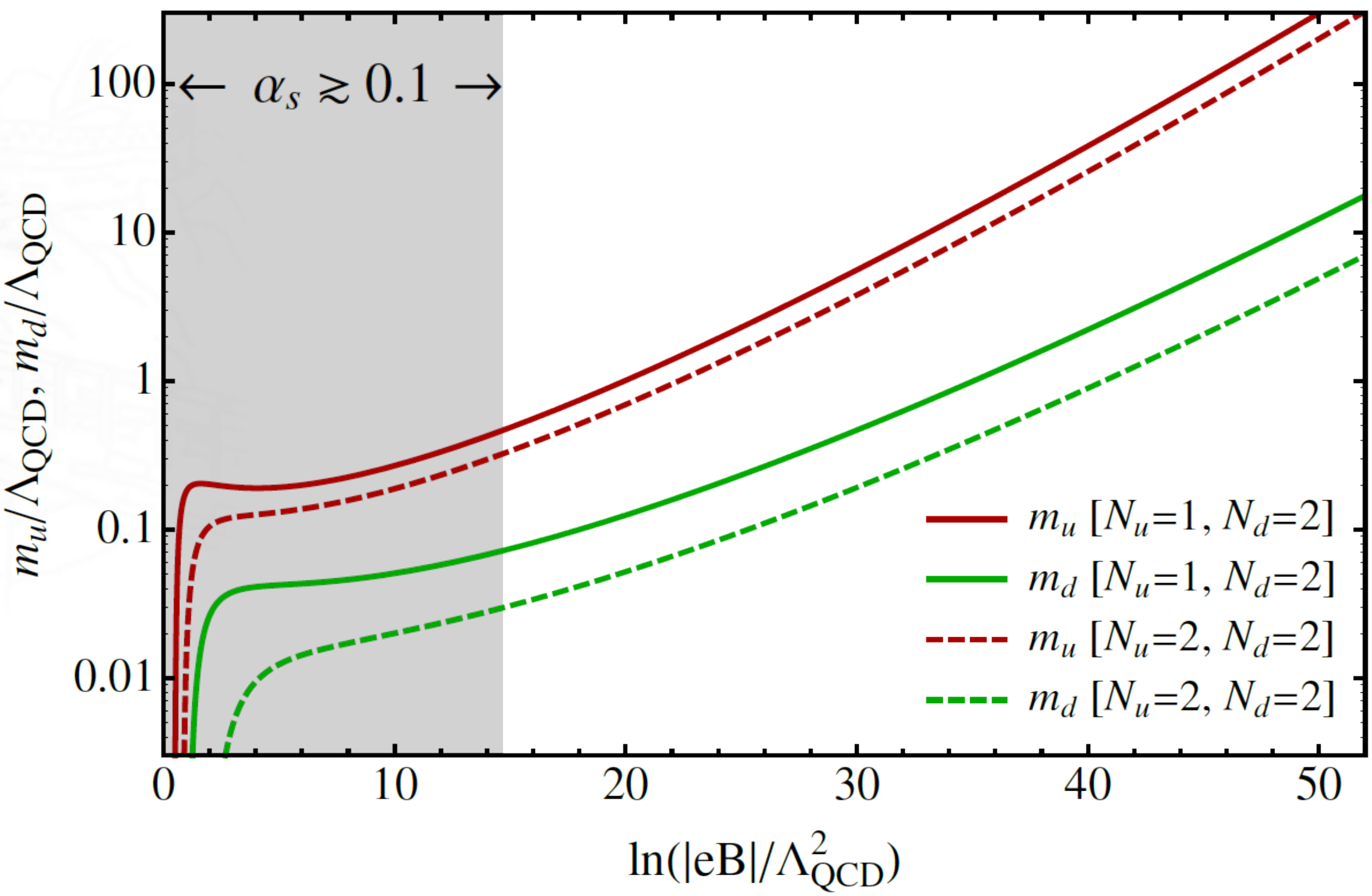
Universality of magnetic catalysis

- **Input**
  - Spin-½ charged particles and $B \neq 0$
  - Attractive particle-antiparticle interaction
- **Output**
  - Dimensional reduction $D \to D-2$ @ low energies
  - particle-antiparticle bound states form
  - Symmetry breaks down
  - Dynamical mass is generated

Magnetic catalysis in QCD

- Approximate dynamical mass ($\sqrt{|eB|} >> \Lambda_{QCD}$):

$$m_f^2 \propto |e_f B| \alpha_s^{3/2} \exp\left(-\frac{4\pi N_c}{\alpha_s (N_c^2 - 1)\ln(C/\alpha_s)}\right)$$



[Miransky & I.S., Phys. Rev. D 66 (2002) 045006]

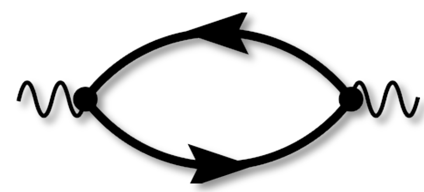
Anisotropic confinement in QCD

- Low-energy gluodynamics ($p << m_{\mathrm{dyn}}$):

$$L \approx \frac{1}{2} \sum_{a=1}^{N_c^2-1} \left(\vec{E}_\perp^a \cdot \vec{E}_\perp^a + \varepsilon E_z^a E_z^a - \vec{B}_\perp^a \cdot \vec{B}_\perp^a - B_z^a B_z^a \right)$$

where the chromo-dielecric constant is given by

$$\varepsilon \approx 1 + \frac{\alpha_s}{6\pi} \sum_{f=1}^{N_f} \frac{|q_f B|}{m_f^2} >> 1$$



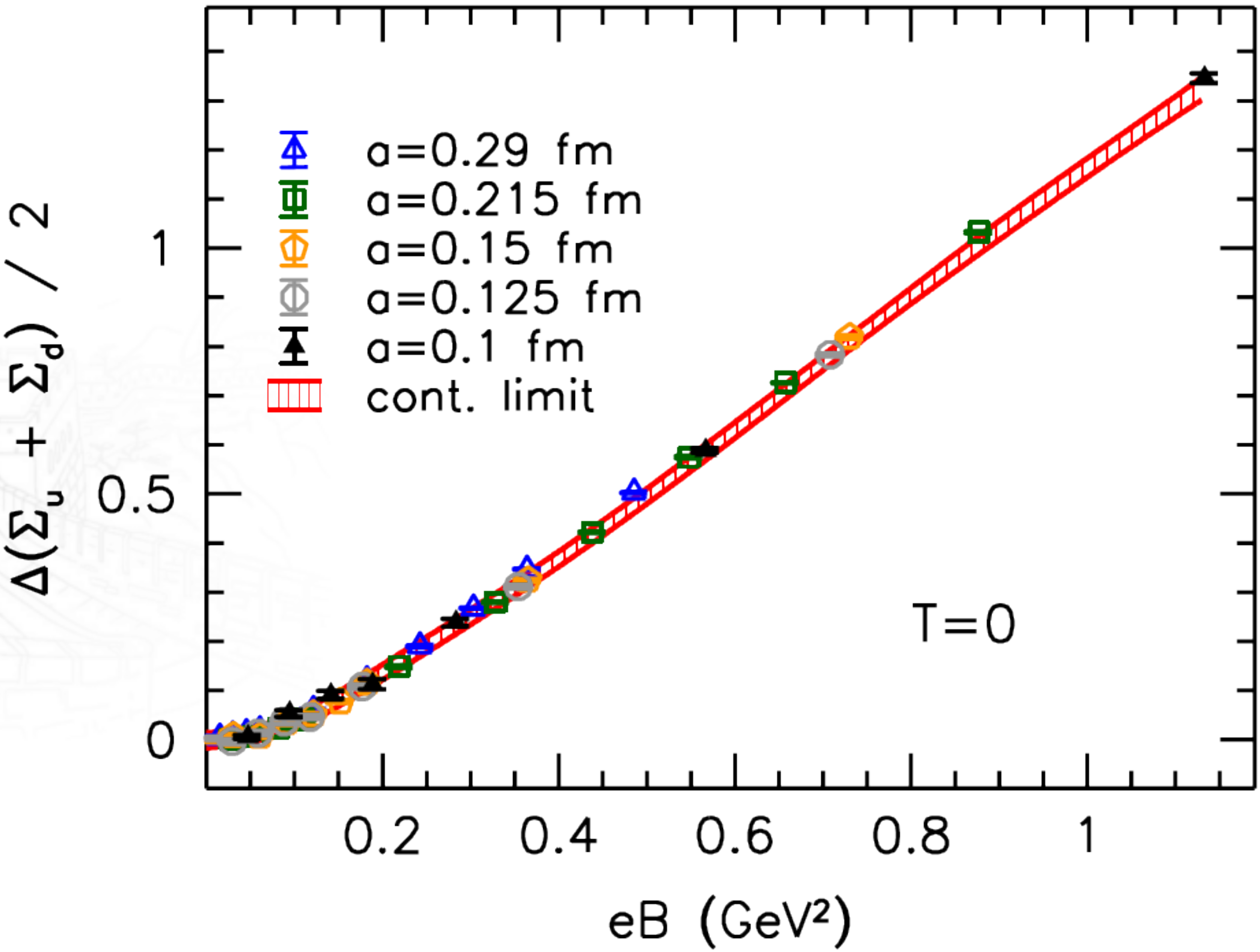
The new confinement scale is

$$\lambda_{\mathrm{QCD}} \approx m_d \left(\frac{\Lambda_{\mathrm{QCD}}}{\sqrt{|eB|}} \right)^{\frac{11N_c - 2N_f}{11N_c}}$$

