ARIZONA STATE UNIVERSITY

Magnetic Fields in Hadron Physics
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Anomalous chiral plasmas: finite size and inhomogeneity effects

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Workshop on Magnetic Fields in Hadron Physics

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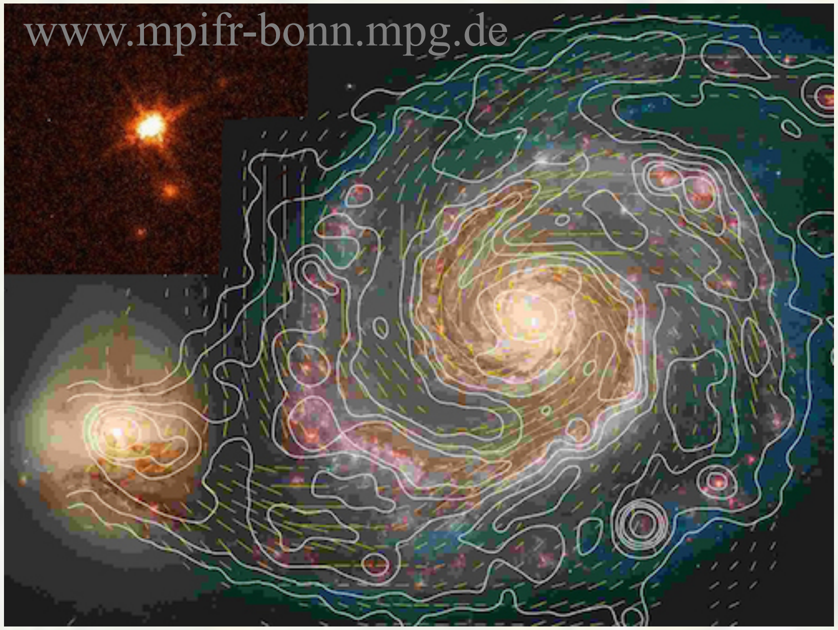
ARIZONA STATE UNIVERSITY

MAGNETIC FIELDS EVERYWHERE

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

Universe



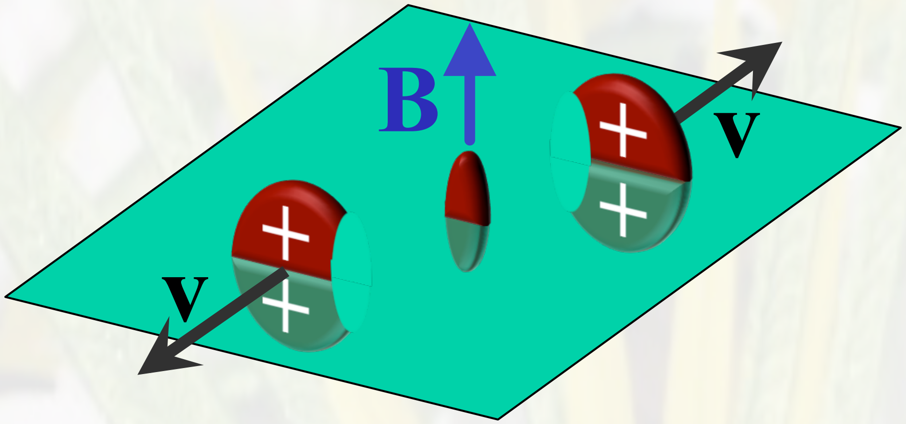
- Current galactic magnetic fields ~ 10^{-6} G
- Current magnetic fields in voids ~ 10^{-15} G
- Problem of magnetogenesis in Early Universe
- Perhaps during the electro-weak phase transition
  - **10^{20} to 10^{24} G (~ 1 GeV to 100 GeV)**

Dense baryonic matter

- Magnetized dense baryonic matter
  - **10^{10} to 10^{18} G (10 keV to 100 MeV)**
- Magnetic field may affect
  - Competition of ground state phases
  - EoS of dense baryonic matter
  - the M-R relation of compact stars
  - Transport and emission properties
  - Evolution of supernovas & protoneutron stars

Little Bangs

- Magnetized QGP at RHIC/LHC
  - **B ~ 10^{18} to 10^{19} G (~ 100 MeV)**



[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108]
…

- Using Lienard-Wiechert potentials,

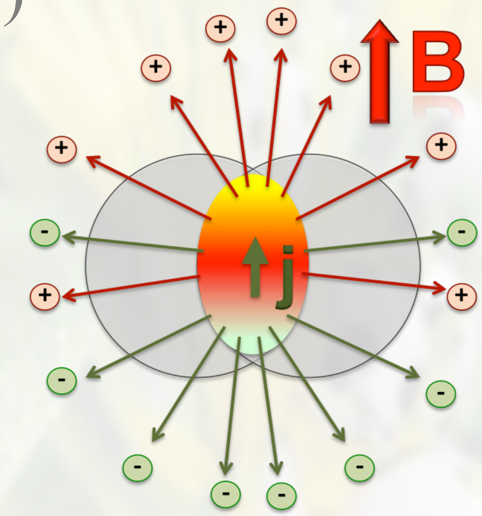
$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n\times\mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

CME, CSE, CMW, etc.

- Chiral magnetic/separation effects, chiral magnetic waves (correlations of charged particle in HIC)

$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5 \quad \& \quad \langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

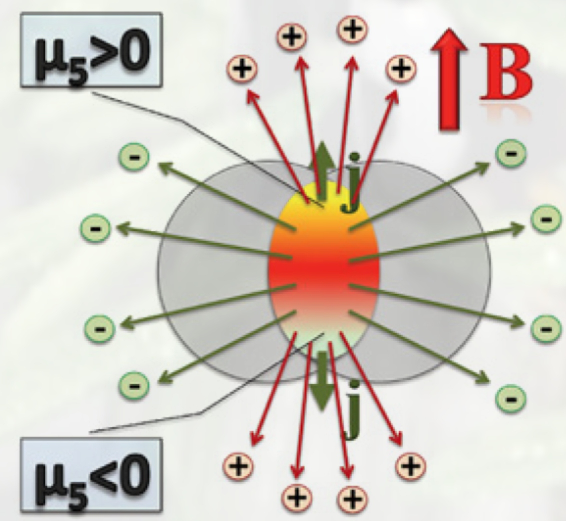


- Signs of local P-violation?

$$\frac{\partial(n_R - n_L)}{\partial t} = -\frac{g^2 N_f}{16\pi^2} F_a^{\mu\nu} \tilde{F}^a_{\mu\nu}$$

[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]

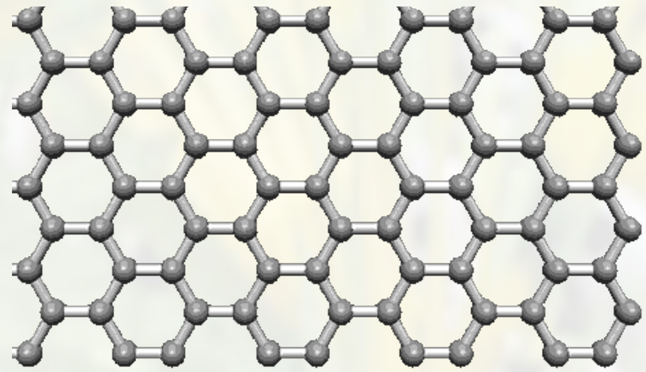
- Signs of a chiral magnetic wave?



