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# Many faces of chiral magnetic effects


Igor Shovkovy
Arizona State University

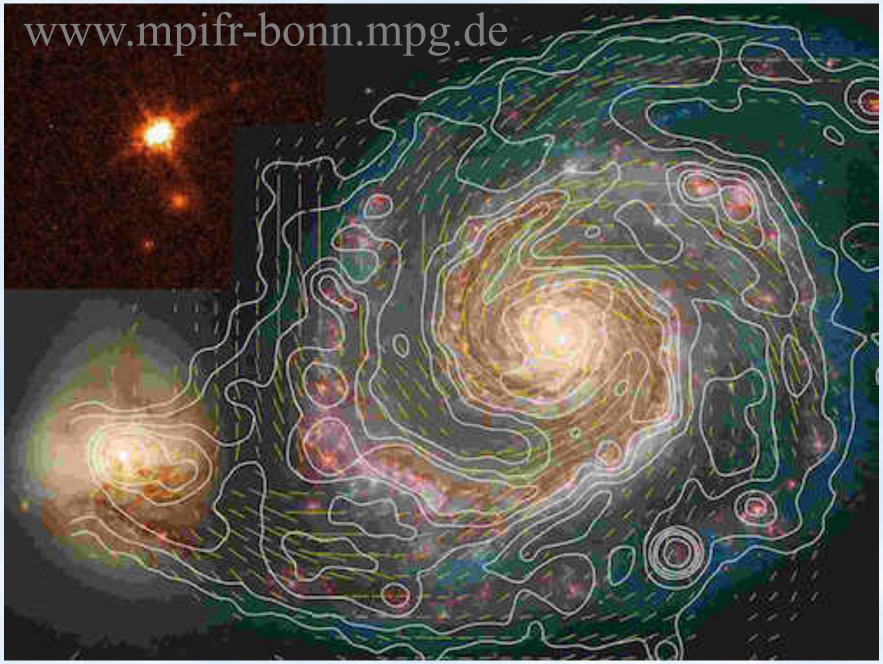
# MAGNETIC FIELDS EVERYWHERE

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

# Universe

- Present day galactic magnetic fields ~ $10^{-6}$ G

- Magnetic fields in voids ~ $10^{-15}$ G




- Problem of magnetogenesis in Early Universe

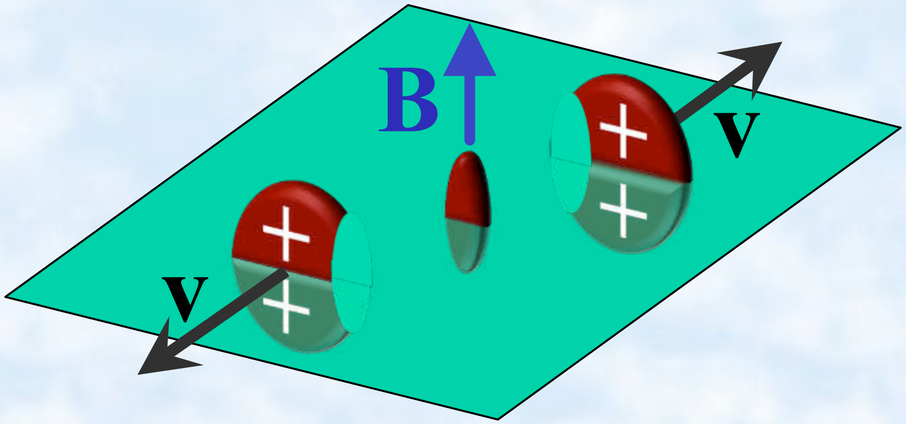
- Perhaps during the electro-weak phase transition
  - **$10^{20}$ to $10^{24}$ G** (~ 1 GeV to 100 GeV)

# Dense baryonic matter

- Magnetized dense baryonic matter
  - **$10^{10}$ to $10^{18}$ G (10 keV to 100 MeV)**
- Magnetic field may affect
  - Competition of ground state phases
  - EoS of dense baryonic matter
  - the M-R relation of compact stars
  - Transport and emission properties
  - Evolution of supernovas & protoneutron stars