Note: $\lambda_{\mathrm{QCD}} << \Lambda_{\mathrm{QCD}}$

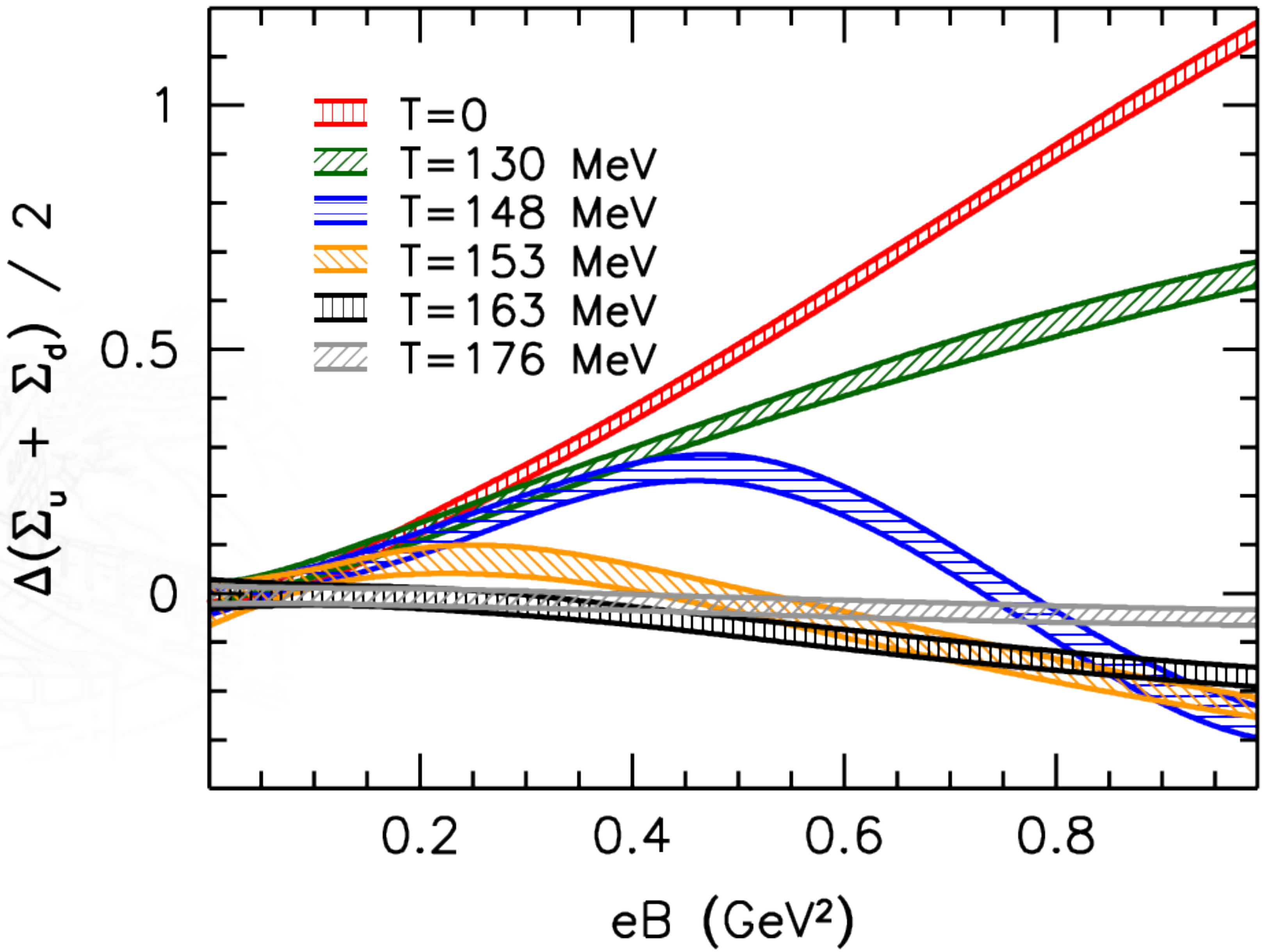
[Miransky & I.S., Phys. Rev. D 66 (2002) 045006]

Magnetic catalysis in QCD (lattice)



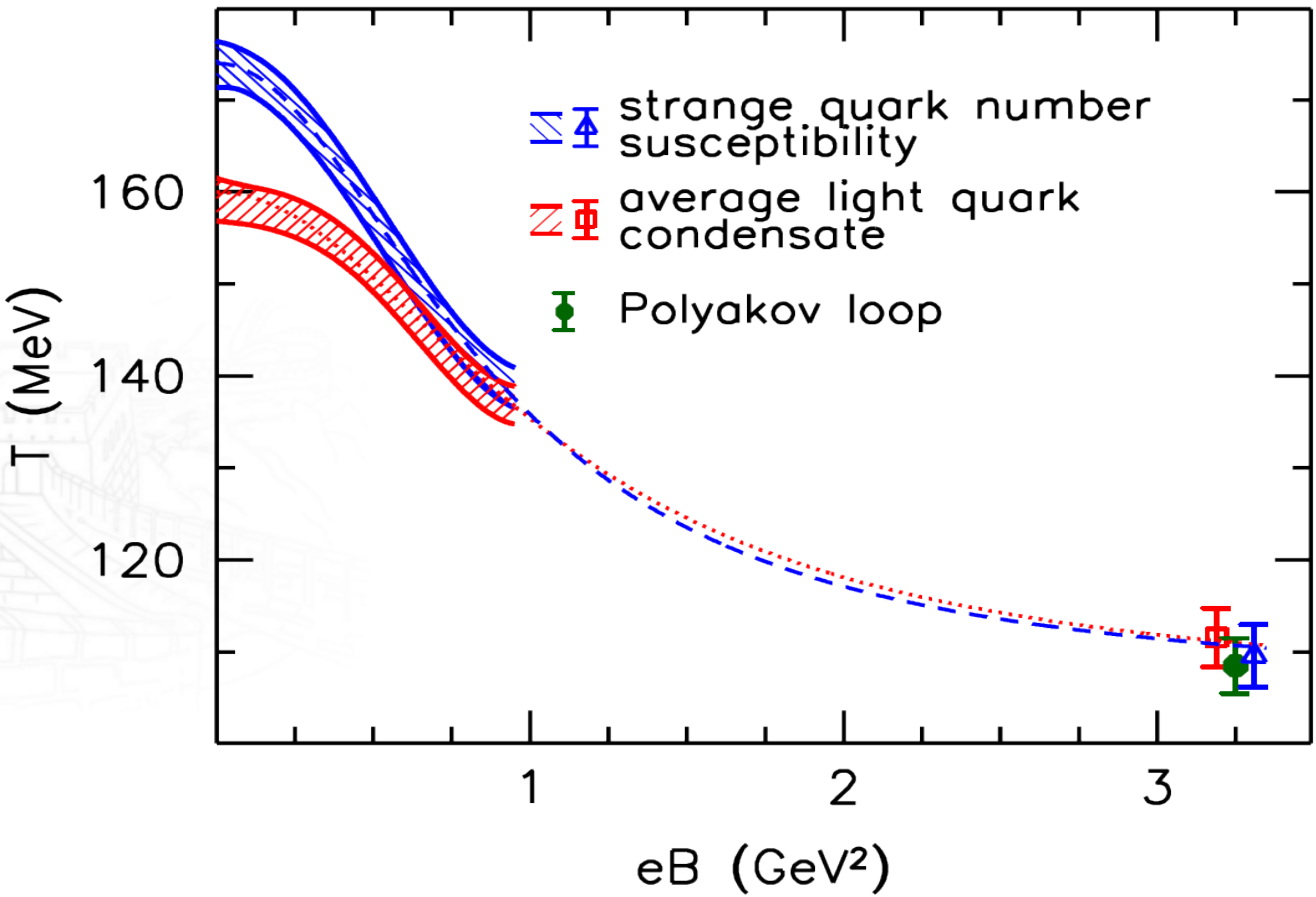
[Bali et al., Phys. Rev. D86, 071502 (2012)]

(Inverse) Catalysis at T≠0?



