ARIZONA STATE UNIVERSITY

QCD Workshop on Chirality, etc.
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UCLA

Anomalous inhomogeneous chiral plasma

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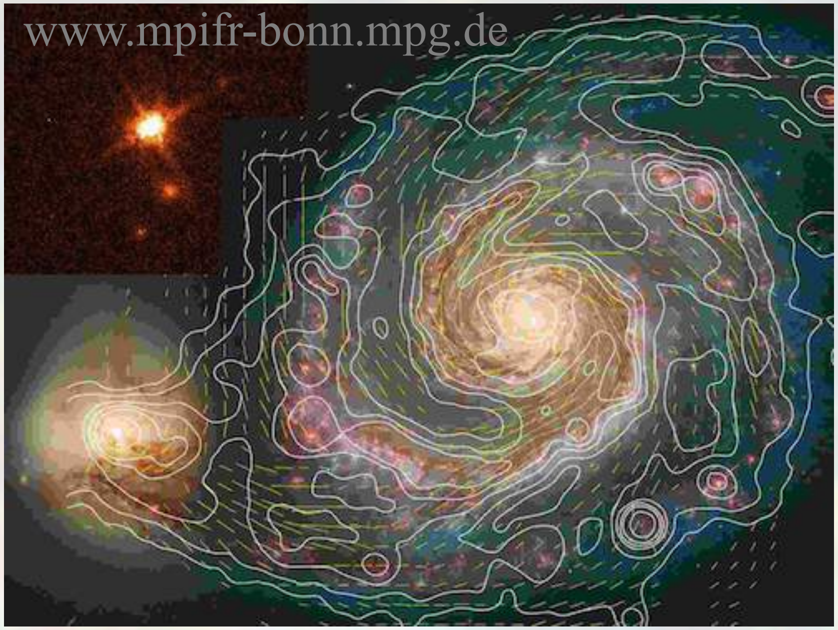
MAGNETIZED CHIRAL PLASMA

- Early Universe
- Heavy-ion collisions
- Dirac semimetals, graphene, etc.
- Compact stars

Universe



- Current galactic magnetic fields ~ 10^{-6} G
- Current magnetic fields in voids ~ 10^{-15} G
- Problem of magnetogenesis (on large scales) in Early Universe
- Perhaps during the electro-weak phase transition
  - **10^{20} to 10^{24} G** (~ 1 GeV to 100 GeV)

Magnetic field/helicity

- Magnetic helicity evolution (inverse cascade) in the Early Universe

$$\nabla \times \vec{B} = \frac{4\pi}{c}\left(\sigma \vec{E} + C_5 \mu_5 \vec{B}\right) + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = 4\pi \rho , \qquad \nabla \cdot \vec{B} = 0$$

[Vilenkin, Phys. Rev. D22, 3080 (1980)]
[Joyce & Shaposhnikov, astro-ph/9703005]
[Giovannini & Shaposhnikov, hep-ph/9710234]

- For specific helicity eigenmodes:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma}\left(4\pi C_5 \mu_5 - ck\right) B_k$$

[Boyarsky et al., arXiv:1109.3350]
[Tashiro et al., arXiv:1206.5549]
[Manuel et al., arXiv:1501.07608]
[Buividovich et al., arXiv:1509.02076]
[Hirono et al., arXiv:1509.07790]

Feedback on $\mu_5(t)$

- Constraint of the total helicity conservation

$$h_{\text{fermion}} = h_0 - h_{\text{gauge}}$$

- Common "homogeneous" approximation:

$$n_5(\vec{x},t) \approx \left\langle n_5(\vec{\mathbf{x}},t) \right\rangle_{\text{space}}, \quad \mu_5(t) \approx \frac{3c^3}{T^2} \left\langle n_5(\vec{\mathbf{x}},t) \right\rangle_{\text{space}}$$

