ARIZONA STATE UNIVERSITY

Magnetic Field Workshop
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Chirality in magnetized relativistic plasma

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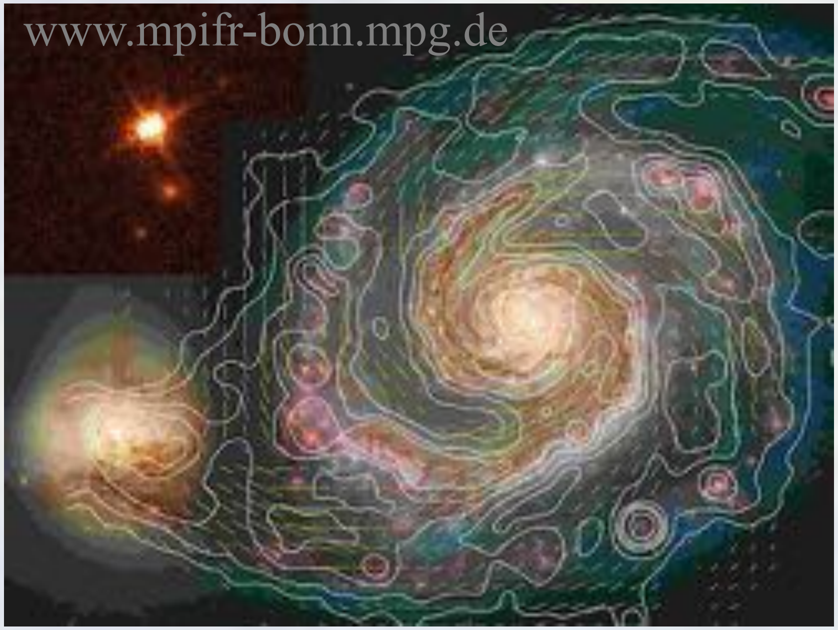
MAGNETIC FIELDS EVERYWHERE

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

- Universe
- Heavy-ion collisions
- Compact stars
- Dirac semimetals, graphene, etc.

Universe

- Current galactic magnetic fields ~ 10^{-6} G
- Current magnetic fields in voids ~ 10^{-15} G



- Problem of magnetogenesis in Early Universe
- Perhaps during the electro-weak phase transition
  - **10^{20} to 10^{24} G** (~ 1 GeV to 100 GeV)

Magnetic field/helicity

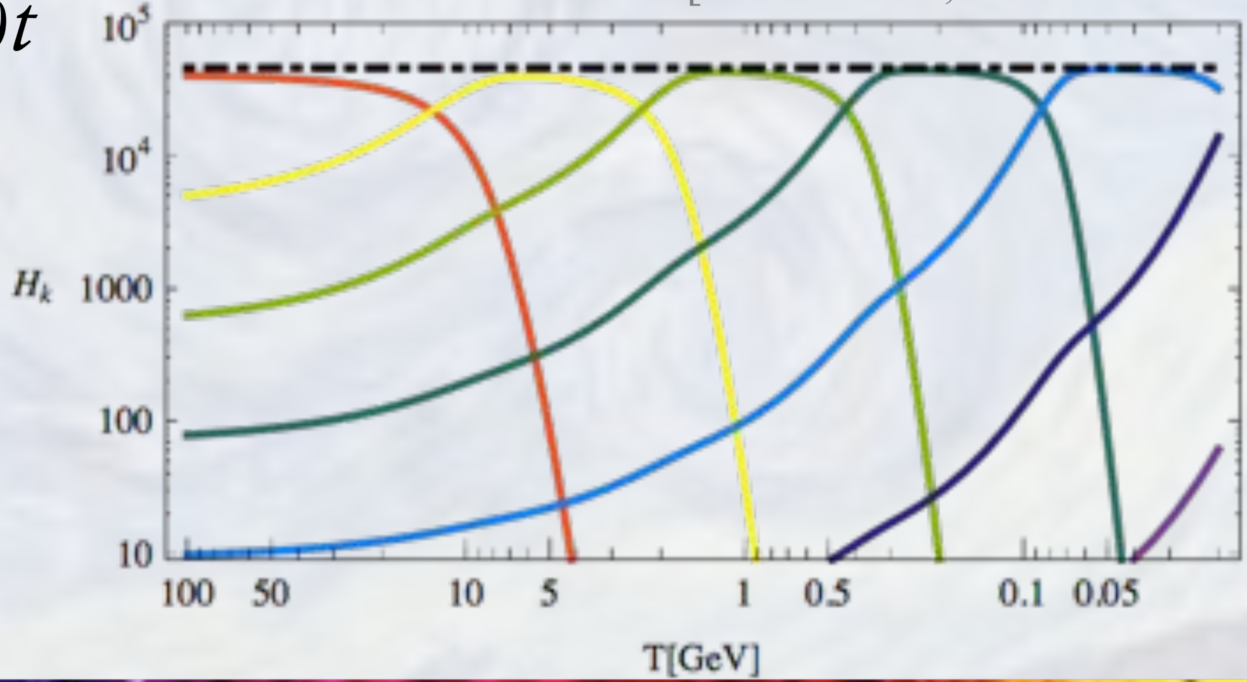
- Magnetic helicity evolution in the Early Universe

$$\frac{d(n_L - n_R)}{dt} = \frac{2\alpha}{\pi}\frac{1}{V}\int d^3x\,(\vec{E}\cdot\vec{B})$$

$$\nabla\times\vec{B} = \sigma\vec{E} + \frac{\alpha}{\pi}\mu_5\vec{B} + \frac{\partial\vec{E}}{\partial t}$$

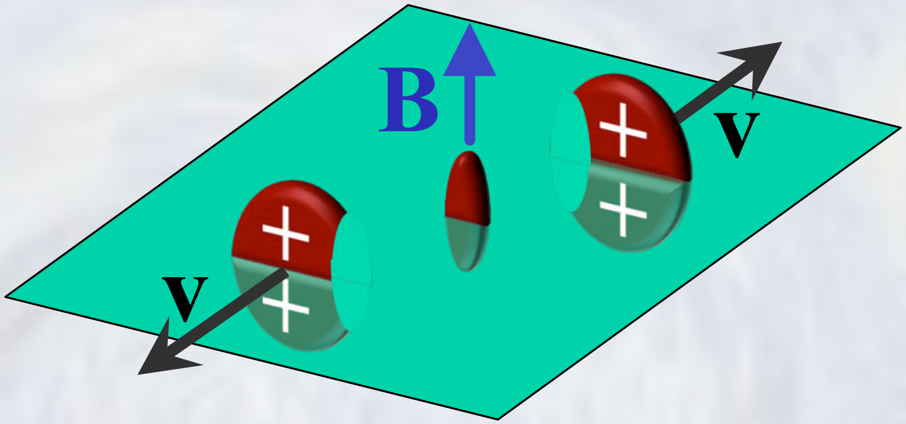
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

[Vilenkin, Phys. Rev. D22, 3080 (1980)]
[Joyce & Shaposhnikov, astro-ph/9703005]
[Giovannini & Shaposhnikov, hep-ph/9710234]
[Boyarsky et al., arXiv:1109.3350]
[Tashiro et al., arXiv:1206.5549]
[Manuel et al., arXiv:1501.07608]
[Buividovich et al., arXiv:1509.02076]
[Hirono et al., arXiv:1509.07790]



Little Bangs

- Magnetized QGP at RHIC/LHC
  - **B ~ 10^{18} to 10^{19} G** (~ 100 MeV)



[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108]
…

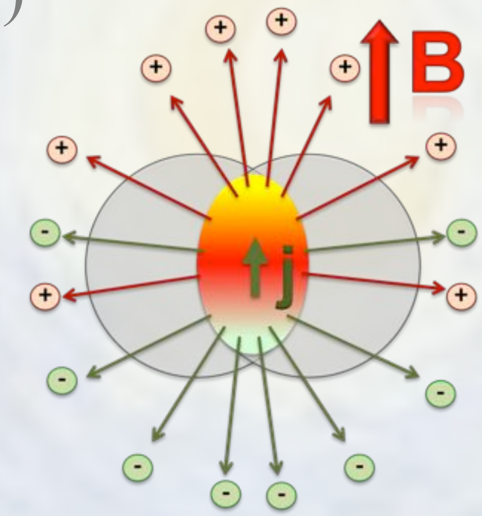
- Using Lienard-Wiechert potentials,

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

CME, CSE, etc.

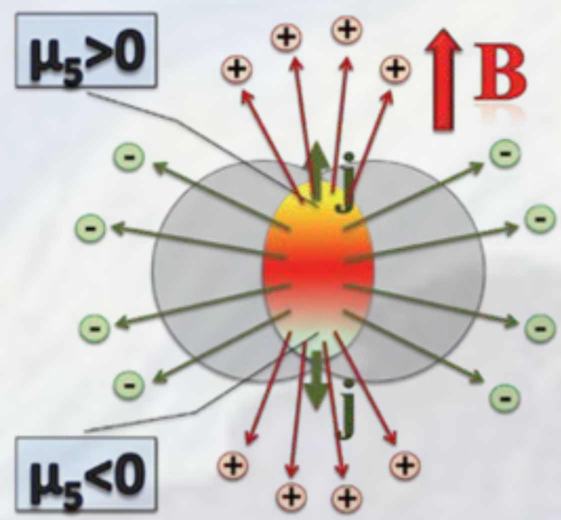
- Chiral magnetic/separation effects, chiral magnetic waves (correlations of charged particle in HIC)



$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5 \quad \& \quad \langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

- Signs of local P-violation?

$$\frac{\partial(n_R - n_L)}{\partial t} = -\frac{g^2 N_f}{16\pi^2} F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$



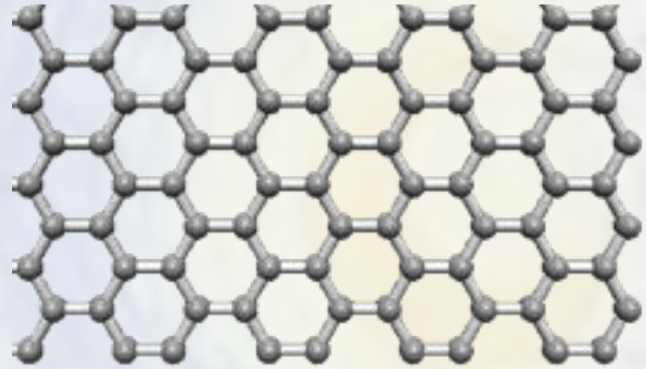
[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]
Review: Kharzeev, Liao, Voloshin, Wang, arXiv:1511.04050 [hep-ph]

- Signs of a chiral magnetic wave?