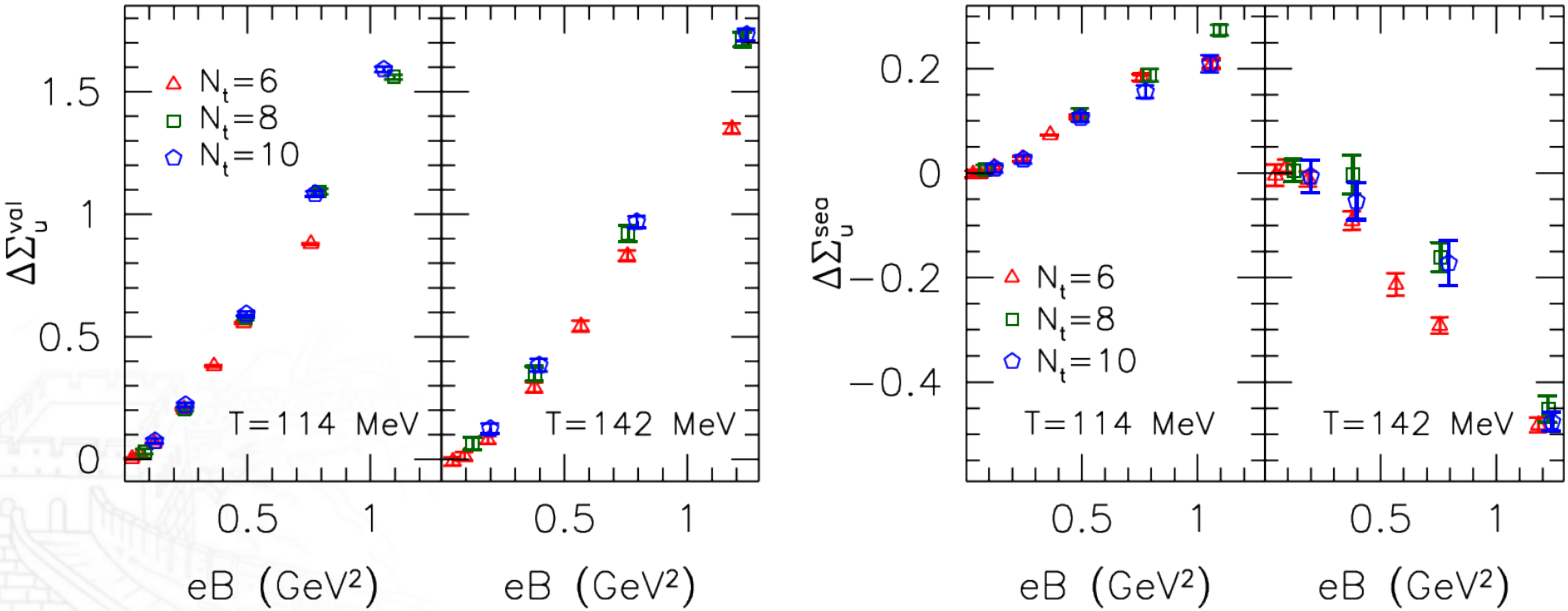
[Bali et al., Phys. Rev. D86, 071502 (2012)]

Inverse catalysis: T_c vs. B



[Bali et al., JHEP 02, 044 (29012)], [Bali et al., PRD86, 071502 (2012)], [G. Endrodi, arXiv:1504.08280]

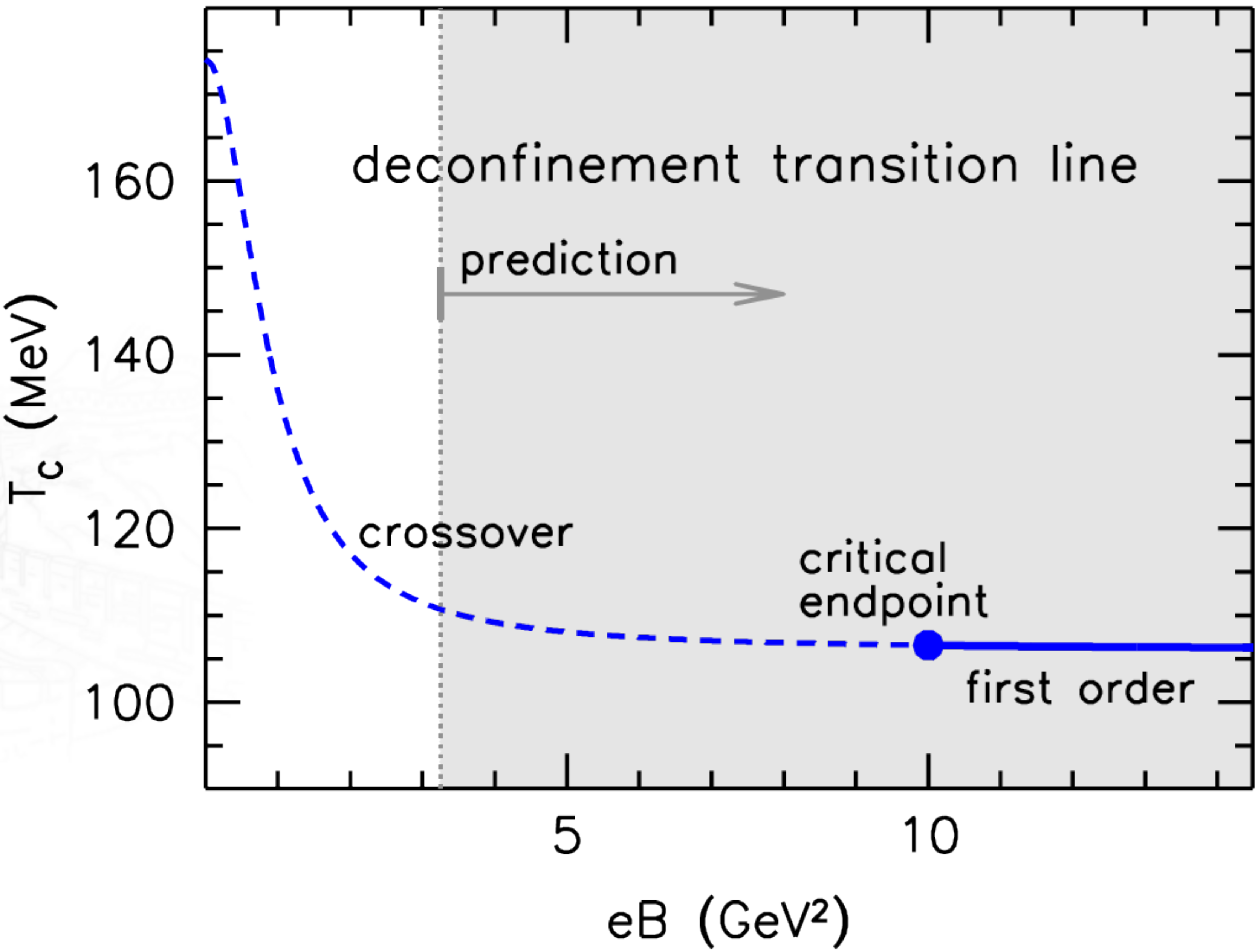
Valence vs. sea



[Bruckmann, G. Endrodi, T. G. Kovacs, arXiv:1303.3972]

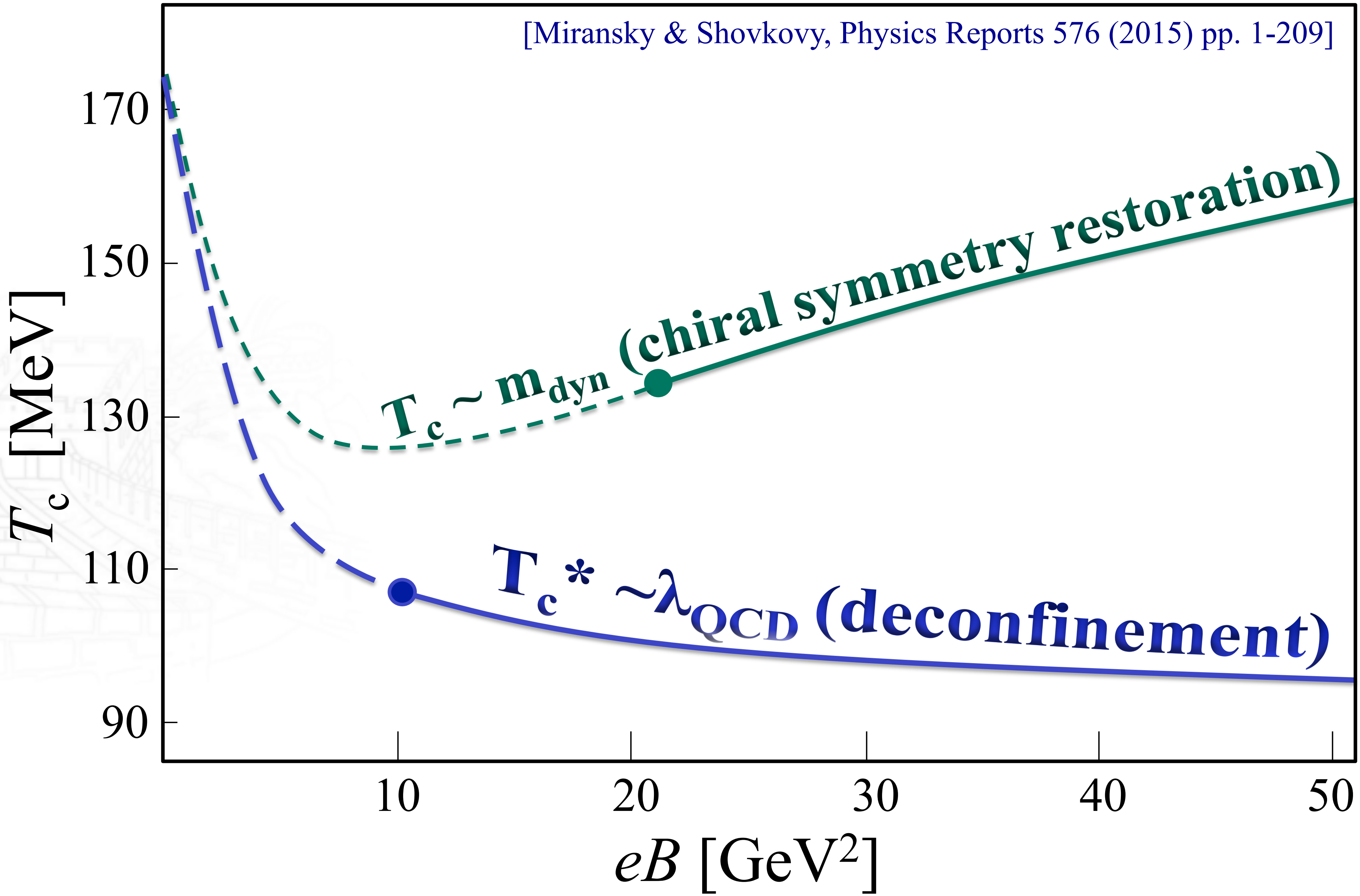
- Gluon screening (?), Polyakov loops (?), …
- See also [Ilgenfritz et al. Phys. Rev. D 89, 054512 (2014)]