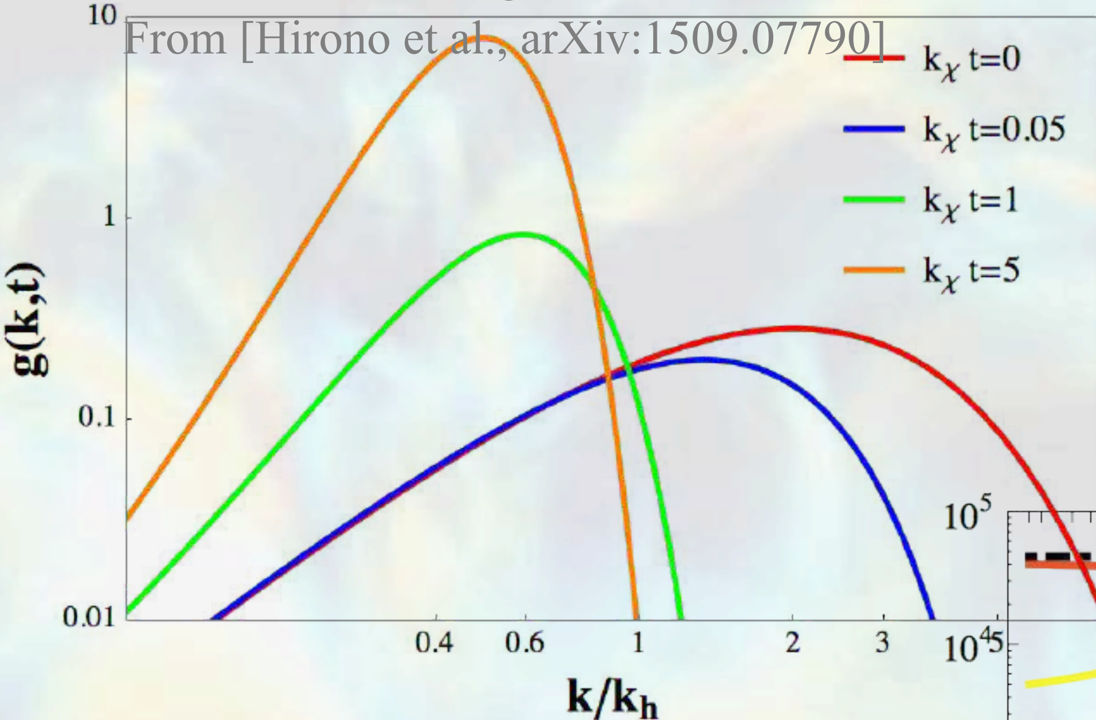
- In other words, the value of μ_5 remains constant on distance scales

$$\Delta x \sim (k_{\text{crit}})^{-1} \sim (\mu_5)^{-1}$$

Magnetic field/helicity

- Magnetic helicity is transferred from short to long-wavelengths modes, while the value of μ_5 decreases

From [Hirono et al., arXiv:1509.07790]