[Yee, Kharzeev, Phys. Rev. D **83**, 085007 (2011)]
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Dirac/Weyl materials

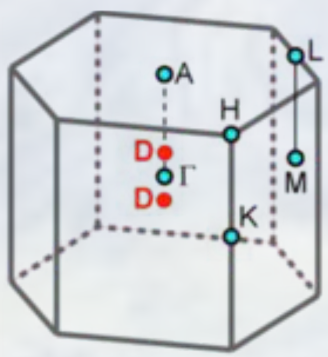
- High magnetic field lab
  - **10^5 G** **(~ 100 meV @ vF=c/300)**

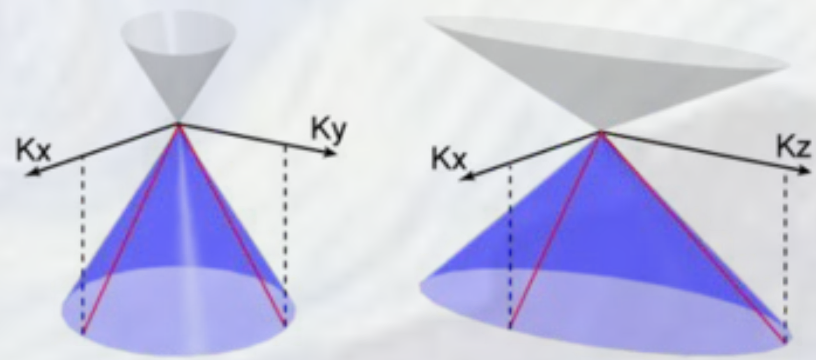


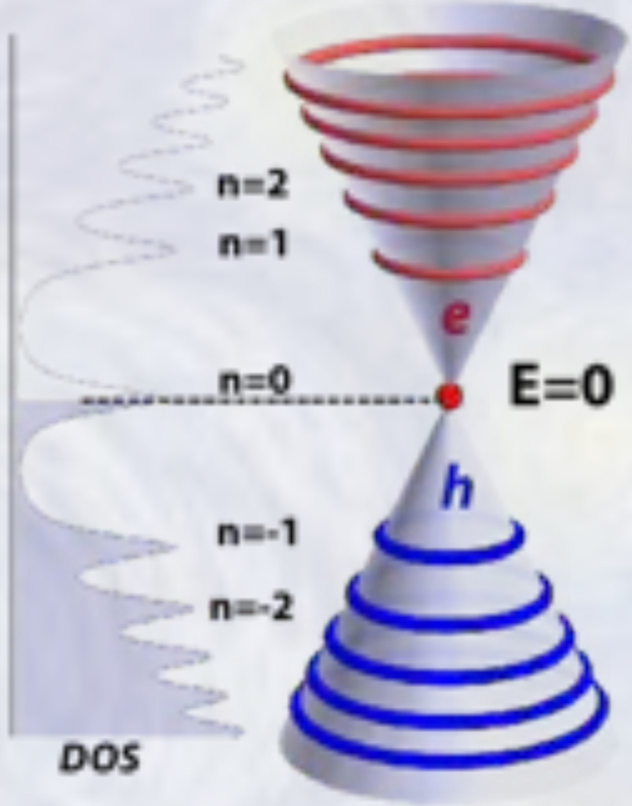
- Graphene
- 3D materials with Dirac/Weyl quasiparticles
  - $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
  - Na_3Bi
  - Cd_3As_2
  - $ZrTe_5$
  - TaAs, NbAs, TaP, …

[Z. K. Liu et al., arXiv:1310.0391]
[M. Neupane et al., arXiv:1309.7892]
[S. Borisenko et al., arXiv:1309.7978]
[X. Li et al., arXiv:1412.6543]

[arXiv:1502.03807, arXiv:1502.04684, arXiv:1504.01350, arXiv:1507.00521]







MAGNETIC CATALYSIS

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

Landau spectrum at B≠0

- Dirac equation with massless fermions (e<0)

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla}+ie\vec{A}\right)\right]\Psi = 0$$

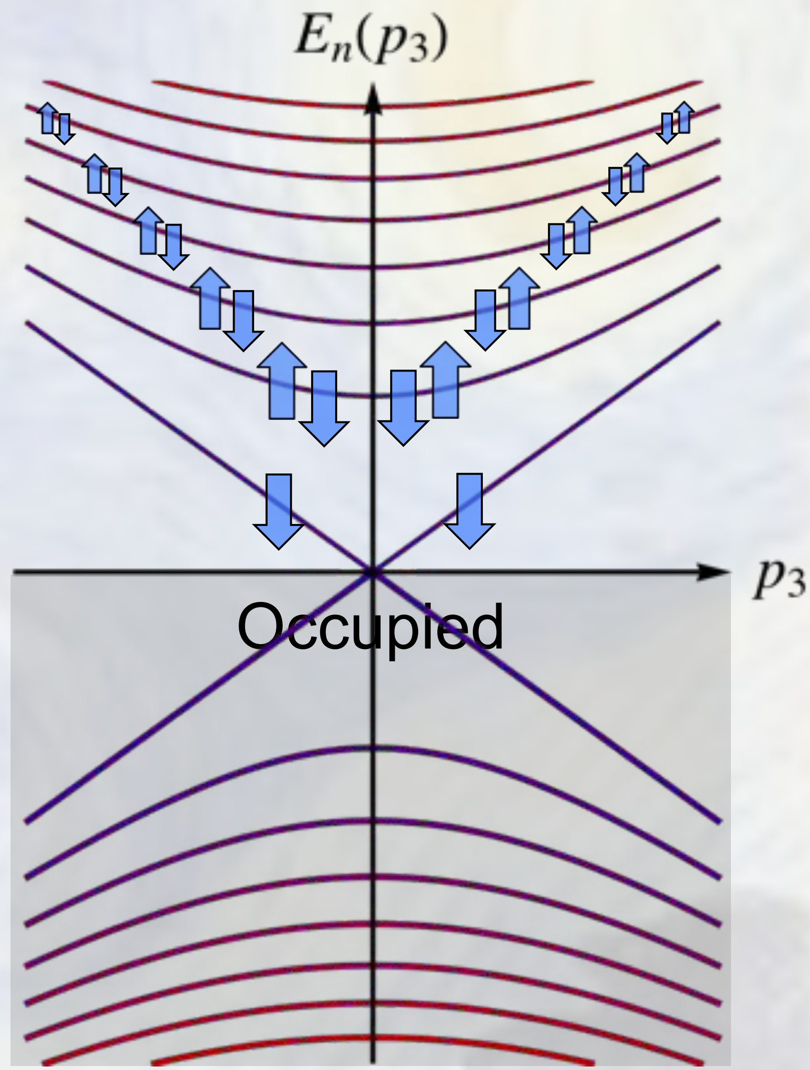
- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where $n = s + k + \frac{1}{2}$

$s = \pm \frac{1}{2}$ (spin)

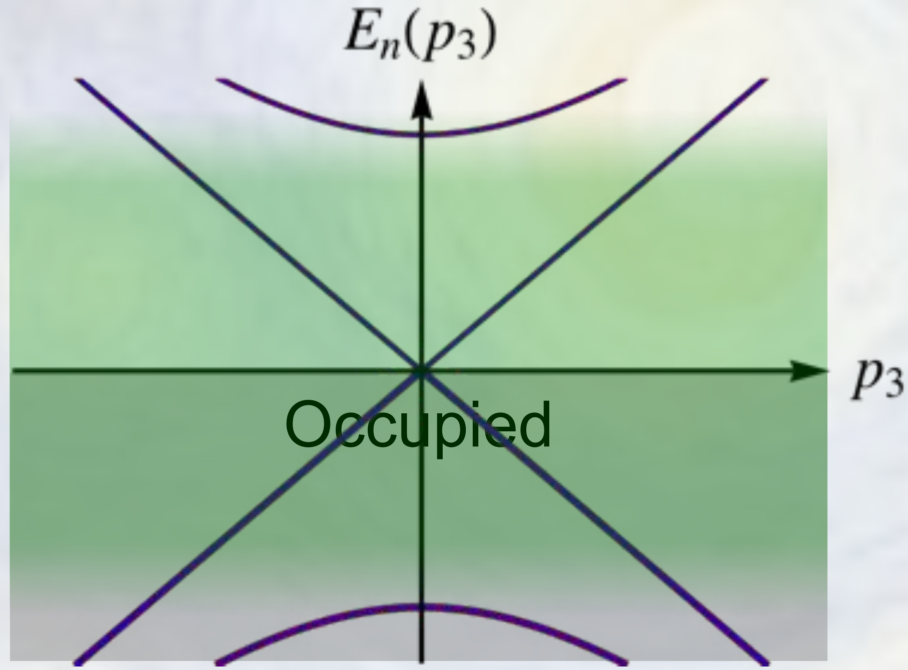
$k = 0, 1, 2, \ldots$ (orbital)



Dimensional reduction

- Low energy theory: *highly degenerate* n=0 Landau level

$$E_0^{(3+1)}(p_3) = \pm p_3$$



- Low-energy excitations are $(1_{space}+1_{time})$-dimensional

- Motion in *xy-plane* is restricted

Essence of magnetic catalysis

- Dimensional *reduction*

$$D \Rightarrow D-2 \quad \text{e.g.,} \quad \begin{cases} 3+1 \to 1+1 \\ 2+1 \to 0+1 \end{cases}$$

- Nonzero density of states at E = 0

$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB| N_f}{4\pi^2}$$

- Symmetry breaking happens at *arbitrarily* weak interaction
- In other words, **B**-field acts as a "catalyst"

[Gusynin, Miransky, Shovkovy, PRL **73** (1994) 3499]

