ARIZONA STATE UNIVERSITY

NUCLEAR THEORY SEMINAR
JUL 31, 2015
BNL

Magnetized Relativistic Matter: Generalized Landau-level representation & its applications

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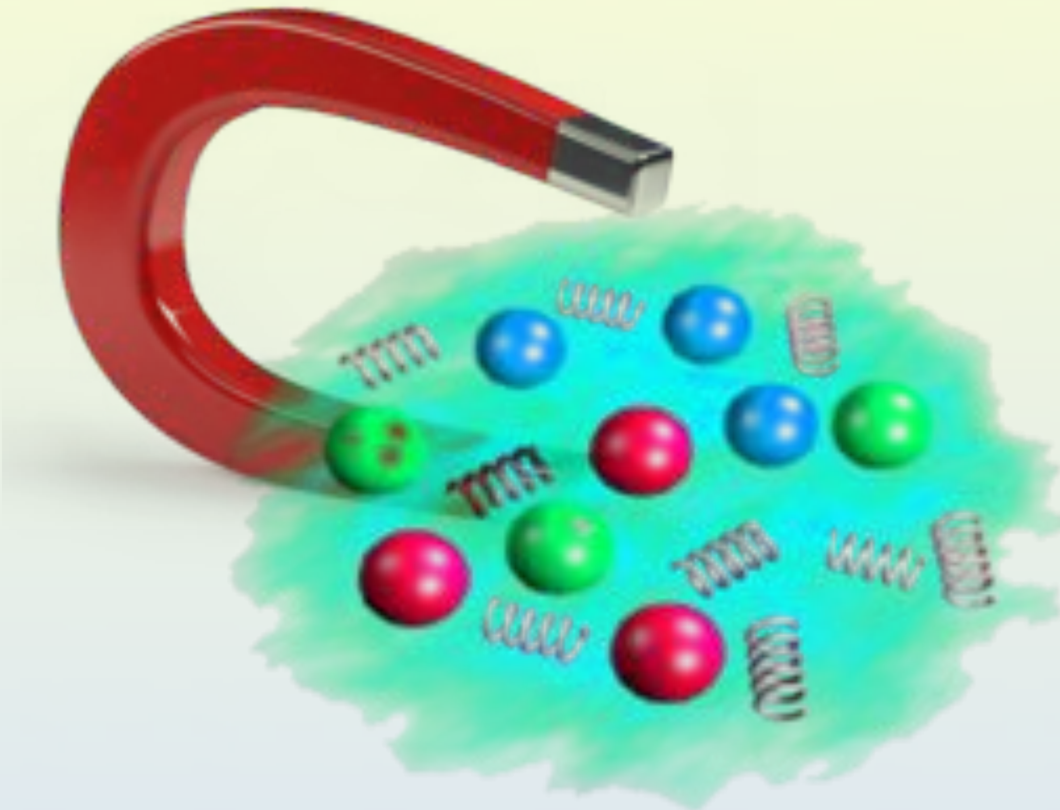
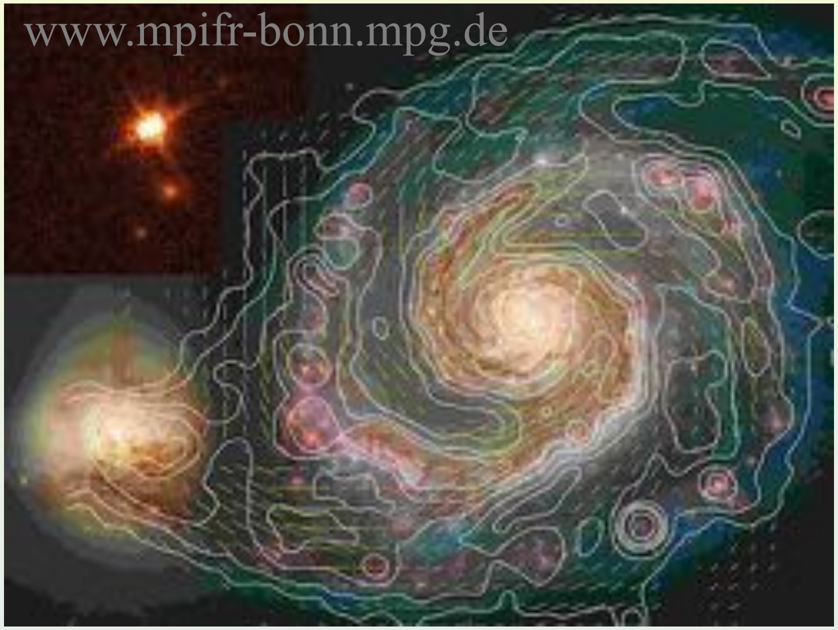


MAGNETIC FIELDS EVERYWHERE

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

Universe



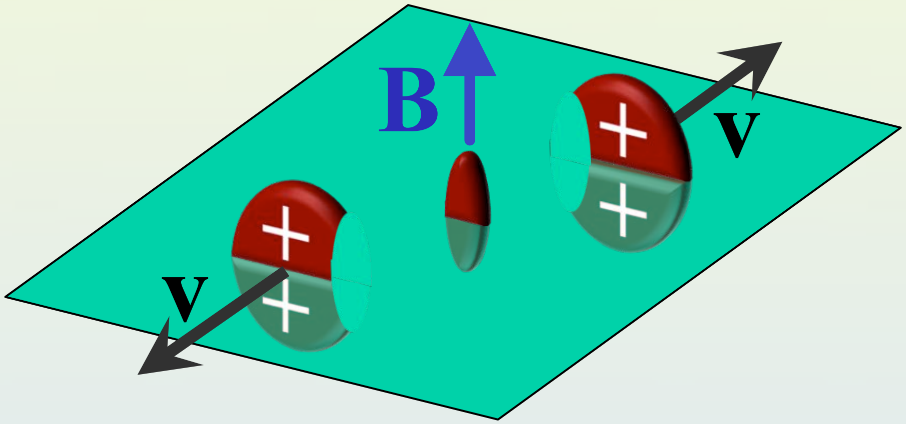
- Current galactic magnetic fields ~ 10^{-6} G
- Current magnetic fields in voids ~ 10^{-15} G
- Problem of magnetogenesis in Early Universe
- Perhaps during the electro-weak phase transition
  - **10^{20} to 10^{24} G** **(~ 1 GeV to 100 GeV)**

Dense baryonic matter

- Magnetized dense baryonic matter
  - **10^{10} to 10^{18} G** **(10 keV to 100 MeV)**
- Magnetic field may affect
  - Competition of ground state phases
  - EoS of dense baryonic matter
  - the M-R relation of compact stars
  - Transport and emission properties
  - Evolution of supernovas & protoneutron stars

Little Bangs

- Magnetized QGP at RHIC/LHC
  - **10^{18} to 10^{19} G (~ 100 MeV)**



[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108]

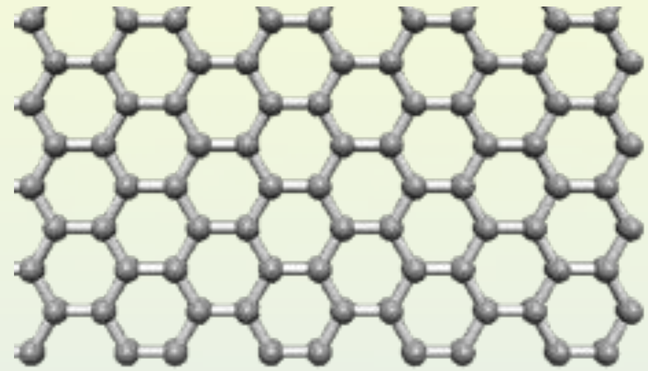
- Using Lienard-Wiechert potentials,

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

Dirac/Weyl materials

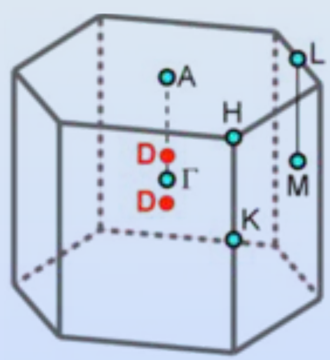
- High magnetic field lab
  - **10^5 G** (~ 100 meV @ vF=c/300)

