



Anomaly-driven chiral magnetic effects

Igor Shovkovy
Arizona State University



1st CORE-U International Conference:
Intense Fields and Extreme Universe
March 7-8, 2016, Higashi Hiroshima Campus of Hiroshima University, Japan

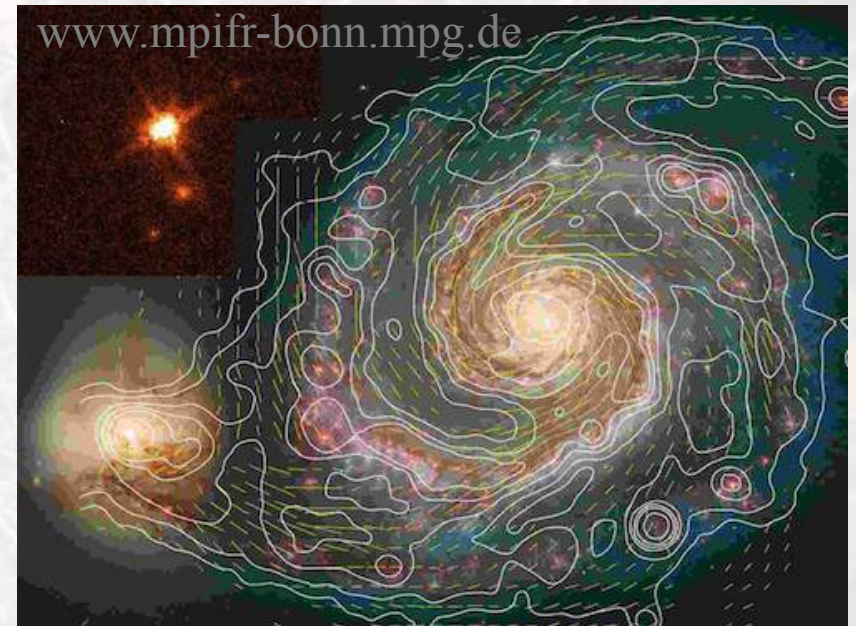
広島大学 極限宇宙研究拠点
CORE U
Core of Research for the Energetic Universe
HIROSHIMA UNIVERSITY



MAGNETIC FIELDS EVERYWHERE

[Miransky & Shovkovy, Physics Reports 576 (2015) pp. 1-209]

- Current galactic magnetic fields $\sim 10^{-6}$ G
- Current magnetic fields in voids $\sim 10^{-15}$ G
- Problem of magnetogenesis in Early Universe
- Perhaps during the electro-weak phase transition
– 10^{20} to 10^{24} G (~ 1 GeV to 100 GeV)



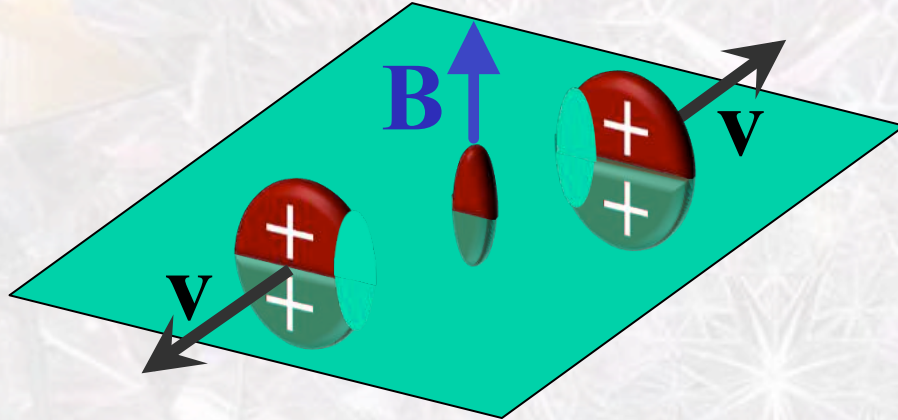
- Magnetized dense baryonic matter
 - 10^{10} to 10^{18} G (10 keV to 100 MeV)
- Magnetic field may affect
 - Competition of ground state phases
 - EoS of dense baryonic matter
 - the M-R relation of compact stars
 - Transport and emission properties
 - Evolution of supernovas & protoneutron stars



[→ talks by Yasufumi Kojima & Teruaki Enoto]

Little Bangs

- Magnetized QGP at RHIC/LHC
 - $B \sim 10^{18}$ to 10^{19} G (~ 100 MeV)



[Rafelski & Müller, PRL, 36, 517 (1976)],
 [Kharzeev et al., arXiv:0711.0950],
 [Skokov et al., arXiv:0907.1396],
 [Voronyuk et al., arXiv:1103.4239],
 [Bzdak & Skokov, arXiv:1111.1949],
 [Deng & Huang, arXiv:1201.5108]

- Using Lienard-Wiechert potentials,

$$e\mathbf{E}(t, \mathbf{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\mathbf{R}_n - R_n \mathbf{v}_n}{(R_n - \mathbf{R}_n \cdot \mathbf{v}_n)^3} (1 - v_n^2)$$

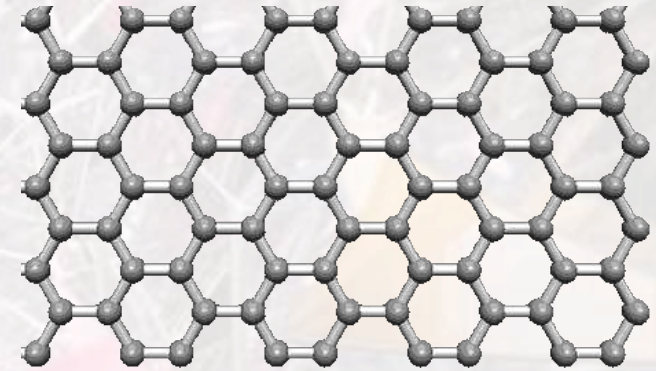
$$e\mathbf{B}(t, \mathbf{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n \cdot \mathbf{v}_n)^3} (1 - v_n^2)$$

[\rightarrow talks by Kazunori Itakura & Ron Belmont]

Dirac/Weyl materials

- High magnetic field lab
 - 10^5 G (~ 100 meV @ $v_F=c/300$)

- Graphene



- 3D materials with Dirac/Weyl quasiparticles

- $\text{Bi}_{1-x}\text{Sb}_x$ alloy (at $x \approx 4\%$)
- Na_3Bi
- Cd_3As_2
- ZrTe_5
- TaAs, NbAs, TaP, ...

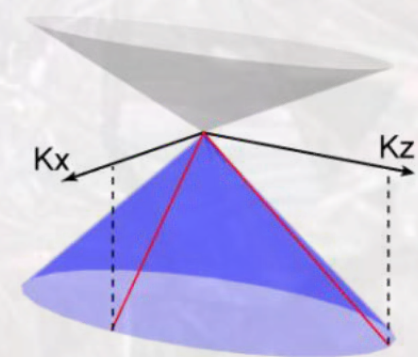
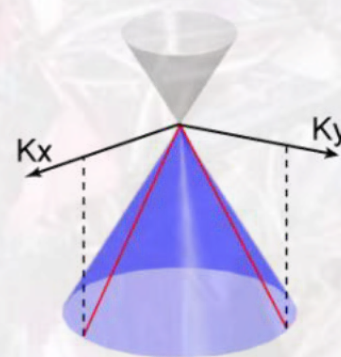
[Z. K. Liu et al., arXiv:1310.0391]

[M. Neupane et al., arXiv:1309.7892]

[S. Borisenko et al., arXiv:1309.7978]

[X. Li et al., arXiv:1412.6543]

[arXiv:1502.03807, arXiv:1502.04684,
arXiv:1504.01350, arXiv:1507.00521]

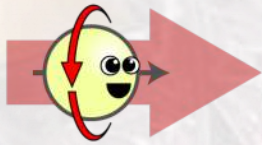




CHIRAL SEPARATION EFFECT

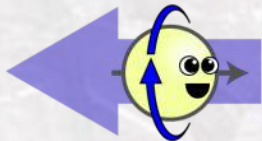
$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2} \mu$$

- Chirality/helicity of a massless (or ultrarelativistic) particle is (approximately) conserved



Right-handed

$$\frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \Psi = \text{sign}(p_0) \gamma^5 \Psi$$



Left-handed

- Conservation of chiral charge is a property of massless Dirac theory (classically)
- The symmetry is anomalous at quantum level

$$\frac{\partial(n_R - n_L)}{\partial t} + \nabla \cdot \vec{j}_5 = -\frac{e^2}{16\pi^2} \epsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda}$$

Chiral separation effect

- Slowly changing electric/chemical potential

$$\mu(z) = e\Phi(z) \Rightarrow eE_z = -\partial_z(e\Phi) = -\partial_z\mu$$

- From the anomaly relation,

$$\partial_z j_5^3 = -\frac{e^2}{2\pi^2} B_z E_z = \frac{e^2}{2\pi^2} B_z \partial_z \mu$$

- Suggesting that for massless fermions,

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2} \mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

[Newman & Son, Phys. Rev. D 73 (2006) 045006]

- This can be easily derived in free theory

Landau spectrum at $B \neq 0$

- Dirac equation with massless fermions

$$\left[i\gamma^0 \partial_0 - i\vec{\gamma} \cdot (\vec{\nabla} + ie\vec{A}) \right] \Psi = 0$$