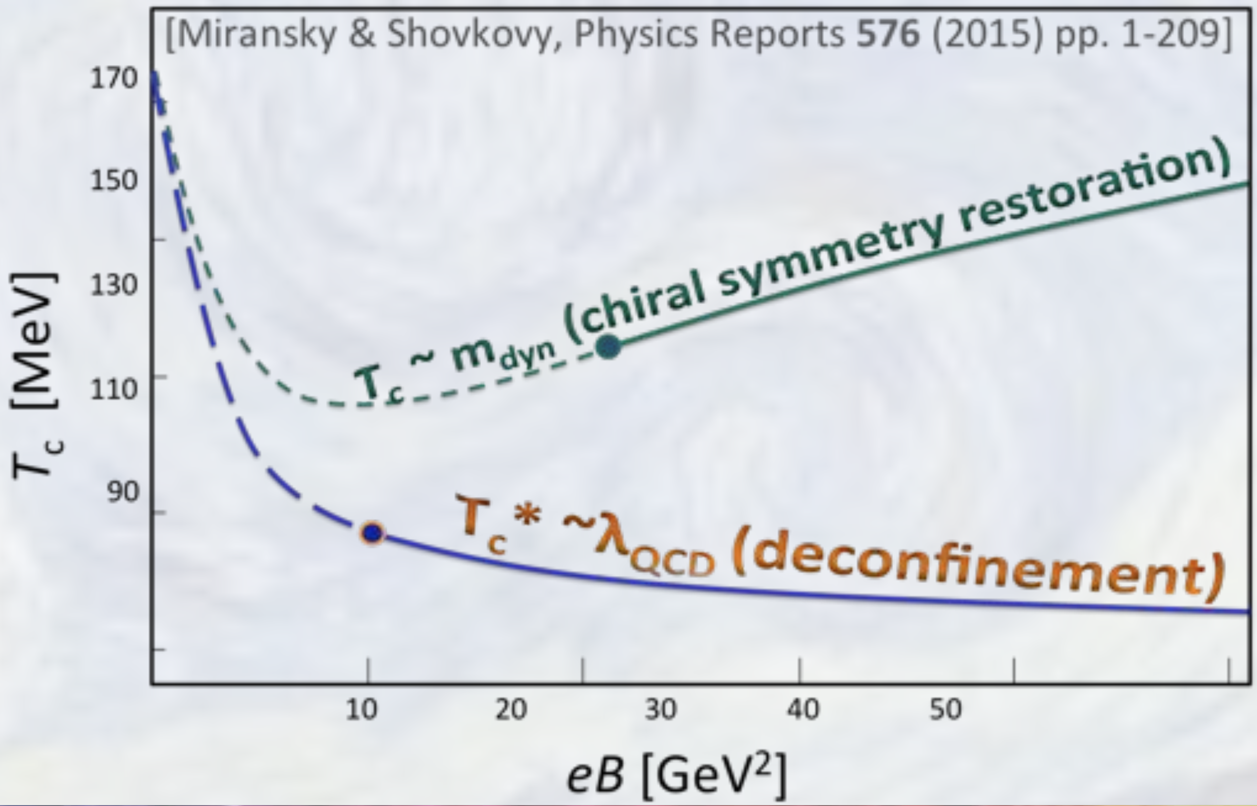
Magnetic catalysis in QCD

- Magnetic catalysis of chiral symmetry breaking & anisotropic confinement (vacuum QCD, QCD @ T≠0)
- T=0: catalysis

[Miransky & I.S., Phys. Rev. D **66** (2002) 045006]
[Bali et al., Phys. Rev. D**86**, 071502 (2012)], …

- $T_c(B)$: inverse catalysis

[Bali et al., JHEP 02, 044 (29012)]
[Bali et al., PRD**86**, 071502 (2012)]
[G. Endrodi, arXiv:1504.08280], …



CHIRAL SHIFT GENERATION

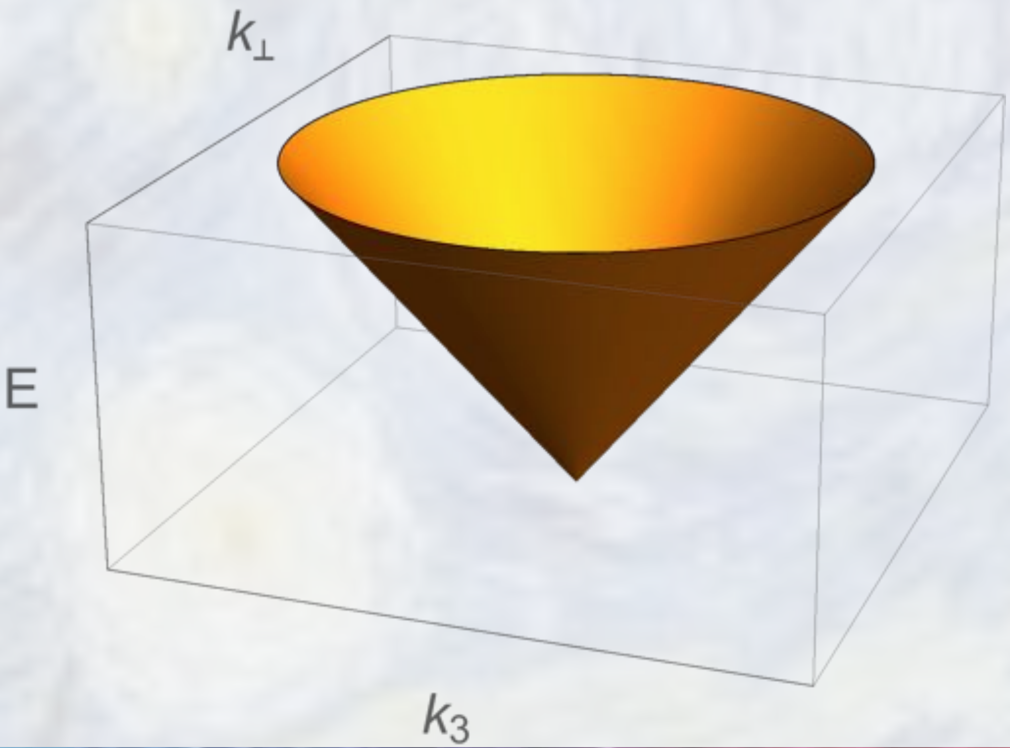
- Dirac + $\vec{B}$ = Weyl

Dirac vs. Weyl (semi-)metal

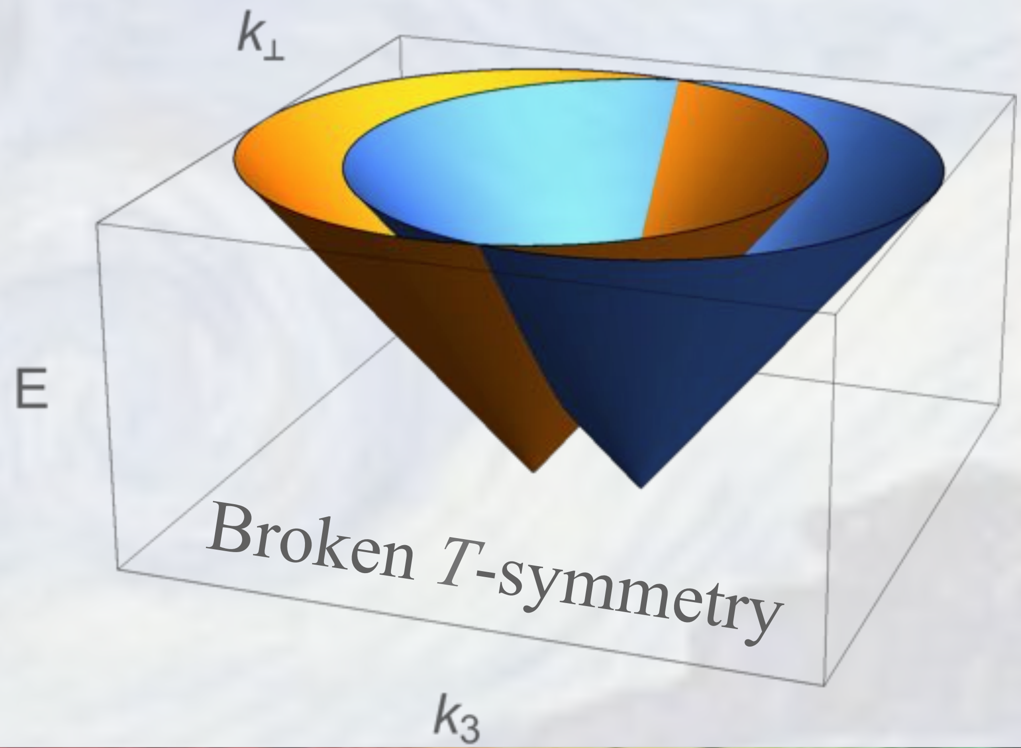
- Low-energy Hamiltonian of Dirac/Weyl semimetal

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5\right]\psi + H_{\text{int}}$$