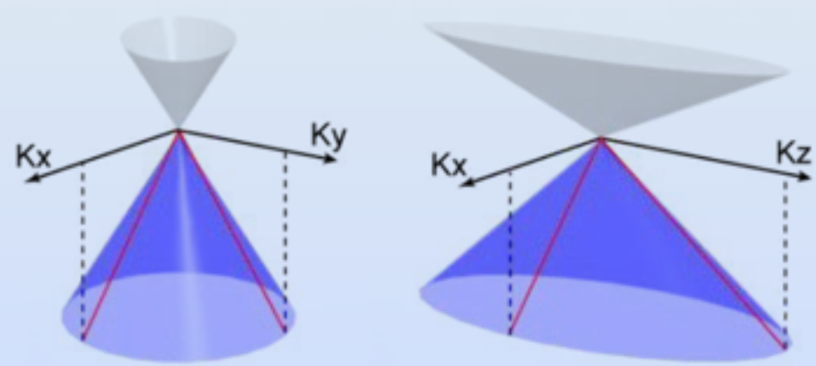


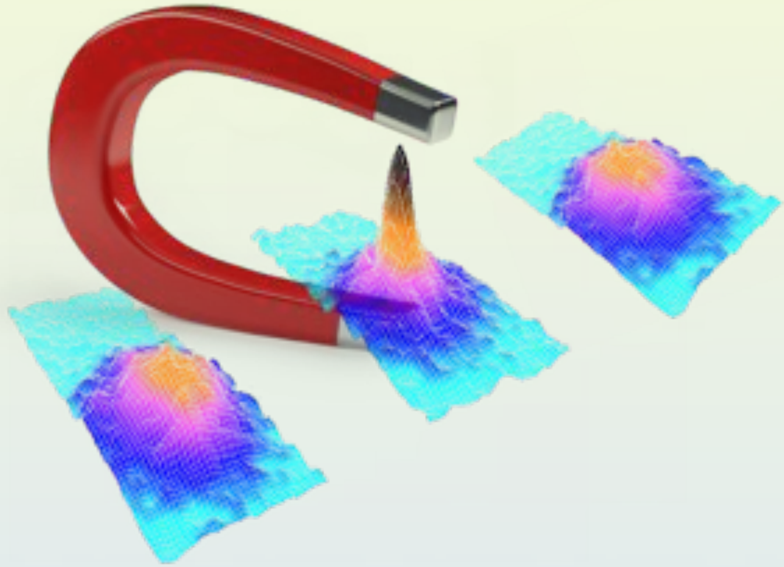
- Graphene
- 3D materials with Dirac/Weyl quasiparticles
  - $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
  - Na_3Bi
  - Cd_3As_2
  - $ZrTe_5$
  - TaAs, NbAs, TaP, …

[Z. K. Liu et al., arXiv:1310.0391]
[M. Neupane et al., arXiv:1309.7892]
[S. Borisenko et al., arXiv:1309.7978]
[X. Li et al., arXiv:1412.6543]

[arXiv:1502.03807, arXiv:1502.04684, arXiv:1504.01350, arXiv:1507.00521]







RESEARCH DIRECTIONS

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

Magnetic catalysis

- Magnetic catalysis of chiral symmetry breaking & anisotropic confinement (vacuum QCD, QCD @ T≠0)

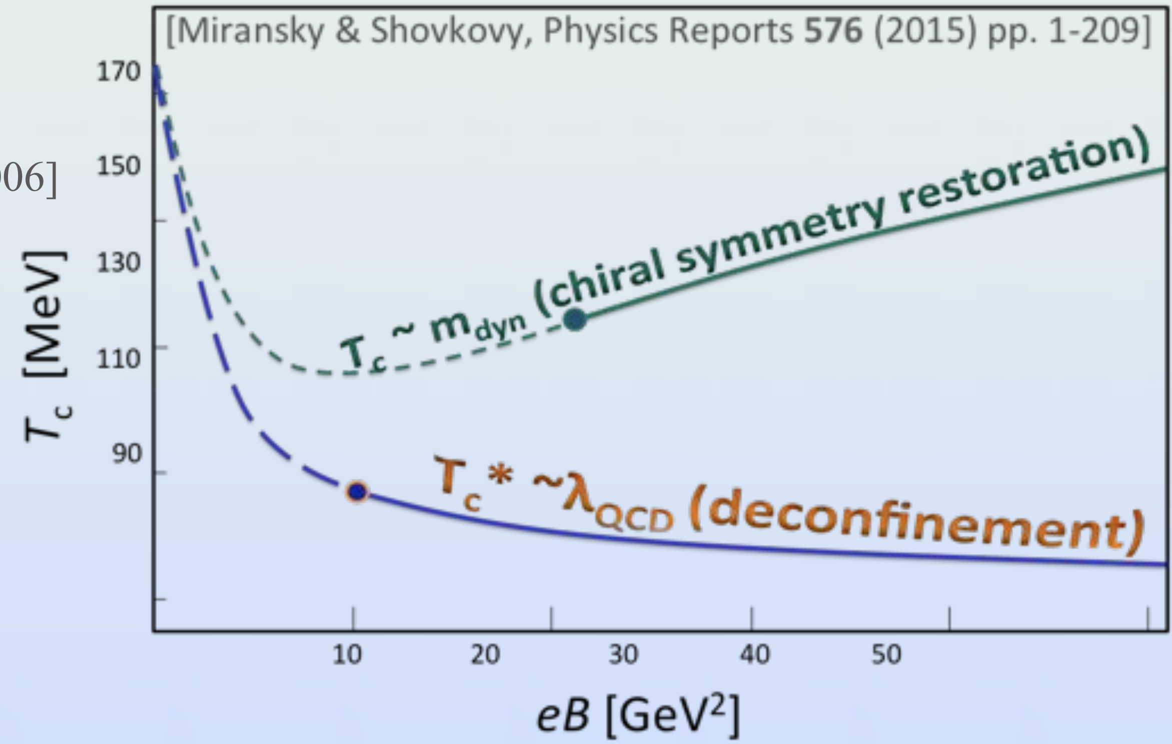
$$\left.\begin{array}{l} E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2} \\ (D \to D-2) \end{array}\right\} \Rightarrow\ m_{\text{dyn}}(B) \neq 0$$

- T=0: catalysis

[Miransky & I.S., Phys. Rev. D **66** (2002) 045006]
[I.S., Lect. Notes Phys. **871**, 13 (2013)]
[Bali et al., Phys. Rev. D**86**, 071502 (2012)]
…

- T_c(B): inverse catalysis

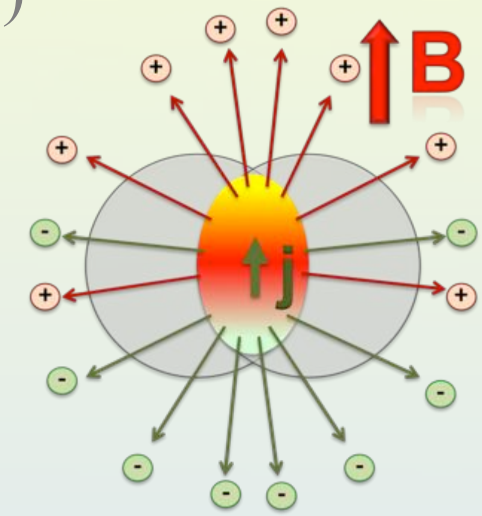
[Bali et al., JHEP 02, 044 (29012)]
[Bali et al., PRD**86**, 071502 (2012)]
[G. Endrodi, arXiv:1504.08280]
…



CME, CSE, etc.

- Chiral magnetic/separation effects, chiral magnetic waves (correlations of charged particle in HIC)

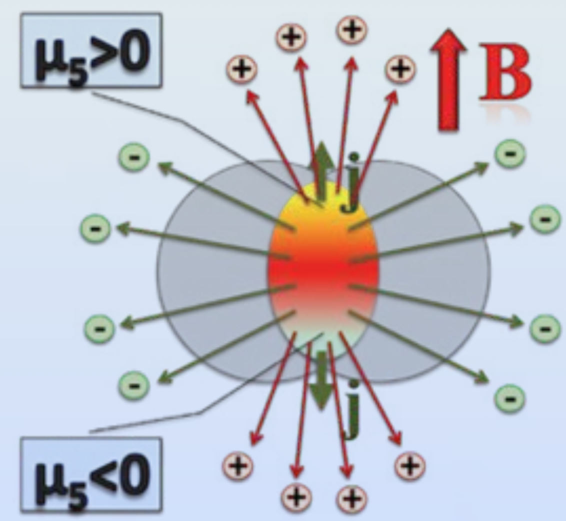
$$\left\langle \vec{j} \right\rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$



- Signs of local P-violation?

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_a^{\mu\nu} \tilde{F}^a_{\mu\nu}$$

[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]
…



- Signs of a chiral magnetic wave?

[Yee, Kharzeev, Phys. Rev. D **83**, 085007 (2011)]
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Magnetic helicity