- Energy spectrum

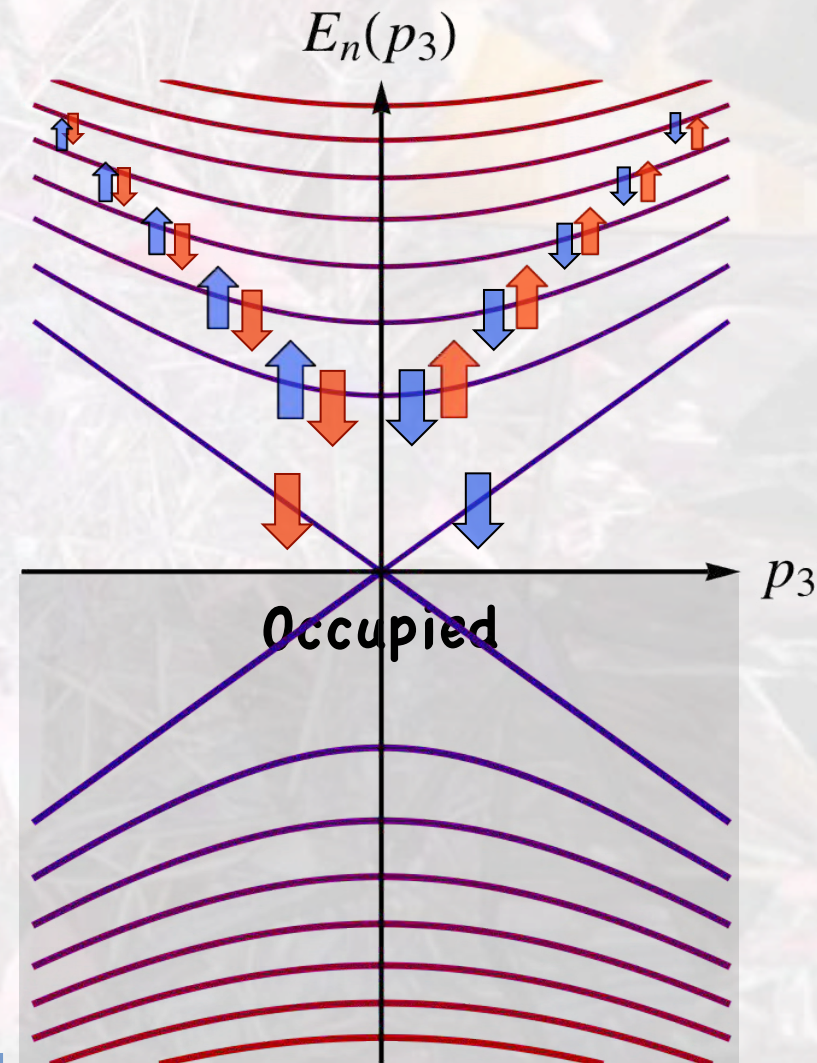
$$E_n^{(3+1)}(p_3) = \pm \sqrt{2n|eB| + p_3^2}$$

$$s = \pm \frac{1}{2} \quad (\text{spin})$$

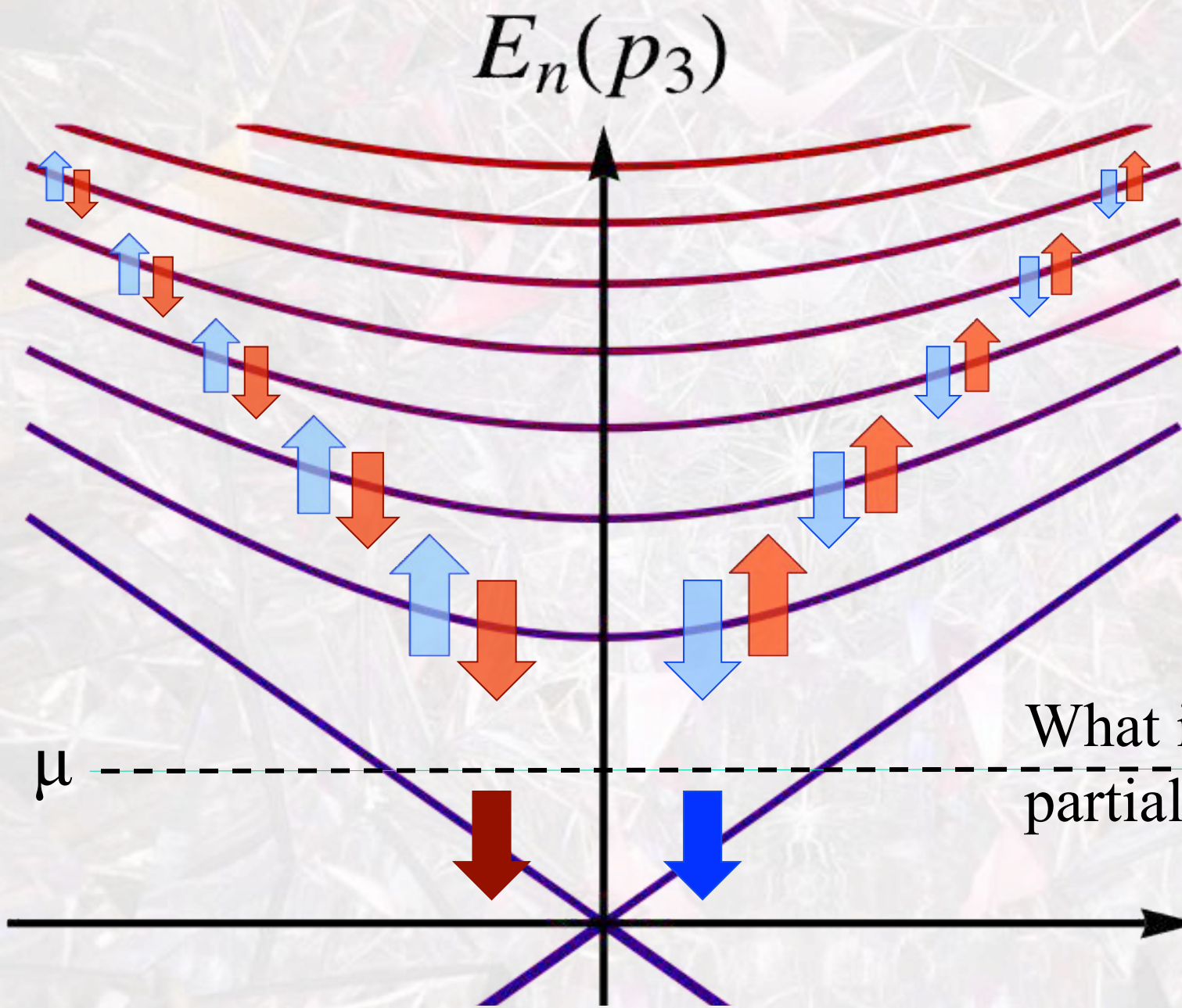
where

$$n = s + k + \frac{1}{2}$$

$$k = 0, 1, 2, \dots (\text{orbital})$$



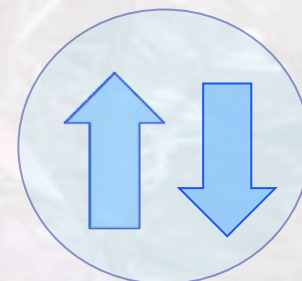
Landau spectrum & $\mu \neq 0$



Right-handed:



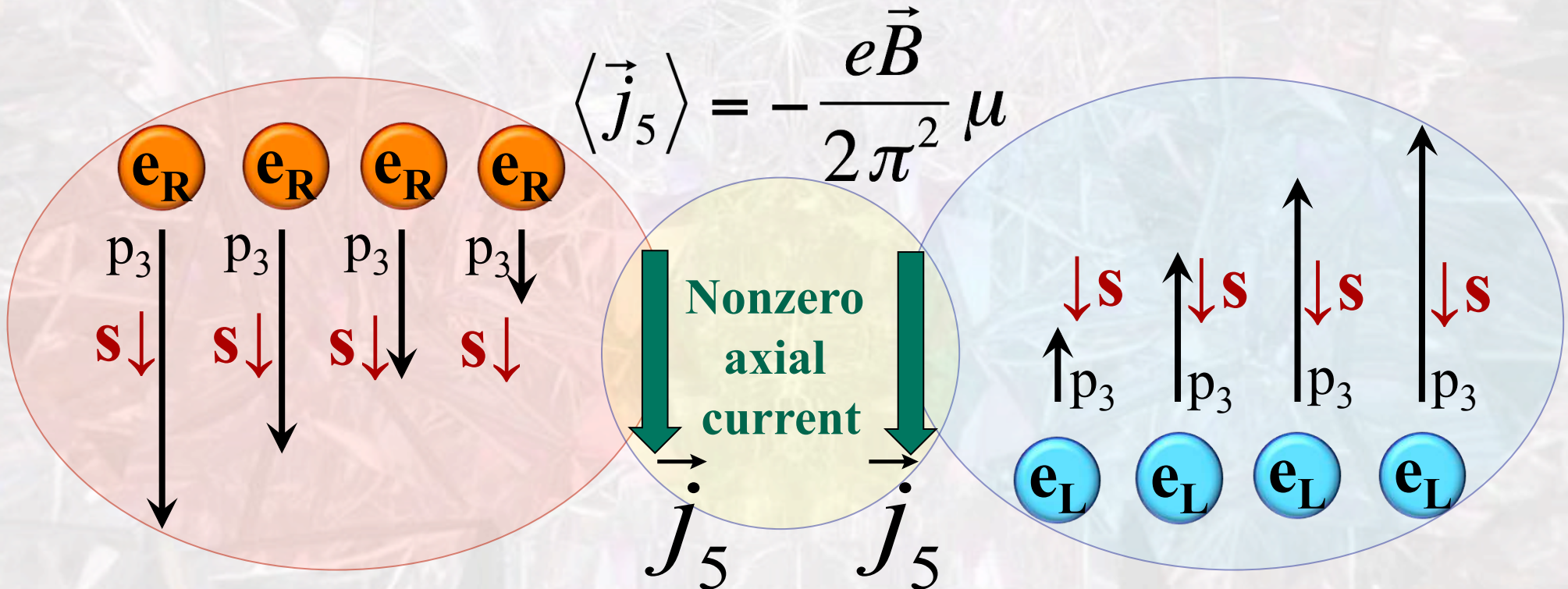
Left-handed:



What if LLL is partially filled?

Partially filled LLL

- **Spin polarized LLL** is chirally asymmetric
 - states with $p_3 < 0$ (and $s = \downarrow$) are R-handed
 - states with $p_3 > 0$ (and $s = \downarrow$) are L-handed
- i.e., a nonzero **axial** current is induced



CSE in Dirac semimetals

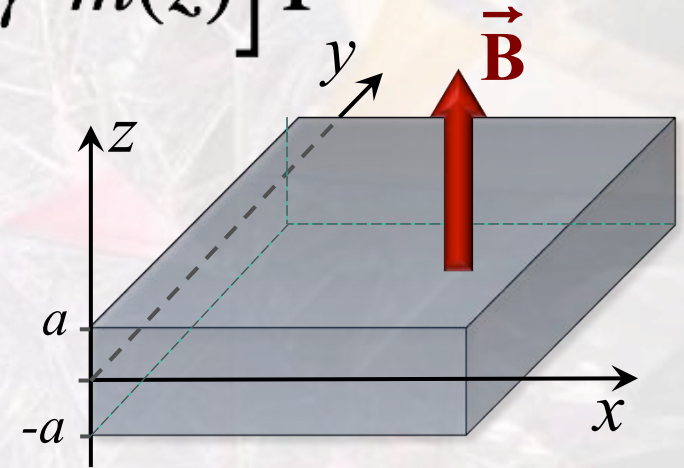
- Model of Dirac semimetal with a slab geometry

$$H = \int d^3r \Psi^\dagger \left[v_F \vec{\alpha} \cdot \left(-i\vec{\nabla} + e\vec{A} \right) + \gamma^0 m(z) \right] \Psi$$

where $\vec{A} = (0, Bx, 0)$ and

$$m(z) = M \theta(z^2 - a^2) + m \theta(a^2 - z^2),$$

with vacuum band gap: $M \rightarrow \infty$ (broken chiral symmetry)



- Boundary conditions:

$$\Psi_{\text{bulk}}(\vec{r}_\perp, a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, a) \quad \text{and} \quad \Psi_{\text{bulk}}(\vec{r}_\perp, -a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, -a)$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