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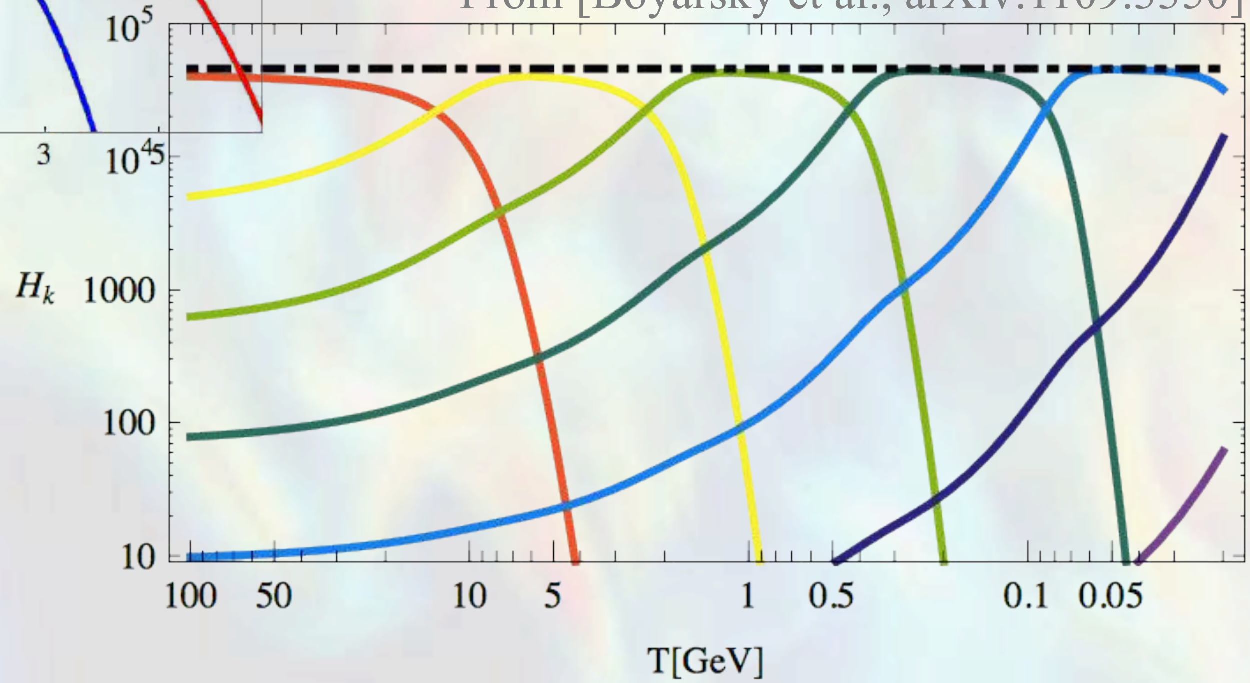


SOME PROBLEMS TO ADDRESS

- Role of inhomogeneities

[Boyarsky, Gorbar, Ruchayskiy, Rudenok, Shovkovy, Vilchinskii, work in progress]

Open questions

- Will the cascade survive if there are variations of order $\delta\mu_5$ on distance scales $(k_{\text{crit}})^{-1}$?
- How large $\delta\mu_5$ can be tolerated?
- Will dynamical fluctuations of μ_5 stay under control?
- How to account for inhomogeneities in the anomalous Maxwell equations?

$$n_\lambda(\vec{x},t) = ? \qquad \vec{j}_\lambda(\vec{x},t) = ?$$

- How to obtain equations for $\mu(t,\mathbf{x})$ and $\mu_5(t,\mathbf{x})$?

Framework

- Chiral kinetic theory as a starting point:

$$\frac{\partial f_\lambda}{\partial t} + \frac{1}{1+\vec{\Omega}_\lambda \cdot \vec{B}}\left[\left(\vec{E} + \vec{v} \times \vec{B} + (\vec{E}\cdot \vec{B})\vec{\Omega}_\lambda\right)\cdot \frac{\partial f_\lambda}{\partial \vec{p}} + \left(\vec{v} + \vec{E} \times \vec{\Omega}_\lambda + (\vec{v}\cdot \vec{\Omega}_\lambda)\vec{B}\right)\cdot \frac{\partial f_\lambda}{\partial \vec{x}}\right] = I_{\mathrm{coll}}$$

where

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{cp-\mu_\lambda}{T}\right)+1}$$

is an expansion in powers of e-m field & $\vec{\nabla}\mu_\lambda, \partial_t \mu_\lambda$

1^{st} order solution

- Equation for $f_\lambda^{(1)}$:

$$\frac{D_\lambda}{T}\frac{\partial \mu_\lambda}{\partial t} - \frac{D_\lambda}{T}\vec{v}\cdot\left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}}\right) = -\frac{\delta f_\lambda^{(1)}}{\tau}$$