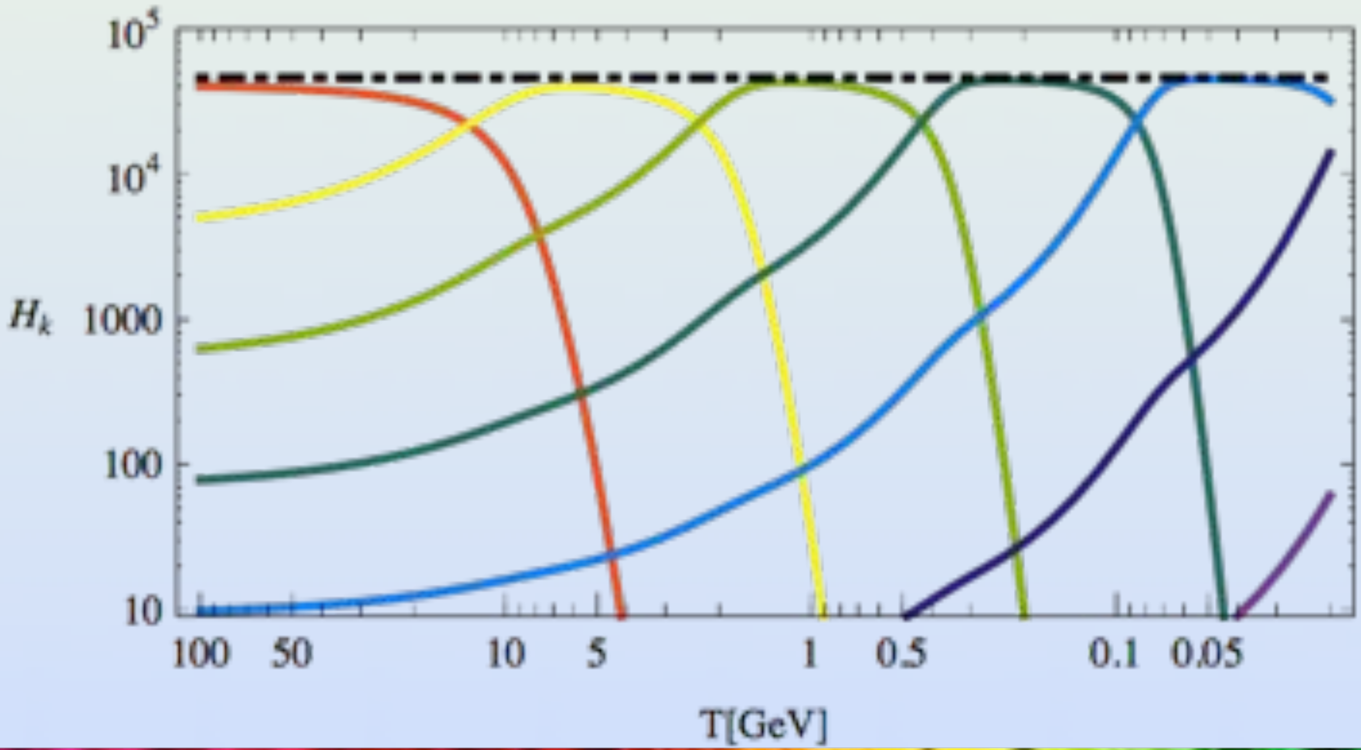
- Magnetic helicity evolution in the Early Universe & in HIC (magnetogenesis, $\mathcal{H}_k \rightarrow \mu_5 \rightarrow \mathcal{H}_k$)

$$\frac{d(n_L - n_R)}{dt} = \frac{2\alpha}{\pi}\frac{1}{V}\int d^3x\,(\vec{E}\cdot\vec{B})$$

$$\nabla \times \vec{B} = \sigma\vec{E} + \frac{\alpha}{\pi}\mu_5\vec{B} + \frac{\partial\vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

[Vilenkin, Phys. Rev. D22, 3080 (1980)]
[Joyce & Shaposhnikov, astro-ph/9703005]
[Giovannini & Shaposhnikov, hep-ph/9710234]
[Boyarsky et al., arXiv:1109.3350]



Hall effect in graphene

- Abnormal quantum Hall effect in graphene (magnetic field induced QH states)

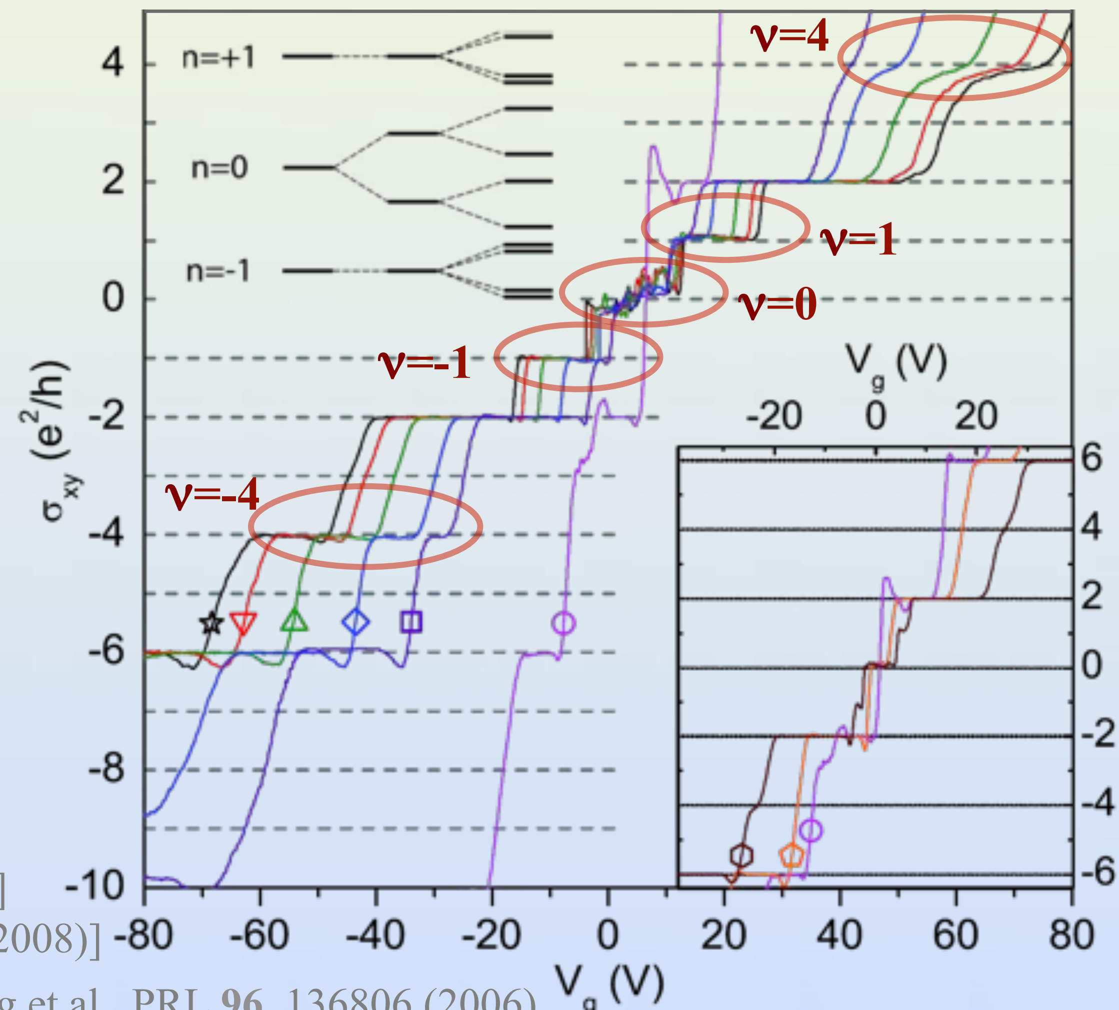
$$H_{\text{low-energy}} = \upsilon_F \int d^2 r\, \bar{\Psi}_s \left(\gamma^1 \pi_x + \gamma^2 \pi_y\right) \Psi_s + H_{\text{int}}$$

- B → 0:

$$\sigma_{xy} = \nu \frac{e^2}{h} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

- Broken symmetry QH states @ large B

$\nu = 0, \pm 1, \pm 3, \pm 4$



[Novoselov et al., Science **315**, 1379 (2007)]
[Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)]
[Checkelsky et al., Phys. Rev. Lett. **100**, 206801 (2008)]
[Xu Du et al., Nature **462**, 192 (2009)]

Zhang et al., PRL **96**, 136806 (2006)

Dirac/Weyl semimetals

- Magnetoresistance in Dirac/Weyl semimetals (signs of chiral anomaly, dHvA oscillations)

"chiral shift"

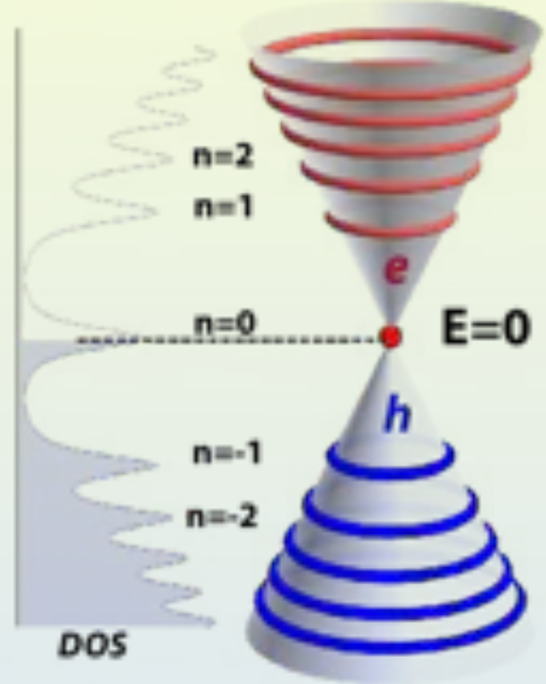
$$H^{(W)} = \int d^3 r \bar{\psi}\left[-i v_F \left(\vec{\gamma}\cdot\mathbf{p}\right) - \left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5 - \mu\gamma^0\right]\psi + H_{\text{int}}$$

- Anomalous contribution to conductivity:

$$\sigma_{33} = \sigma_{33}^{(LLL)} + \sigma_{33}^{(HLL)}, \quad \text{where} \quad \sigma_{33}^{(LLL)} = \frac{e^2 v_F |eB|}{4\pi^2 c\Gamma_0}$$

$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\text{anom}}, \quad \text{where} \quad \sigma_{12,\text{anom}} = \frac{e^2 b}{2\pi^2}$$

[Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]
[Burkov & Balents, Phys. Rev. Lett. 107, 127205 (2011)]
[V. Aji, Phys. Rev. B 85, 241101 (2012)]
[Son & Spivak, Phys. Rev. B 88, 104412 (2013)]
[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89, 085126 (2014)]
[Lu, Shen, Phys. Rev. B 92, 035203 (2015)]
[Klier, Gornyi, Mirlin, arXiv:1507.03481]
[Monteiro, Abanov, Kharzeev, arXiv:1507.05077]



SOME TECHNICAL PROBLEM

Schwinger propagator

- Proper-time representation in (2+1)-D

$$S(x,y)=\exp\left(-e\int_y^x A_\mu\, dz^\mu\right)\bar{S}(x-y)$$

Schwinger phase

$$\bar{S}(x)=\int\frac{d^3k}{(2\pi)^3}e^{-ikx}\bar{S}(k)$$

where

$$\bar{S}(k)=\int_0^\infty ds\exp\left(-is[m^2-k_0^2-\mathbf{k}^2\frac{\tan(eBs)}{eBs}]\right)$$

$$\times\left[k_\nu\gamma^\nu+m-(k^2\gamma^1-k^1\gamma^2)\tan(eBs)\right]\left[1-\gamma^1\gamma^2\tan(eBs)\right]$$

- Landau levels?