[Yee, Kharzeev, Phys. Rev. D **83**, 085007 (2011)]
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Dirac/Weyl materials

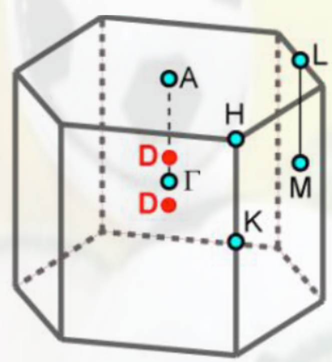
- High magnetic field lab
  - **10^5 G** (~ 100 meV @ vF=c/300)

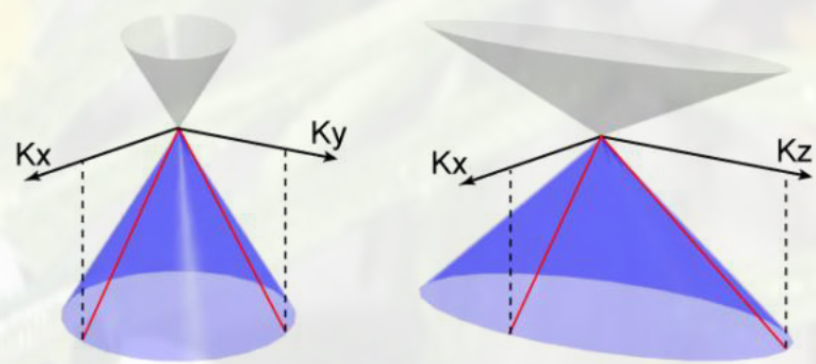


- Graphene
- 3D materials with Dirac/Weyl quasiparticles
  - $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
  - Na_3Bi
  - Cd_3As_2
  - $ZrTe_5$
  - TaAs, NbAs, TaP, …

[Z. K. Liu et al., arXiv:1310.0391]
[M. Neupane et al., arXiv:1309.7892]
[S. Borisenko et al., arXiv:1309.7978]
[X. Li et al., arXiv:1412.6543]

[arXiv:1502.03807, arXiv:1502.04684, arXiv:1504.01350, arXiv:1507.00521]



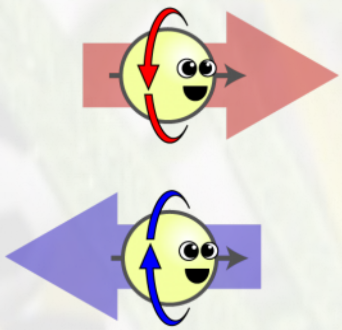


CHIRAL SEPARATION EFFECT

$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

Chirality & Anomaly

- Chirality/helicity of a massless (or ultrarelativistic) particle is (approximately) conserved



Right-handed

Left-handed

$$\frac{\vec{\Sigma}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

- Conservation of chiral charge is a property of massless Dirac theory (classically)

- The symmetry is anomalous at quantum level

$$\frac{\partial(n_R - n_L)}{\partial t} + \nabla\cdot\vec{j}_5 = -\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}F_{\kappa\lambda}$$

Chiral separation effect

- Slowly changing electric/chemical potential

$$\mu(z) = e\Phi(z) \quad \Rightarrow \quad eE_z = -\partial_z\left(e\Phi\right) = -\partial_z\mu$$

- From the anomaly relation,

$$\partial_z\, j_5^3 = -\frac{e^2}{2\pi^2} B_z\, E_z = \frac{e^2}{2\pi^2} B_z\, \partial_z\mu$$

- Suggesting that, for massless fermions,

$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Landau spectrum at B≠0

- Dirac equation with massless fermions

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum

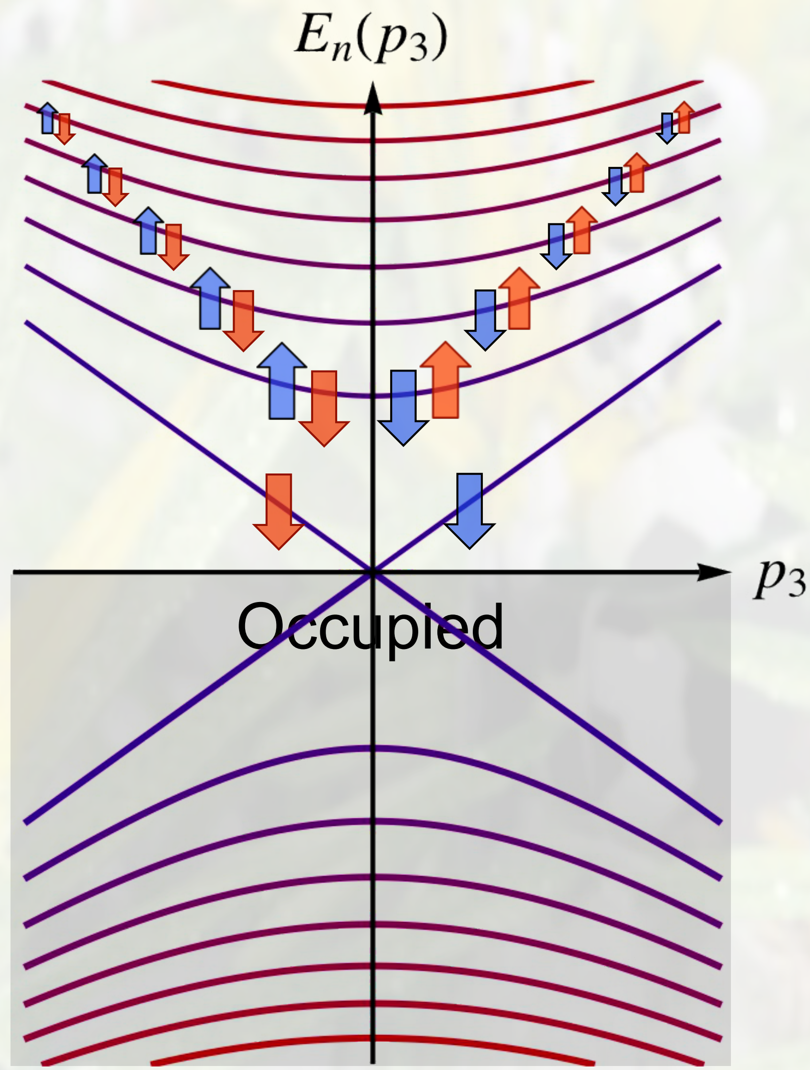
$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