- Corresponding currents & densities

$$n_\lambda = \frac{\mu_\lambda\left(\mu_\lambda^2 + \pi^2 T^2\right)}{6\pi^2 c^3} - \frac{\tau\left(3\mu_\lambda^2 + \pi^2 T^2\right)}{6\pi^2 c^3}\frac{\partial \mu_\lambda}{\partial t}$$

$$\vec{j}_\lambda = \frac{\lambda e \mu_\lambda \vec{B}}{4\pi^2 c} + \frac{\tau\left(3\mu_\lambda^2 + \pi^2 T^2\right)}{18\pi^2 c}\left(e\vec{E} - \frac{\partial \mu}{\partial \vec{x}}\right)$$

- Continuity equation gives a constraint: $\frac{\partial \mu_\lambda}{\partial t} = 0$

2^{nd} order solution

- Equation for $f_\lambda^{(2)}$ is slightly more complicated…
- The currents & densities are

$$n_\lambda^{(2)} = \frac{\lambda e^2 \tau}{4\pi^2 c} \vec{B} \cdot \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right) - \frac{c^2 \tau^2}{3} \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \vec{\nabla} \cdot \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right)$$

$$- \frac{e\tau^2 \mu_\lambda}{3\pi^2 c} \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right) \cdot \frac{\partial \mu_\lambda}{\partial \vec{x}}$$

$$\vec{j}_\lambda^{(2)} = -\frac{ec^2 \tau^2}{3} \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \frac{\partial \vec{E}}{\partial t} - \frac{e\tau^2 \mu_\lambda}{6\pi^2} \vec{B} \times \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right)$$

- Continuity equations give should be enforced

Equations for chemical potentials

- Resulting equation of motion for μ_λ:

$$\frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(\frac{\partial \mu_\lambda}{\partial t}+\frac{e\tau c^2}{3}\vec{\nabla}\cdot\vec{E}_\lambda\right)+\frac{e\tau\mu_\lambda}{3\pi^2 c}\left(\vec{E}_\lambda\cdot\frac{\partial \mu_\lambda}{\partial \vec{x}}\right)=\frac{\lambda e^2}{4\pi^2 c}\left(\vec{E}_\lambda\cdot\vec{B}\right)$$

where $n_\lambda^{(0)}=\frac{\mu_\lambda^3+\pi^2T^2\mu_\lambda}{3\pi^2c^3}$ and $\vec{E}_\lambda=\vec{E}-\frac{1}{e}\frac{\partial\mu_\lambda}{\partial\vec{x}}$

The corresponding equations for the currents:

$$\vec{j}=\underbrace{\frac{e\mu_5\vec{B}}{2\pi^2c}}_{\text{CME}}+\underbrace{\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{drift \& diffusion}}+\underbrace{\frac{e\tau^2\mu}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}_{\text{Hall type}}+\vec{j}_{\text{new}}$$

$$\vec{j}_5=\underbrace{\frac{e\mu\vec{B}}{2\pi^2c}}_{\text{CSE}}-\underbrace{\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{diffusion}}+\underbrace{\frac{2e\tau\mu_5\mu}{3\pi^2c}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{CESE}}+\vec{j}_{5,\text{new}}$$

New types of currents

- New contribution to the electric current:

$$\vec{j}_{\text{new}} = -\underbrace{\frac{2\tau\mu\mu_5}{3\pi^2 c}\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{Chiral diffusion}} - \underbrace{\frac{e\tau^2\mu_5}{3\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right)}_{\text{Hall diffusion}} - \frac{e\tau^2\left(3\mu^2+3\mu_5^2+\pi^2T^2\right)}{9\pi^2 c}\frac{\partial\vec{E}}{\partial t}$$

- New contribution to the chiral current:

$$\vec{j}_{5,\text{new}} = -\overbrace{\frac{e\tau^2\mu}{3\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right)}^{\text{Chiral Hall diffusion}} + \overbrace{\frac{e\tau^2\mu_5}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}^{\text{Chiral Hall effect}} - \frac{2e\tau^2\mu\mu_5}{3\pi^2 c}\frac{\partial\vec{E}}{\partial t}$$