[Schwinger, Phys. Rev. 82, 664 (1951)]

Landau level expansion

- Schwinger propagator is equivalent to

$$\bar{S}(k) = i\exp\left(-\frac{\mathbf{k}^2}{|eB|}\right)\sum_{n=0}^{\infty}\frac{(-1)^n D_n(k)}{k_0^2 - m^2 - 2n|eB|}$$

where

$$D_n(k) = (k_0\gamma^0 + m)\left[P_+ L_n\left(\frac{2\mathbf{k}^2}{|eB|}\right) - P_- L_{n-1}\left(\frac{2\mathbf{k}^2}{|eB|}\right)\right] + 4(\mathbf{k}\cdot\boldsymbol{\gamma})L_{n-1}^1\left(\frac{2\mathbf{k}^2}{|eB|}\right)$$

Free propagator → full dynamical propagator?

[Chodos, Everding, Owen, Phys. Rev. D42, 2881 (1990)]

Running dynamical parameters?

- Ritus method

$$\bar{S}(x,y)=\sum_{n=0}^{\infty}\int\frac{dp_0dp_2}{(2\pi)^3}E_p(x)\frac{1}{(\bar{\mathbf{p}}\cdot\gamma)+\Sigma(p)}\bar{E}_p(y)$$

where $\bar{\mathbf{p}}=\left(p_0,0,-\sqrt{2|eB|n}\right)$ and $\bar{E}_p(x)=\gamma^0E_p^+(x)\gamma^0$

$$(\pi_\nu\gamma^\nu)E_p(x)=E_p(x)(\bar{\mathbf{p}}\cdot\gamma)$$

- Shortcomings:
  - Schwinger phase is never fully factorized
  - Eigenstates $E_p(x)$ are Dirac matrices
  - Obscure meaning of parameters in Σ(p)

A better representation?

- Full propagator

$$G(x,y) = i\left\langle x \left\| \left[k_0\gamma^0 + \hat{F}^+ \left(\boldsymbol{\pi}\cdot\boldsymbol{\gamma} \right) + \hat{\Sigma}^+ \right]^{-1} \right| y \right\rangle$$

Here $\hat{F}^\pm$ & $\hat{\Sigma}^\pm$ are functions of $\underbrace{\gamma^0, i\gamma^1\gamma^2, \& \left(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}\right)^2}$

3 mutually commuting operators

$$\hat{F}^\pm = f \pm \gamma^0 g \pm i\gamma^1\gamma^2 \tilde{g} + i\gamma^0\gamma^1\gamma^2 \tilde{f}$$

$$\hat{\Sigma}^\pm = m \pm \gamma^0 \mu \pm i\gamma^1\gamma^2 \tilde{\mu} + i\gamma^0\gamma^1\gamma^2 \Delta$$

Dirac mass

Haldane mass

where $f, g, \tilde{g}, \tilde{f}, m, \mu, \tilde{\mu}, \Delta$ depend on $\left(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}\right)^2$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

Complete set of eigenstates

- Eigenstates for orbital motion:

$$\boldsymbol{\pi}^2 \left| N\, p \right\rangle = \left| eB \right| (2N+1) \left| N\, p \right\rangle$$

E.g., in the Landau gauge $\mathbf{A}=(0,Bx)$,

$$\left\langle \mathbf{r} \middle| N\, p \right\rangle = C_N H_N \left(\frac{x}{l} + pl \right) \exp\left(-\frac{(x+pl^2)^2}{2l^2} + ipy \right)$$

They satisfy

$$\int d^2\mathbf{r} \left\langle N\, p \middle| \mathbf{r} \right\rangle \left\langle \mathbf{r} \middle| N'\, p' \right\rangle = \delta_{NN'} \delta\left(p - p' \right)$$

$$\sum_{N=0}^{\infty} \int_{-\infty}^{\infty} dp \left\langle \mathbf{r} \middle| N\, p \right\rangle \left\langle N\, p \middle| \mathbf{r}' \right\rangle = \delta\left(\mathbf{r} - \mathbf{r}' \right)$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

New LL representation

- Proper-time representation in (2+1)-D

$$G(x,y)=\exp\left(-e\int_y^x A_\mu\, dz^\mu\right)\bar{G}(x-y)$$

Schwinger phase

$$\bar{G}(t,\mathbf{r})=\int\frac{dk_0}{2\pi}e^{-ik_0 t}\bar{G}(k_0,\mathbf{r})$$

where

$$\bar{G}(k_0,\mathbf{r})=i\frac{e^{-\xi/2}}{2\pi l^2}\sum_{n,\sigma,s}\left(\frac{s(k_0+\mu_{n,\sigma})-m_{n,\sigma}}{(k_0+\mu_{n,\sigma})^2-E_{n,\sigma}^2}\left[\delta_{-\sigma}^s L_n(\xi)+\delta_\sigma^s L_{n-1}(\xi)\right]\right.$$

$$\left.+\frac{i(f_{n,\sigma}-s\,g_{n,\sigma})}{(k_0+\mu_{n,\sigma})^2-E_{n,\sigma}^2}\frac{(\gamma\cdot\mathbf{r})}{l^2}\right)P_{s,s\sigma}, \qquad \xi=\mathbf{r}^2/(2l^2)$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

LL energies

- Landau-level energies

Haldane mass

Dirac mass

$$n=0: \quad E_{0,\sigma} = \Delta_0 + \sigma m_0$$

$$n \geq 1: \quad E_{n,\sigma} = \sqrt{2(f_{n,\sigma}^2 - g_{n,\sigma}^2) n |eB| + m_{n,\sigma}^2}$$

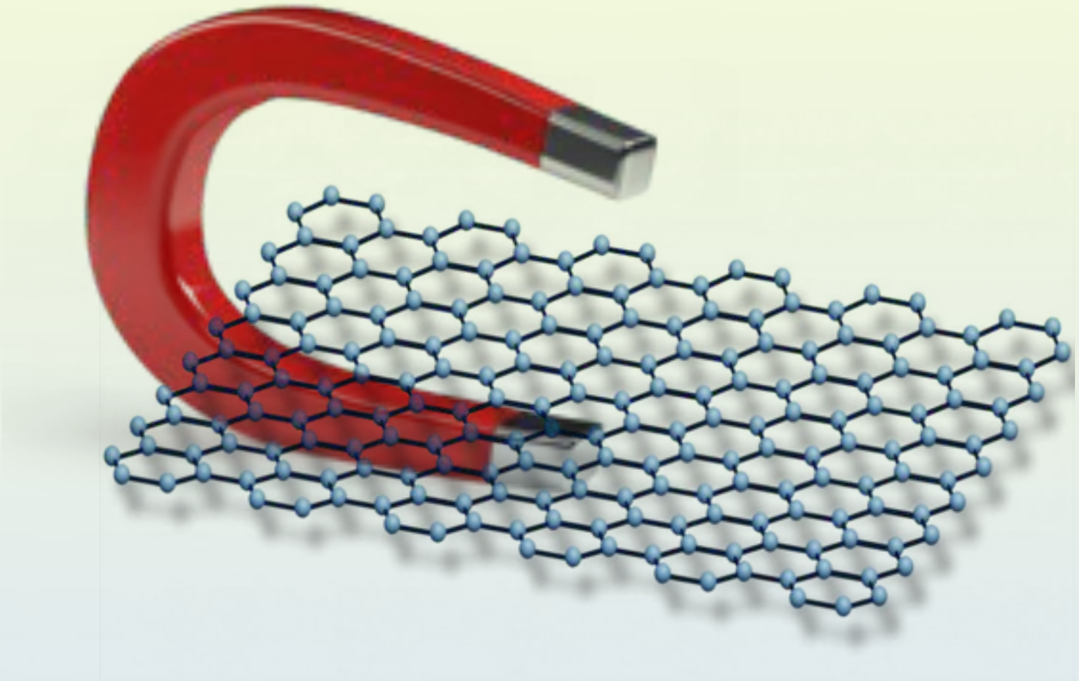
where $\quad m_{n,\sigma} = m_n + \sigma \Delta_n, \quad \mu_{n,\sigma} = \mu_n + \sigma \tilde{\mu}_n, \quad$ etc.