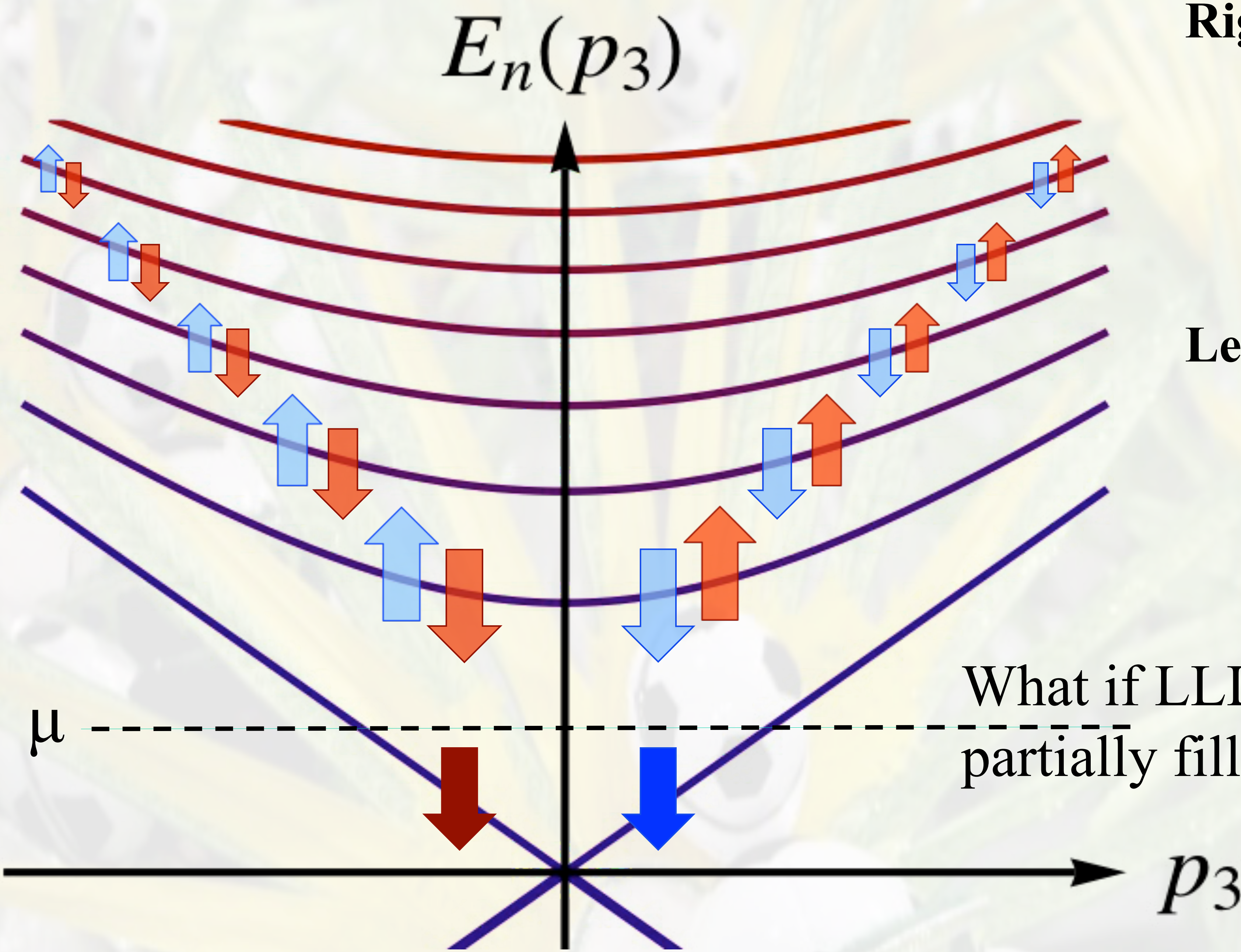
$$n = s + k + \frac{1}{2}$$

$s = \pm \frac{1}{2}$ (spin)

$k = 0, 1, 2, \ldots$ (orbital)

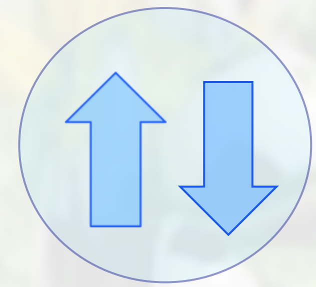


Landau spectrum & $\mu \neq 0$



Right-handed:

Left-handed:



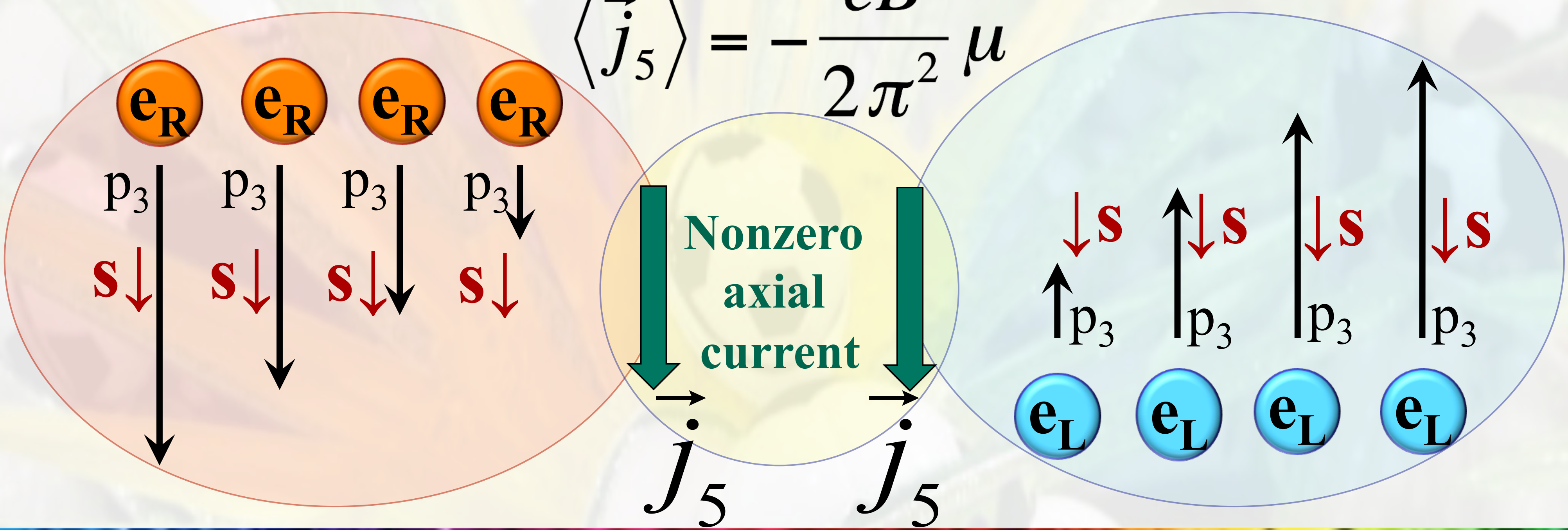
What if LLL is partially filled?

Partially filled LLL

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed

  i.e., a nonzero **axial** current is induced

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$



MOTIVATION

Real systems are

— finite in size

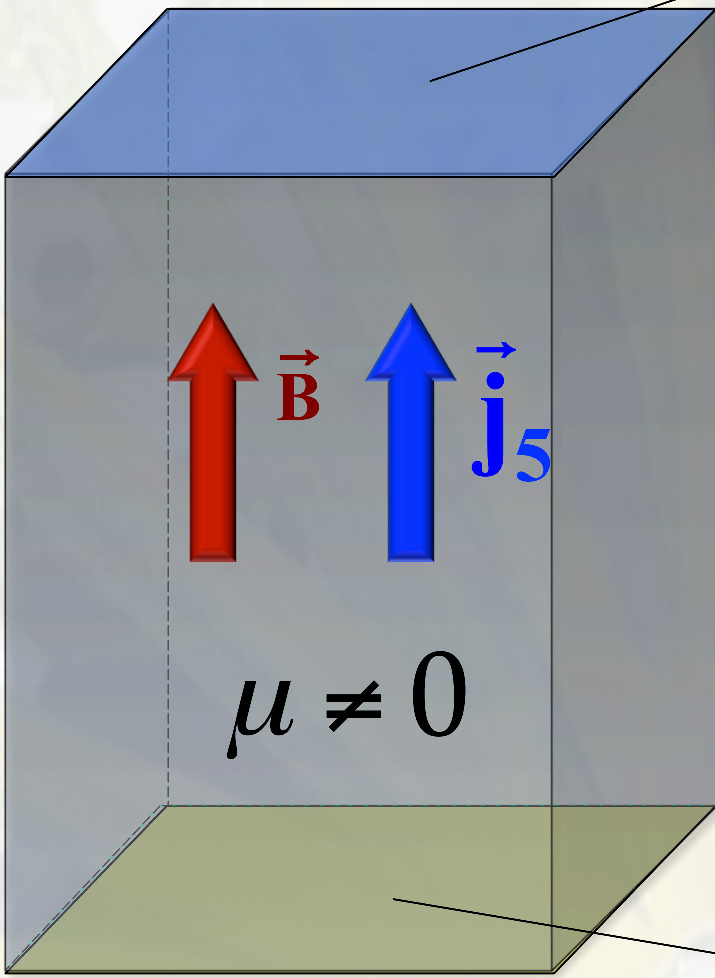
— inhomogeneous

— time-dependent

Any effects of finite size?

- Magnetic field + electric chemical potential = chiral current

Positive chiral charge?



$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

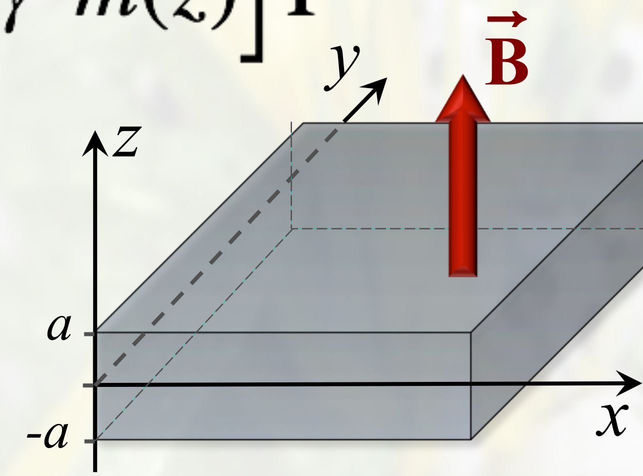
- Is the chiral charge truly separated?