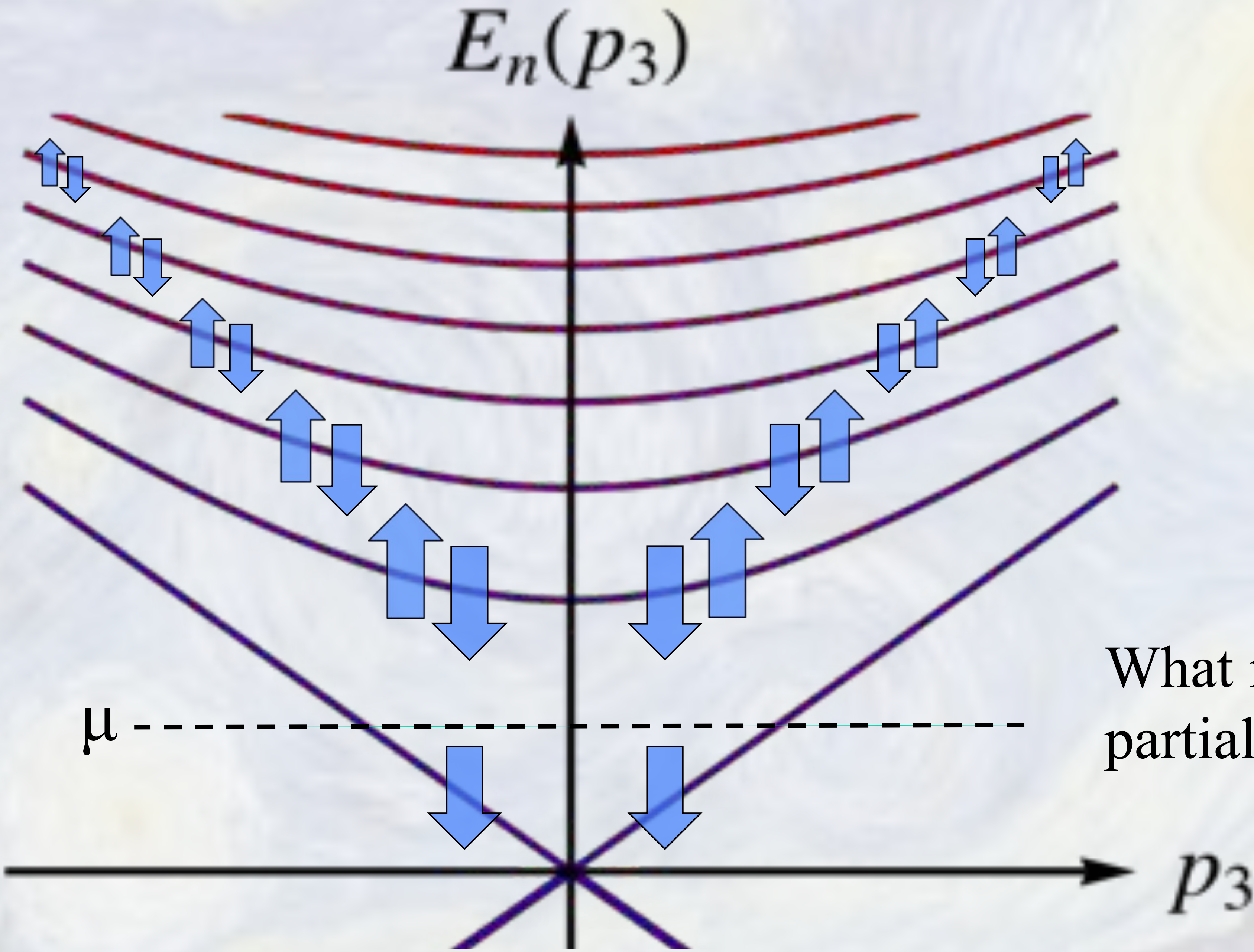
Dirac semimetal



Weyl semimetal



Landau spectrum

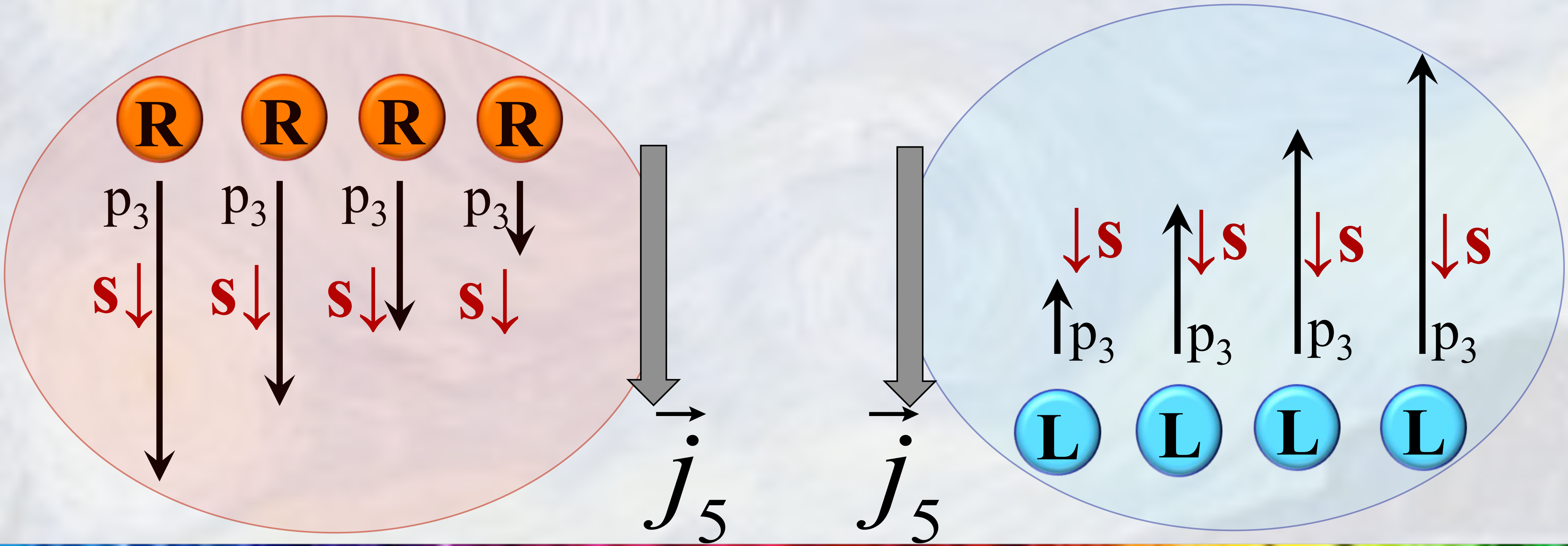


What if LLL is partially filled?

Partially filled LLL

- LLL is spin polarized and chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed

- This indeed implies axial current density



Chiral shift at B≠0

- The axial current (CSE) in free theory

$$\left\langle \bar{\psi}\gamma^3\gamma^5\psi \right\rangle = \frac{eB}{2\pi^2}\mu \quad \text{with} \quad \vec{B} = (0,0,B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

+ **interactions** should induce a chiral shift parameter Δ associated with the condensate,

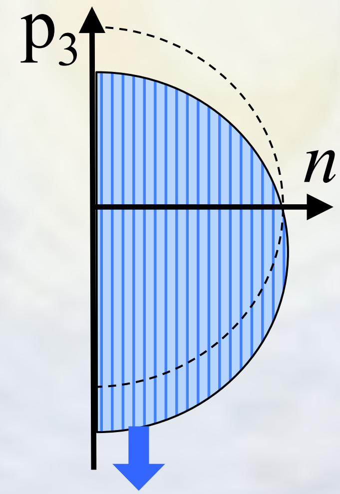
$$\delta L = \Delta\, \bar{\psi}\gamma^3\gamma^5\psi$$

This is a perturbative effect: there is no symmetry to protect Δ =0

[Gorbar, Miransky, Shovkovy, Phys. Rev. C **80**, 032801(R) (2009)]

Chiral shift @ Fermi surface

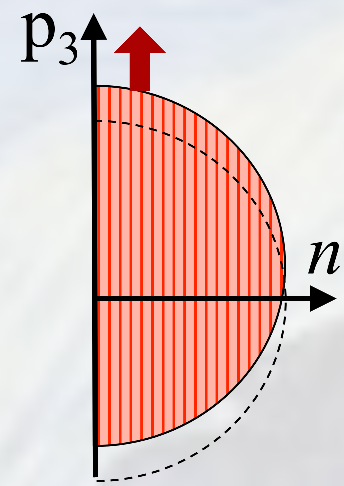
- Chirality is $\approx$ well-defined at Fermi surface ($|p_3| \gg m$)
- L-handed Fermi surface:



$$n>0: \quad p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:



$$n>0: \quad p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

QED in weak field (B→0)

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta(p) + \gamma^0\gamma^5\mu_5(p)$$

Near Fermi surface ($p_0 \to 0$, $|\mathbf{p}| \to p_F$)

$$\Delta(p) \approx \frac{\alpha eB\mu}{\pi m^2}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

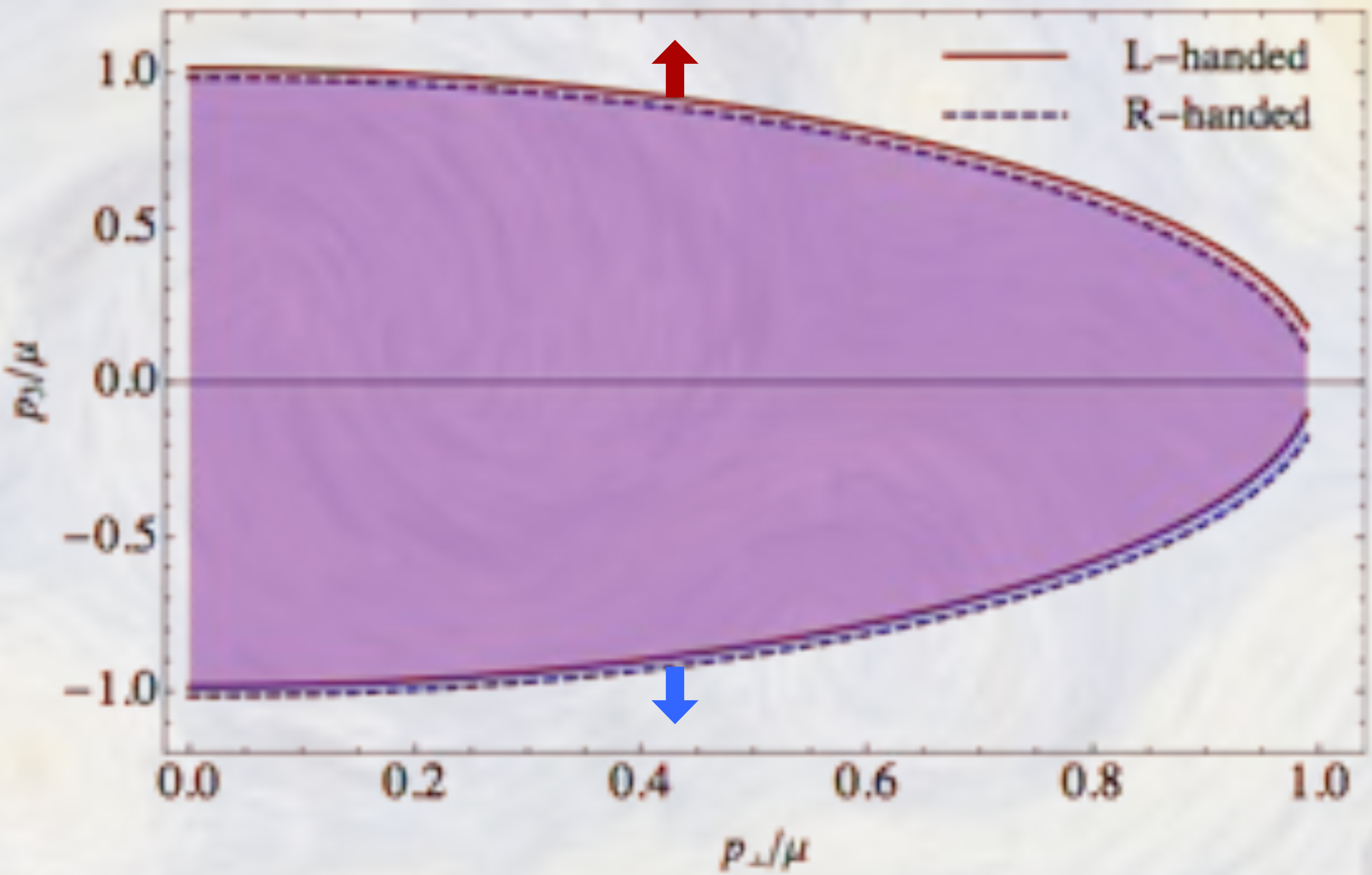
$$\mu_5(p) \approx -\frac{\alpha eB\mu}{\pi m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu\left(|\mathbf{p}| - p_F\right)} - 1\right)$$