# Little Bangs

- Magnetized QGP at RHIC/LHC
  - **B ~ $10^{18}$ to $10^{19}$ G (~ 100 MeV)**




[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108]
…

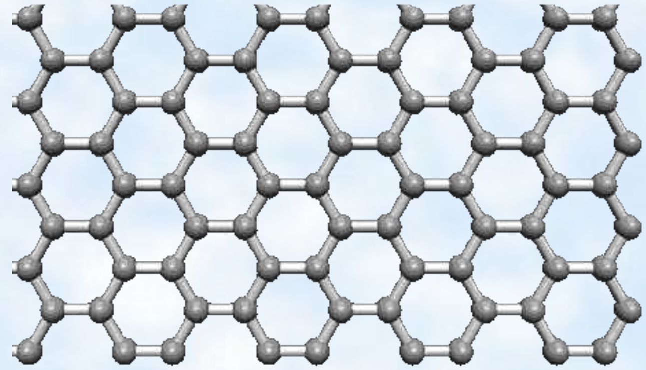
- Using Lienard-Wiechert potentials,

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

# Dirac/Weyl materials

- High magnetic field lab
  - **$10^5$ G** (~ 100 meV @ vF=c/300)



- Graphene
- 3D materials with Dirac/Weyl quasiparticles
  - $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
  - $Na_3Bi$
  - $Cd_3As_2$
  - $ZrTe_5$
  - TaAs, NbAs, TaP, …

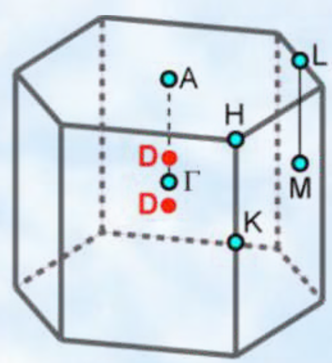
[Z. K. Liu et al., arXiv:1310.0391]

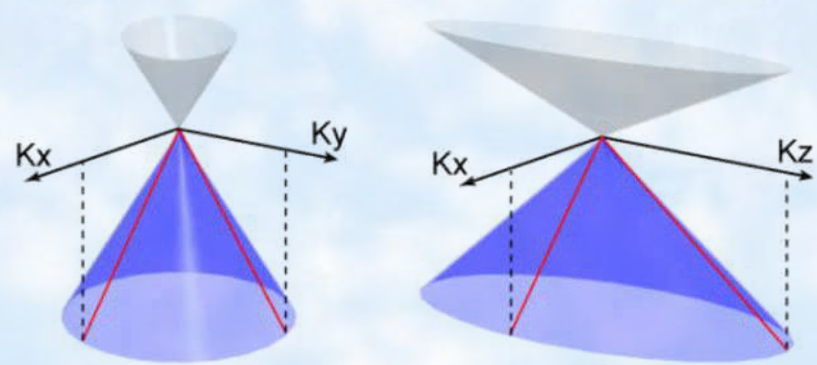
[M. Neupane et al., arXiv:1309.7892]

[S. Borisenko et al., arXiv:1309.7978]

[X. Li et al., arXiv:1412.6543]

[arXiv:1502.03807, arXiv:1502.04684, arXiv:1504.01350, arXiv:1507.00521]



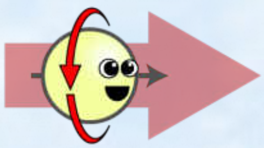



# CHIRAL SEPARATION EFFECT

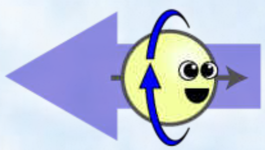
$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

# Chirality & Anomaly

- Chirality/helicity of a massless (or ultrarelativistic) particle is (approximately) conserved



**Right-handed**



**Left-handed**

$$\frac{\vec{\Sigma}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

- Chiral charge conservation is a property of massless Dirac theory (classically)

- The symmetry is anomalous at quantum level

$$\frac{\partial(n_R - n_L)}{\partial t} + \nabla\cdot\vec{j}_5 = -\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}F_{\kappa\lambda}$$

# Chiral separation effect

- Slowly changing electric/chemical potential

$$\mu(z) = e\Phi(z) \quad \Rightarrow \quad eE_z = -\partial_z\left(e\Phi\right) = -\partial_z\mu$$

- From the (steady state) anomaly relation,

$$\partial_z j_5^3 = -\frac{e^2}{2\pi^2} B_z E_z = \frac{e^2}{2\pi^2} B_z \partial_z \mu$$

- Suggesting that, for massless fermions,

$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- How is this possible?