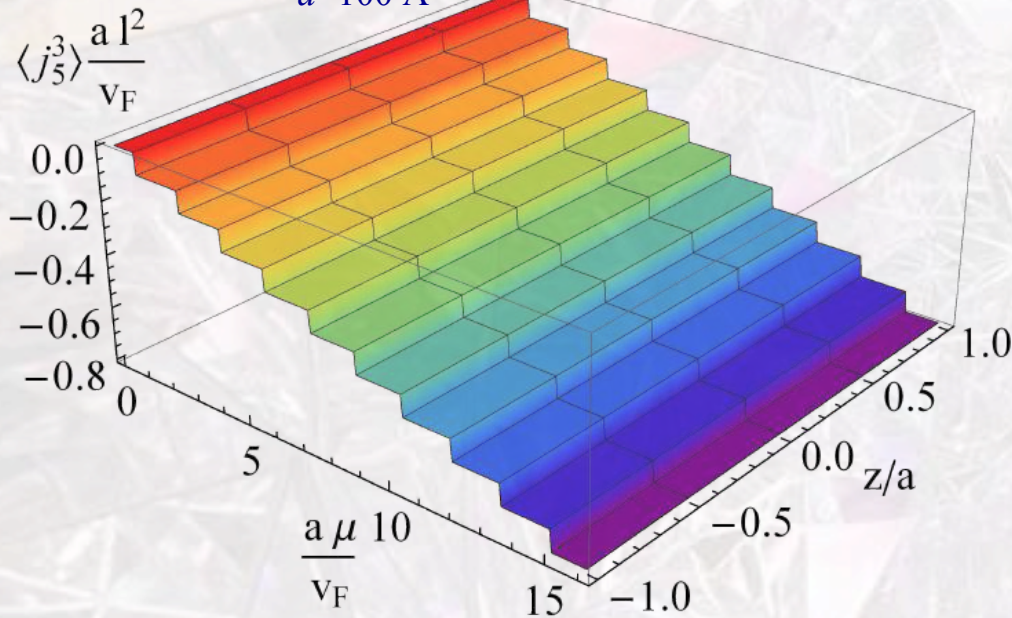
Quantization of axial current

- Axial current density is non-uniform when $m \neq 0$

$m=0$

$v_F=2.5 \text{ eV \AA}$

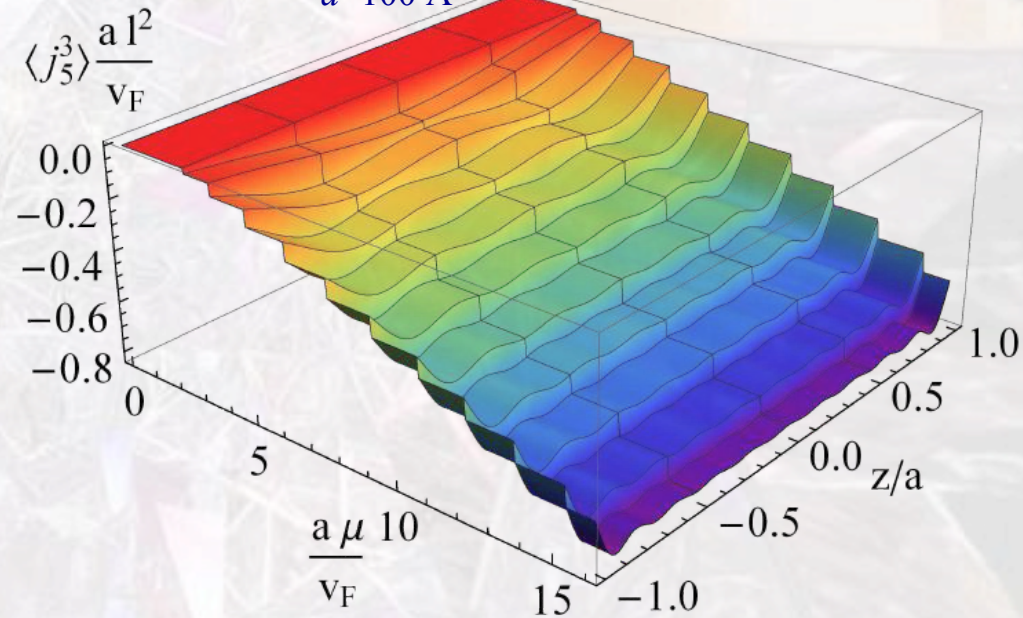
$a=100 \text{ \AA}$



$m=0.05 \text{ eV}$

$v_F=2.5 \text{ eV \AA}$

$a=100 \text{ \AA}$

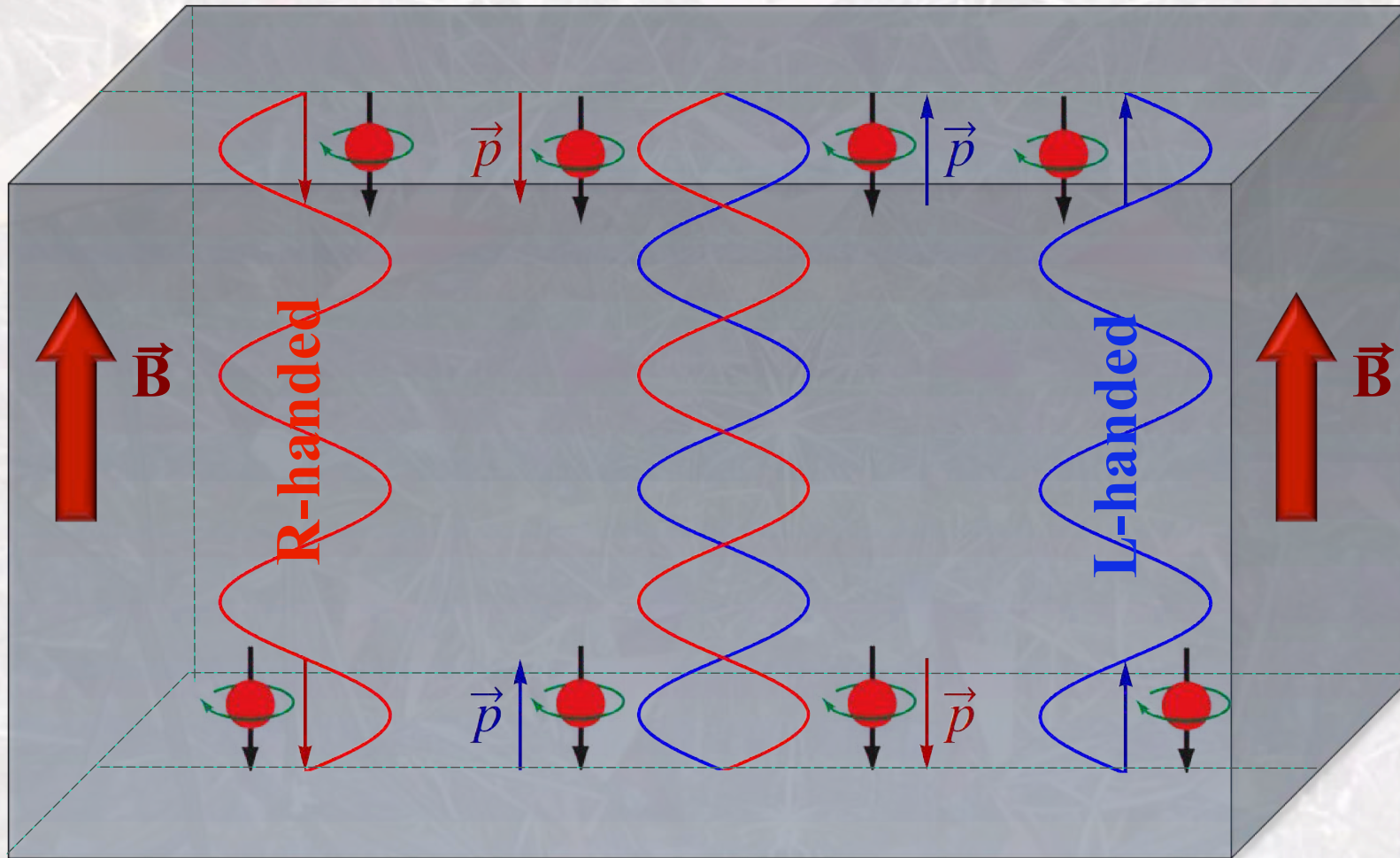


- Note that axial charge density vanishes: $\langle j_5^0 \rangle = 0$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

ASU Axial current as a standing wave?

- Recall that LLL is spin polarized



- A perfect chirality flip at the boundary

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]



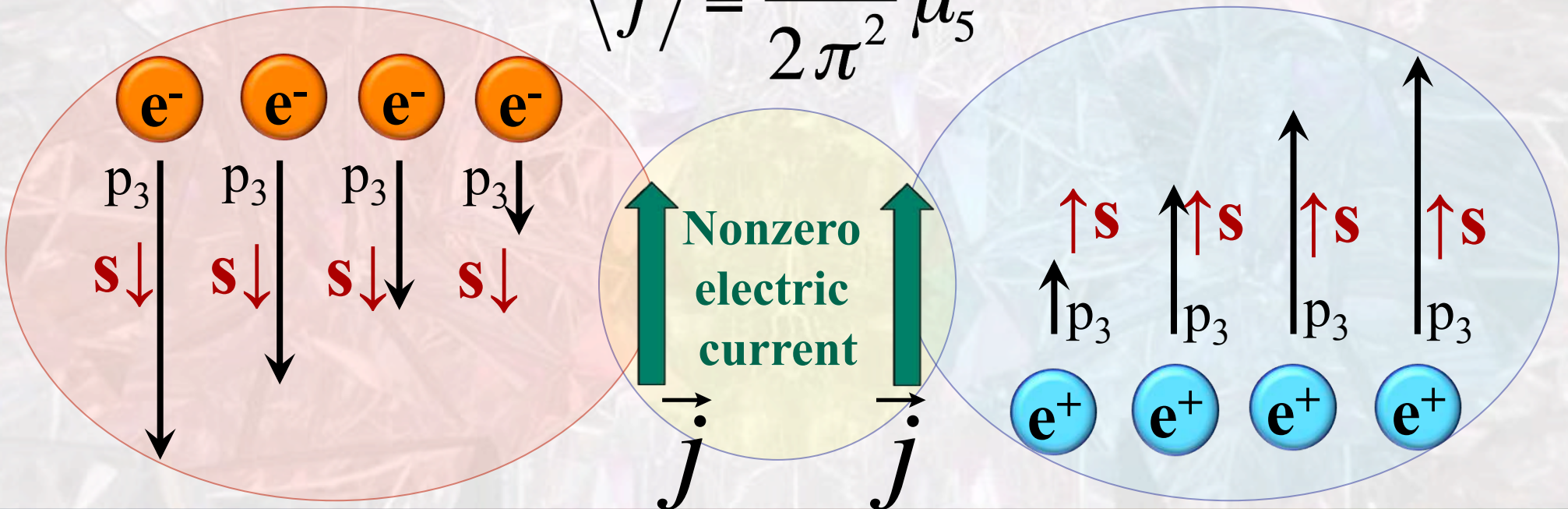
CHIRAL MAGNETIC EFFECT

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

Partially filled LLL @ $\mu_5 \neq 0$

- **Spin polarized LLL** is chirally asymmetric
 - states with $p_3 < 0$ (and $s = \downarrow$) are R-handed **electrons**
 - states with $p_3 > 0$ (and $s = \downarrow$) are L-handed **positrons**
- i.e., a nonzero **electric** current is induced