- However, there is no term $\propto \frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu}{\partial\vec{x}}\right)$

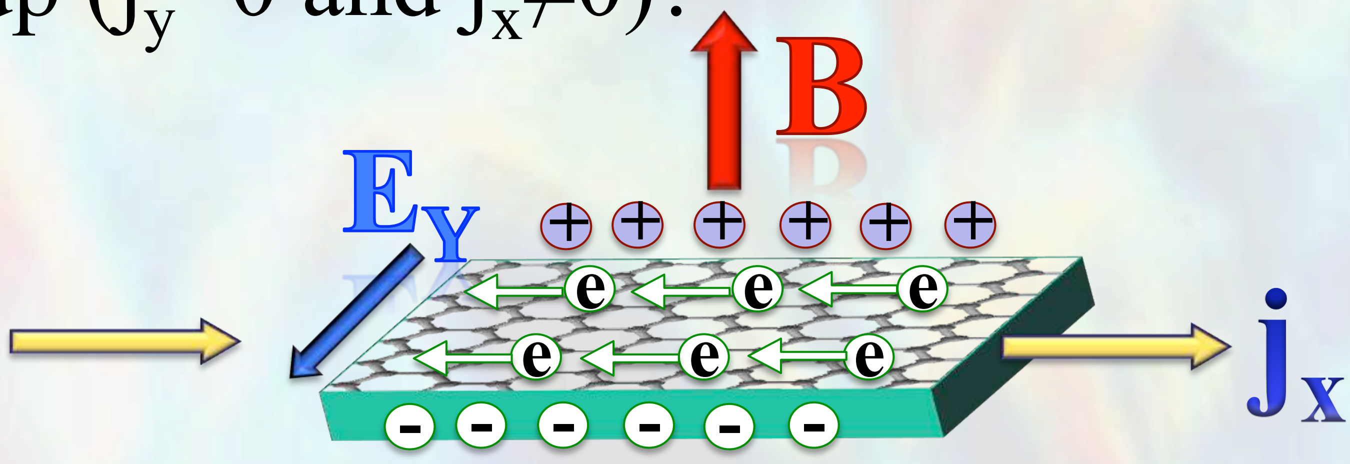
Question about Hall current

- Note that we had

$$\vec{j}^{\,(\text{Hall})} = \frac{e^2 \tau^2 \mu}{3\pi^2} \vec{E} \times \vec{B}$$

- Why is this proportional to τ^2 ?

- What do you observe in the usual experimental setup (j_y=0 and $j_x \neq 0$)?