[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Dispersion relations in QED

- Let us use the condition (for a small B)

$$\mathrm{Det}\left[i\bar{S}^{-1}(p)+\bar{\Sigma}^{(1)}(p)\right]=0$$



[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

QED in strong field

Self-energy in the Landau-level representation:

$$\bar{\Sigma} = 2e^{-\mathbf{k}_\perp^2 l^2} \sum_{n=0}^{\infty} (-1)^n \left(m_n + \gamma^3\gamma^5\Delta_n - \gamma^0\gamma^5\mu_{5,n} + \ldots\right)\left[P_+ L_n - P_- L_{n-1}\right] + \ldots$$

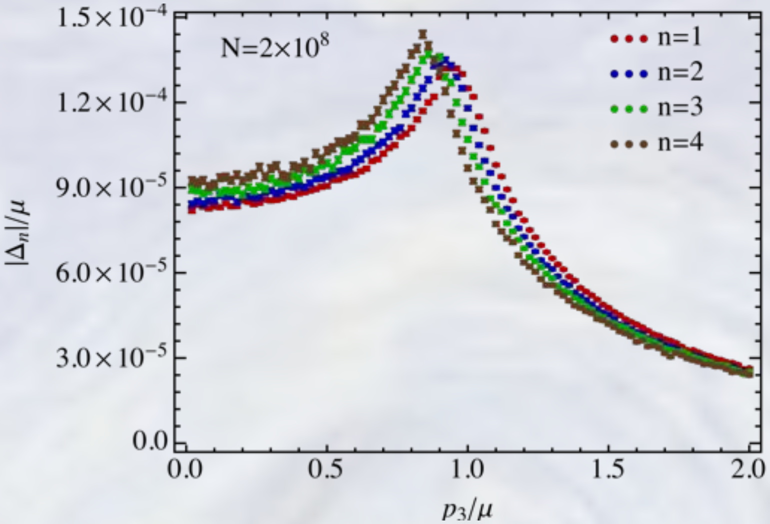
where m_n, Δ_n, $\mu_{5,n}$, … are “projections” of the self-energy on the nth Landau level,

$$\Delta_n(k_0,k_3) = \frac{(-1)^n l^2}{8\pi} \int d^2k_\perp e^{-k_\perp^2 l^2} \left[L_n + L_{n+1}\right] Tr\left[\gamma^0 \bar{\Sigma}(k)\right]$$

$$\mu_{5,n}(k_0,k_3) = \frac{(-1)^n l^2}{8\pi} \int d^2k_\perp e^{-k_\perp^2 l^2} \left[L_n - L_{n+1}\right] Tr\left[\gamma^0\gamma^5 \bar{\Sigma}(k)\right]$$

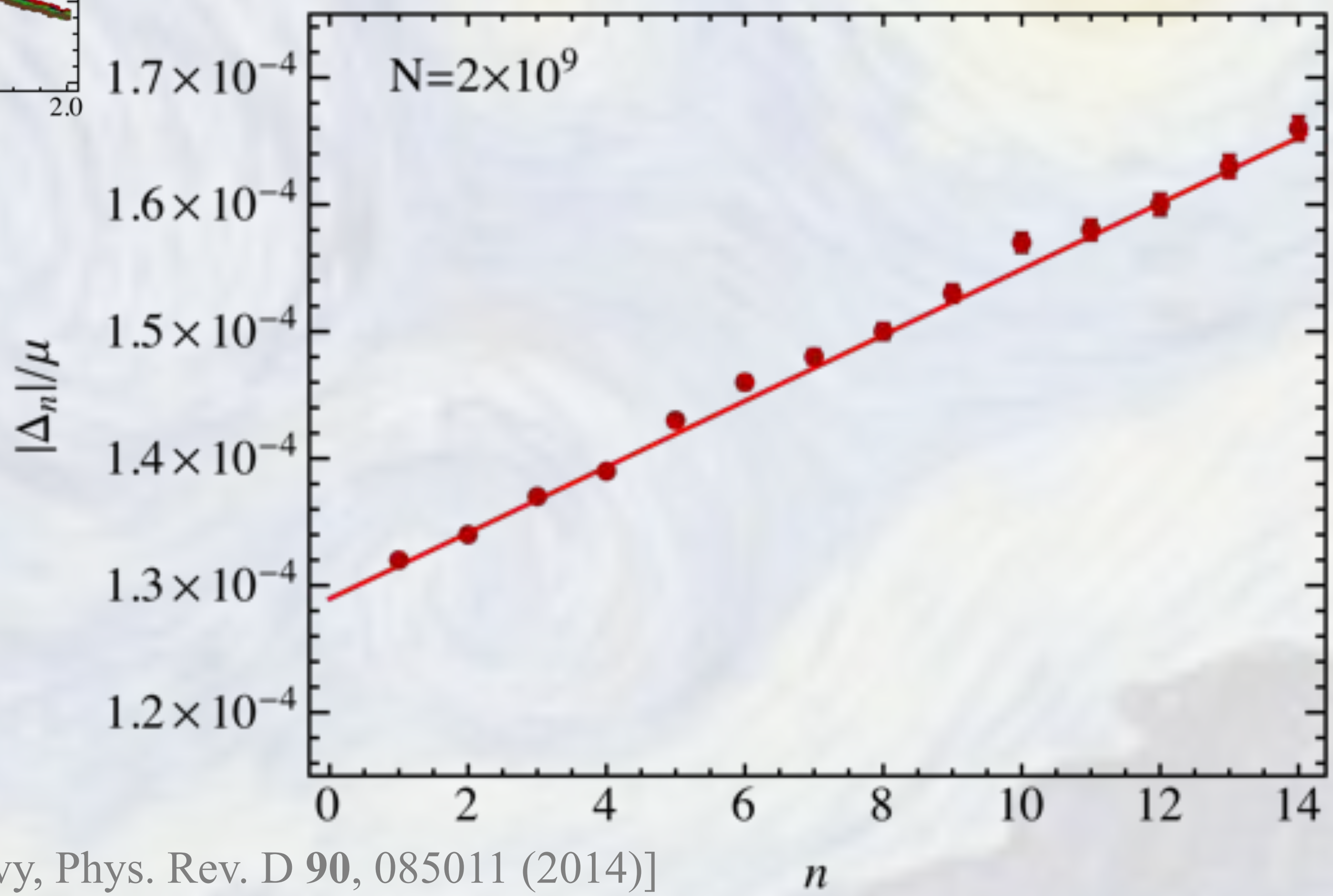
[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025025 (2013)]

Chiral shift QED @ B≠0



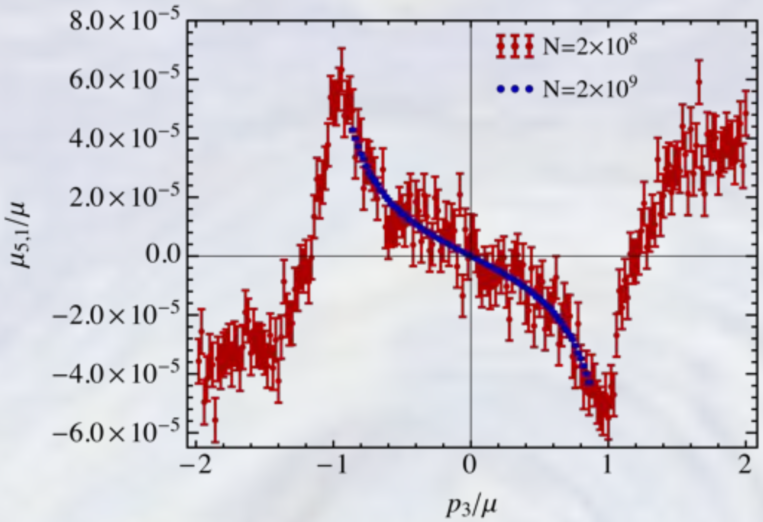
Model fit:

$$\Delta_n = -\frac{\alpha|eB|}{\mu}\left(0.53 + 0.32\frac{|eB|n}{\mu^2}\right)$$



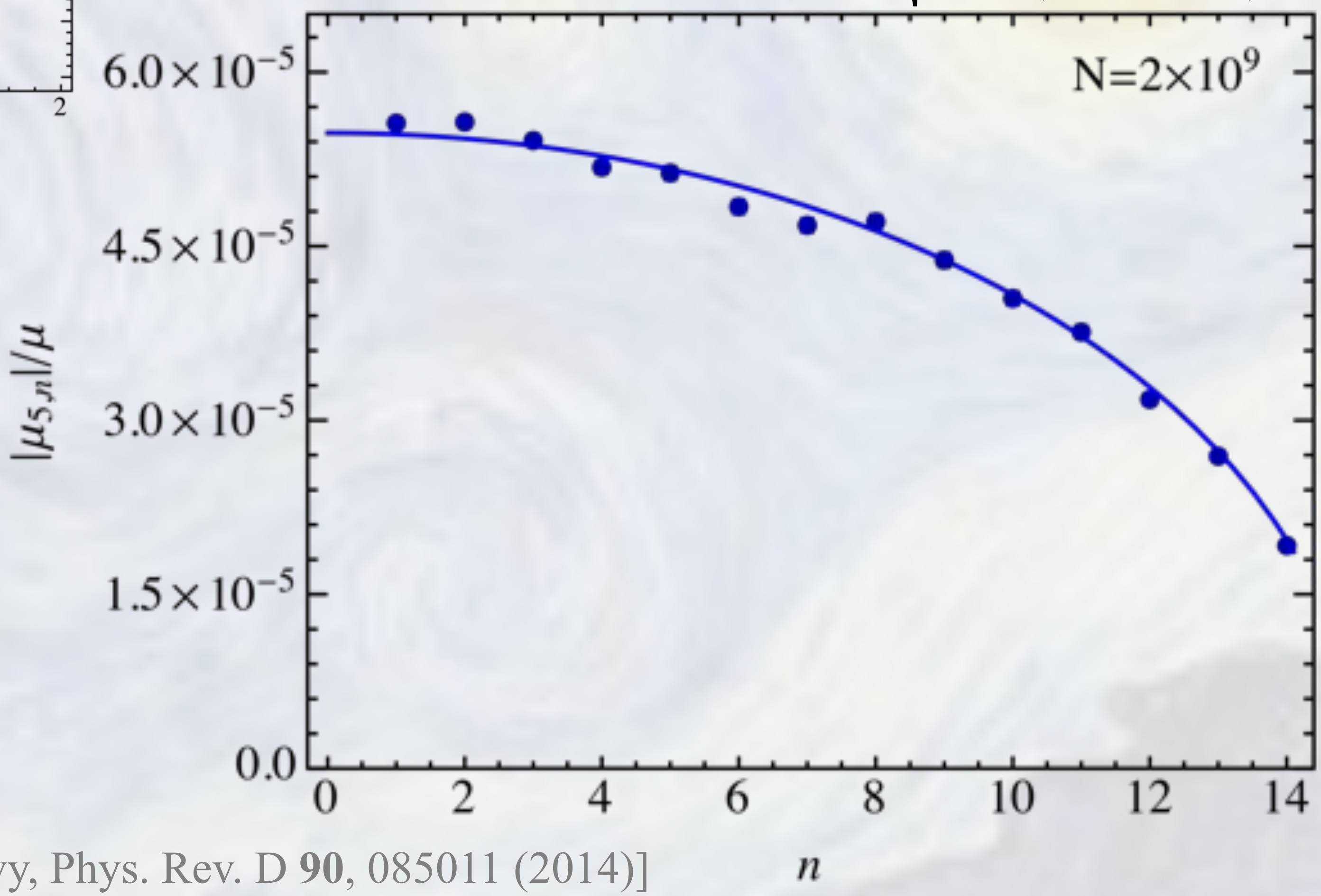
[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