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$



- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$

- A random fluctuation with nonzero chirality could result in

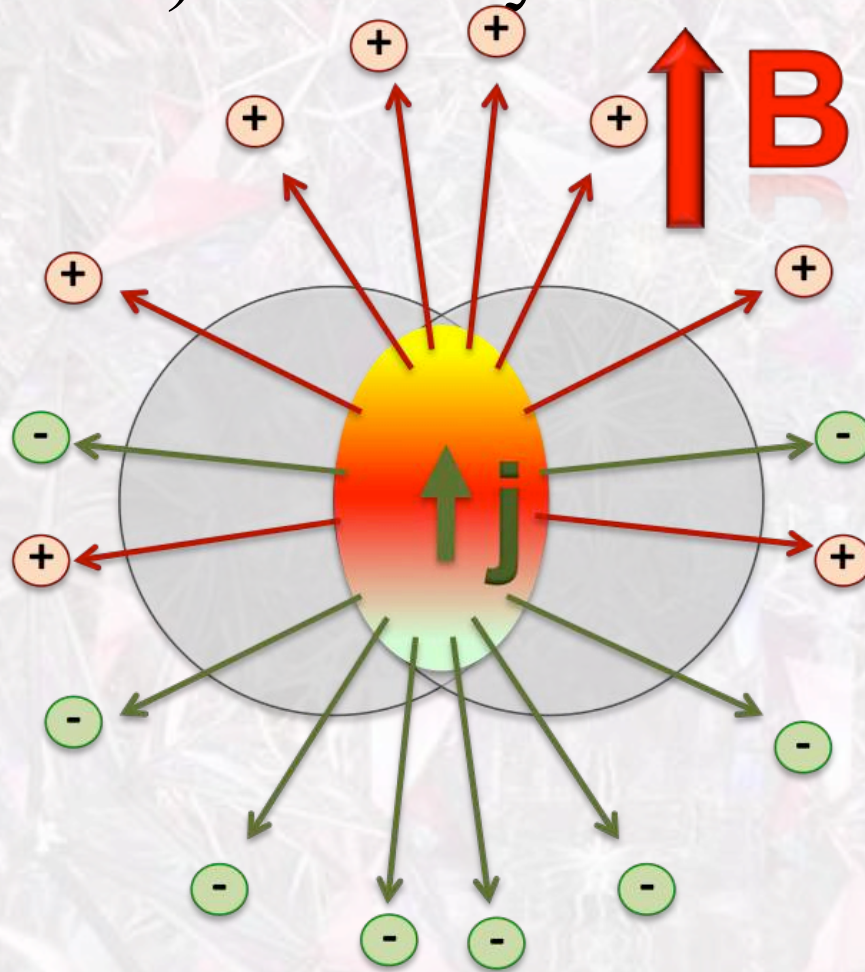
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- This should lead to an electric current

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

Dipole CME

- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions

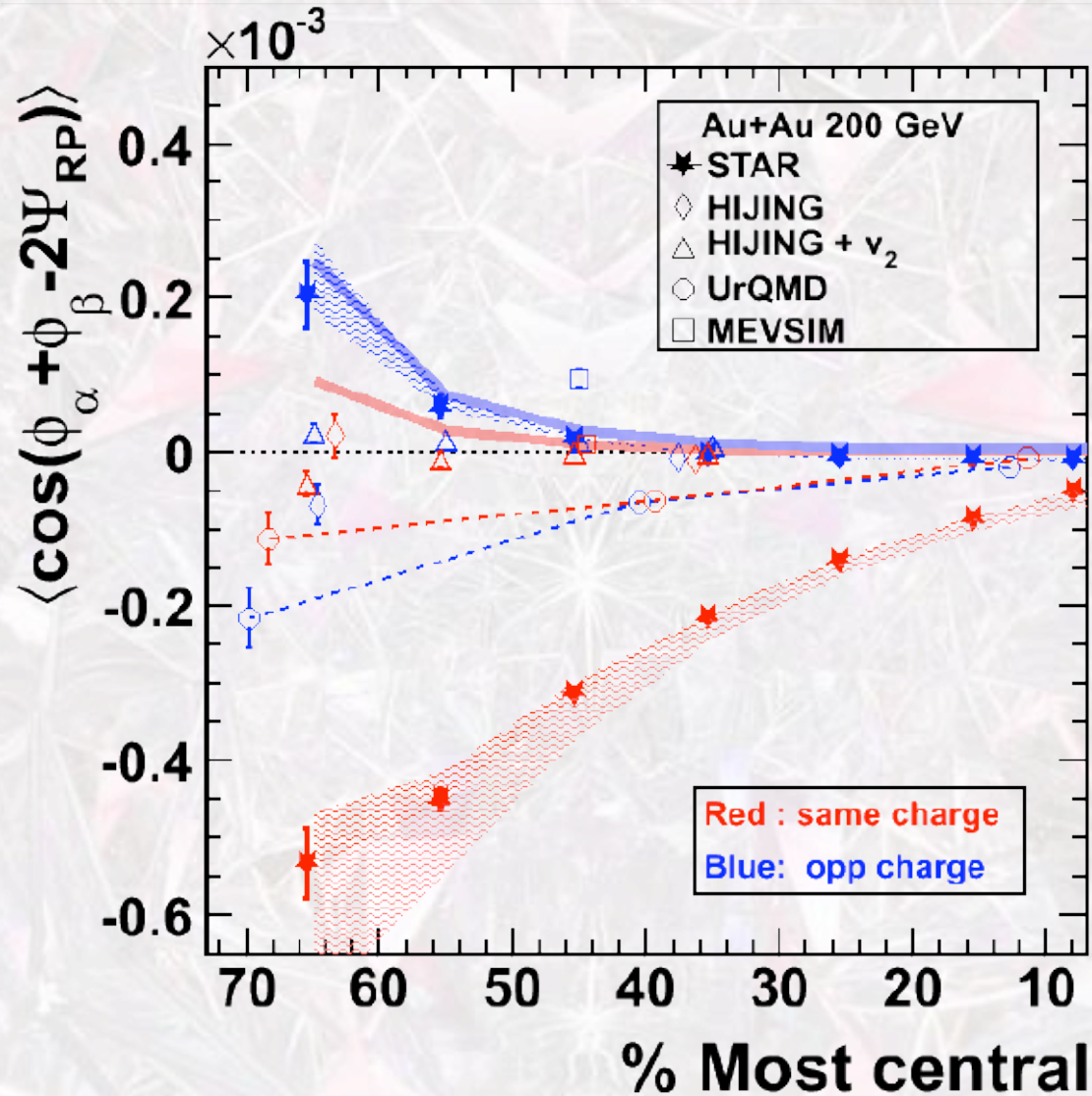


[→ talk Ron Belmont]

[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Experimental evidence



[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. **103**, 251601 (2009)]

[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C **81**, 054908 (2010)]

[Adamczyk et al. (STAR Collaboration), Phys. Rev. C **88**, 064911 (2013)]



CHIRAL MAGNETIC WAVE

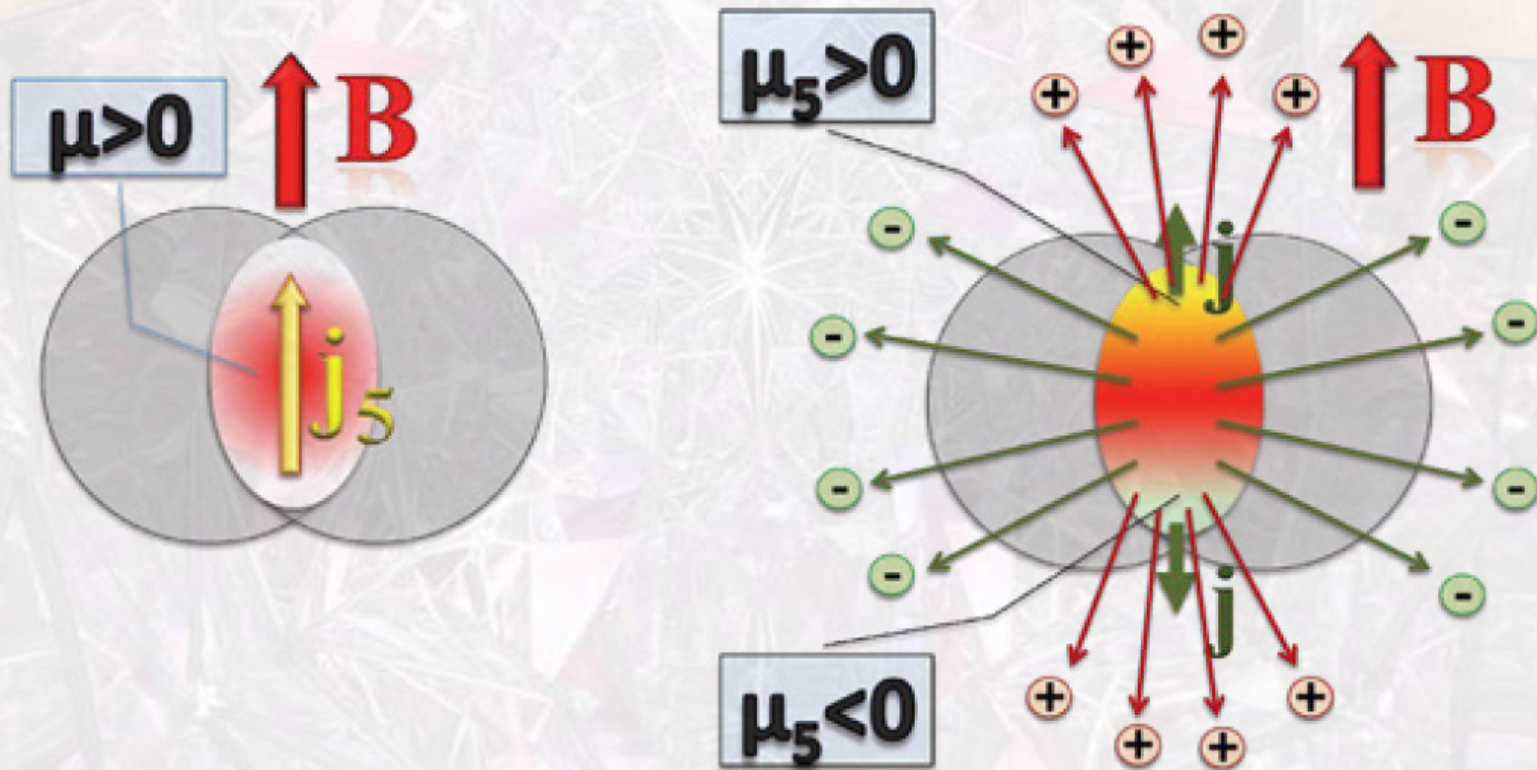
$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2} \mu \quad \langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2} \mu_5$$