By definition, $n = N + (1+s_{12})/2$ and

$$m\left((\boldsymbol{\pi}\cdot\boldsymbol{\gamma})^2\right)\left|N, s_{12}\right\rangle = m_n \left|N, s_{12}\right\rangle,$$

$$f\left((\boldsymbol{\pi}\cdot\boldsymbol{\gamma})^2\right)\left|N, s_{12}\right\rangle = f_n \left|N, s_{12}\right\rangle, \quad \text{etc.}$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]



(2+1)-D GENERALIZED LANDAU-LEVEL REPRESENTATION

Coulomb interaction in graphene

- Model Hamiltonian

U(4)

$\mathrm{SU(2)}_{\uparrow}\times\mathrm{SU(2)}_{\downarrow}$

$$H = H_0 + H_{\text{Coulomb}} + \mu_B B \int d^2 r \Psi^{+} \sigma_3 \Psi$$

- MC and QHF order parameters

$m_{n,s}$	:	$\bar{\Psi}_s \Psi_s$	Dirac mass (CDW)
$\Delta_{n,s}$	:	$\bar{\Psi}_s (i\gamma^0\gamma^1\gamma^2)\Psi_s$	Haldane mass
$\tilde{\mu}_{n,s}$	:	$\bar{\Psi}_s (i\gamma^1\gamma^2)\Psi_s$	pseudospin density
$\mu_{n,3}$	:	$\bar{\Psi}_{\uparrow}\gamma^0\Psi_{\uparrow} - \bar{\Psi}_{\downarrow}\gamma^0\Psi_{\downarrow}$	spin polarization

- Note that $\Psi_s^T = \left(\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, \psi_{K'As}\right)$

Normal ground state (B→0)

- No symmetry breaking parameters
- Renormalization of v_F:

$$n \geq 1: \quad E_n = v_F \sqrt{2n|eB|} \quad \longrightarrow \quad E_n = f_n v_F \sqrt{2n|eB|}$$

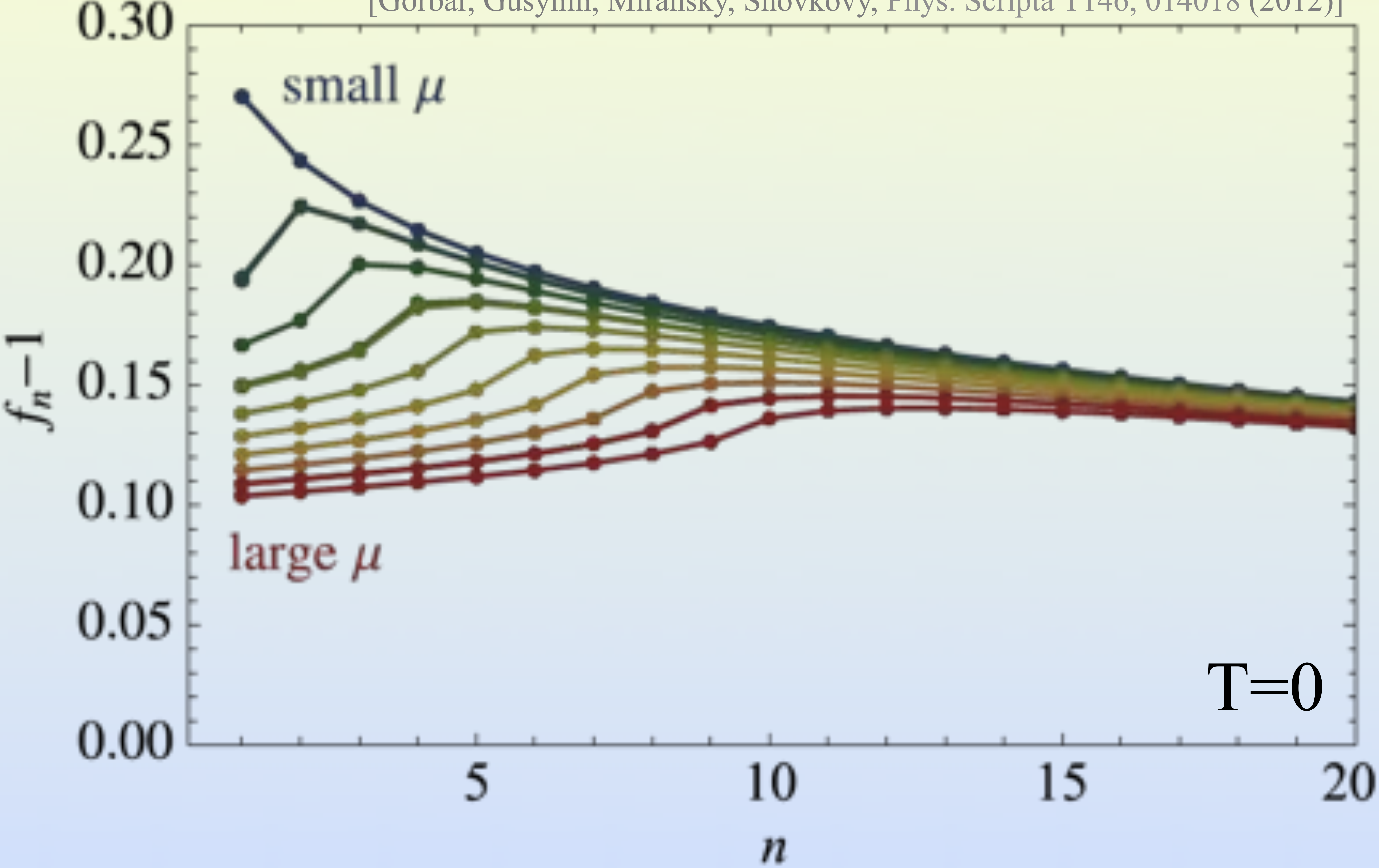
- Schwinger-Dyson equations for f_n

$$f_n = 1 + \frac{\alpha}{2} \sum_{n=1}^{n_{\max}} \frac{\kappa^{(1)}_{n'-1,n-1}}{n\sqrt{2n'}} \left[1 - n_F\left(E_{n'} - \mu\right) - n_F\left(E_{n'} + \mu\right)\right]$$

$$\kappa^{(i)}_{m,n} = \int_0^\infty \frac{dk}{2\pi} \frac{kl\ L^{(i)}_{m,n}(kl)}{k + \Pi(0,k)}, \quad \text{where} \quad l = 1/\sqrt{|eB|}$$