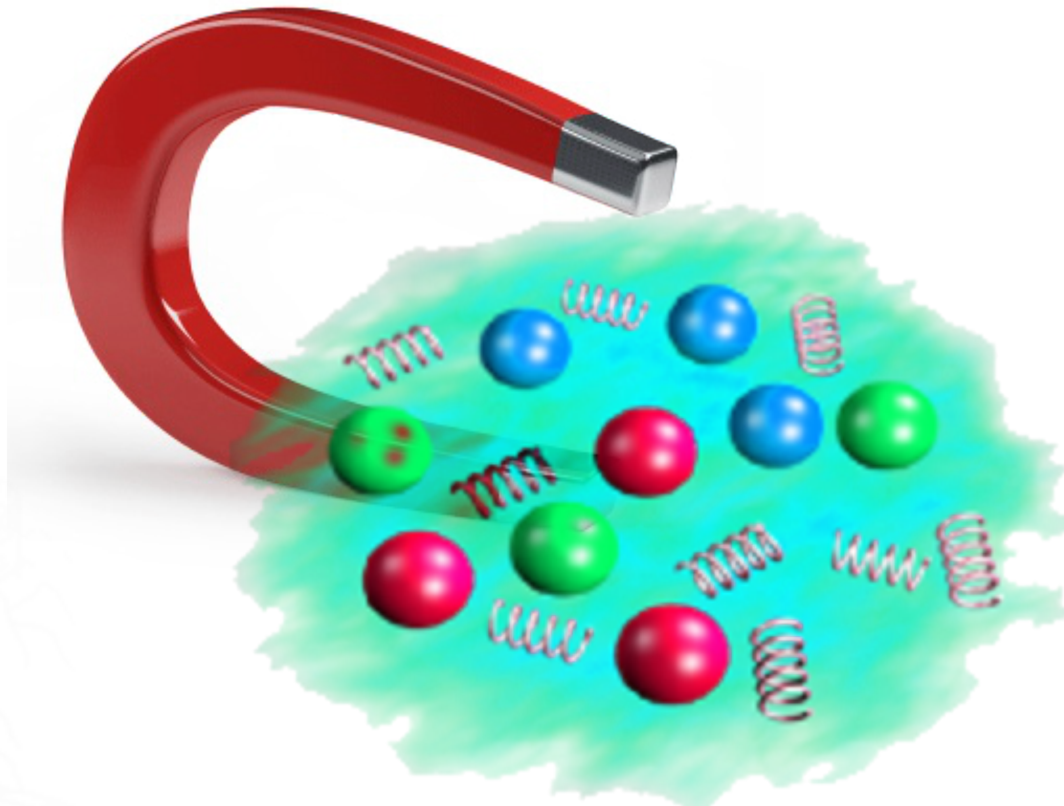
Super-strong *B*: prediction



[Cohen & Yamamoto, PRD89, 054029 (2014)], [G. Endrodi, arXiv:1504.08280]

Complete diagram: prediction





MAGNETIZED QCD MATTER

[Miransky & Shovkovy, Physics Reports 576 (2015) pp. 1-209]

B≠0: Chiral separation effect

- Axial current density induced by the chemical potential

$$\left\langle \vec{j}_5 \right\rangle_{\text{free}} = \frac{e\vec{B}}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- This result is connected to the chiral anomaly relation ($\mu \to e\Phi$)

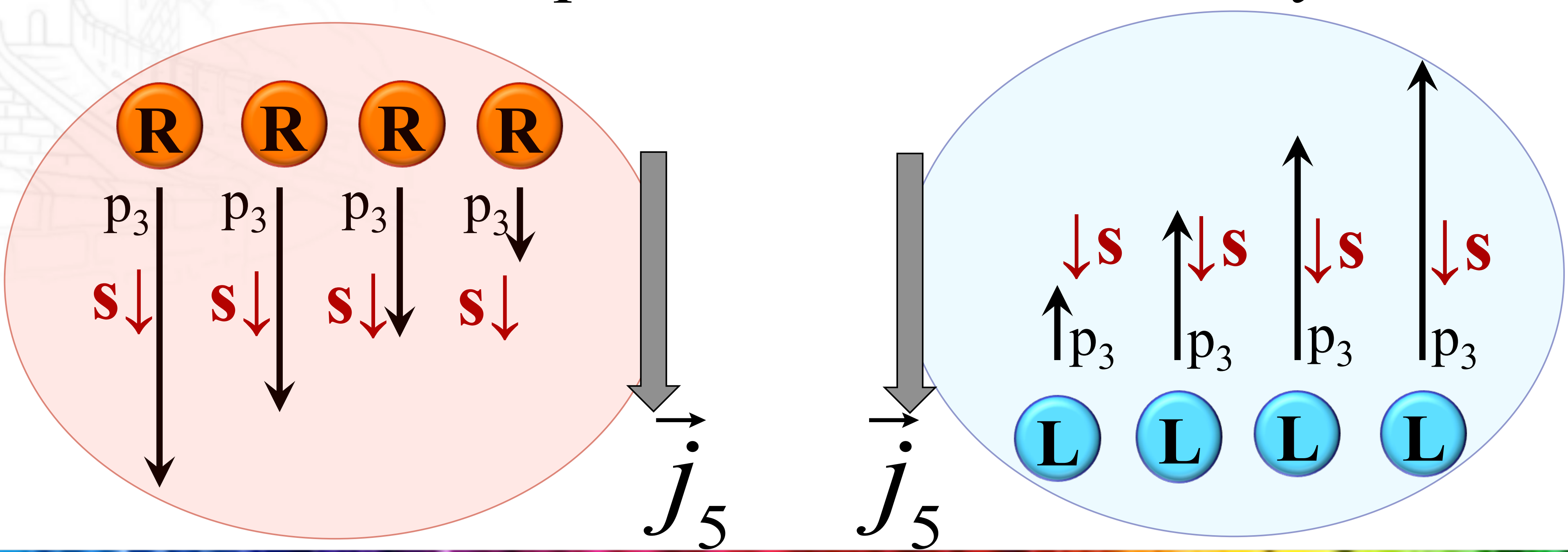
$$\partial_z \left\langle j_5^z \right\rangle_{\text{free}} = \frac{e}{2\pi^2} B_z \partial_z \left(e\Phi \right) = -\frac{e^2}{2\pi^2} B_z E_z$$

- However, axial current gets radiative corrections

Asymmetry: LLL ➟ hLLs

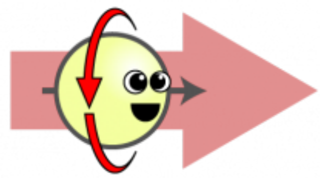
- LLL is spin polarized and chirally asymmetric
  - states with $p_3<0$ (and s=↓) are R-handed
  - states with $p_3>0$ (and s=↓) are L-handed

- This indeed implies axial current density

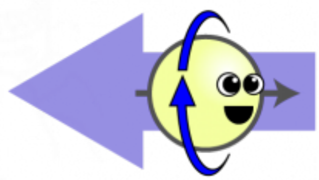


Spin vs. orbital motion

- Helicity/chirality of massless (ultrarelativistic) fermions is (≈) conserved

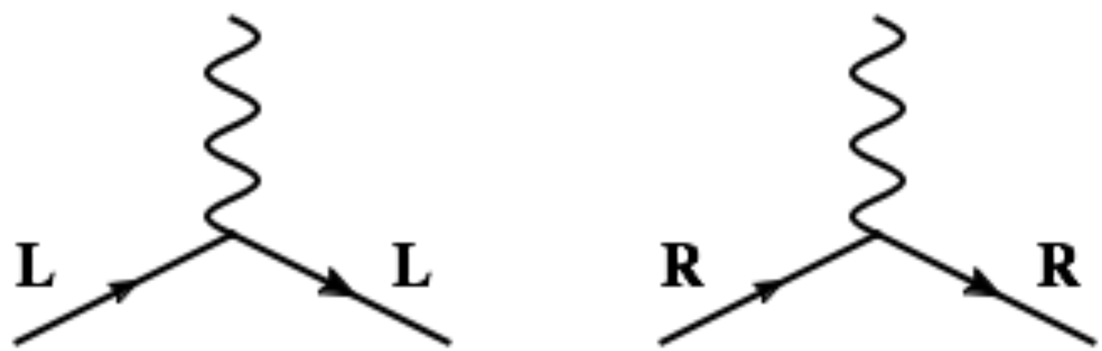


R-handed



L-handed

- Chirality does not change in elementary QED interactions



Asymmetry: LLL ⇒ hLLs

- What is the effect of interactions?
- To preserve chirality, particle momenta have to "flip" whenever the spin "flips"
- B-field ⇒ preferred spin orientation $s=\downarrow$
- Interactions ⇒ chiral asymmetry in hLLs