# Landau spectrum at B≠0

- Dirac equation with massless fermions

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum

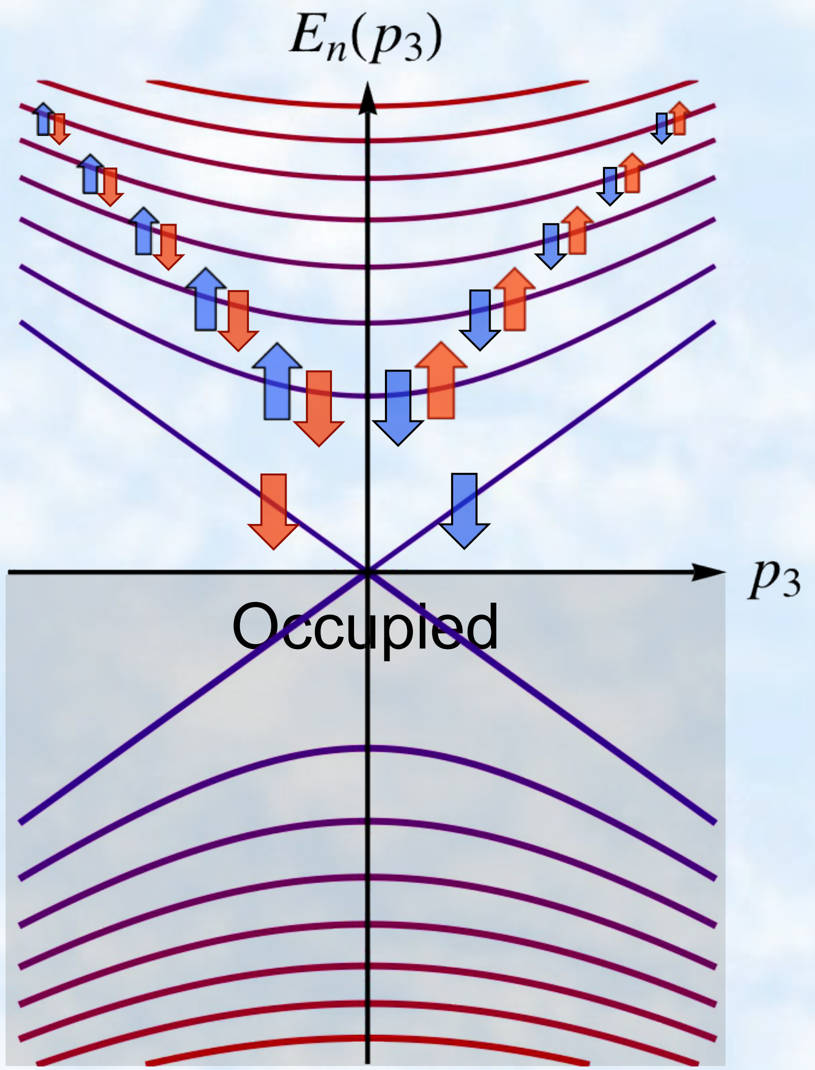
$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