$\mu_{5,n}$ in QED @ B≠0



Model fit:

$$\mu_{5,n} = -0.225\frac{\alpha|eB|}{\mu}\sqrt{1-\left(\frac{2n|eB|}{\mu^2}\right)^2}$$

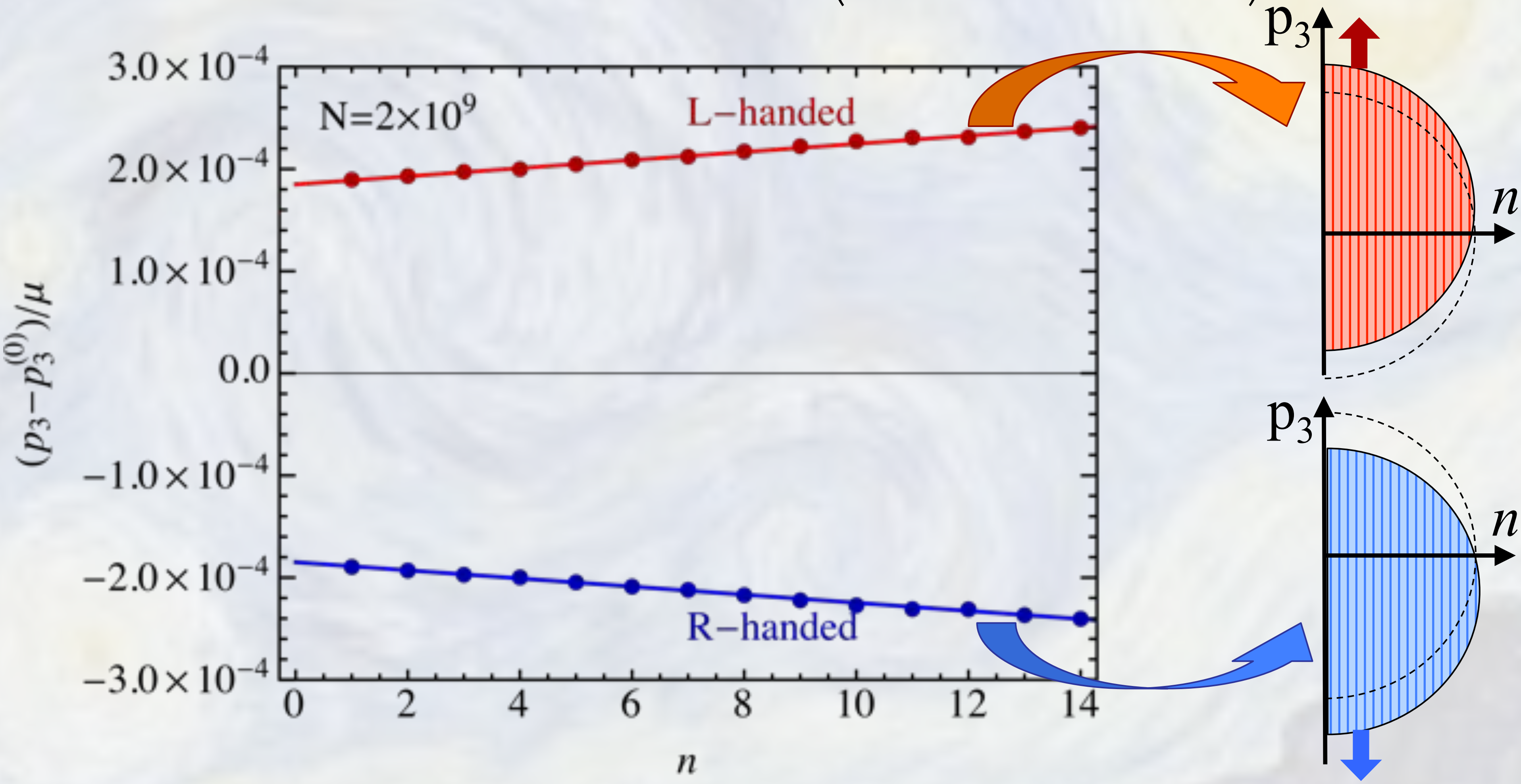


[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

QED in strong field: δp_3

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu}\left(0.76 + 0.49\frac{|eB| n}{\mu^2}\right)$$



[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

How large is the asymmetry?

In QED:

$$\frac{\alpha|eB|}{\mu} \approx 0.4 \left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{100\,\mathrm{MeV}}{\mu}\right) \mathrm{MeV/c}$$

In QCD:

$$\frac{\alpha_s|eB|}{\mu} \approx 10 \left(\frac{B}{10^{18}\,\mathrm{G}}\right)\left(\frac{400\,\mathrm{MeV}}{\mu}\right) \mathrm{MeV/c}$$

may have some observable consequences…

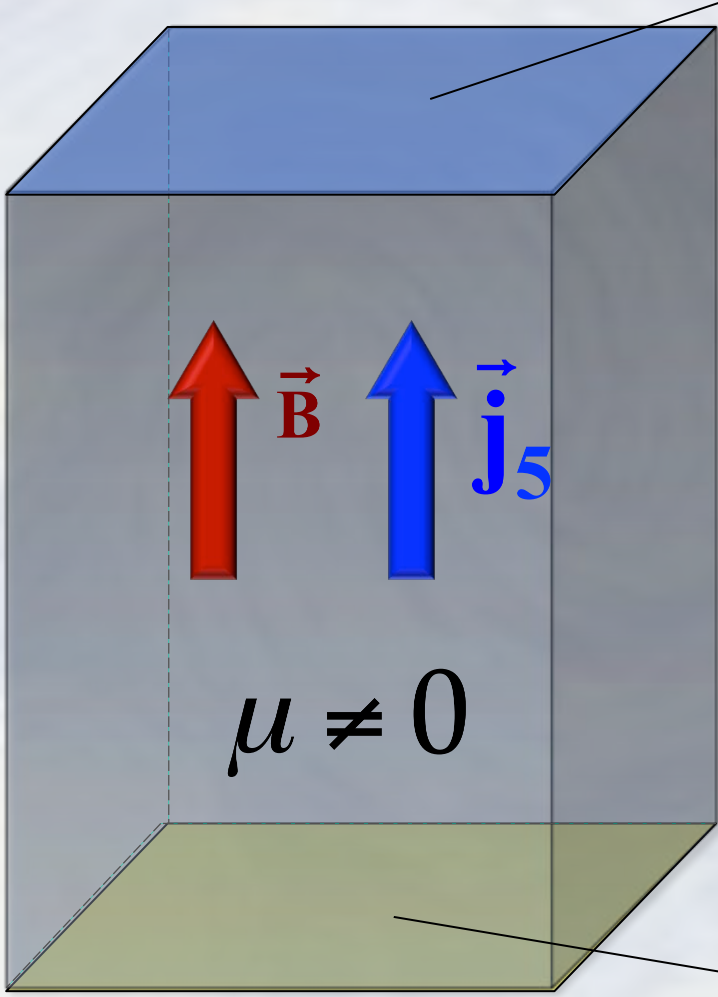
SOME PROBLEMS TO ADDRESS

- Effects of finite size
- Role of inhomogeneities

Effects of finite size

- Magnetic field + electric chemical potential = chiral current

Positive chiral charge?



$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

- Is the chiral charge truly separated?

Negative chiral charge?

Dirac semimetal

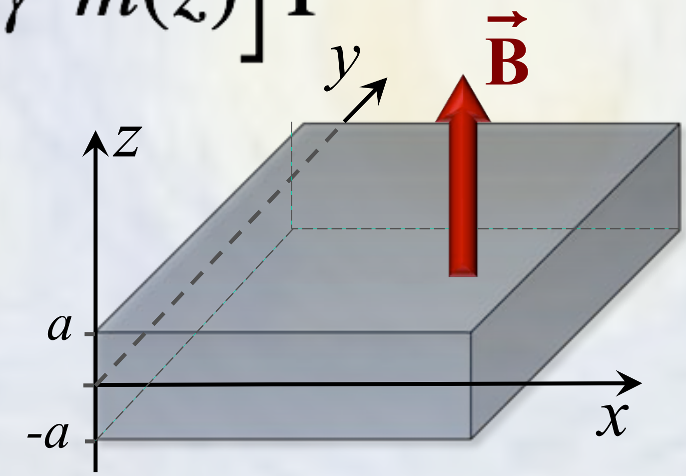
- Model of Dirac semimetal with a slab geometry

$$H = \int d^3 r \Psi^{+} \left[v_F \vec{\alpha} \cdot \left(-i\vec{\nabla} + e\vec{A} \right) + \gamma^0 m(z) \right] \Psi$$

where $\vec{A} = (0, Bx, 0)$ and

$$m(z) = M\,\theta(z^2 - a^2) + m\,\theta(a^2 - z^2),$$

with vacuum band gap: $M \to \infty$ (broken chiral symmetry)



- Boundary conditions:

$$\Psi_{\text{bulk}}(\vec{r}_\perp, a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, a) \quad \text{and} \quad \Psi_{\text{bulk}}(\vec{r}_\perp, -a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, -a)$$

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1509.06769]