L-handed prefer $s=\downarrow$ and, thus, $p_3<0$

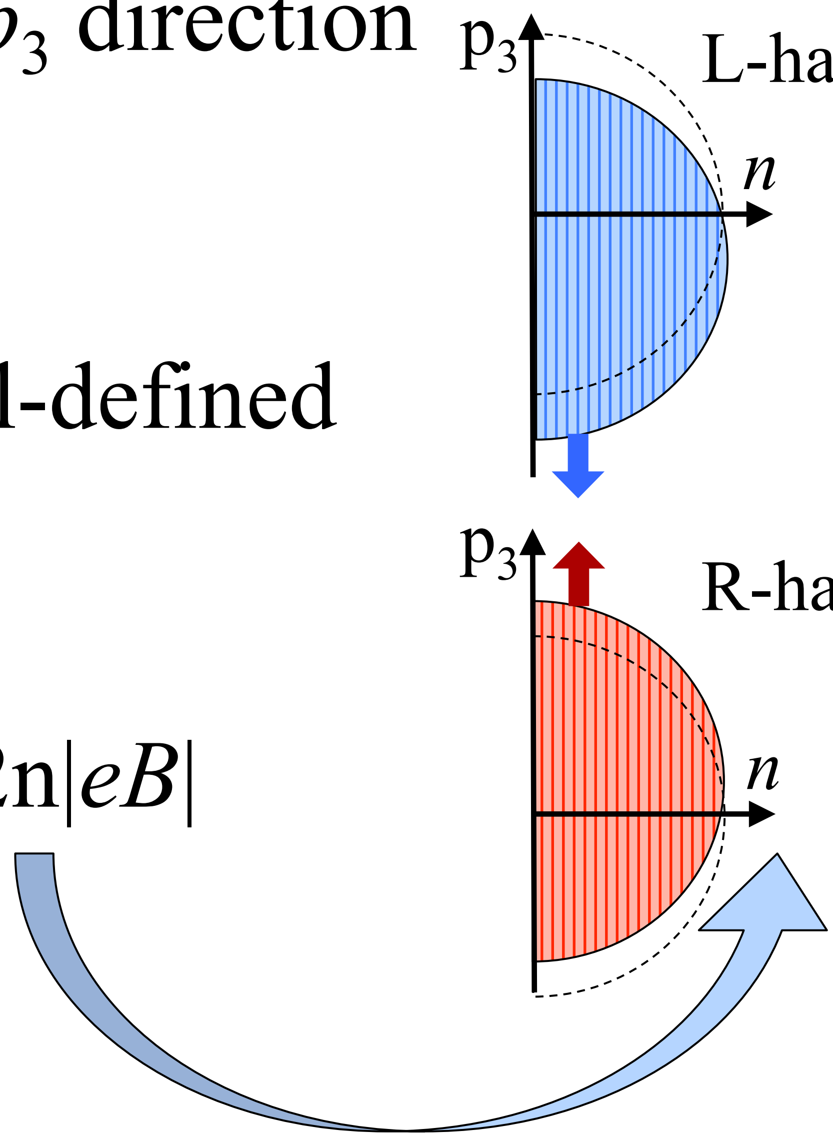
R-handed prefer $s=\downarrow$ and, thus, $p_3>0$

Chiral asymmetry

- Anticipated outcome: L- & R-handed Fermi surfaces shift in p_3 direction

Note: $\mathbf{p}_\perp$ is not well-defined

$p_\perp^2$ is replaced by $2n|eB|$



Chiral shift at low energies

- Ground state expectation value of the axial current (CSE)

$$\left\langle \bar{\psi}\gamma^3\gamma^5\psi \right\rangle = \frac{eB}{2\pi^2}\mu \quad \text{with} \quad \vec{B} = (0,0,B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

should induce a dynamical (chiral shift) parameter Δ associated with the condensate,

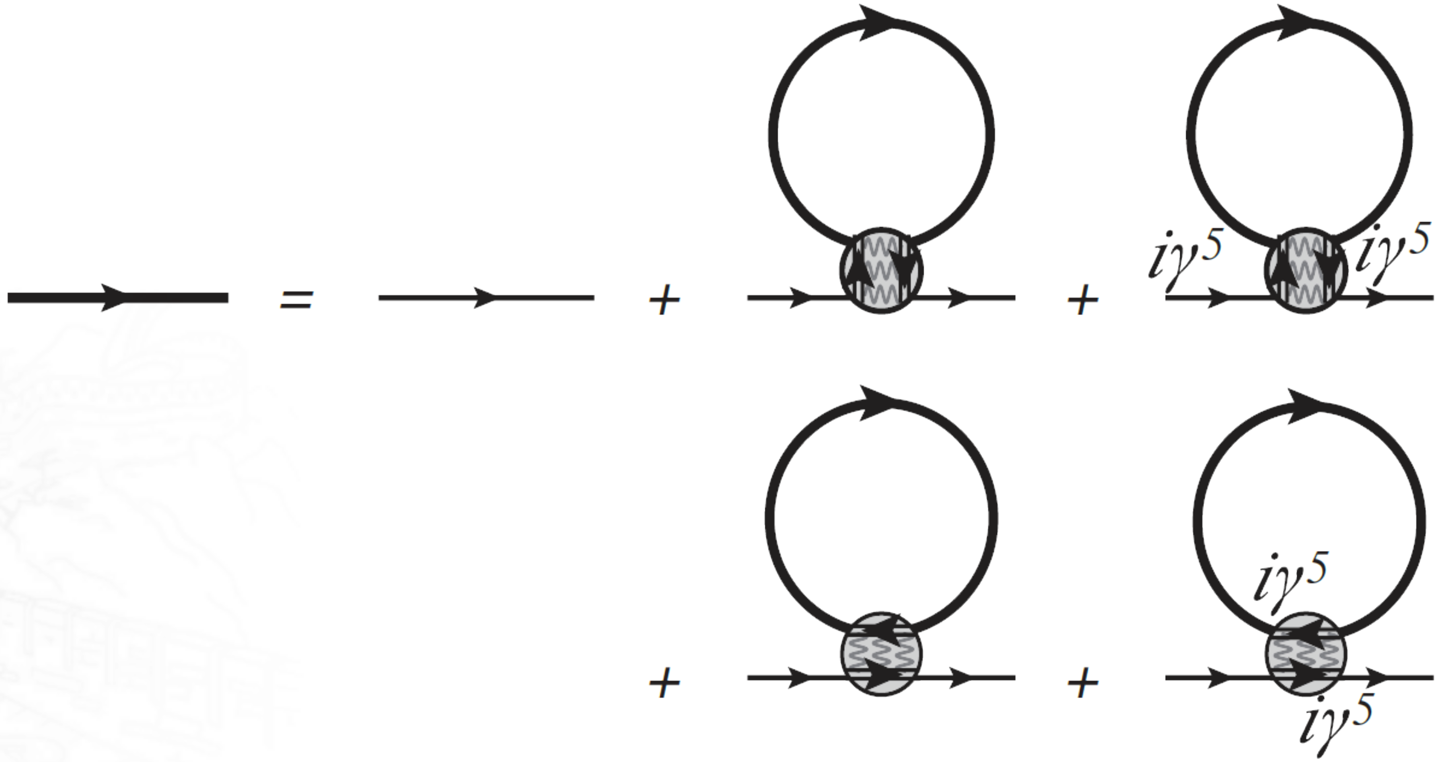
$$\delta L = \Delta\, \bar{\psi}\gamma^3\gamma^5\psi$$

[Gorbar, Miransky, I.S., Phys. Rev. C **80**, 032801(R) (2009)]

(Δ=0 is not protected by any symmetry)

NJL model: quick check

- NJL model (local interaction)



- The equation for the chiral shift

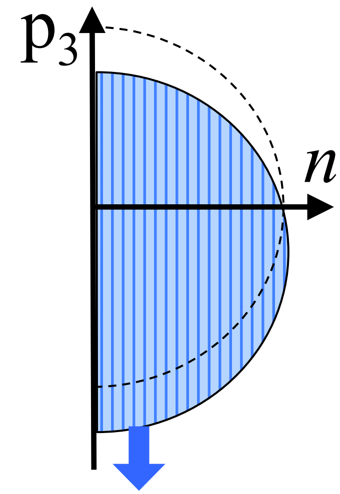
$$\Delta = -\frac{1}{2} G_{\text{int}} \left\langle \bar{\psi} \gamma^3 \gamma^5 \psi \right\rangle \approx -\frac{G_{\text{int}} eB}{4\pi^2} \mu$$

Chiral shift @ Fermi surface

- Chirality is $\approx$ well-defined at Fermi surface ($|p_3| \gg m$)
- L-handed Fermi surface:

$$n>0: \quad p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

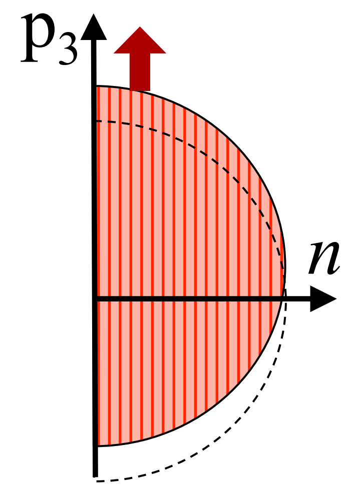
$$p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



- R-handed Fermi surface:

$$n>0: \quad p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

QED in weak field (B→0)

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta(p) + \gamma^0\gamma^5\mu_5(p)$$