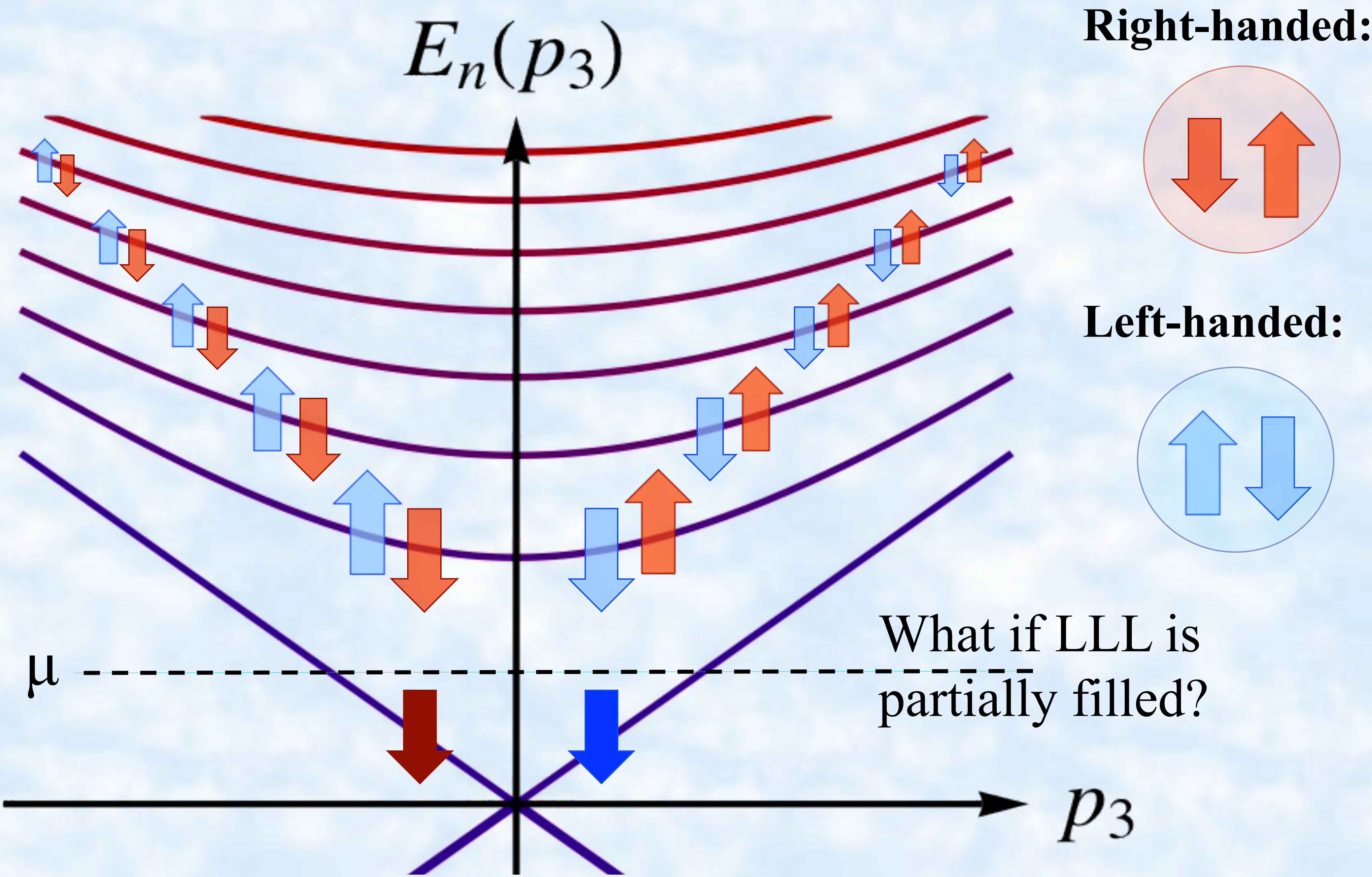
$$n = s + k + \frac{1}{2}$$

$s = \pm 1/2$ (spin)

$k = 0, 1, 2, \ldots$ (orbital)