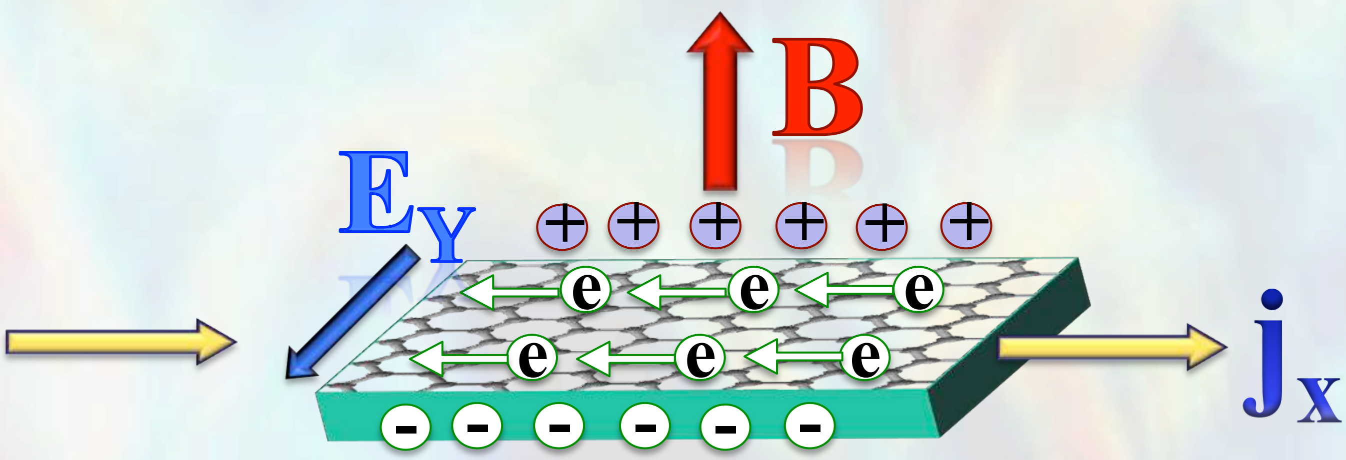
Question about Hall current

- Enforcing j_y=0 gives

$$a\tau E_y = b\tau^2 E_x B_z$$

Then, in the approximation used,

$$j_x = a\tau E_x + b\tau^2 E_y B_z = \frac{(a\tau)^2}{b\tau^2 B_z} E_y + b\tau^2 E_y B_z \approx \frac{a^2}{b\,B_z} E_y$$