Near Fermi surface ($p_0 \to 0$, $|\mathbf{p}| \to p_F$)

$$\Delta(p) \approx \frac{\alpha eB\mu}{\pi m^2}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

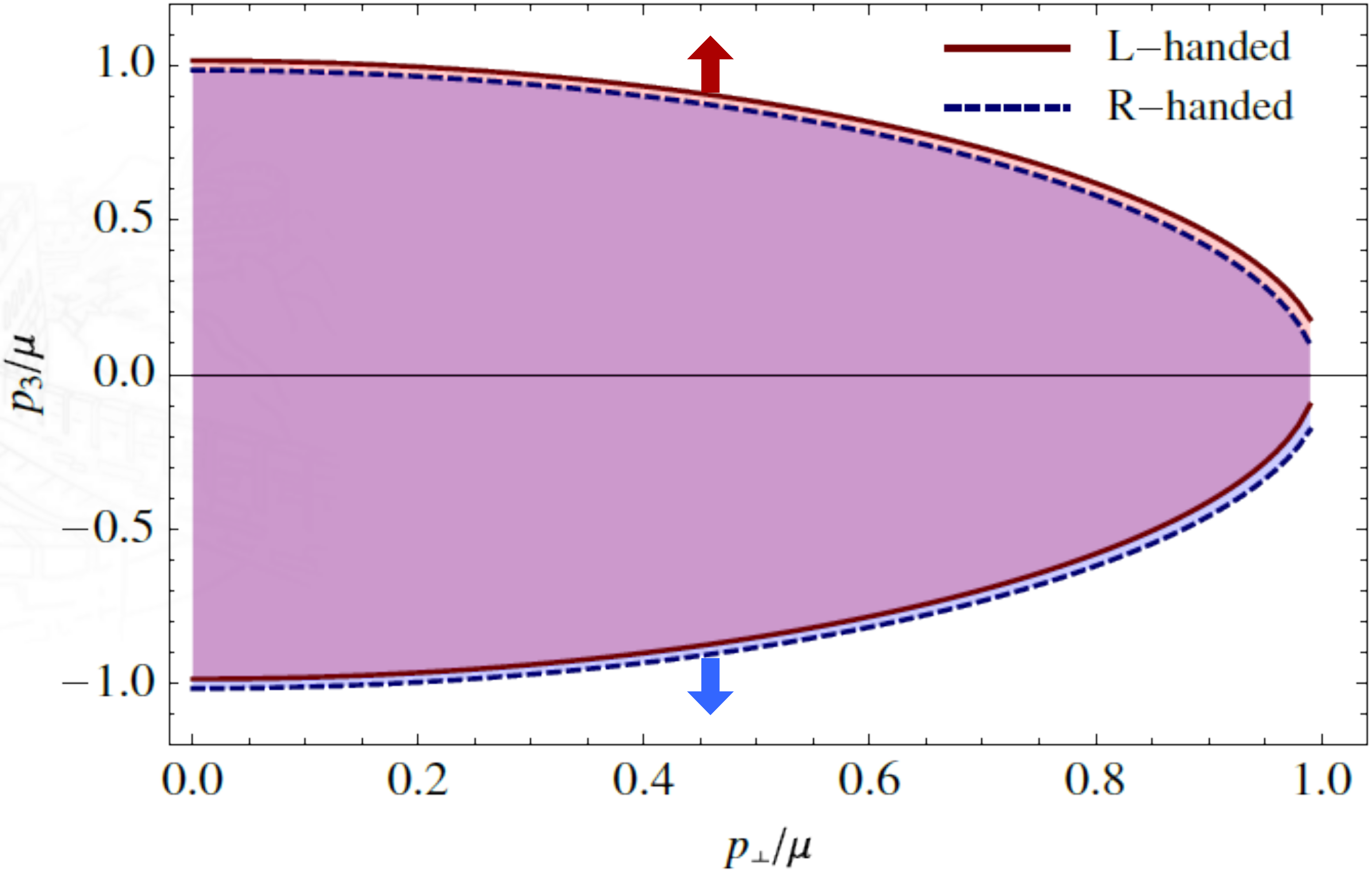
$$\mu_5(p) \approx -\frac{\alpha eB\mu}{\pi m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Dispersion relations in QED

- Let us use the condition (for a small *B*)

$$\mathrm{Det}\left[i\,\overline{S}^{-1}(p) + \overline{\Sigma}^{(1)}(p) \right] = 0$$



[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

QED in strong field

Self-energy in the Landau-level representation:

$$\bar{\Sigma}(p) = 2e^{-p_\perp^2 l^2} \sum_{n=0}^{\infty} (-1)^n \left(-\gamma^0 \delta\mu_n - \gamma^3\gamma^5 \Delta_n - \gamma^0\gamma^5 \mu_{5,n} + m_n + \ldots\right)\left[P_- L_n - P_+ L_{n-1}\right] - \ldots$$

where $\delta\mu_n$, Δ_n, $\mu_{5,n}$, … are "projections" of the self-energy on the *n*th Landau level,

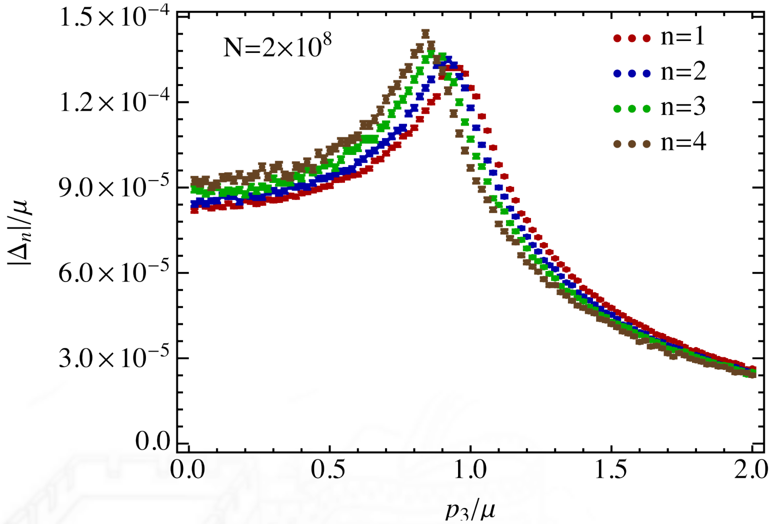
$$\Delta_n(p_0, p_3) = \frac{(-1)^n l^2}{8\pi} \int d^2 p_\perp e^{-p_\perp^2 l^2} \left[L_n + L_{n+1}\right] Tr\left[\gamma^0 \bar{\Sigma}(p)\right]$$

where

$$\bar{\Sigma}(p) = -4i\pi\alpha \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \bar{S}(k) \gamma^\nu D_{\mu\nu}(k-p)$$

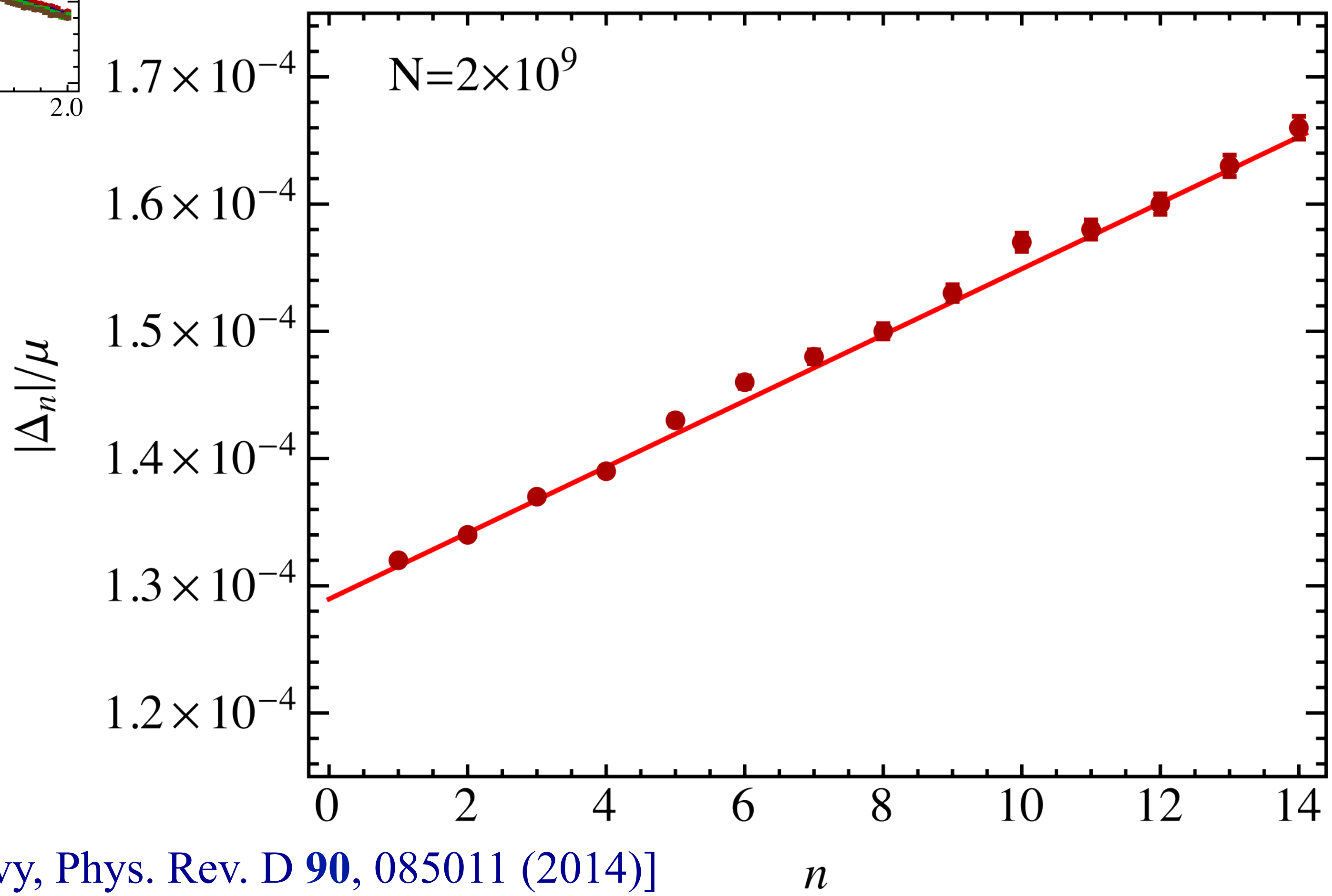
[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