# Landau spectrum & $\mu \neq 0$



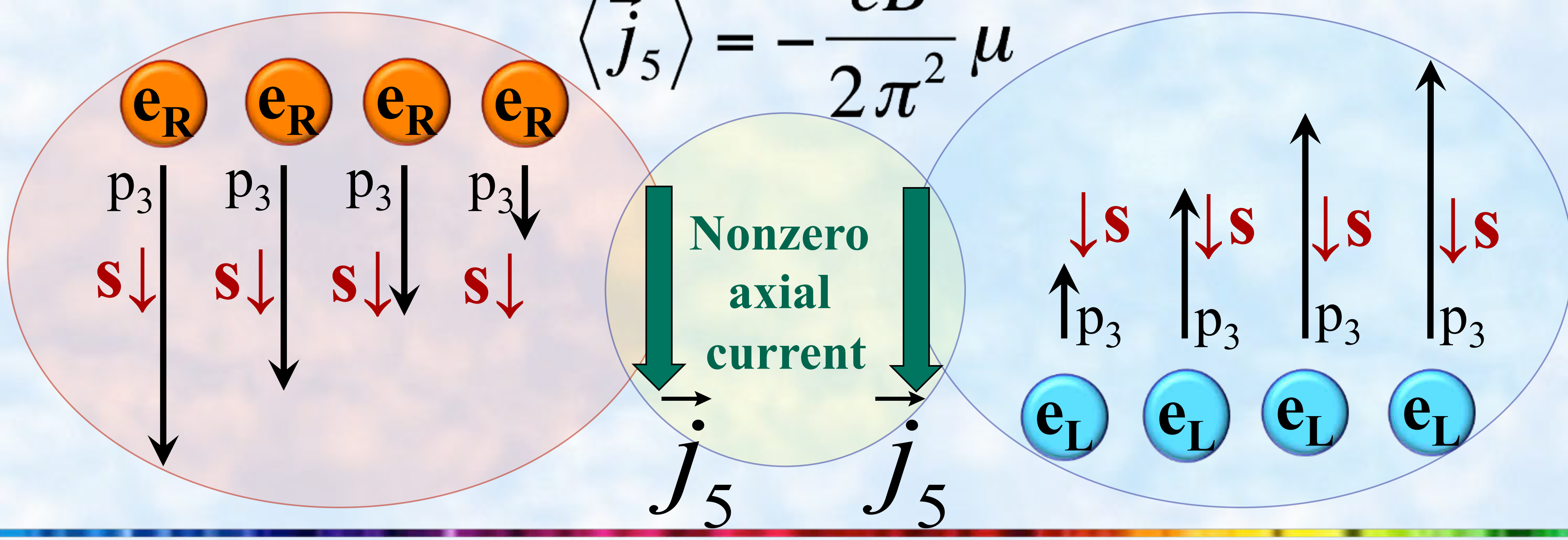

What if LLL is partially filled?

# Partially filled LLL

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed

i.e., a nonzero **axial** current is induced

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$



# CHIRAL MAGNETIC EFFECT

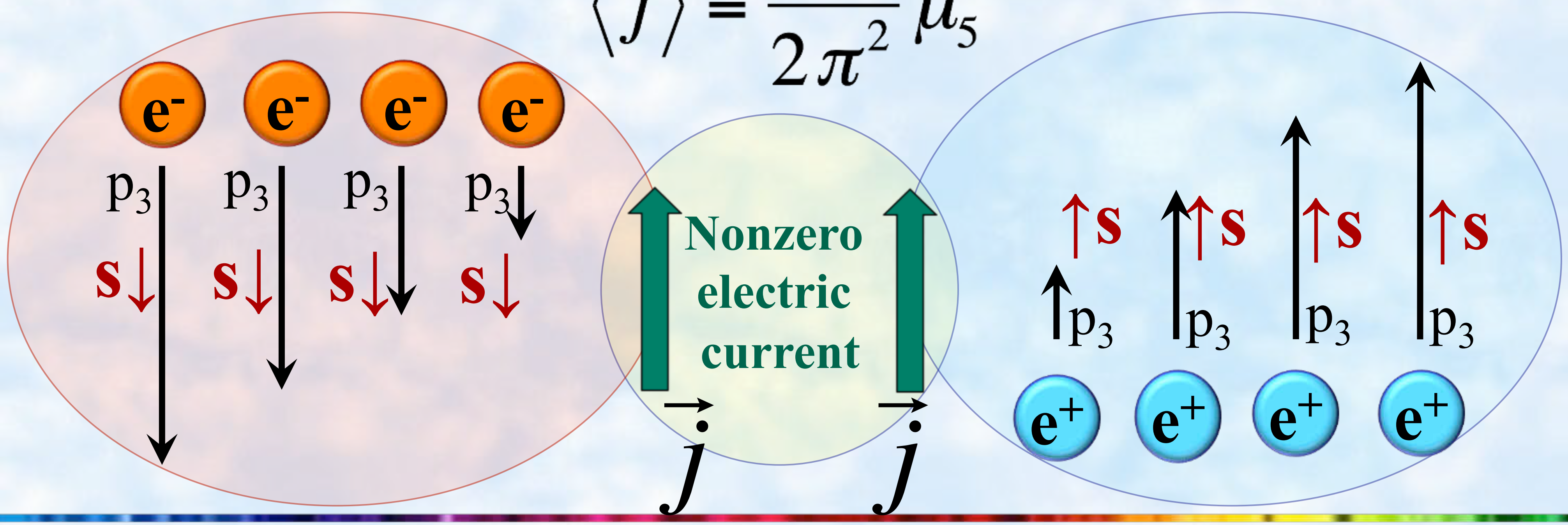
$$\left\langle \vec{j} \right\rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