Negative chiral charge?

CSE in finite system

- Model of Dirac semimetal with a slab geometry

$$H = \int d^3 r \Psi^{+} \left[v_F \vec{\alpha} \cdot \left(-i\vec{\nabla} + e\vec{A} \right) + \gamma^0 m(z) \right] \Psi$$



where $\vec{A} = (0, Bx, 0)$ and

$$m(z) = M\theta(z^2 - a^2) + m\theta(a^2 - z^2),$$

with vacuum band gap: $M \to \infty$ (broken chiral symmetry)

- Boundary conditions:

$$\Psi_{\text{bulk}}(\vec{r}_\perp, a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, a) \quad \text{and} \quad \Psi_{\text{bulk}}(\vec{r}_\perp, -a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, -a)$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

Wave functions

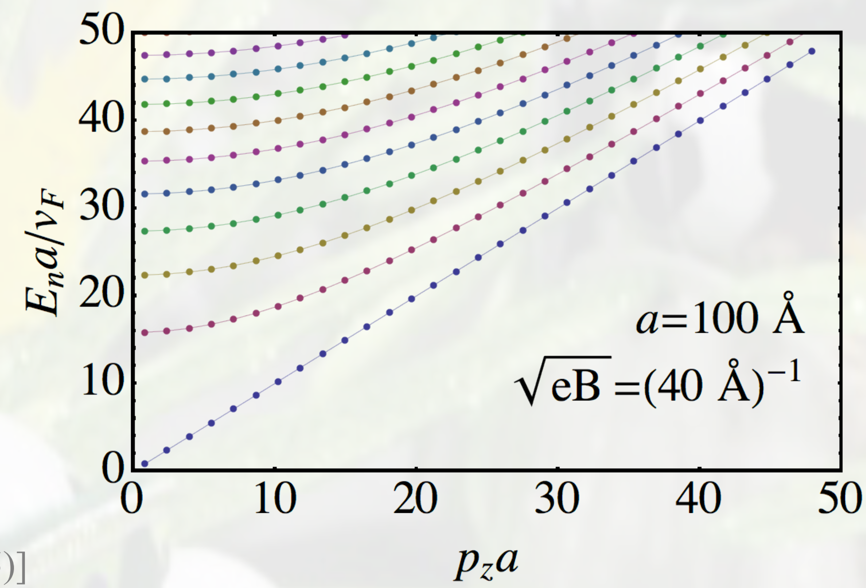
- Wave functions are standing waves, e.g.,

$$\text{LLL}: \quad \Psi_{\text{slab},n=0} = C_0 e^{-\frac{1}{2}(x/l+p_y l)^2} e^{i(p_y y + p_z a)} \begin{pmatrix} 0 \\ \dfrac{v_F p_z \cos\big(p_z(z-a)\big) - (m+iE_0)\sin\big(p_z(z-a)\big)}{im + v_F p_z - E_0} \\ 0 \\ -i\dfrac{v_F p_z \cos\big(p_z(z-a)\big) - (m-iE_0)\sin\big(p_z(z-a)\big)}{im + v_F p_z - E_0} \end{pmatrix}$$

where the wave vector p_z is determined by the spectral equation

$$v_F p_z \cos(2a\, p_z) + m \sin(2a\, p_z) = 0$$

$$\Rightarrow \quad p_{z,k}^{(m)} \approx \frac{\pi(2k-1)}{4a} + \frac{2m}{\pi v_F (2k-1)} + \ldots$$



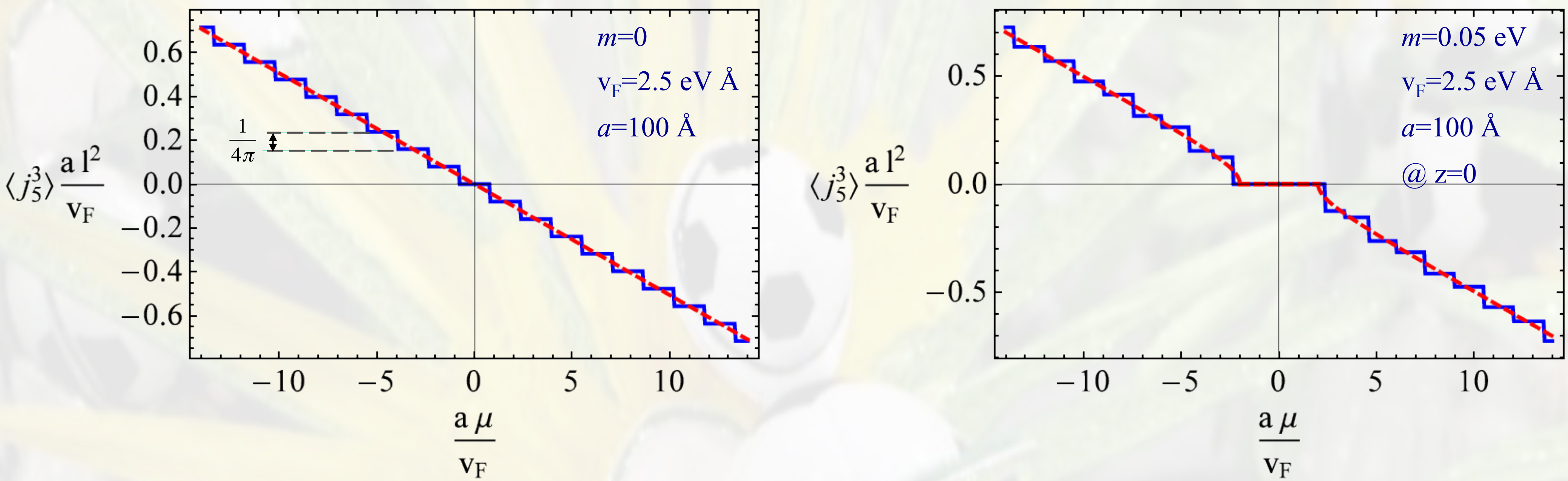
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

Discretized CSE

- Only LLL contributes

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B} v_F \,\mathrm{sign}(\mu)}{2\pi a} \sum_{p_z} \theta(\mu^2 - m^2 - v_z^2 p_z^2) \frac{(m^2 + v_z^2 p_z^2)[1 - \cos(2z\, p_z)\cos(2a\, p_z)]}{2(m^2 + v_z^2 p_z^2) + m v_F / a}$$

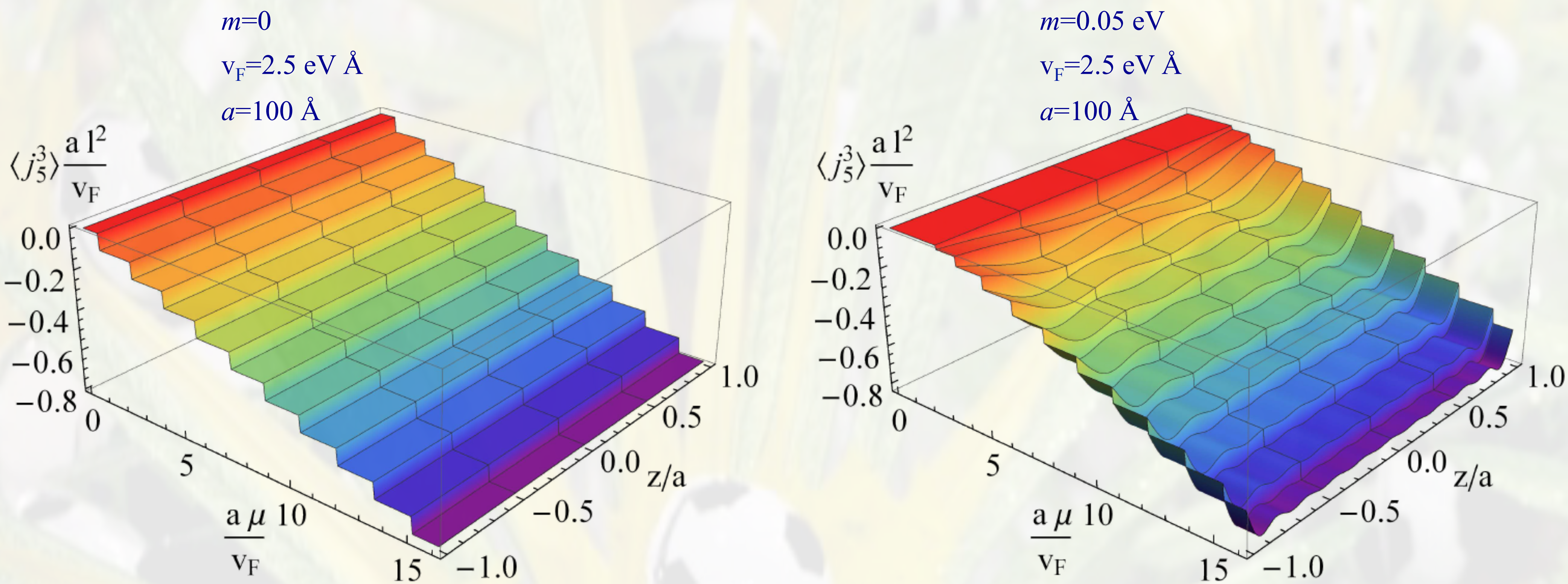
- For $m \to 0$: $\langle \vec{j}_5 \rangle = -\frac{e\vec{B} v_F \,\mathrm{sign}(\mu)}{4\pi a} k_{\max}$, where $k_{\max} = \left[\frac{2a|\mu|}{\pi v_F} + \frac{1}{2} \right]$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