QED in strong field: Δ_n



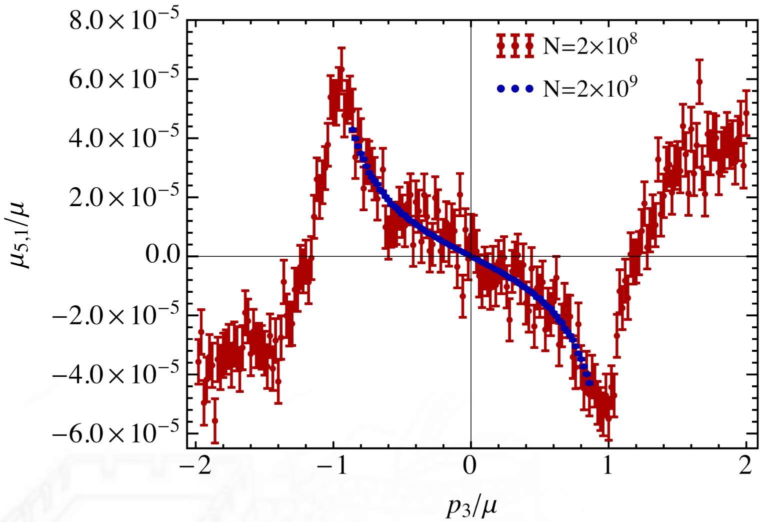
Model fit:

$$\Delta_n = -\frac{\alpha|eB|}{\mu}\left(0.53 + 0.32\frac{|eB|n}{\mu^2}\right)$$



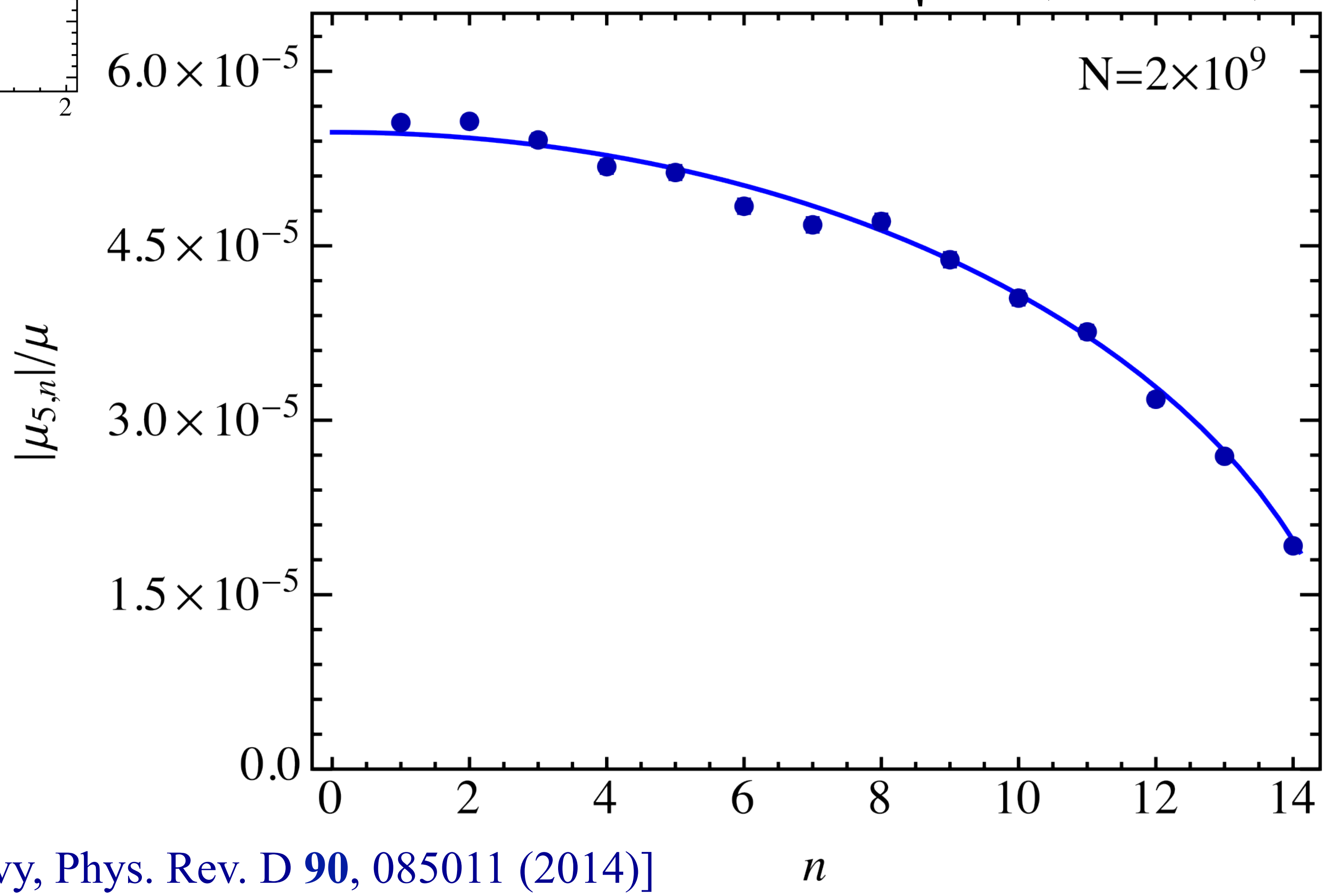
[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: $\mu_{5,n}$



Model fit:

$$\mu_{5,n} = -0.225\frac{\alpha|eB|}{\mu}\sqrt{1-\left(\frac{2n|eB|}{\mu^2}\right)^2}$$

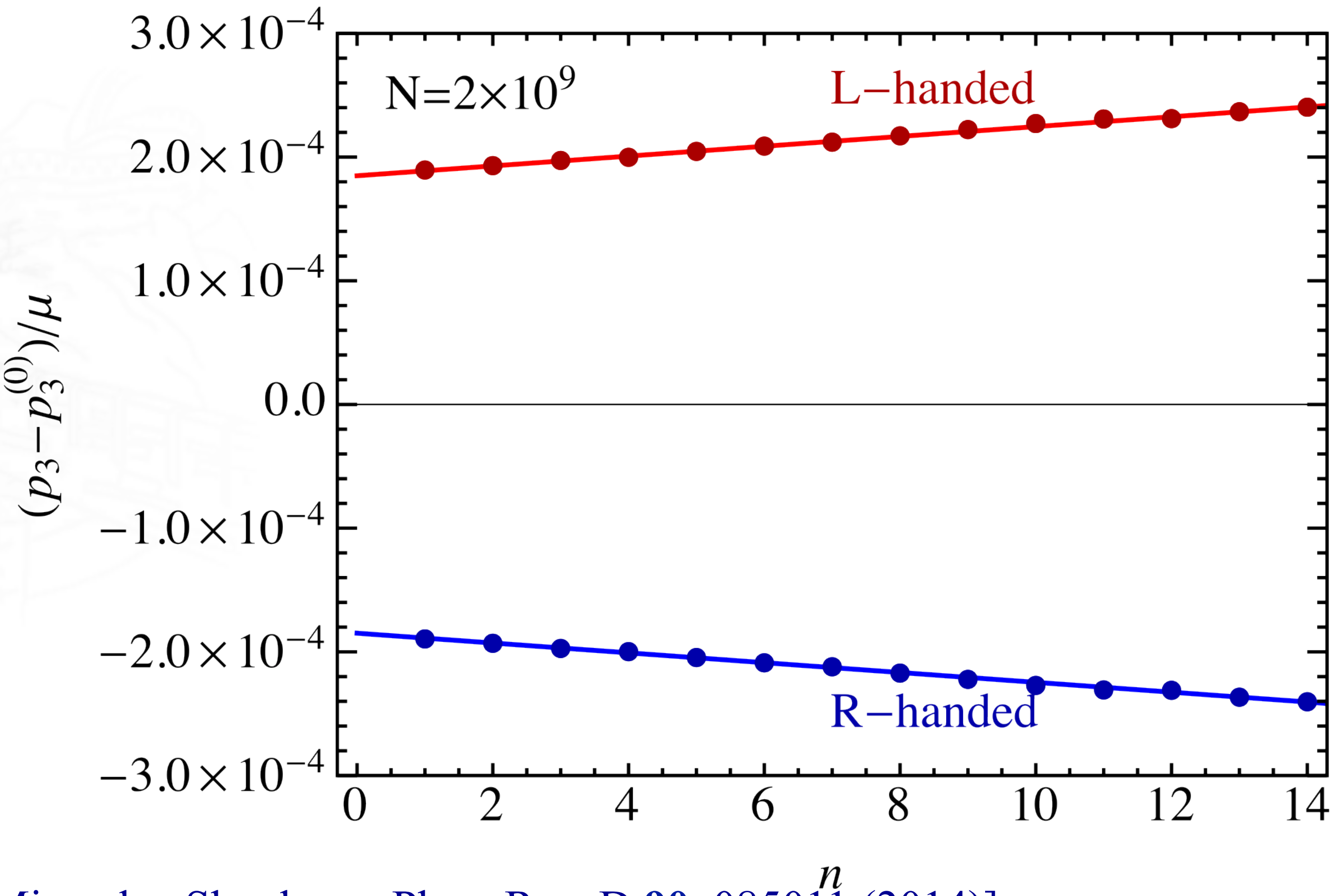


[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: δp_3

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu} \left(0.76 + 0.49 \frac{|eB| n}{\mu^2} \right)$$



[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

How large is the asymmetry?