# Partially filled LLL @ $\mu_5 \neq 0$

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed **electrons**
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed **positrons**

i.e., a nonzero **electric** current is induced

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$



# CME in heavy ion collisions?

- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$

- A random fluctuation with nonzero chirality could result in

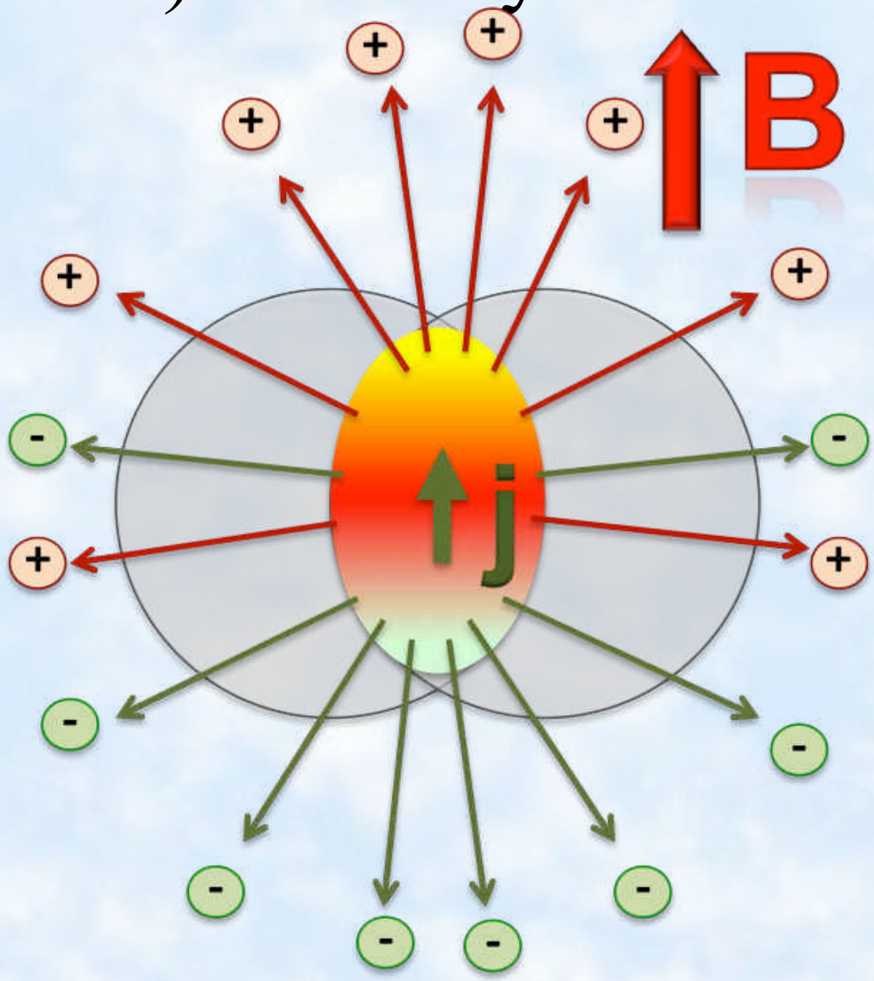
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- This should lead to an electric current

$$\left\langle \vec{j} \right\rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