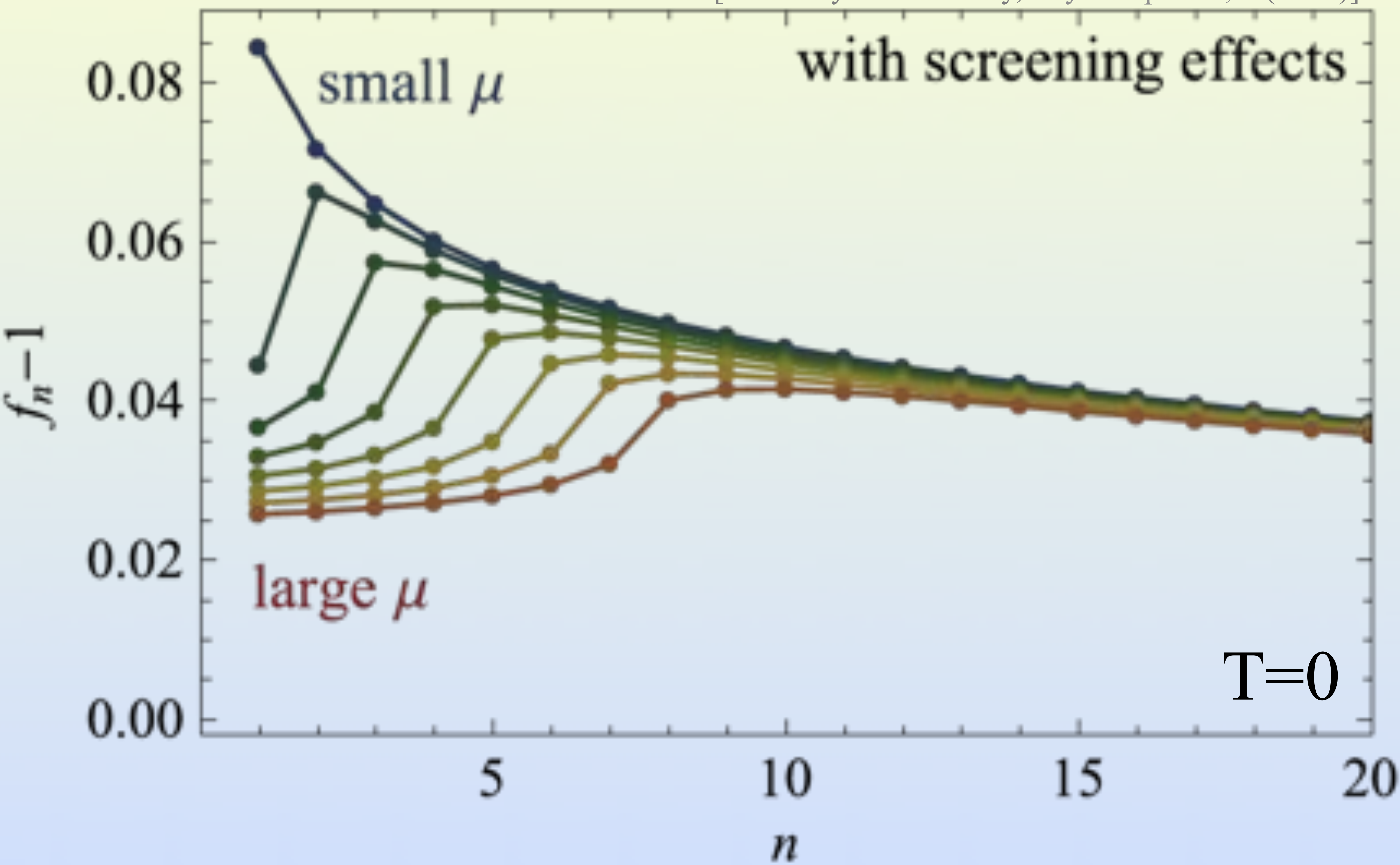
Renormalization of v_F

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

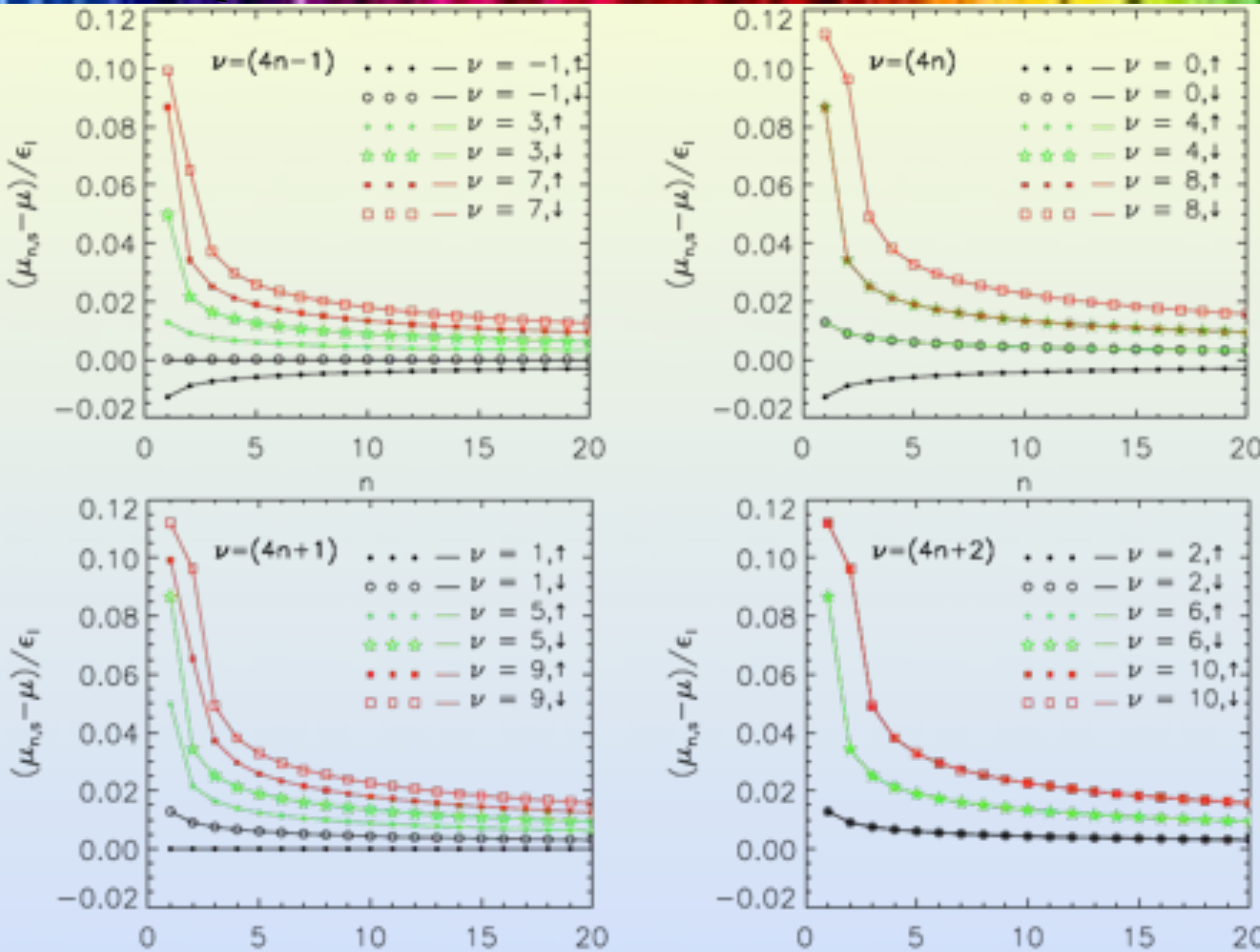


Renormalization of v_F

[Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]

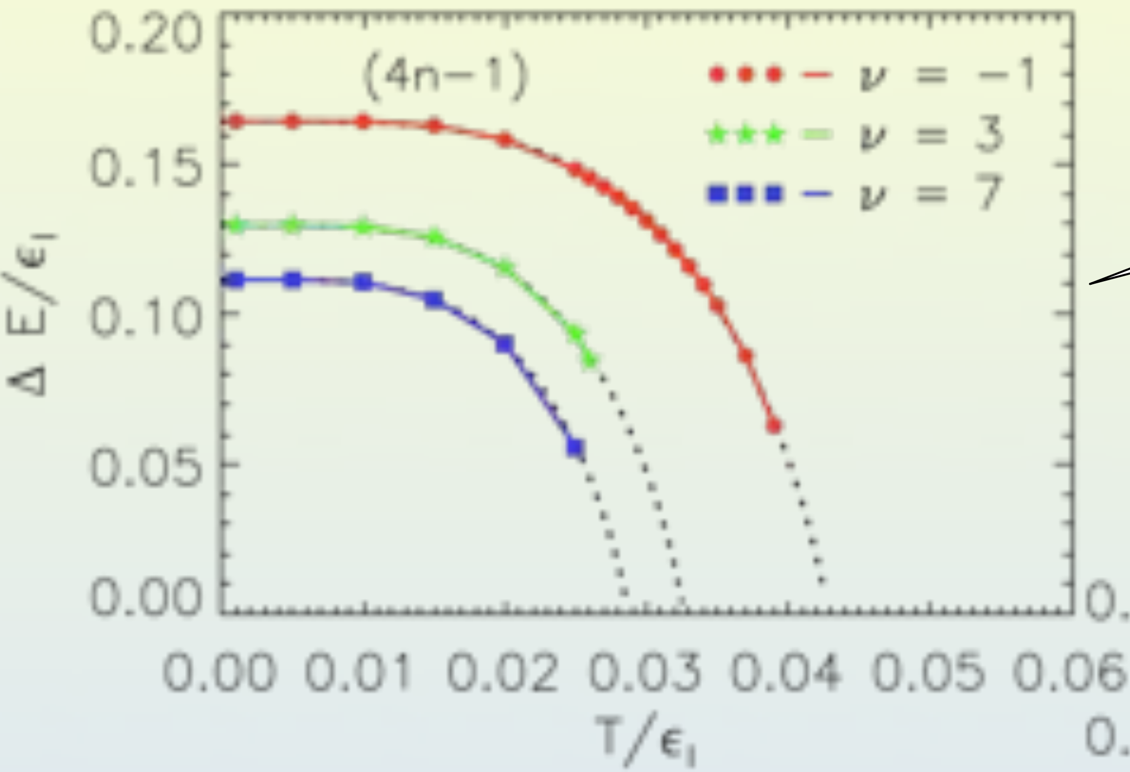


QH states with different ν



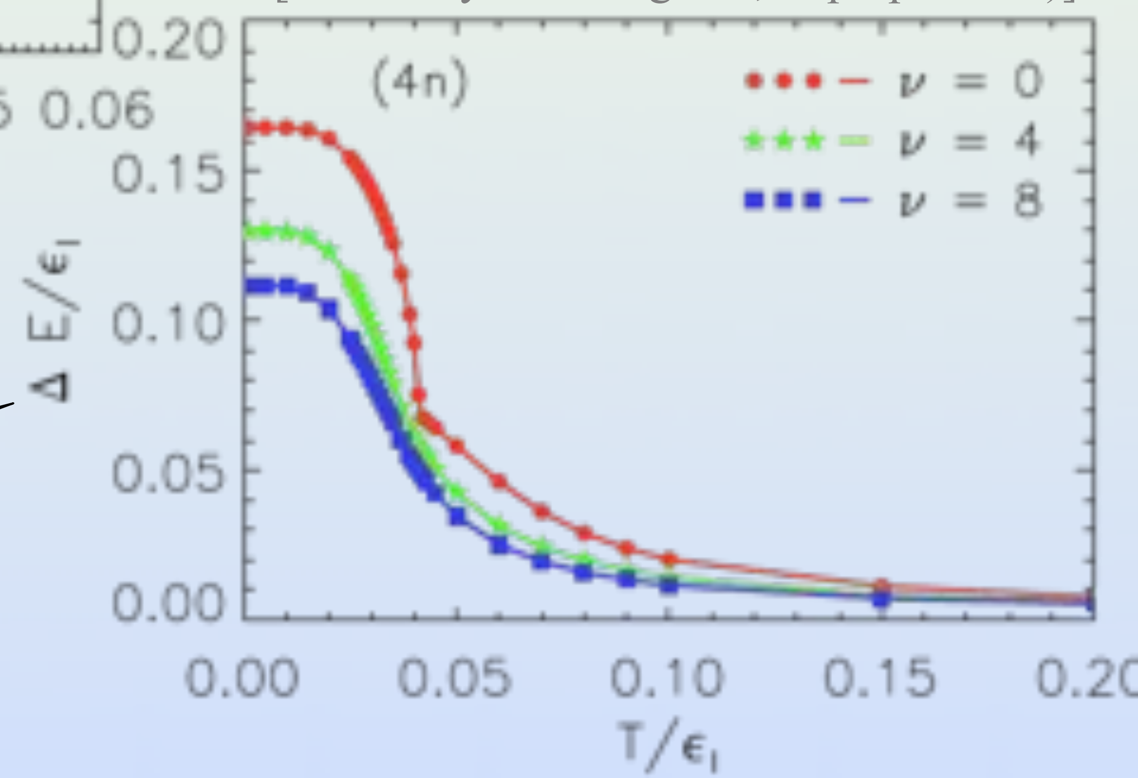
[Shovkovy & Lifang Xia, in preparation)]