Wave functions

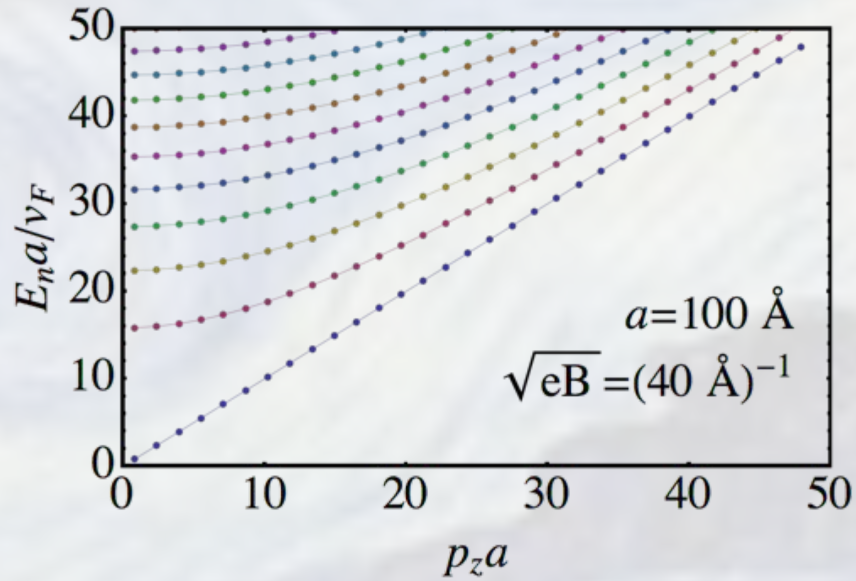
- Wave functions are standing waves, e.g.,

$$\text{LLL}: \quad \Psi_{\text{slab},n=0} = C_0 e^{-\frac{1}{2}(x/l + p_y l)^2} e^{i(p_y y + p_z a)} \begin{pmatrix} 0 \\ \frac{v_F p_z \cos\left(p_z(z-a)\right) - (m + iE_0)\sin\left(p_z(z-a)\right)}{im + v_F p_z - E_0} \\ 0 \\ -i\frac{v_F p_z \cos\left(p_z(z-a)\right) - (m - iE_0)\sin\left(p_z(z-a)\right)}{im + v_F p_z - E_0} \end{pmatrix}$$

where the wavevector p_z is determined by the spectral equation

$$v_F p_z \cos\left(2a\, p_z\right) + m \sin\left(2a\, p_z\right) = 0$$

$$\Rightarrow \quad p_{z,k}^{(m)} \approx \frac{\pi(2k-1)}{4a} + \frac{2m}{\pi v_F (2k-1)} + \ldots$$



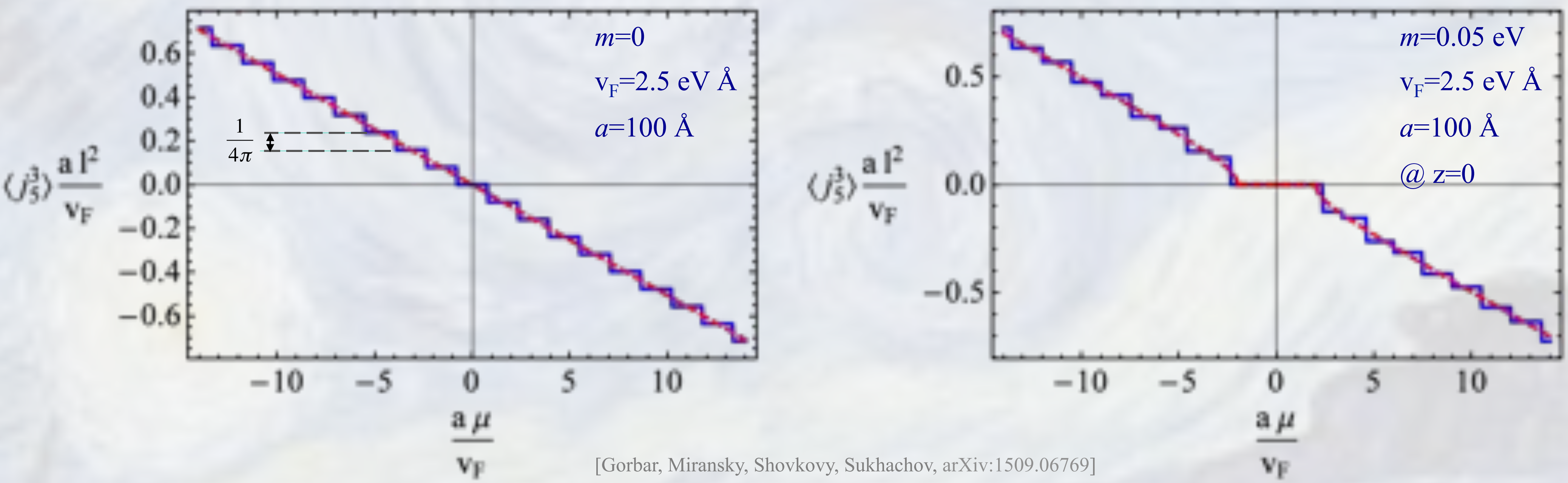
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1509.06769]

Discretized CSE

- Only LLL contributes

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B} v_F \,\mathrm{sign}(\mu)}{2\pi a} \sum_{p_z} \theta\left(\mu^2 - m^2 - v_z^2 p_z^2\right) \frac{\left(m^2 + v_z^2 p_z^2\right)\left[1 - \cos(2z\, p_z)\cos(2a\, p_z)\right]}{2\left(m^2 + v_z^2 p_z^2\right) + m v_F / a}$$

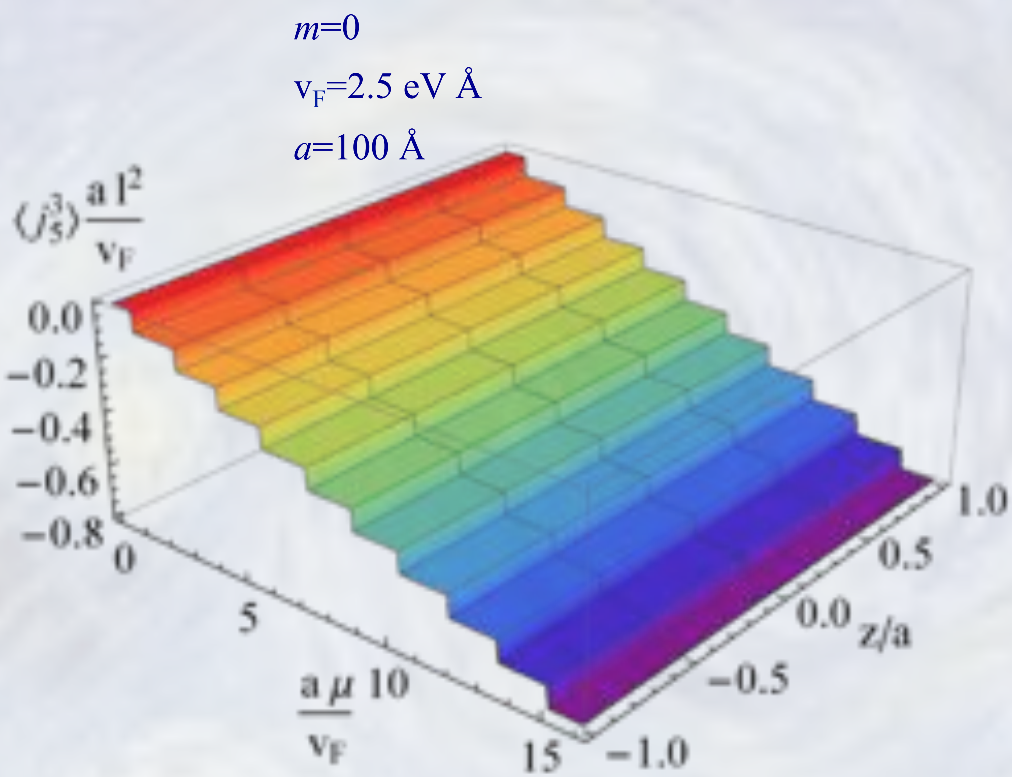
- For $m \to 0$: $\langle \vec{j}_5 \rangle = -\frac{e\vec{B} v_F \,\mathrm{sign}(\mu)}{4\pi a} k_{\max}$, where $k_{\max} = \left[\frac{2a|\mu|}{\pi v_F} + \frac{1}{2}\right]$

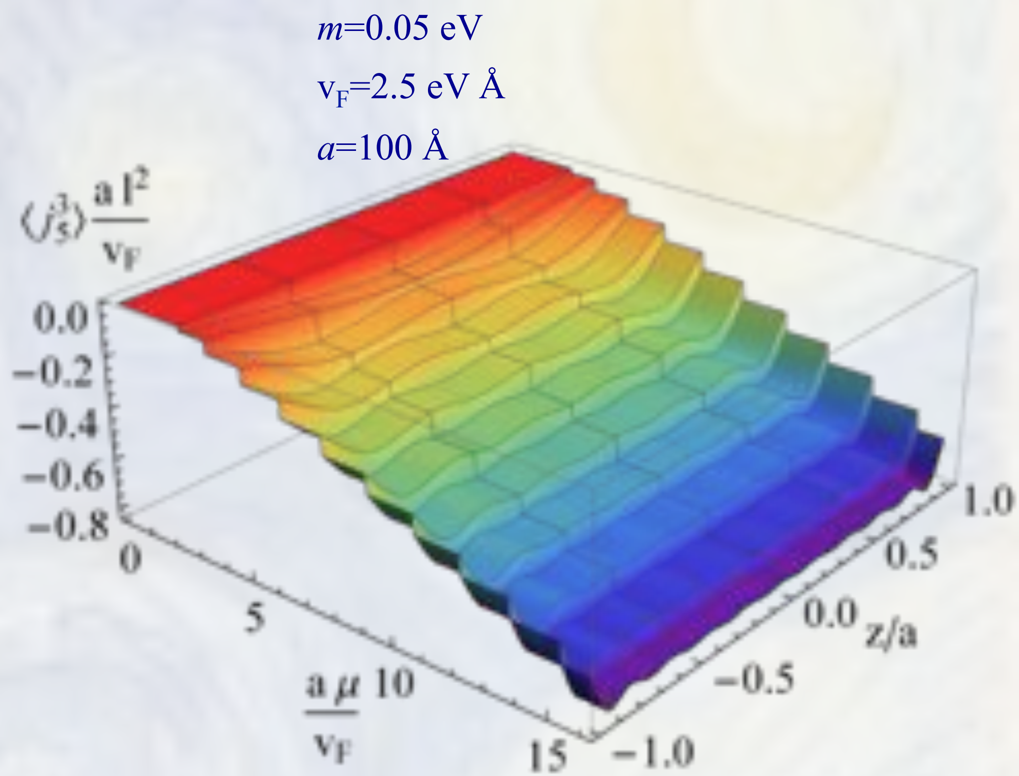


[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1509.06769]