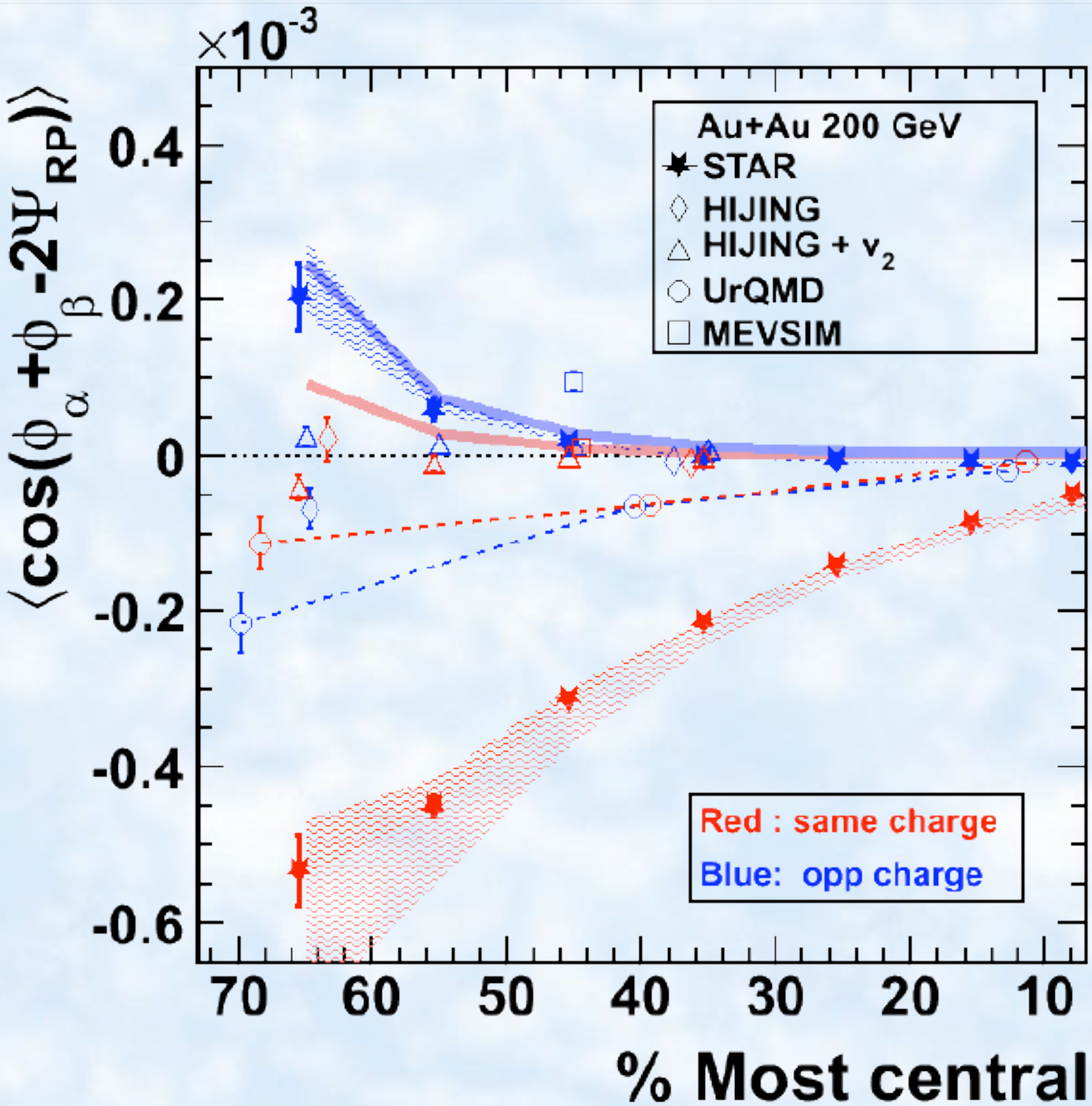
# Dipole CME

- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions




[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

# Experimental evidence




[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. **103**, 251601 (2009)]
[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C **81**, 054908 (2010)]
[Adamczyk et al. (STAR Collaboration), Phys. Rev. C **88**, 064911 (2013)]