CMW/Quadrupole CME

- Start from a small baryon density and $B \neq 0$

$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2} \mu$$

$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2} \mu_5$$



- Produce back-to-back electric currents

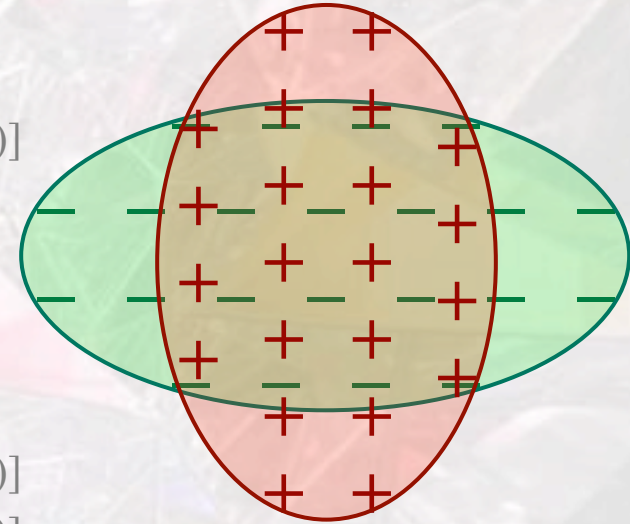
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
 [Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Experimental evidence

- Elliptic flows of π^+ and π^- depend on charge asymmetry:

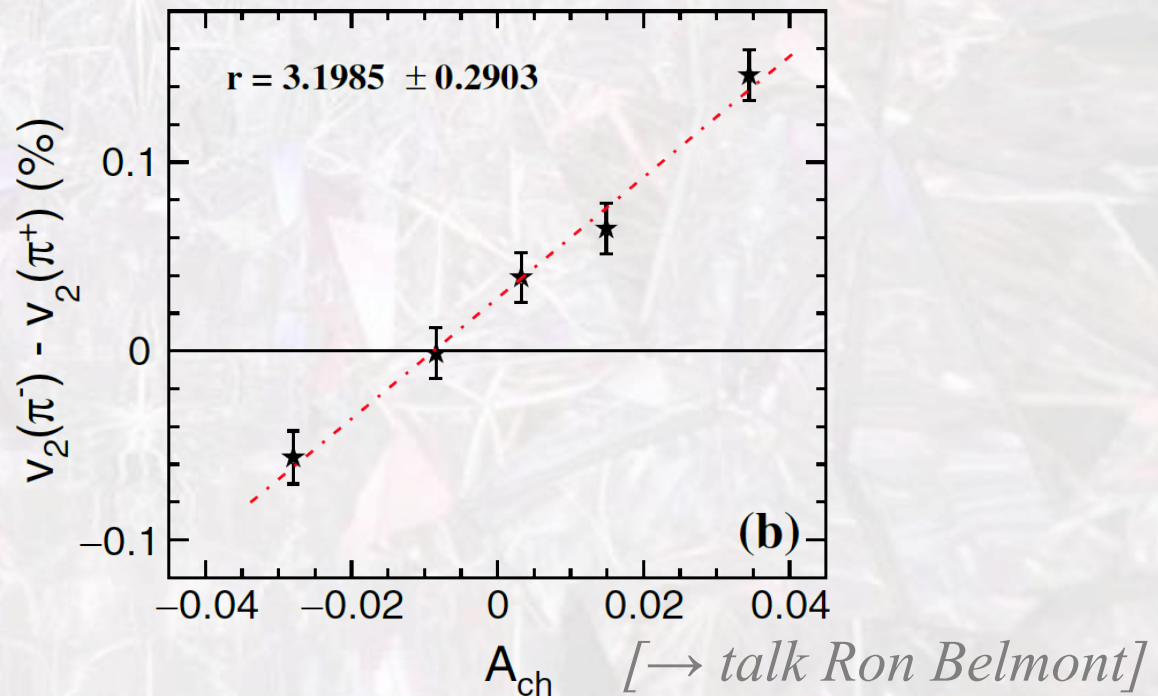
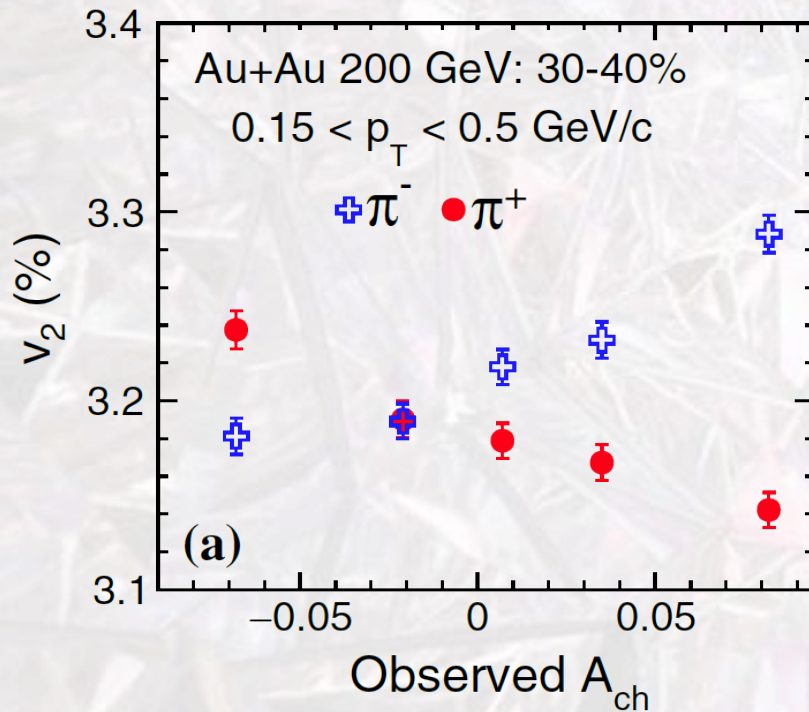
[Burnier, Kharzeev, Liao, Yee, PRL **107**, 052303 (2011)]

$$\frac{dN_{\pm}}{d\phi} \approx \bar{N}_{\pm} \left[1 + 2v_2 \cos(2\phi) \mp A_{\pm} r \cos(2\phi) \right]$$



[H. Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]





FURTHER DEVELOPMENTS

- Dynamical chiral shift

- The axial current (CSE) in free theory

$$\langle \bar{\psi} \gamma^3 \gamma^5 \psi \rangle = \frac{eB}{2\pi^2} \mu \quad \text{with} \quad \vec{B} = (0, 0, B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

+ **interactions** should induce a chiral shift parameter Δ associated with the condensate,

$$\delta L = \Delta \bar{\psi} \gamma^3 \gamma^5 \psi$$

This is a perturbative effect: there is no symmetry to protect $\Delta = 0$

[Gorbar, Miransky, Shovkovy, Phys. Rev. C **80**, 032801(R) (2009)]

Chiral shift @ Fermi surface

- Chirality is \approx well-defined at Fermi surface ($|p_3| \gg m$)
- L-handed Fermi surface:

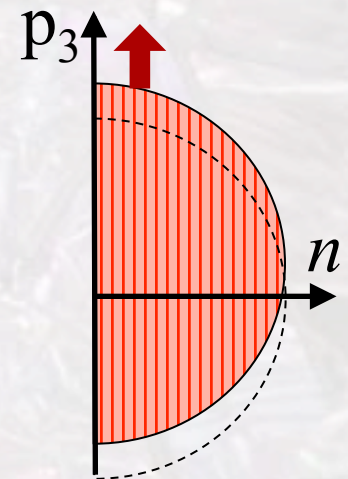
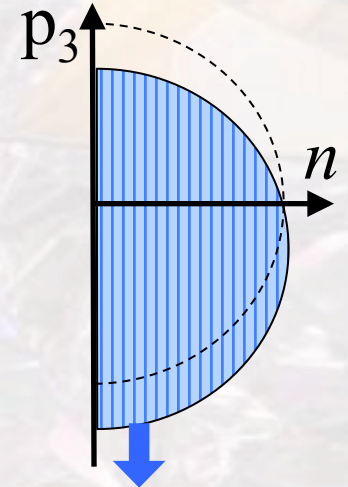
$$n > 0: \quad p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_{\perp} \Delta\right)^2 - m^2}$$

$$p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_{\perp} \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n > 0: \quad p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_{\perp} \Delta\right)^2 - m^2}$$

$$p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_{\perp} \Delta\right)^2 - m^2}$$



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- The result has the form

$$\bar{\Sigma}^{(1)}(p) = \gamma^3 \gamma^5 \Delta(p) + \gamma^0 \gamma^5 \mu_5(p)$$

Near Fermi surface ($p_0 \rightarrow 0$, $|\mathbf{p}| \rightarrow p_F$)

$$\Delta(p) \approx \frac{\alpha e B \mu}{\pi m^2} \left(\ln \frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1 \right)$$