Quantization of axial current

- Axial current density is non-uniform when *m≠0*

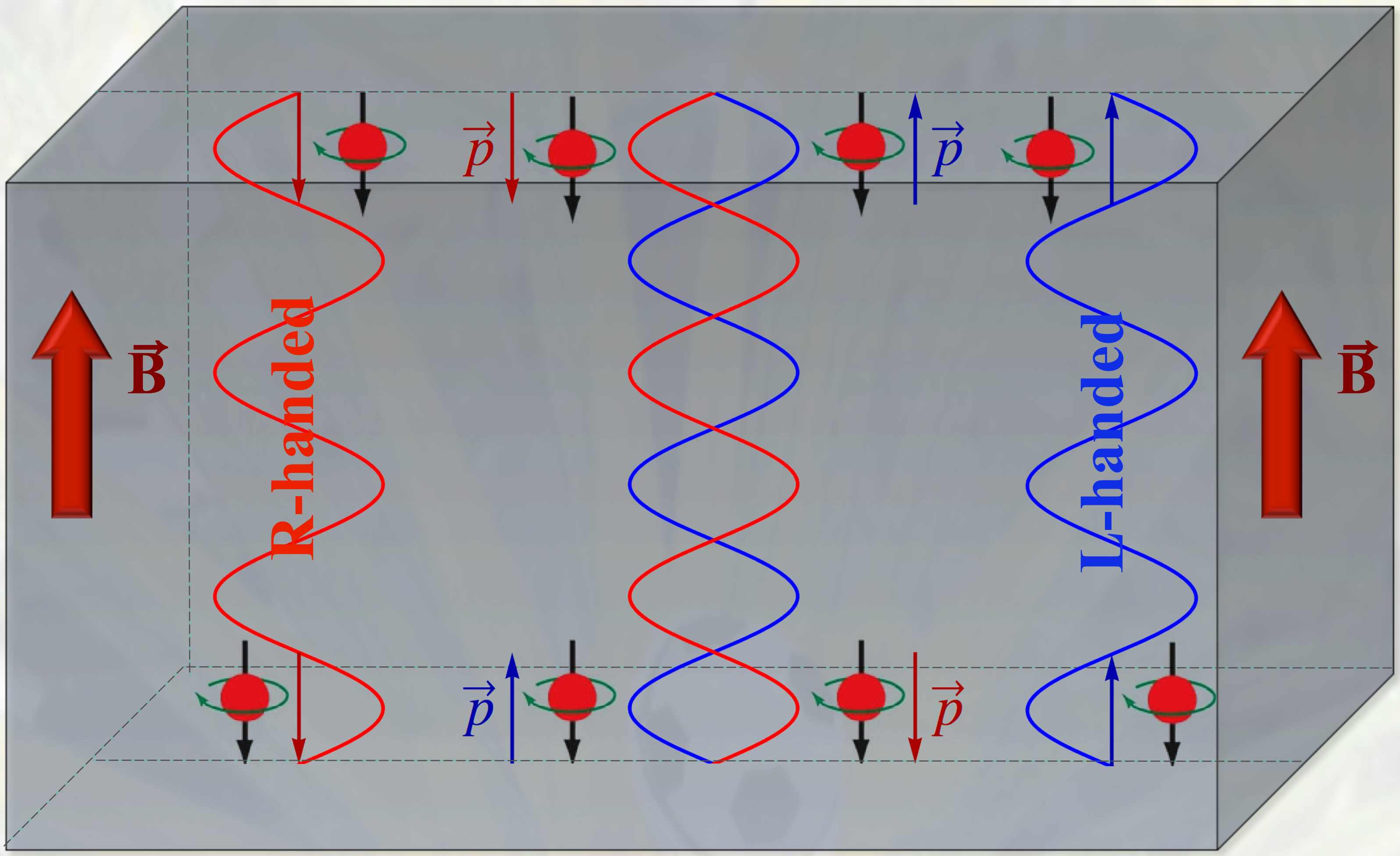


- Note that axial charge density vanishes: $\left\langle j_5^0 \right\rangle = 0$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

Axial current as a standing wave?

- Recall that LLL is spin polarized



- A perfect chirality flip at the boundary

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

Key observations

- Chiral current in the CSE is discretized
- *m≠0:* chiral current density is non-uniform
- *m=0:* chiral current density is uniform
- Chiral current is **not** necessarily connected with a "flow" of chiral charge
- Chiral current need **not** lead to chiral charge accumulation on the boundary

FURTHER DEVELOPMENTS

- Anomalous Maxwell equations for chiral plasmas

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, arXiv:1603.03442]

Magnetic field/helicity

- Magnetic helicity evolution (inverse cascade) in the Early Universe

$$\nabla \times \vec{B} = \frac{4\pi}{c}\left(\sigma \vec{E} + C_5 \mu_5 \vec{B}\right) + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = 4\pi \rho , \qquad \nabla \cdot \vec{B} = 0$$

[Vilenkin, Phys. Rev. D22, 3080 (1980)]
[Joyce & Shaposhnikov, astro-ph/9703005]
[Giovannini & Shaposhnikov, hep-ph/9710234]

- For specific helicity eigenmodes:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma}\left(C_5 \mu_5 - \frac{ck}{4\pi}\right) B_k$$

[Boyarsky et al., arXiv:1109.3350]
[Tashiro et al., arXiv:1206.5549]
[Manuel et al., arXiv:1501.07608]
[Buividovich et al., arXiv:1509.02076]
[Hirono et al., arXiv:1509.07790]

Feedback on μ_5(t)

- Constraint of the total helicity conservation

$$h_{\text{fermion}} = h_0 - h_{\text{gauge}}$$

- Common "homogeneous" approximation:

$$n_5(\vec{x},t) \approx \left\langle n_5(\vec{\mathbf{x}},t)\right\rangle_{\text{space}}, \quad \mu_5(t) \approx \frac{3c^3}{T^2}\left\langle n_5(\vec{\mathbf{x}},t)\right\rangle_{\text{space}}$$