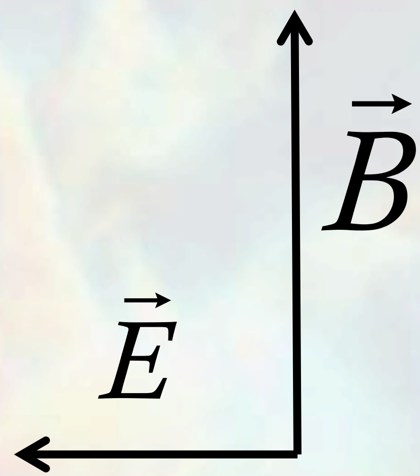


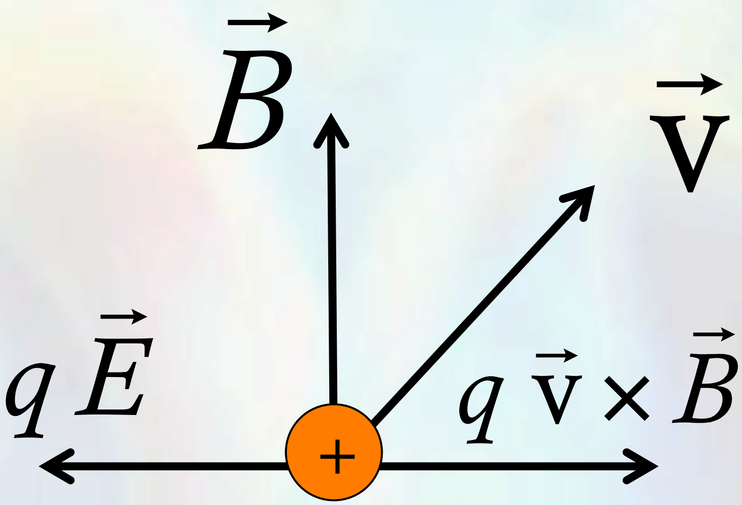
Plasma drift

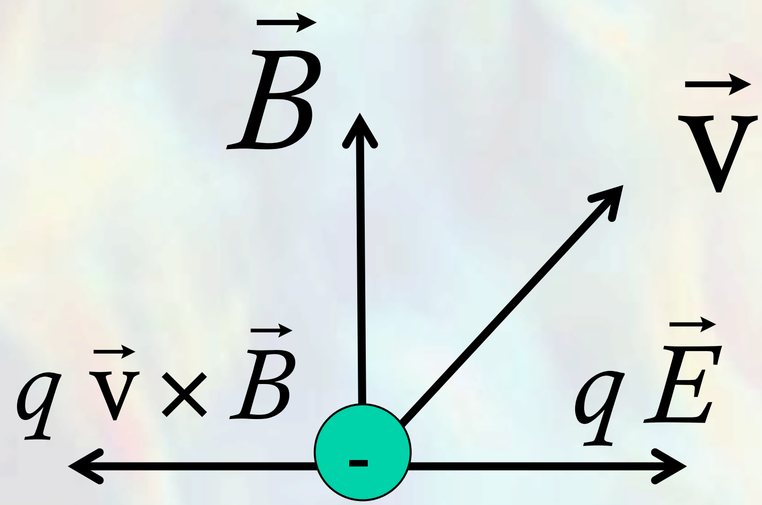
Plasma as a whole experiences a non-dissipative drift. Can this be included in $f_\lambda^{(0)}$?

Why should plasma drift?

Consider $\vec{E} \perp \vec{B}$ (with $E < B$):