Some results at T≠0



U(1)$_\uparrow$×SU(2)$_\downarrow$

[Shovkovy & Lifang Xia, in preparation)]



SU(2)$_\uparrow$×SU(2)$_\downarrow$

(3+1)-D GENERALIZED LANDAU-LEVEL REPRESENTATION

Self-energy, B≠0

- Same as (inverse) propagator

$$\Sigma(x,y) = \exp\left(-e\int_y^x A_\mu\, dz^\mu\right)\bar{\Sigma}(x-y)$$

$$\bar{\Sigma}(x) = \int \frac{d^4k}{(2\pi)^2} e^{-ikx}\bar{\Sigma}(k)$$

where

$$\bar{\Sigma} = 2e^{-\mathbf{k}_\perp^2 l^2}\sum_{n=0}^{\infty}(-1)^n\left(m_n + \gamma^3\gamma^5\Delta_n - \gamma^0\gamma^5\mu_{5,n} + \ldots\right)\left[P_+L_n - P_-L_{n-1}\right] + \ldots$$

LL projections

Landau-level parameters m_n, Δ_n, $\mu_{5,n}$, … are given by LL-"projections", e.g.,

$$\Delta_n(k_0,k_3) = \frac{(-1)^n l^2}{8\pi}\int d^2k_\perp e^{-k_\perp^2 l^2}\left[L_n + L_{n+1}\right]Tr\left[\gamma^0\overline{\Sigma}(k)\right]$$

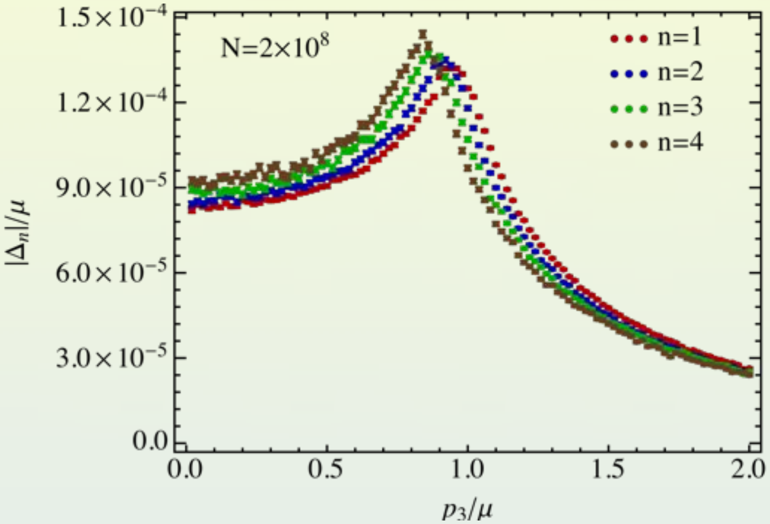
$$\mu_{5,n}(k_0,k_3) = \frac{(-1)^n l^2}{8\pi}\int d^2k_\perp e^{-k_\perp^2 l^2}\left[L_n - L_{n+1}\right]Tr\left[\gamma^0\gamma^5\overline{\Sigma}(k)\right]$$

where, at 1-loop order,

$$\overline{\Sigma}(p) = -4i\pi\alpha\int\frac{d^4k}{(2\pi)^4}\gamma^\mu\,\overline{S}(k)\gamma^\nu D_{\mu\nu}(k-p)$$

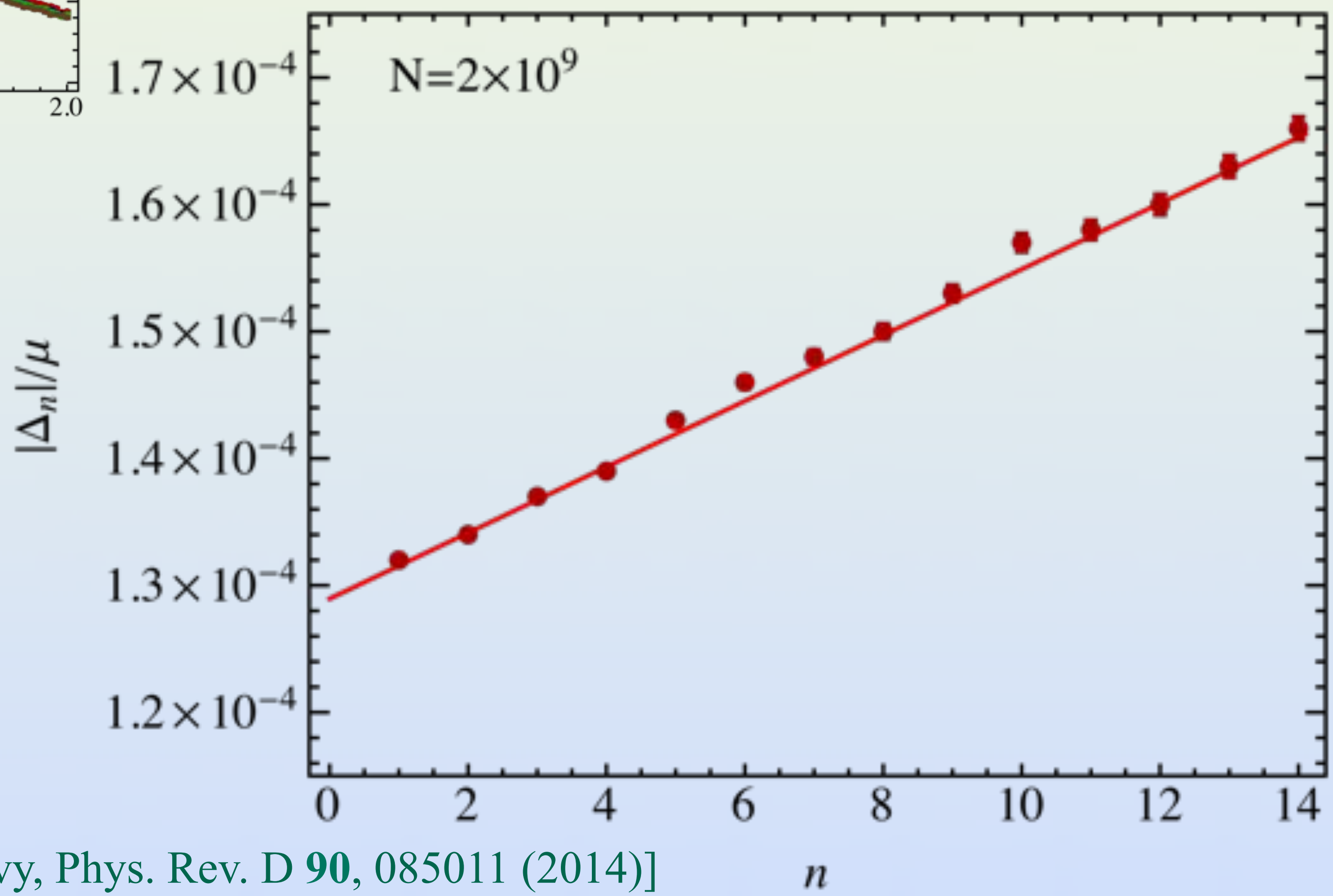
[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Chiral shift QED @ B≠0



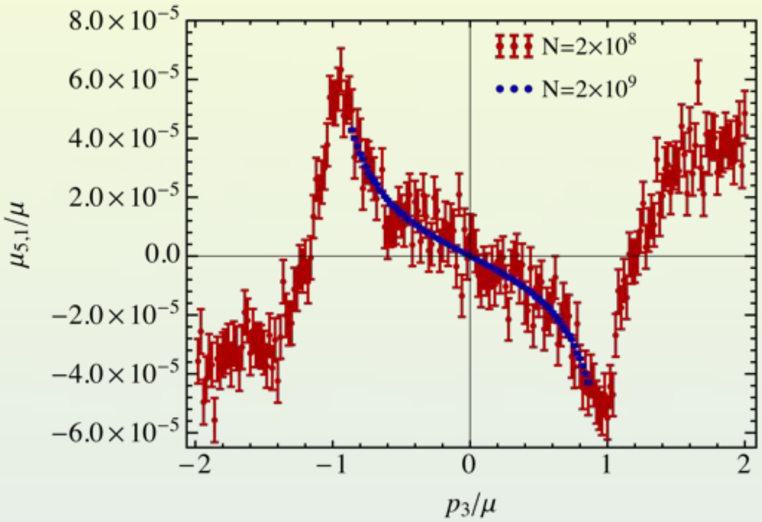
Model fit:

$$\Delta_n = -\frac{\alpha|eB|}{\mu}\left(0.53 + 0.32\frac{|eB|n}{\mu^2}\right)$$