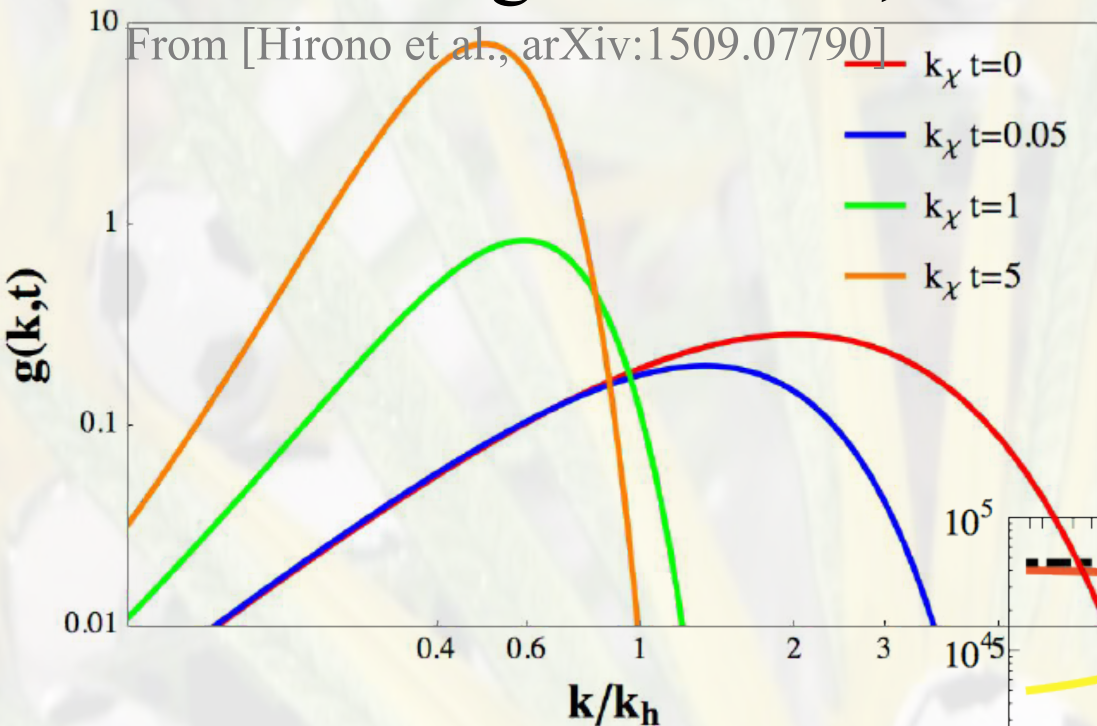
- In other words, the value of μ_5 remains constant on distance scales

$$\Delta x \sim (k_{\text{crit}})^{-1} \sim (\mu_5)^{-1}$$

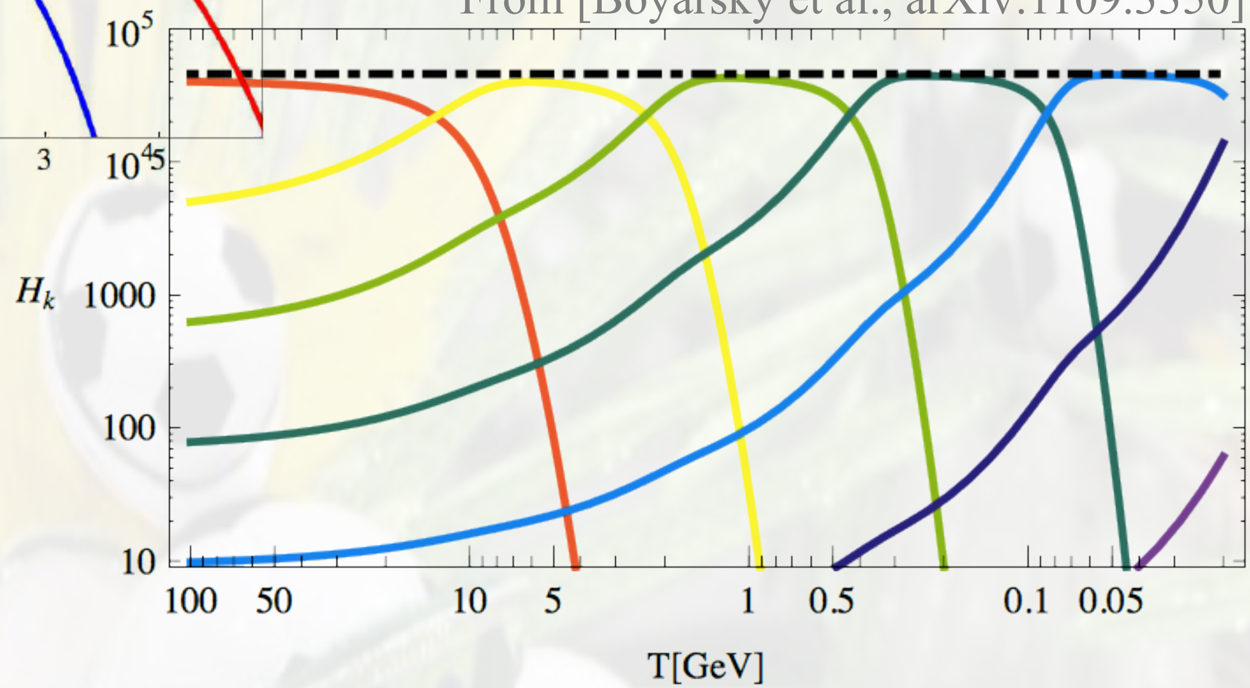
Magnetic field/helicity

- Magnetic helicity is transferred from short to long-wavelengths modes, while the value of μ_5 decreases

From [Hirono et al., arXiv:1509.07790]



From [Boyarsky et al., arXiv:1109.3350]



Open questions

- Will the cascade survive if there are variations of order $\delta\mu_5$ on distance scales $(k_{crit})^{-1}$?
- How large $\delta\mu_5$ can be tolerated?
- Will dynamical fluctuations of μ_5 stay under control?
- How to account for inhomogeneities in the anomalous Maxwell equations?

$$n_\lambda(\vec{x},t) = ? \qquad \vec{j}_\lambda(\vec{x},t) = ?$$

- How to obtain equations for $\mu(t,\mathbf{x})$ and $\mu_5(t,\mathbf{x})$?

Framework

- Chiral kinetic theory as a starting point:

$$\frac{\partial f_\lambda}{\partial t} + \frac{1}{1+\vec{\Omega}_\lambda\cdot\vec{B}}\left[\left(\vec{E}+\vec{v}\times\vec{B}+(\vec{E}\cdot\vec{B})\vec{\Omega}_\lambda\right)\cdot\frac{\partial f_\lambda}{\partial\vec{p}} + \left(\vec{v}+\vec{E}\times\vec{\Omega}_\lambda+(\vec{v}\cdot\vec{\Omega}_\lambda)\vec{B}\right)\cdot\frac{\partial f_\lambda}{\partial\vec{x}}\right] = I_{\text{coll}}$$

where

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{cp-\mu_\lambda}{T}\right)+1}$$

is an expansion in powers of e-m field & $\vec{\nabla}\mu_\lambda, \partial_t\mu_\lambda$

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, arXiv:1603.03442]

Equations for chemical potentials

- Resulting equation of motion for μ_λ:

$$\frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(\frac{\partial \mu_\lambda}{\partial t}+\frac{e\tau c^2}{3}\vec{\nabla}\cdot\vec{E}_\lambda\right)+\frac{e\tau\mu_\lambda}{3\pi^2 c}\left(\vec{E}_\lambda\cdot\frac{\partial \mu_\lambda}{\partial \vec{x}}\right)=\frac{\lambda e^2}{4\pi^2 c}\left(\vec{E}_\lambda\cdot\vec{B}\right)$$

where $n_\lambda^{(0)}=\dfrac{\mu_\lambda^3+\pi^2T^2\mu_\lambda}{3\pi^2c^3}$ and $\vec{E}_\lambda=\vec{E}-\dfrac{1}{e}\dfrac{\partial\mu_\lambda}{\partial\vec{x}}$

The corresponding equations for the currents:

CME — drift & diffusion — Hall type

$$\vec{j}=\underbrace{\frac{e\mu_5\vec{B}}{2\pi^2c}}_{\text{CME}}+\underbrace{\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{drift \& diffusion}}+\underbrace{\frac{e\tau^2\mu}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}_{\text{Hall type}}+\vec{j}_{\text{new}}$$

$$\vec{j}_5=\underbrace{\frac{e\mu\vec{B}}{2\pi^2c}}_{\text{CSE}}-\underbrace{\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{diffusion}}+\underbrace{\frac{2e\tau\mu_5\mu}{3\pi^2c}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{CESE}}+\vec{j}_{5,\text{new}}$$