In QED:

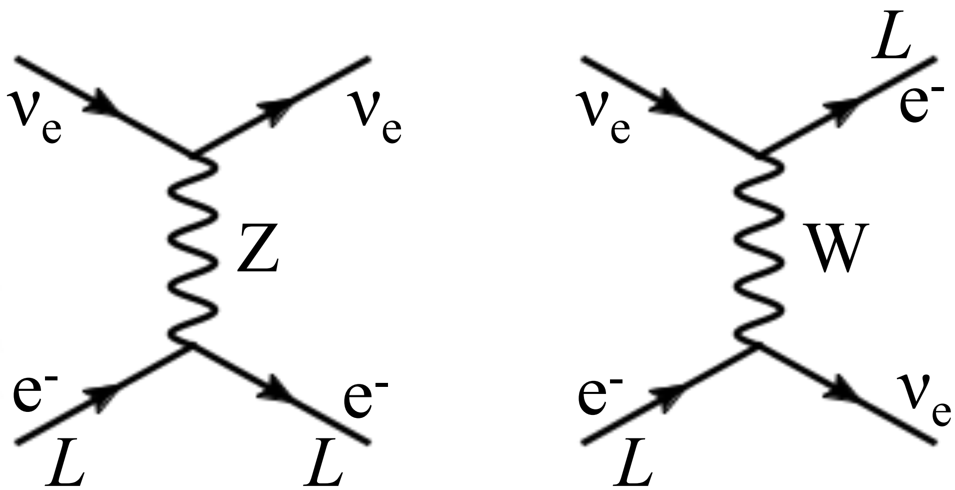
$$\frac{\alpha|eB|}{\mu} \approx 0.4 \left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{100\,\mathrm{MeV}}{\mu}\right) \mathrm{MeV/c}$$

In QCD:

$$\frac{\alpha_s|eB|}{\mu} \approx 10 \left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{400\,\mathrm{MeV}}{\mu}\right) \mathrm{MeV/c}$$

Neutrino asymmetry

- Neutrinos equilibrate with the "flow" of L-handed fermions via



- An asymmetric L-handed Fermi surface with

$$\delta p_3 \sim \alpha\, |eB|/\mu$$

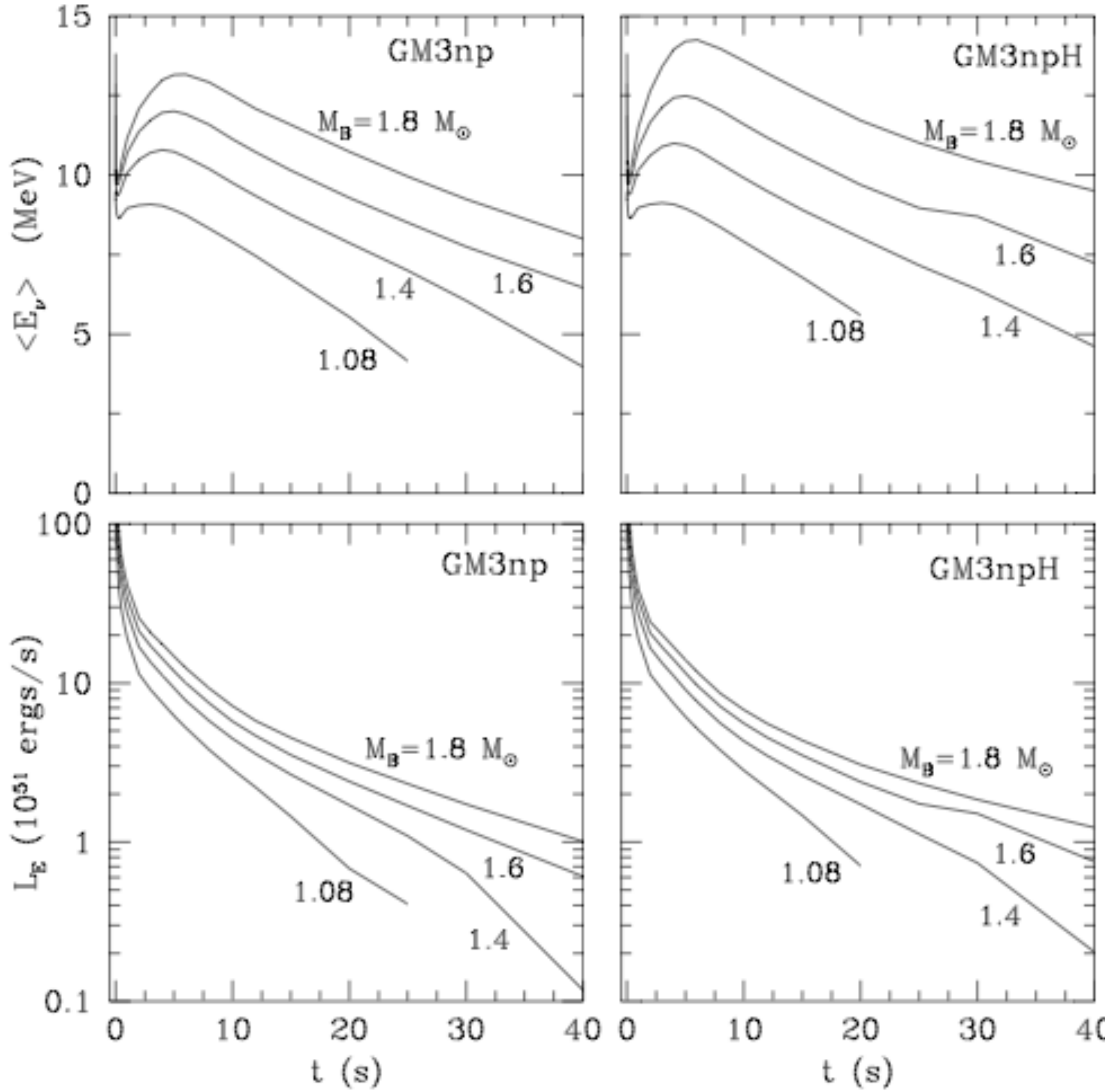
should scatter ν_e's more preferably in the direction of the field

Neutrinos from protoneutron star

$$\langle E_\nu \rangle \approx 5 \text{ to } 10 \text{ MeV}$$

$$L_E \approx 2\times10^{51} \text{ erg/s}$$
$$\approx 10^{57} \text{ MeV/s}$$

$$N_{\text{tot}} \approx \frac{10^{57} \text{ MeV/s}}{5 \text{ MeV}} 20 \text{ s}$$
$$\approx 4\times10^{57}$$



[Pons, Reddy, Prakash, Lattimer, Miralles, Astrophys.J. 513 (1999) 780]

Pulsar kicks

- Sizeable kick carry momenta of order

$$
\begin{aligned}
(1000 \text{ km/s}) \times 1.4 M_{\text{Sun}} &\approx 3 \times 10^{36} \text{ kg} \cdot \text{m/s} \\
&\approx 9 \times 10^{44} \text{ J/c} \\
&\approx 6 \times 10^{57} \text{ MeV/c}
\end{aligned}
$$

i.e., average momentum asymmetry per neutrino should be about

$$
\frac{6 \times 10^{57} \text{ MeV/c}}{4 \times 10^{57}} \approx 1.5 \text{ MeV/c}
$$

Total momentum estimate

- Total momentum carrier by L-handed fermions
  - QED (B=10^{18} G and μ=100 MeV):

$$P \sim N\,\delta p \sim 10^{57}\,\frac{\alpha\left|eB\right|}{\mu} \sim (70\ \text{km/s}) \times 1.4 M_{\text{Sun}}$$

  - QCD (B=10^{18} G and μ=400 MeV):

$$P \sim N\,\delta p \sim 10^{57}\,\frac{\alpha_s\left|eB\right|}{\mu} \sim (1700\ \text{km/s}) \times 1.4 M_{\text{Sun}}$$

- Pulsar kicks? Possible, but questions remain…

Summary

- Magnetism profoundly affects chiral properties of QCD
- $T=0$ & $\mu=0$: Magnetic catalysis (lattice)
- $T\neq0$ & $\mu=0$: Inverse magnetic catalysis (lattice)
- $T=0$ & $\mu\neq0$: Chiral shift (compact stars)
- $T\neq0$ & $\mu\neq0$: CME, CSE, … (relativistic HIC)