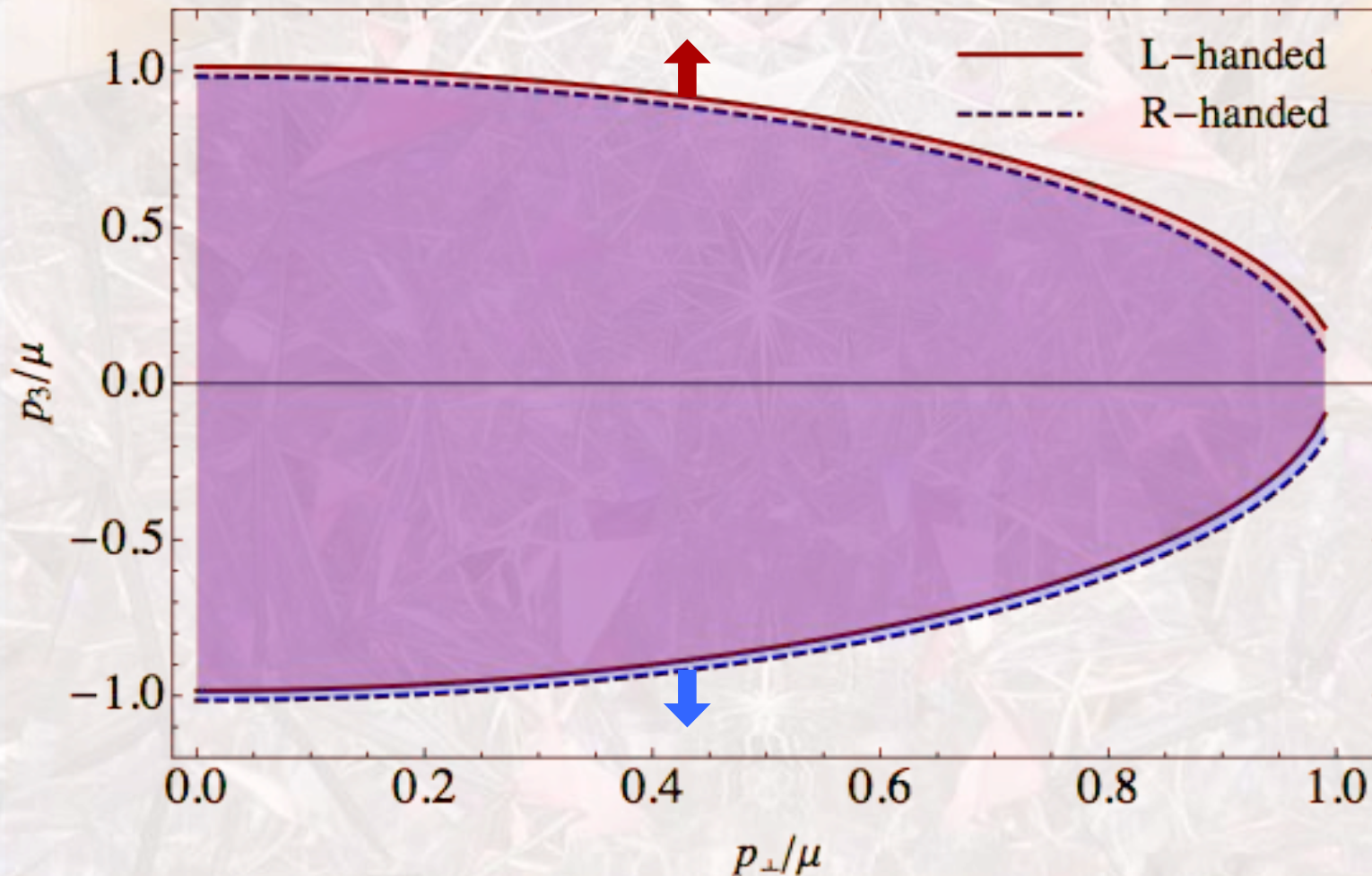
$$\mu_5(p) \approx -\frac{\alpha e B \mu}{\pi m^2} \frac{p_3}{p_F} \left(\ln \frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1 \right)$$

[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Dispersion relations in QED

- Let us use the condition (for a small B)

$$\text{Det}\left[i\bar{S}^{-1}(p) + \bar{\Sigma}^{(1)}(p)\right] = 0$$

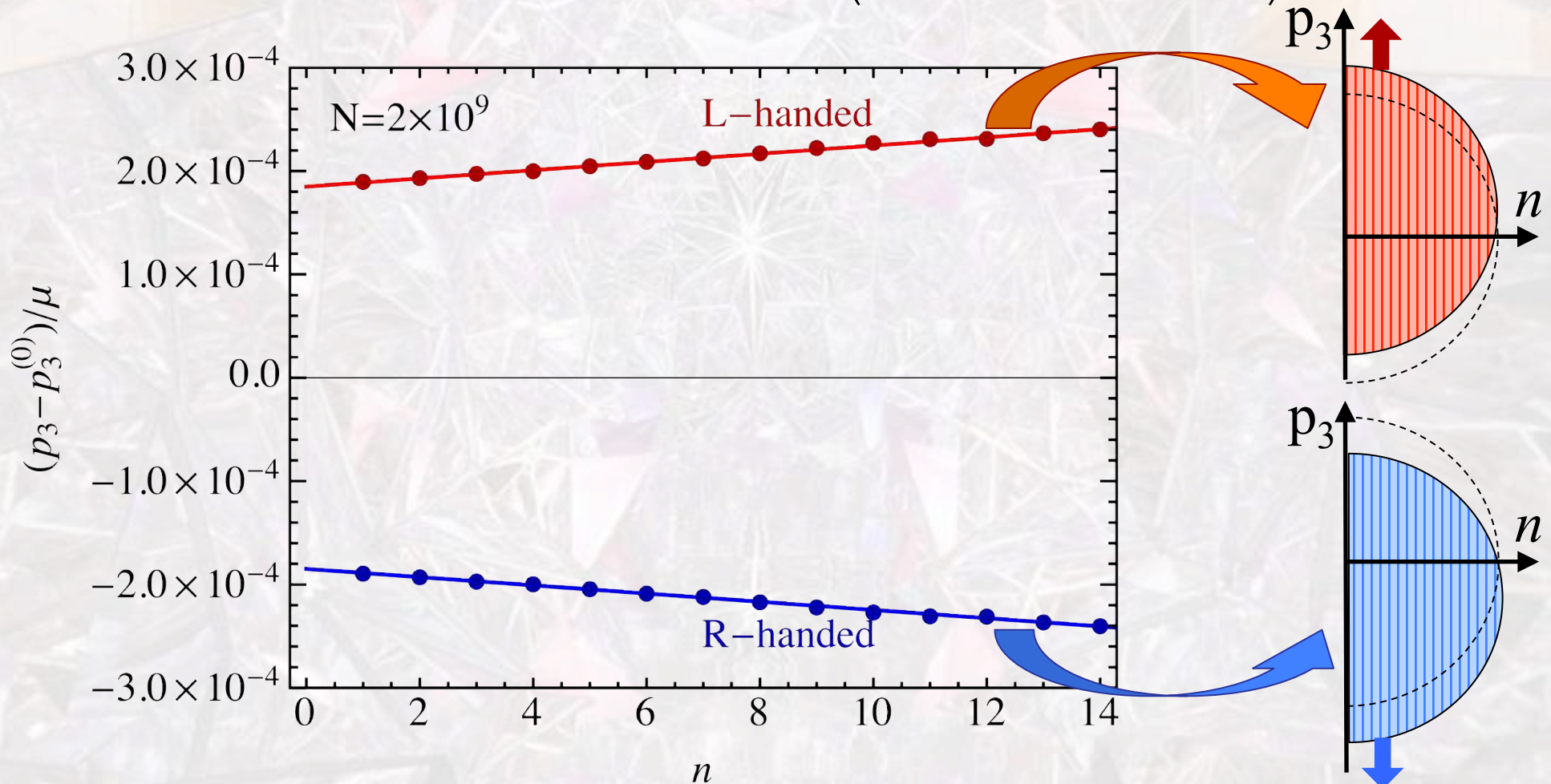


[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

QED in strong field: δp_3

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu} \left(0.76 + 0.49 \frac{|eB|n}{\mu^2} \right)$$



[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D **90**, 085011 (2014)]

How large is the asymmetry?

In QED:

$$\frac{\alpha |eB|}{\mu} \approx 0.4 \left(\frac{B}{10^{18} \text{ G}} \right) \left(\frac{100 \text{ MeV}}{\mu} \right) \text{ MeV}/c$$

In QCD:

$$\frac{\alpha_s |eB|}{\mu} \approx 10 \left(\frac{B}{10^{18} \text{ G}} \right) \left(\frac{400 \text{ MeV}}{\mu} \right) \text{ MeV}/c$$

may have some observable consequences...



FURTHER DEVELOPMENTS

- Anomalous Maxwell equations for chiral plasmas

[Boyarsky, Gorbar, Ruchayskiy, Rudenok, Shovkovy, Vilchinskii, to appear]

Magnetic field/helicity

- Magnetic helicity evolution (inverse cascade) in the Early Universe

$$\nabla \times \vec{B} = \frac{4\pi}{c} \left(\sigma \vec{E} + C_5 \mu_5 \vec{B} \right) + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = 4\pi \rho, \quad \nabla \cdot \vec{B} = 0$$

[Vilenkin, Phys. Rev. D22, 3080 (1980)]

[Joyce & Shaposhnikov, astro-ph/9703005]

[Giovannini & Shaposhnikov, hep-ph/9710234]

- For specific helicity eigenmodes:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma} \left(4\pi C_5 \mu_5 - ck \right) B_k$$

[Boyarsky et al., arXiv:1109.3350]

[Tashiro et al., arXiv:1206.5549]

[Manuel et al., arXiv:1501.07608]

[Buividovich et al., arXiv:1509.02076]

[Hirono et al., arXiv:1509.07790]

Feedback on $\mu_5(t)$

- Constraint of the total helicity conservation

$$h_{\text{fermion}} = h_0 - h_{\text{gauge}}$$

- Common “homogeneous” approximation:

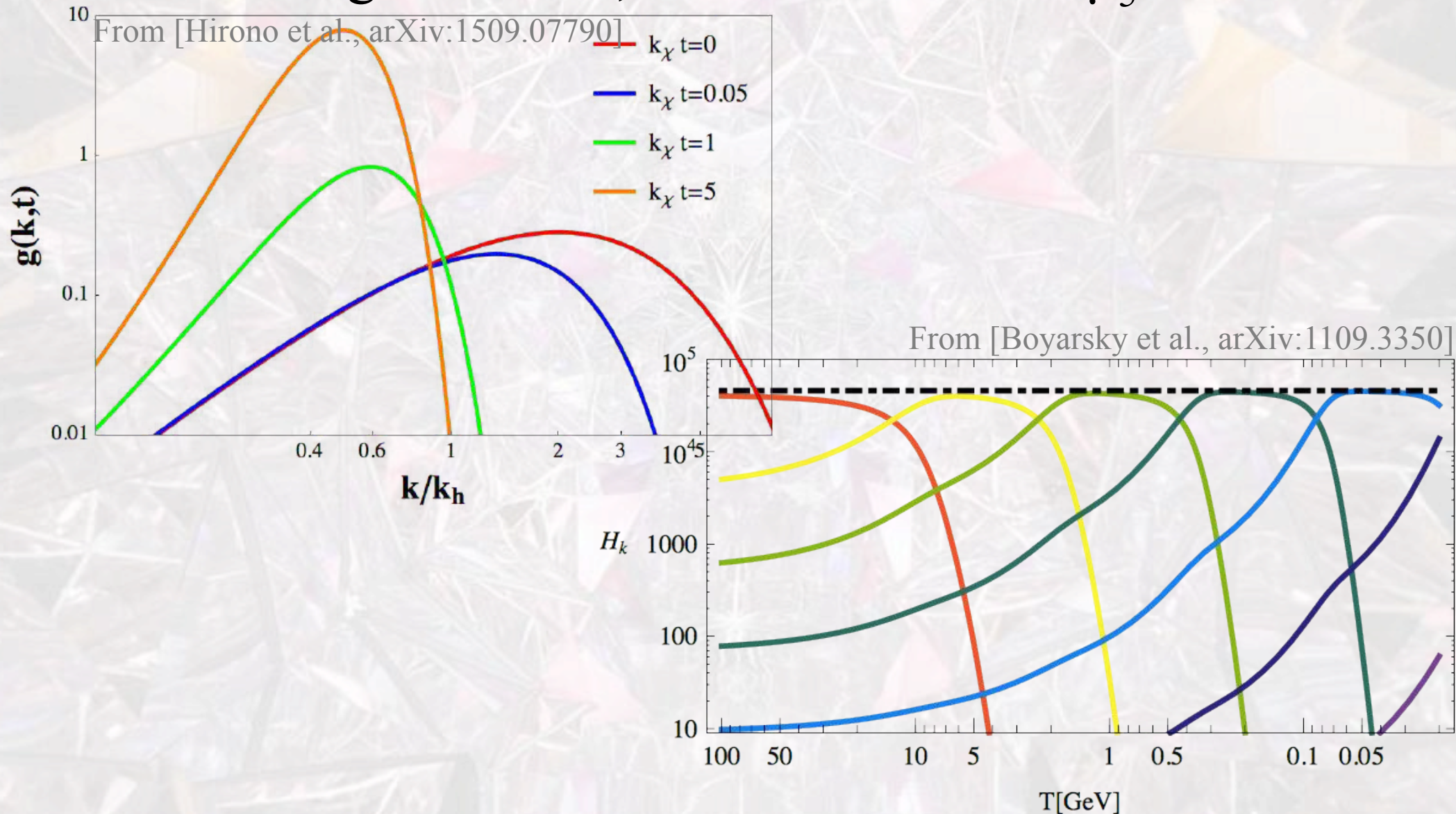
$$n_5(\vec{x}, t) \approx \left\langle n_5(\vec{\mathbf{X}}, t) \right\rangle_{\text{space}}, \quad \mu_5(t) \approx \frac{3c^3}{T^2} \left\langle n_5(\vec{\mathbf{X}}, t) \right\rangle_{\text{space}}$$