CSE — diffusion — CESE

New types of currents

- New contribution to the electric current:

$$\vec{j}_{\text{new}} = \underbrace{-\frac{2\tau\mu\mu_5}{3\pi^2 c}\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{Chiral diffusion}} \underbrace{-\frac{e\tau^2\mu_5}{3\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right)}_{\text{Hall diffusion}} - \frac{e\tau^2\left(3\mu^2+3\mu_5^2+\pi^2T^2\right)}{9\pi^2 c}\frac{\partial\vec{E}}{\partial t}$$

- New contribution to the chiral current:

$$\vec{j}_{5,\text{new}} = \overbrace{-\frac{e\tau^2\mu}{3\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right)}^{\text{Chiral Hall diffusion}} \overbrace{+\frac{e\tau^2\mu_5}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}^{\text{Chiral Hall effect}} - \frac{2e\tau^2\mu\mu_5}{3\pi^2 c}\frac{\partial\vec{E}}{\partial t}$$

- There is also a term $\propto \frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu}{\partial\vec{x}}\right)$

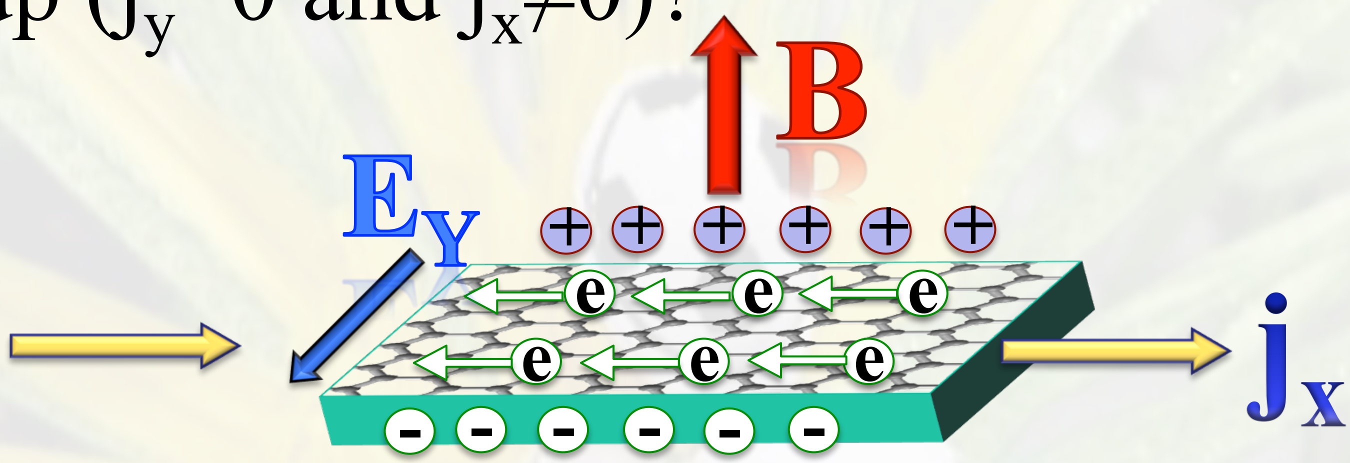
Question about Hall current

- Note that we had

$$\vec{j}^{\,(\mathrm{Hall})} = \frac{e^2 \tau^2 \mu}{3\pi^2} \vec{E} \times \vec{B}$$

- Why is this proportional to τ^2 ?

- What do you observe in the usual experimental setup (j_y=0 and $j_x \neq 0$)?



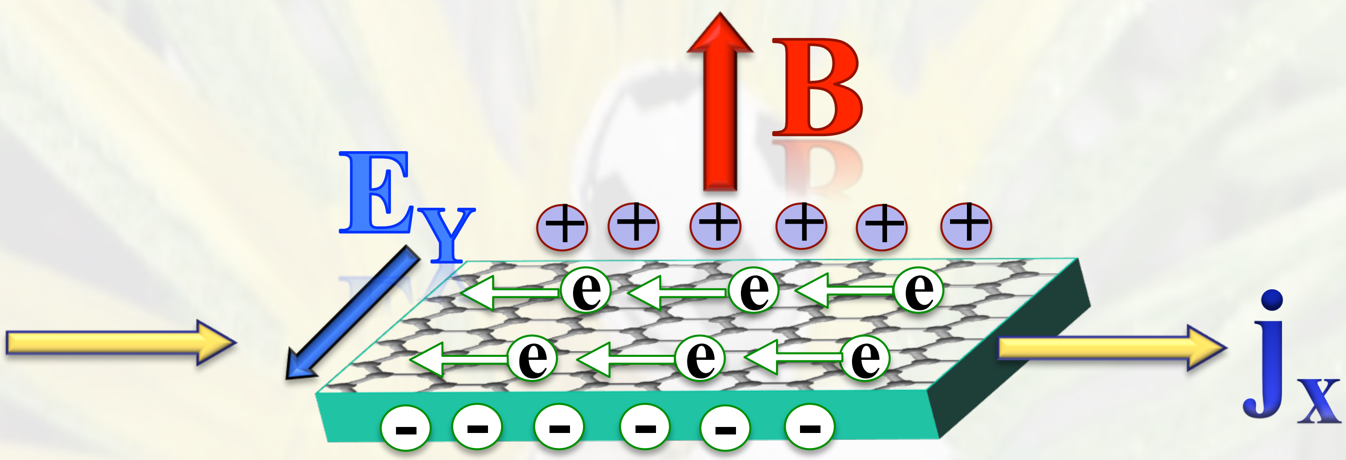
Question about Hall current

- Enforcing $j_y=0$ gives

$$a\tau E_y = b\tau^2 E_x B_z$$

Then, in the approximation used,

$$j_x = a\tau E_x + b\tau^2 E_y B_z = \frac{(a\tau)^2}{b\tau^2 B_z} E_y + b\tau^2 E_y B_z \approx \frac{a^2}{b\,B_z} E_y$$