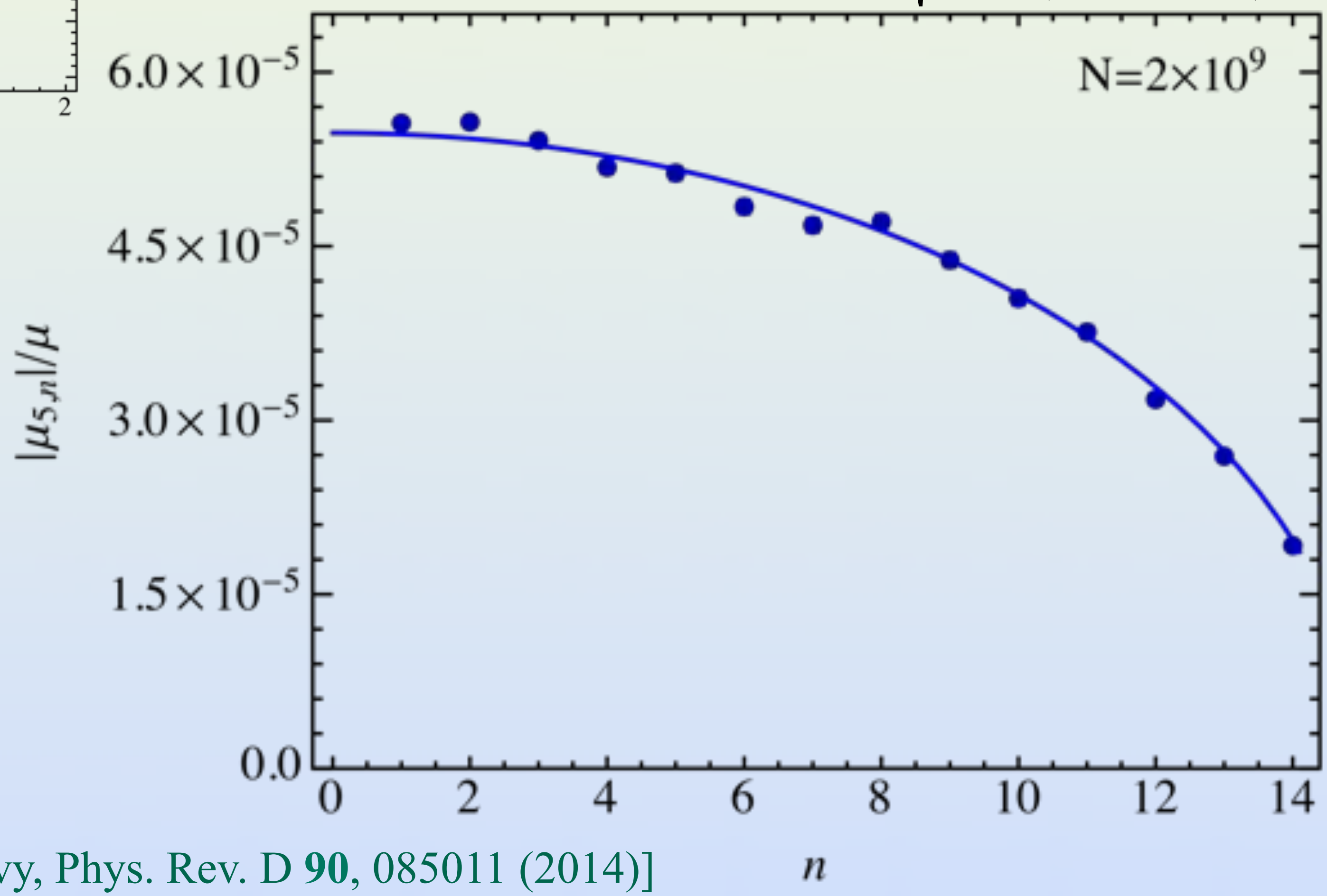
[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

$\mu_{5,n}$ in QED @ B≠0



Model fit:

$$\mu_{5,n} = -0.225\frac{\alpha|eB|}{\mu}\sqrt{1-\left(\frac{2n|eB|}{\mu^2}\right)^2}$$

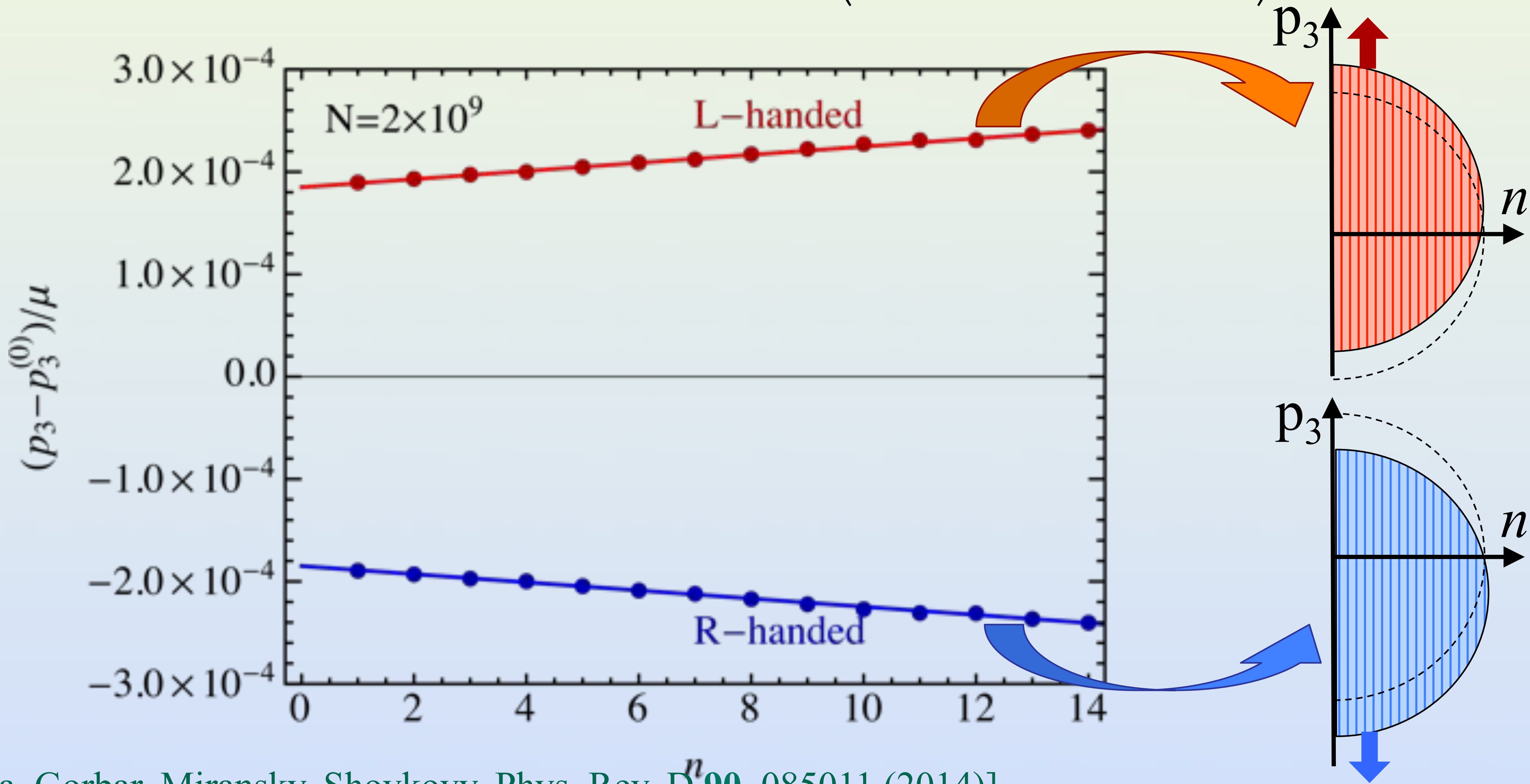


[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: δp_3

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu}\left(0.76 + 0.49 \frac{|eB| n}{\mu^2}\right)$$



[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

How large is the asymmetry?

In QED:

$$\frac{\alpha\left|eB\right|}{\mu} \approx 0.4\left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{100\,\mathrm{MeV}}{\mu}\right)\mathrm{MeV/c}$$

In QCD:

$$\frac{\alpha_s\left|eB\right|}{\mu} \approx 10\left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{400\,\mathrm{MeV}}{\mu}\right)\mathrm{MeV/c}$$

may have some consequences for compact stars…

Summary

- Many relativistic systems where nonzero B plays an essential role
- New generalized LL-representation is proposed
- Works in 2+1 as well as in 3+1 dimensions
- QH states in graphene can be described with highly flexible parametrization
- Chiral asymmetry in QED at $B \neq 0$