Axial current vs. μ and z

- Axial current density is non-uniform when $m \neq 0$



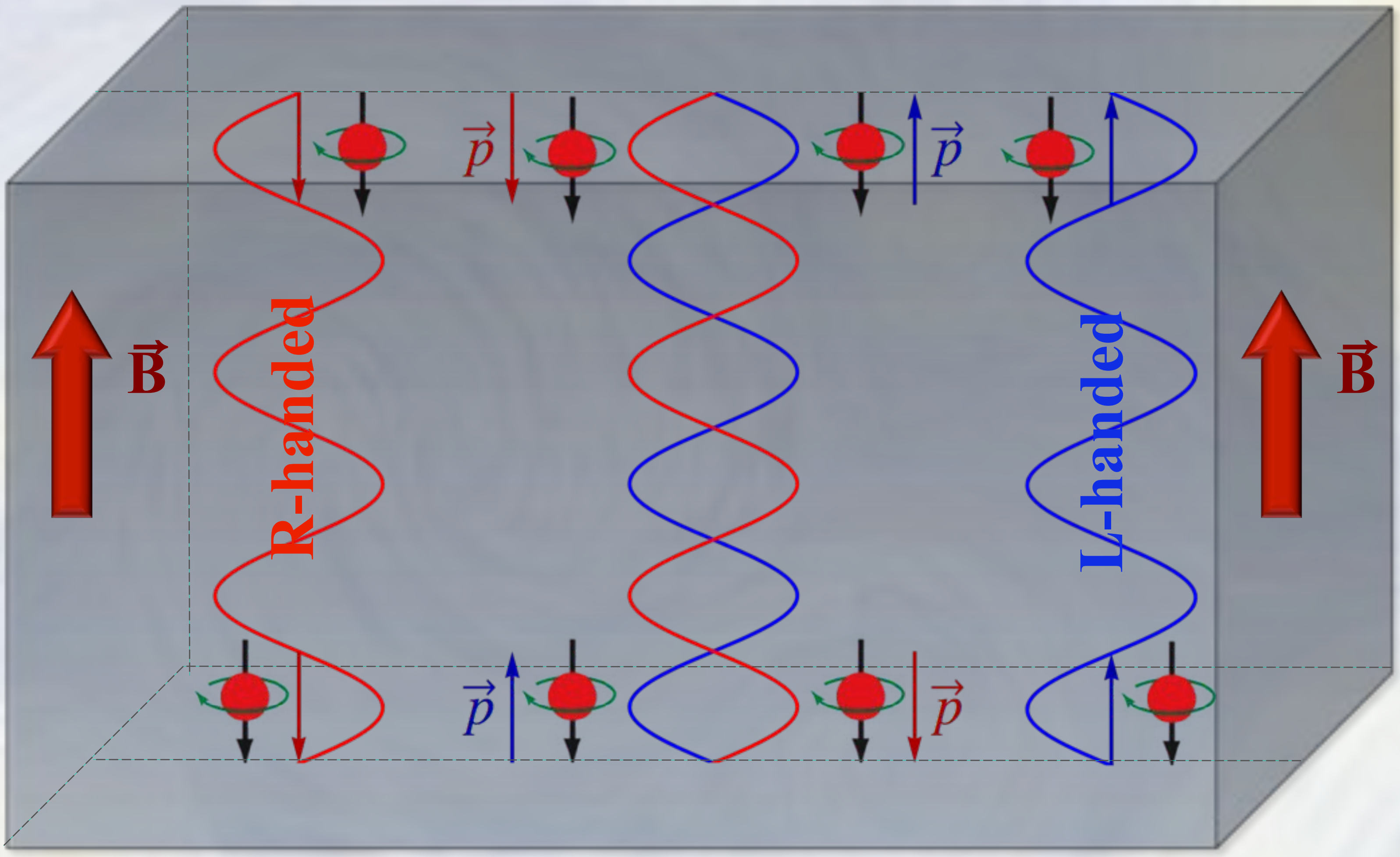


- Note that axial charge density vanishes: $\left\langle j_5^0 \right\rangle = 0$

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1509.06769]

Axial current as a standing wave?

- Recall that LLL is spin polarized



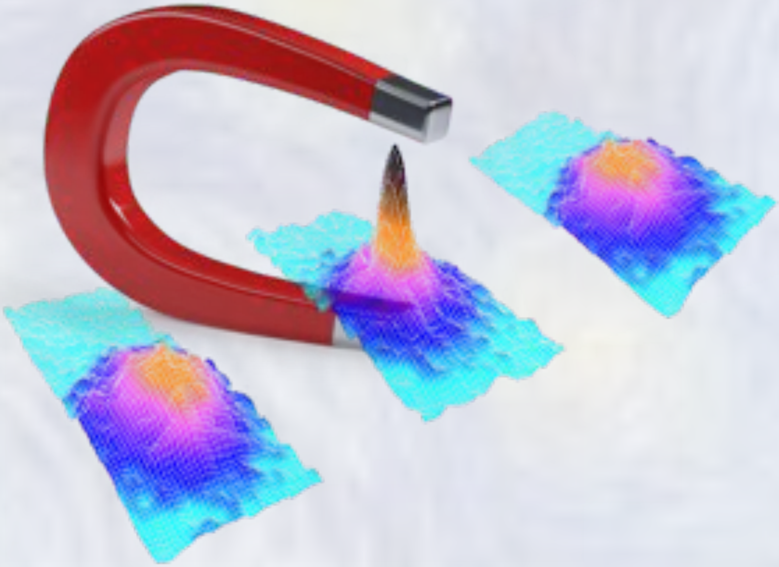
- A perfect chirality flip at the boundary

Key observations

- Chiral current in the CSE is discretized
- $m \neq 0$: chiral current density is non-uniform
- $m=0$: chiral current density is uniform
- Chiral current is **not** necessarily connected with a "flow" of chiral charge
- Chiral current need **not** lead to chiral charge accumulation on the boundary

Role of inhomogeneities?

- When $\mu_5 \neq 0$, there is no CME in a slab
- What about chiral magnetic waves?
- What about inverse cascade in cosmology?
- Reality
  - Inhomogeneous
  - Non-static
- What is the systematic approach to use?



ANOMALOUS MAXWELL EQUATIONS

Anomalous Maxwell equations

- Common "homogeneous" approximation:

$$\nabla \times \vec{B} = \sigma \vec{E} + \frac{2\alpha}{\pi} \mu_5 \vec{B} + \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\frac{d}{dt}\Big(\langle n_5(\vec{\mathbf{x}},t) \rangle_{\text{space}} \Big) = \frac{\alpha}{2\pi^2} \langle \vec{E} \cdot \vec{B} \rangle_{\text{space}} \qquad \mu_5(t) \approx \frac{3c^3}{T^2} \langle n_5(\vec{\mathbf{x}},t) \rangle_{\text{space}}$$

- How to generalize these equations to include inhomogeneities?

Chiral kinetic theory

- Starting point:

$$\frac{\partial f_\lambda}{\partial t} + \frac{1}{1+\vec{\Omega}_\lambda\cdot\vec{B}}\left[\left(\vec{E}+\vec{v}\times\vec{B}+(\vec{E}\cdot\vec{B})\vec{\Omega}_\lambda\right)\cdot\frac{\partial f_\lambda}{\partial\vec{p}} + \left(\vec{v}+\vec{E}\times\vec{\Omega}_\lambda+(\vec{v}\cdot\vec{\Omega}_\lambda)\vec{B}\right)\cdot\frac{\partial f_\lambda}{\partial\vec{x}}\right] = I_{\text{coll}}$$

where

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{cp-\mu_\lambda}{T}\right)+1}$$

is an expansion in powers of E-M field and $\vec{\nabla}\mu_\lambda$

Equations for chemical potentials

- Resulting equation of motion for μ_λ:

$$\left(1+\frac{3\mu_\lambda^2}{\pi^2T^2}\right)\frac{\partial\mu_\lambda}{\partial t}+\frac{3\lambda c^2 e}{2\pi^2T^2}\left(\vec{B}\cdot\frac{\partial\mu_\lambda}{\partial\vec{x}}\right)-\frac{\tau c^2}{3}\left(1+\frac{3\mu_\lambda^2}{\pi^2T^2}\right)\nabla^2\mu_\lambda+\frac{2\tau c^2\mu_\lambda}{\pi^2T^2}\left(e\vec{E}\cdot\frac{\partial\mu_\lambda}{\partial\vec{x}}\right)=\frac{3\lambda c^2e^2}{2\pi^2T^2}\left(\vec{E}\cdot\vec{B}\right)$$

The corresponding equations for the currents:

CME — drift & diffusion — Hall type

$$\vec{j}=\frac{e\mu_5}{2\pi^2c}\vec{B}+\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)+\frac{2\tau\mu}{\text{[illegible]}}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}+\vec{j}_{\text{new}}$$

$$\vec{j}_5=\frac{e\mu}{2\pi^2c}\vec{B}-\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\frac{\partial\mu_5}{\partial\vec{x}}+\frac{2e\tau\mu_5\mu}{3\pi^2c}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)+\vec{j}_{5,\text{new}}$$

CSE — diffusion — CESE

New types of currents

- New contribution to the electric current:

$$\vec{j}_{\text{new}} = -\frac{2\tau\mu\mu_5}{3\pi^2 c}\frac{\partial\mu}{\partial\vec{x}} - \frac{e\tau^2\mu}{6\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right) - \frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu_5}{\partial\vec{x}}\right)$$

- New contribution to the chiral current:

$$\vec{j}_{5,\text{new}} = -\frac{e\tau^2\mu}{6\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right) - \frac{e\tau^2\mu_5}{6\pi^2}\left(\vec{E} - \frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B} - \frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu}{\partial\vec{x}}\right)$$

Preliminary

Instead of summary

- We now have Anomalous Maxwell equations for inhomogeneous chiral plasmas
- All previously known currents are reproduced
- A whole set of new currents obtained
- Observational signatures?

e.g., how about $\vec{j}_{5,\mathrm{new}} = -\frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu}{\partial\vec{x}}\right)$?