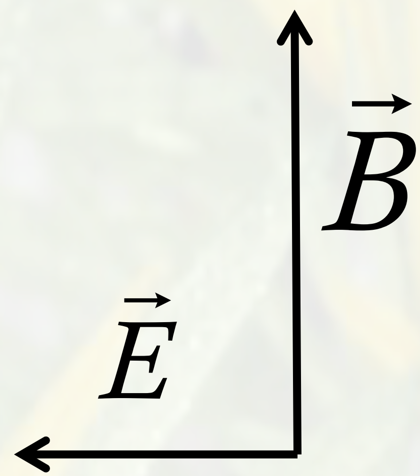


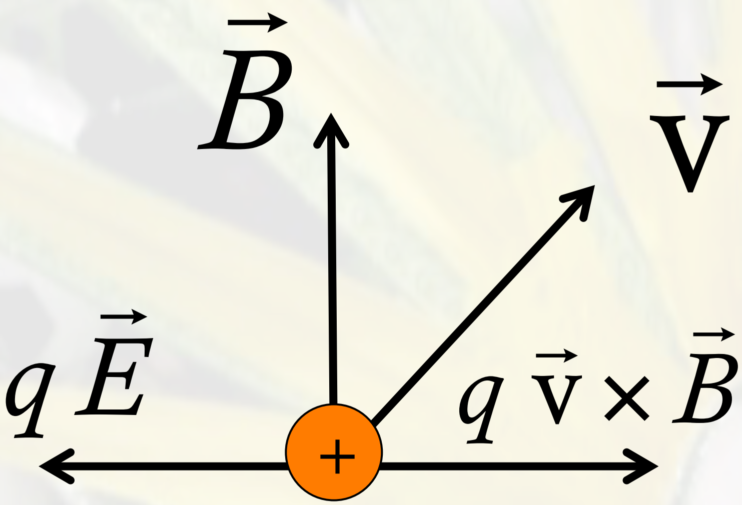
Plasma drift

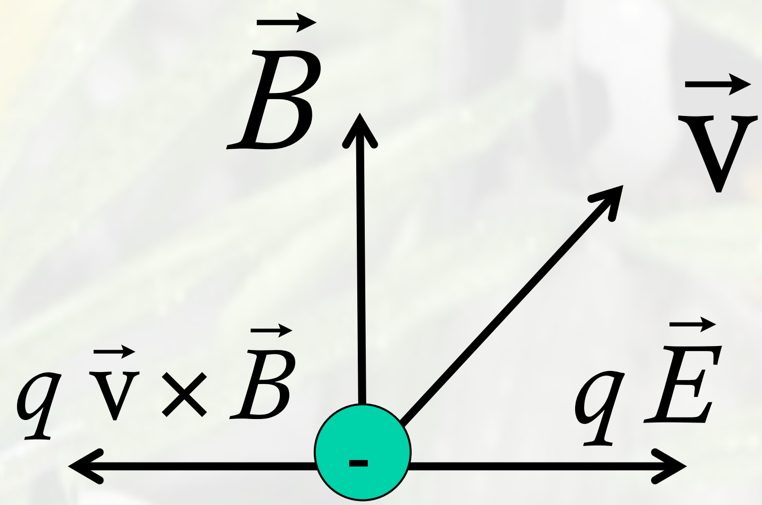
Plasma as a whole experiences a non-dissipative drift. Can this be included in $f_\lambda^{(0)}$?

Why should plasma drift?

Consider $\vec{E} \perp \vec{B}$ (with $E < B$):







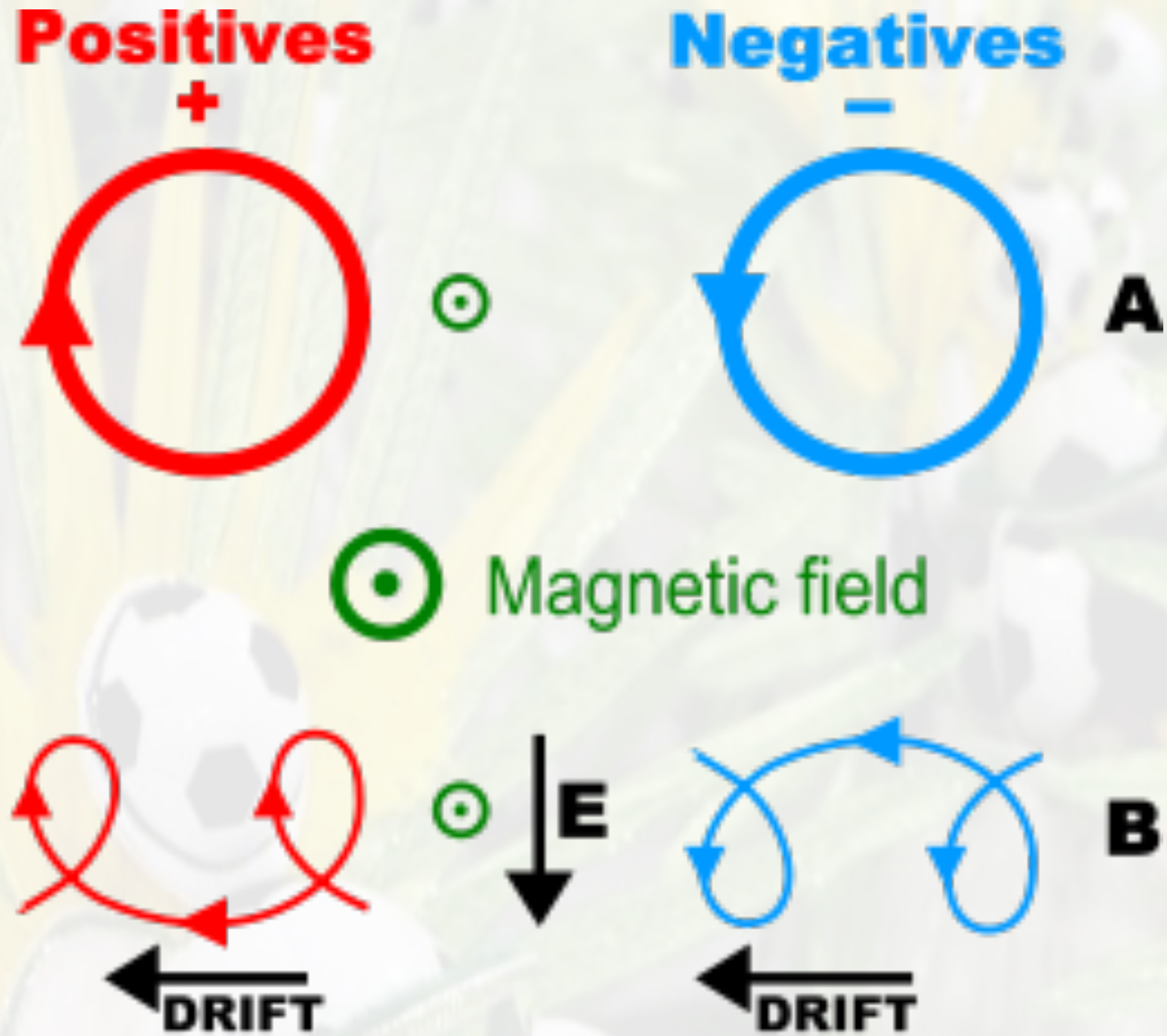
Frames of reference

Another viewpoint

$f_\lambda^{(0)}$

Drifting frame:

Lab frame:



$f_\lambda^{(0)}$ for magnetized plasma

- Consider a special case
  - Plasma consists of only e-m charged degrees of freedom
  - Fields so that $\vec{E} \perp \vec{B}$ (with $E < B$)
- No electric field in the drift frame
- Lab frame: use boosted Fermi-Dirac distribution

$$f_\lambda^{(\text{lab})} = \frac{1}{\exp\left(\dfrac{c\,p - \vec{p}\cdot\vec{\text{v}}_{\text{drift}} - \mu_\lambda}{T}\right) + 1}$$

with

$$\vec{\text{v}}_{\text{drift}} = c\,\frac{\vec{E}\times\vec{B}}{B^2}$$

1^{st} Perturbation in drifting plasma

- Charge density

$$n_\lambda^{(\mathrm{lab})} = n_\lambda^{(0)} - \tau \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \left(\frac{\partial \mu_\lambda}{\partial t} + \vec{v}_{\mathrm{drift}} \cdot \nabla \mu_\lambda \right) + \tau n_\lambda^{(0)} \left(\nabla \cdot \vec{v}_{\mathrm{drift}} \right) + \ldots$$

- Current density

$$\vec{j}_\lambda^{(\mathrm{lab})} = c n_\lambda^{(0)} \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\lambda e \mu_\lambda}{4\pi^2 c} \vec{B} + \frac{\lambda e \mu_\lambda}{4\pi^2 c} \vec{E}_\perp \frac{\left(\vec{E} \cdot \vec{B}\right)}{E_\perp^2} \left(\frac{B}{2E_\perp} ln \frac{B + E_\perp}{B - E_\perp} - 1 \right)$$

$$- \tau \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \left(g_1 \left(\frac{e\left(\vec{E} \cdot \vec{B}\right)}{B^2} \vec{B} - \nabla \mu_\lambda \right) + g_2 \vec{v}_{\mathrm{drift}} \left(\vec{v}_{\mathrm{drift}} \cdot \nabla \mu_\lambda \right) + g_3 \frac{\partial \mu_\lambda}{\partial t} \vec{v}_{\mathrm{drift}} \right) + \ldots$$

Drift in QGP plasma?

- In QGP, gluons play a profound role
  - Gluons are neutral and, thus, are not drifting
  - The zeroth approximation is the usual Fermi-Dirac distribution

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{cp - \mu_\lambda}{T}\right) + 1}$$

- Expansion in small e-m fields and gradients is well define

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

- Expansion of the 1st type (no drift) may be better

Summary

- Chiral plasmas have widespread applications
- Anomaly plays a profound role in such plasmas
- Many interesting chiral/anomalous effects are triggered by a magnetic field
- There are nontrivial effects due to a finite size
- Accounting for inhomogeneities is possible in a systematic way
- Consequences of these effects in physical systems are still to be fully investigated