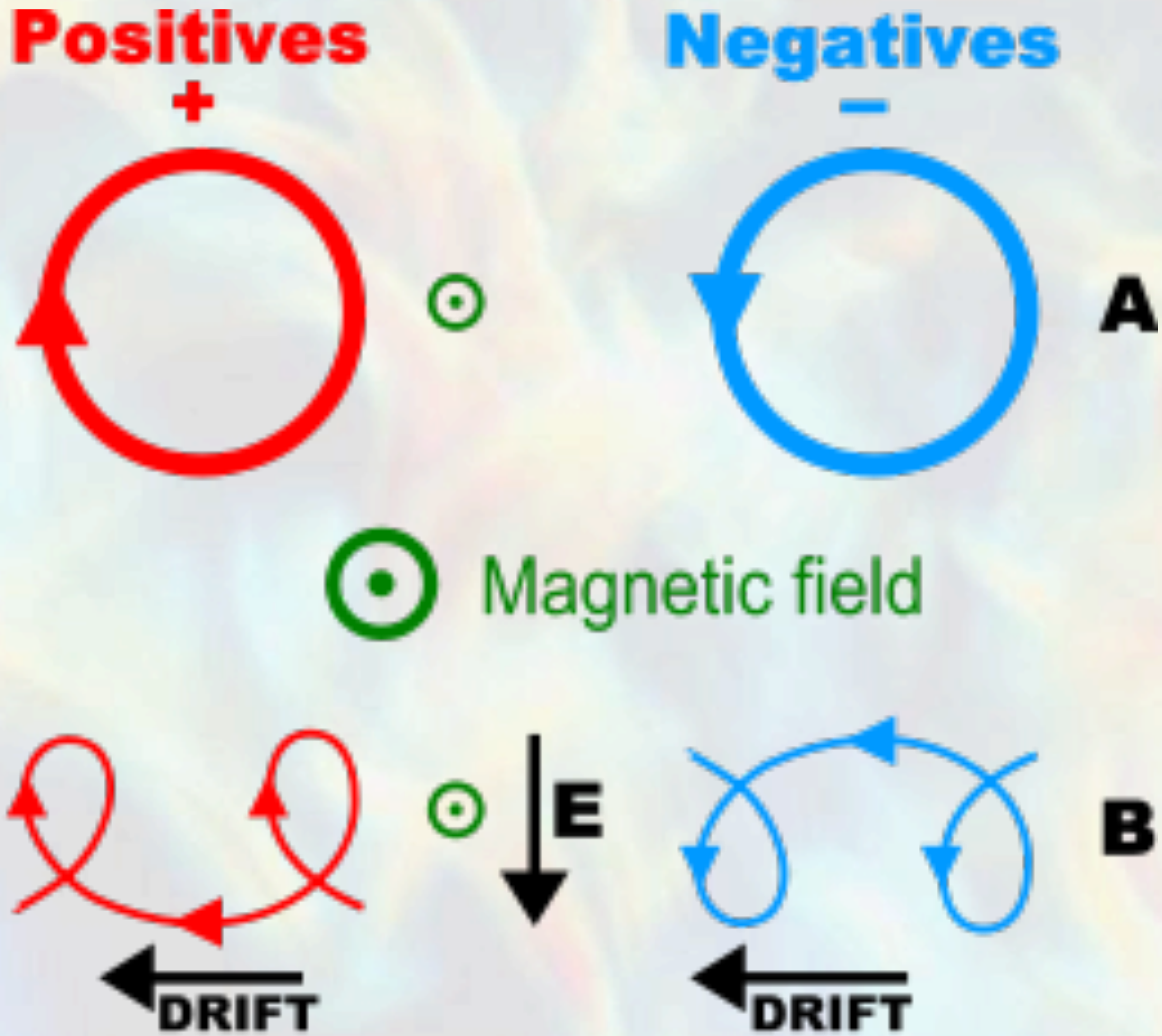
Frames of reference

Another viewpoint

$f_\lambda^{(0)}$

Drifting frame:

Lab frame:



$f_\lambda^{(0)}$ for magnetized plasma

- Consider a special case
  - Plasma consists of only e-m charged degrees of freedom
  - Fields so that $\vec{E} \perp \vec{B}$ (with $E < B$)
- No electric field in the drift frame
- Lab frame: use boosted Fermi-Dirac distribution

$$f_\lambda^{(\text{lab})} = \frac{1}{\exp\left(\dfrac{c\,p - \vec{p}\cdot\vec{\text{v}}_{\text{drift}} - \mu_\lambda}{T}\right)+1}$$

with

$$\vec{\text{v}}_{\text{drift}} = c\,\frac{\vec{E}\times\vec{B}}{B^2}$$

1st Perturbation in drifting plasma

- Charge density

$$n_\lambda^{(\mathrm{lab})} = n_\lambda^{(0)} - \tau \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(\frac{\partial \mu_\lambda}{\partial t} + \vec{v}_{\mathrm{drift}} \cdot \nabla \mu_\lambda\right) + \tau n_\lambda^{(0)}\left(\nabla \cdot \vec{v}_{\mathrm{drift}}\right) + \ldots$$

- Current density

$$\vec{j}_\lambda^{(\mathrm{lab})} = c n_\lambda^{(0)} \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\lambda e \mu_\lambda}{4\pi^2 c}\vec{B} + \frac{\lambda e \mu_\lambda}{4\pi^2 c}\vec{E}_\perp \frac{\left(\vec{E}\cdot\vec{B}\right)}{E_\perp^2}\left(\frac{B}{2E_\perp} ln \frac{B+E_\perp}{B-E_\perp} - 1\right)$$

$$- \tau \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(g_1\left(\frac{e\left(\vec{E}\cdot\vec{B}\right)}{B^2}\vec{B} - \nabla\mu_\lambda\right) + g_2 \vec{v}_{\mathrm{drift}}\left(\vec{v}_{\mathrm{drift}}\cdot\nabla\mu_\lambda\right) + g_3 \frac{\partial \mu_\lambda}{\partial t}\vec{v}_{\mathrm{drift}}\right) + \ldots$$

Drift in QGP plasma?

- In QGP, gluons play a profound role
  - Gluons are neutral and, thus, are not drifting
  - The zeroth approximation is the usual Fermi-Dirac distribution

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{c\,p - \mu_\lambda}{T}\right) + 1}$$

- Expansion in small e-m fields and gradients is well define

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

- Expansion of the 1st type (no drift) may be better

Summary (1)

- We now have Anomalous Maxwell equations for inhomogeneous chiral plasmas
- All previously known currents are reproduced
- A whole set of new currents obtained
- Equations for $\mu(t,\mathbf{x})$ and $\mu_5(t,\mathbf{x})$ are derived
- Effects of drift give new non-dissipative terms