- In other words, the value of μ_5 remains constant on distance scales

$$\Delta x \sim (k_{\text{crit}})^{-1} \sim (\mu_5)^{-1}$$

Magnetic field/helicity

- Magnetic helicity is transferred from short to long-wavelengths modes, while the value of μ_5 decreases



Open questions

- Will the cascade survive if there are variations of order $\delta\mu_5$ on distance scales $(k_{\text{crit}})^{-1}$?
- How large $\delta\mu_5$ can be tolerated?
- Will dynamical fluctuations of μ_5 stay under control?
- How to account for inhomogeneities in the anomalous Maxwell equations?

$$n_\lambda(\vec{x}, t) = ? \quad \vec{j}_\lambda(\vec{x}, t) = ?$$

- How to obtain equations for $\mu(t, \mathbf{x})$ and $\mu_5(t, \mathbf{x})$?

- Chiral kinetic theory as a starting point:

$$\frac{\partial f_\lambda}{\partial t} + \frac{1}{1 + \vec{\Omega}_\lambda \cdot \vec{B}} \left[(\vec{E} + \vec{v} \times \vec{B} + (\vec{E} \cdot \vec{B}) \vec{\Omega}_\lambda) \cdot \frac{\partial f_\lambda}{\partial \vec{p}} + (\vec{v} + \vec{E} \times \vec{\Omega}_\lambda + (\vec{v} \cdot \vec{\Omega}_\lambda) \vec{B}) \cdot \frac{\partial f_\lambda}{\partial \vec{x}} \right] = I_{\text{coll}}$$

where

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \dots$$

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{c p - \mu_\lambda}{T}\right) + 1}$$

is an expansion in powers of e-m field & $\vec{\nabla} \mu_\lambda, \partial_t \mu_\lambda$

[Boyarsky, Gorbar, Ruchayskiy, Rudenok, Shovkovy, Vilchinskii, to appear]

ASU Equations for chemical potentials

- Resulting equation of motion for μ_λ :

$$\frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \left(\frac{\partial \mu_\lambda}{\partial t} + \frac{e\tau c^2}{3} \vec{\nabla} \cdot \vec{E}_\lambda \right) + \frac{e\tau \mu_\lambda}{3\pi^2 c} \left(\vec{E}_\lambda \cdot \frac{\partial \mu_\lambda}{\partial \vec{x}} \right) = \frac{\lambda e^2}{4\pi^2 c} (\vec{E}_\lambda \cdot \vec{B})$$

where $n_\lambda^{(0)} = \frac{\mu_\lambda^3 + \pi^2 T^2 \mu_\lambda}{3\pi^2 c^3}$ and $\vec{E}_\lambda = \vec{E} - \frac{1}{e} \frac{\partial \mu_\lambda}{\partial \vec{x}}$

The corresponding equations for the currents:

CME

drift & diffusion

Hall type

$$\vec{j} = \underbrace{\frac{e\mu_5 \vec{B}}{2\pi^2 c}}_{\text{CME}} + \underbrace{\frac{e\tau T^2}{9c} \left(1 + \frac{3(\mu^2 + \mu_5^2)}{\pi^2 T^2} \right) \left(e\vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right)}_{\text{drift \& diffusion}} + \underbrace{\frac{e\tau^2 \mu}{3\pi^2} \left(e\vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right) \times \vec{B}}_{\text{Hall type}} + \vec{j}_{\text{new}}$$

$$\vec{j}_5 = \underbrace{\frac{e\mu \vec{B}}{2\pi^2 c}}_{\text{CSE}} - \underbrace{\frac{e\tau T^2}{9c} \left(1 + \frac{3(\mu^2 + \mu_5^2)}{\pi^2 T^2} \right) \frac{\partial \mu_5}{\partial \vec{x}}}_{\text{diffusion}} + \underbrace{\frac{2e\tau \mu_5 \mu}{3\pi^2 c} \left(e\vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right)}_{\text{CESE}} + \vec{j}_{5,\text{new}}$$

CSE

diffusion

CESE

New types of currents

- New contribution to the electric current:

$$\vec{j}_{\text{new}} = \underbrace{-\frac{2\tau\mu\mu_5}{3\pi^2c} \frac{\partial\mu_5}{\partial\vec{x}}}_{\text{Chiral diffusion}} - \underbrace{\frac{e\tau^2\mu_5}{3\pi^2} \left(\frac{\partial\mu_5}{\partial\vec{x}} \times \vec{B} \right)}_{\text{Hall diffusion}} - \frac{e\tau^2(3\mu^2 + 3\mu_5^2 + \pi^2T^2)}{9\pi^2c} \frac{\partial\vec{E}}{\partial t}$$

- New contribution to the chiral current:

$$\vec{j}_{5,\text{new}} = \underbrace{-\frac{e\tau^2\mu}{3\pi^2} \left(\frac{\partial\mu_5}{\partial\vec{x}} \times \vec{B} \right)}_{\text{Chiral Hall diffusion}} + \underbrace{\frac{e\tau^2\mu_5}{3\pi^2} \left(e\vec{E} - \frac{\partial\mu}{\partial\vec{x}} \right) \times \vec{B}}_{\text{Chiral Hall effect}} - \frac{2e\tau^2\mu\mu_5}{3\pi^2c} \frac{\partial\vec{E}}{\partial t}$$

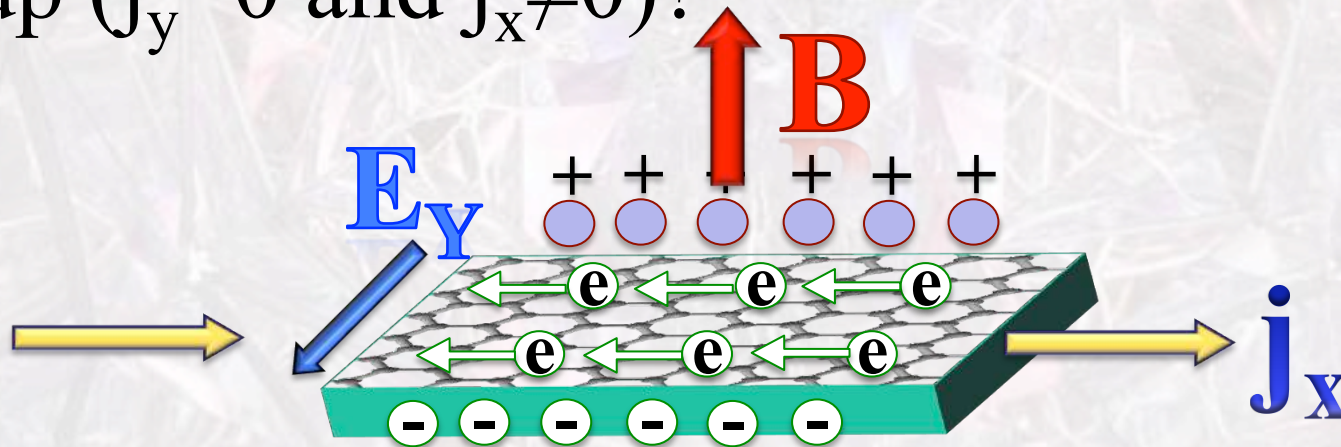
- There is also a term $\propto \frac{e\tau}{6\pi^2} \left(\vec{E} \times \frac{\partial\mu}{\partial\vec{x}} \right)$

Question about Hall current

- Note that we had

$$\vec{j}^{(\text{Hall})} = \frac{e^2 \tau^2 \mu}{3\pi^2} \vec{E} \times \vec{B}$$

- Why is this proportional to τ^2 ?
- What do you observe in the usual experimental setup ($j_y=0$ and $j_x \neq 0$)?



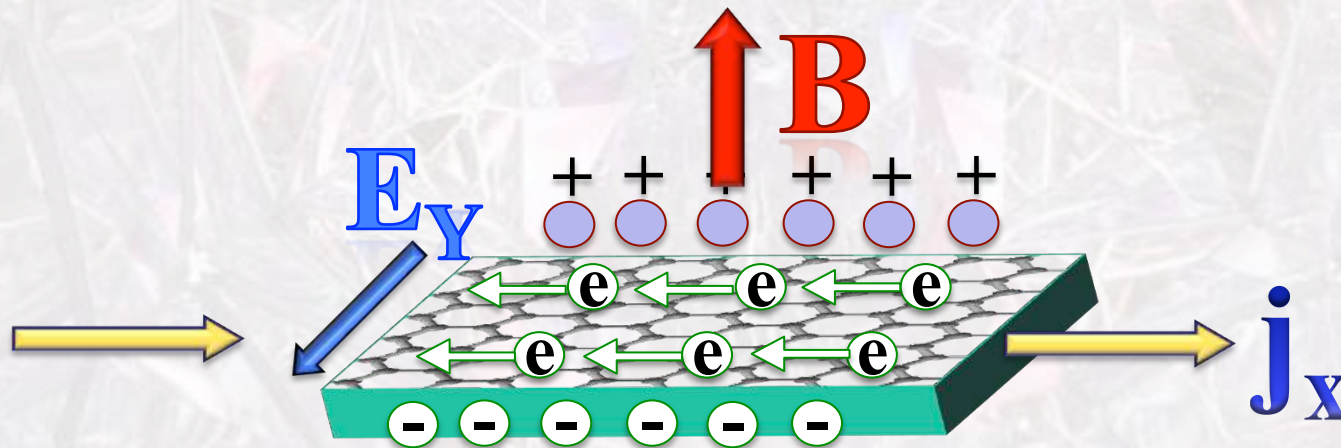
Question about Hall current

- Enforcing $j_y=0$ gives

$$a\tau E_y = b\tau^2 E_x B_z$$

Then, in the approximation used,

$$j_x = a\tau E_x + b\tau^2 E_y B_z = \frac{(a\tau)^2}{b\tau^2 B_z} E_y + b\tau^2 E_y B_z \approx \frac{a^2}{b B_z} E_y$$

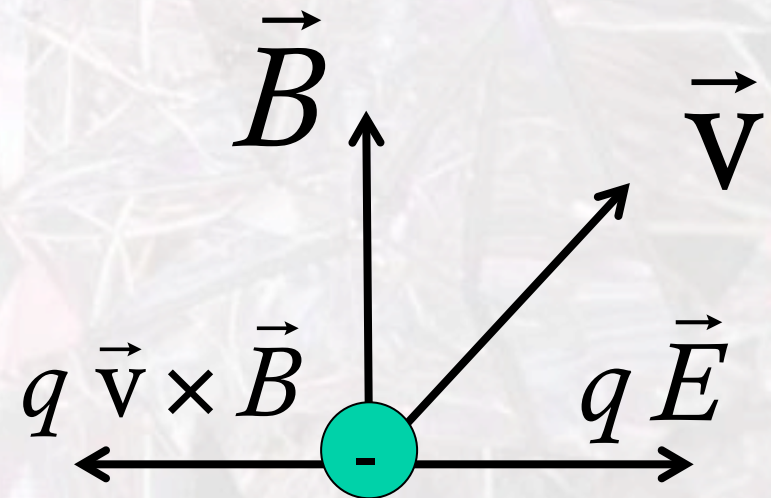
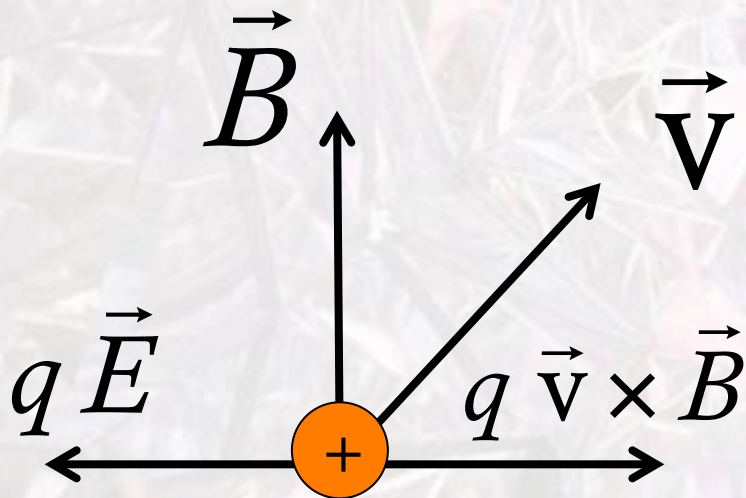
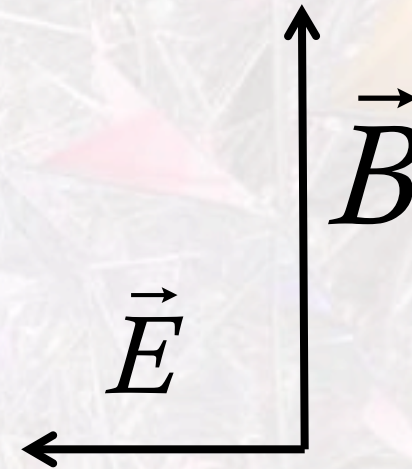


Plasma drift

Plasma as a whole experiences a non-dissipative drift. Can this be included in $f_\lambda^{(0)}$?

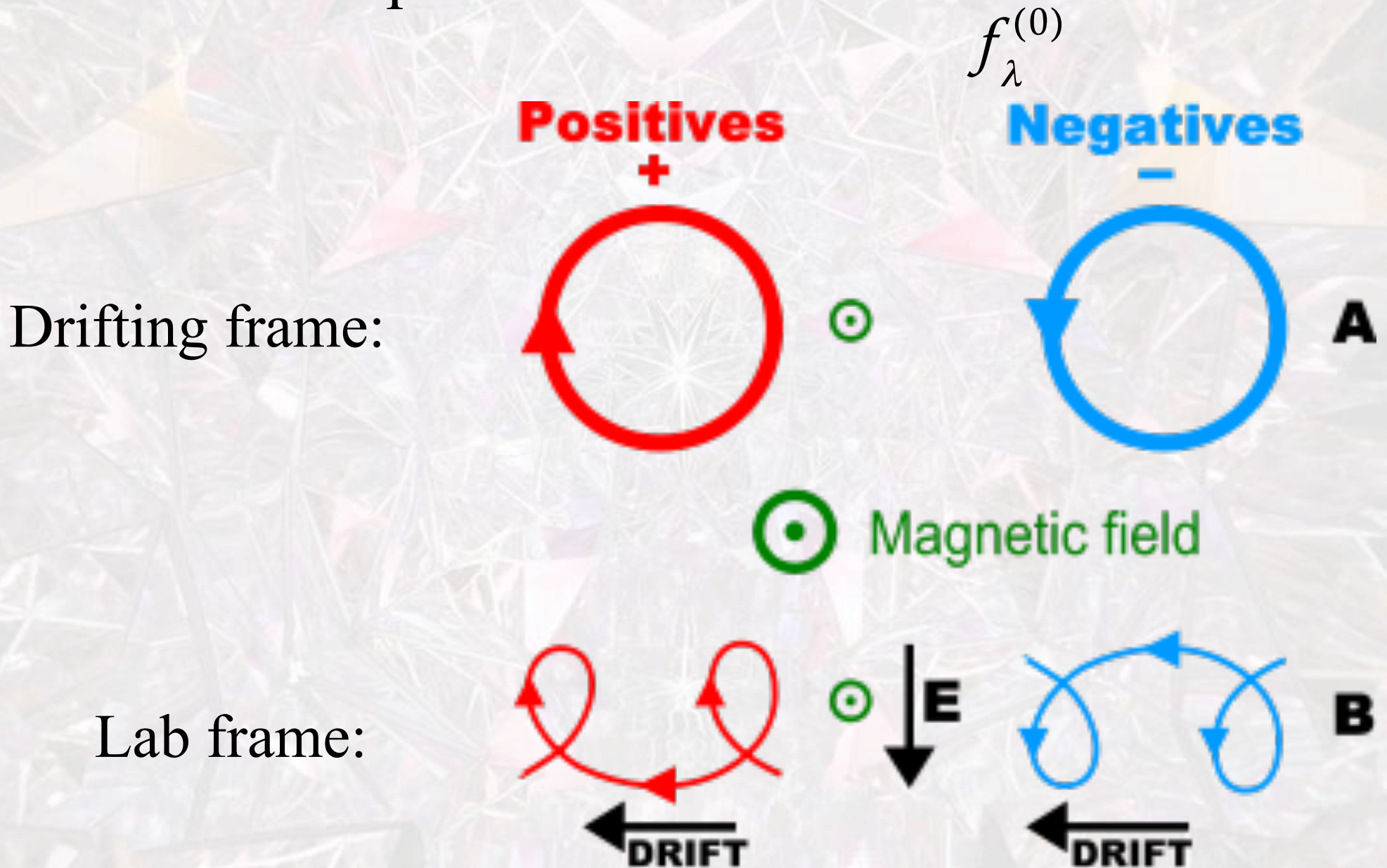
Why should plasma drift?

Consider $\vec{E} \perp \vec{B}$ (with $E < B$):



Frames of reference

Another viewpoint



- Consider a special case
 - Plasma consists of only e-m charged degrees of freedom
 - Fields so that $\vec{E} \perp \vec{B}$ (with $E < B$)
- No electric field in the drift frame
- Lab frame: use boosted Fermi-Dirac distribution

$$f_{\lambda}^{(\text{lab})} = \frac{1}{\exp\left(\frac{c p - \vec{p} \cdot \vec{v}_{\text{drift}} - \mu_{\lambda}}{T}\right) + 1}$$

with

$$\vec{v}_{\text{drift}} = c \frac{\vec{E} \times \vec{B}}{B^2}$$

- Charge density

$$n_{\lambda}^{(\text{lab})} = n_{\lambda}^{(0)} - \tau \frac{\partial n_{\lambda}^{(0)}}{\partial \mu_{\lambda}} \left(\frac{\partial \mu_{\lambda}}{\partial t} + \vec{v}_{\text{drift}} \cdot \nabla \mu_{\lambda} \right) + \tau n_{\lambda}^{(0)} \left(\nabla \cdot \vec{v}_{\text{drift}} \right) + \dots$$

- Current density

$$\vec{j}_{\lambda}^{(\text{lab})} = c n_{\lambda}^{(0)} \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\lambda e \mu_{\lambda}}{4\pi^2 c} \vec{B} + \frac{\lambda e \mu_{\lambda}}{4\pi^2 c} \vec{E}_{\perp} \frac{(\vec{E} \cdot \vec{B})}{E_{\perp}^2} \left(\frac{B}{2E_{\perp}} \ln \frac{B + E_{\perp}}{B - E_{\perp}} - 1 \right) - \tau \frac{\partial n_{\lambda}^{(0)}}{\partial \mu_{\lambda}} \left(g_1 \left(\frac{e(\vec{E} \cdot \vec{B})}{B^2} \vec{B} - \nabla \mu_{\lambda} \right) + g_2 \vec{v}_{\text{drift}} \left(\vec{v}_{\text{drift}} \cdot \nabla \mu_{\lambda} \right) + g_3 \frac{\partial \mu_{\lambda}}{\partial t} \vec{v}_{\text{drift}} \right) + \dots$$

Drift in QGP plasma?

- In QGP, gluons play a profound role
 - Gluons are neutral and, thus, are not drifting
 - The zeroth approximation is the usual Fermi-Dirac distribution

$$f_{\lambda}^{(0)} = \frac{1}{\exp\left(\frac{c p - \mu_{\lambda}}{T}\right) + 1}$$

- Expansion in small e-m fields and gradients is well define

$$f_{\lambda} = f_{\lambda}^{(0)} + f_{\lambda}^{(1)} + f_{\lambda}^{(2)} + \dots$$

- Expansion of the 1st type (no drift) may be better

- Chiral plasmas have widespread applications
 - Heavy-ion collisions
 - Cosmology
 - Dirac/Weyl semimetals
 - Neutron stars
- Anomaly plays a profound role in such plasmas
- Many interesting chiral/anomalous effects are triggered by a magnetic field
- Experimental search for signatures is underway