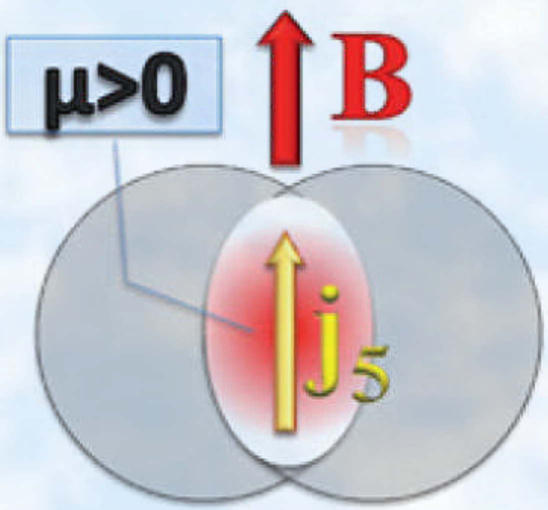
# CHIRAL MAGNETIC WAVE

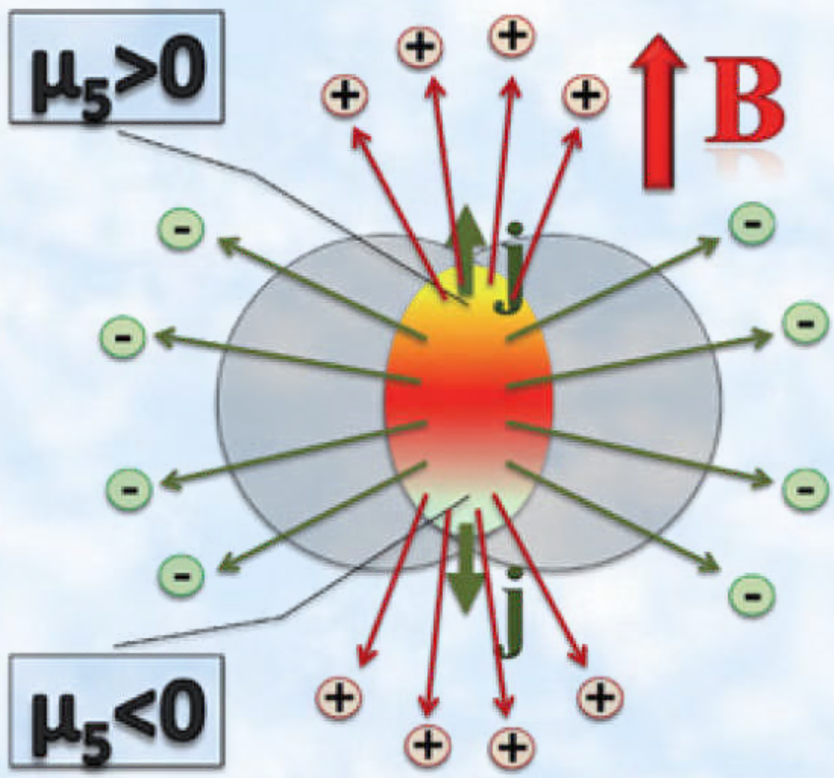
$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

# CMW/Quadrupole CME

- Start from a small baryon density and B≠0

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$




- Produce back-to-back electric currents

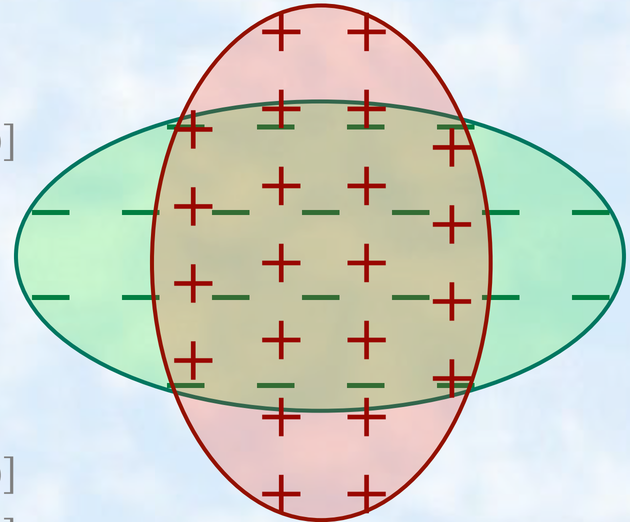
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

# Experimental evidence

- Elliptic flows of $\pi^+$ and $\pi^-$ depend on charge asymmetry:

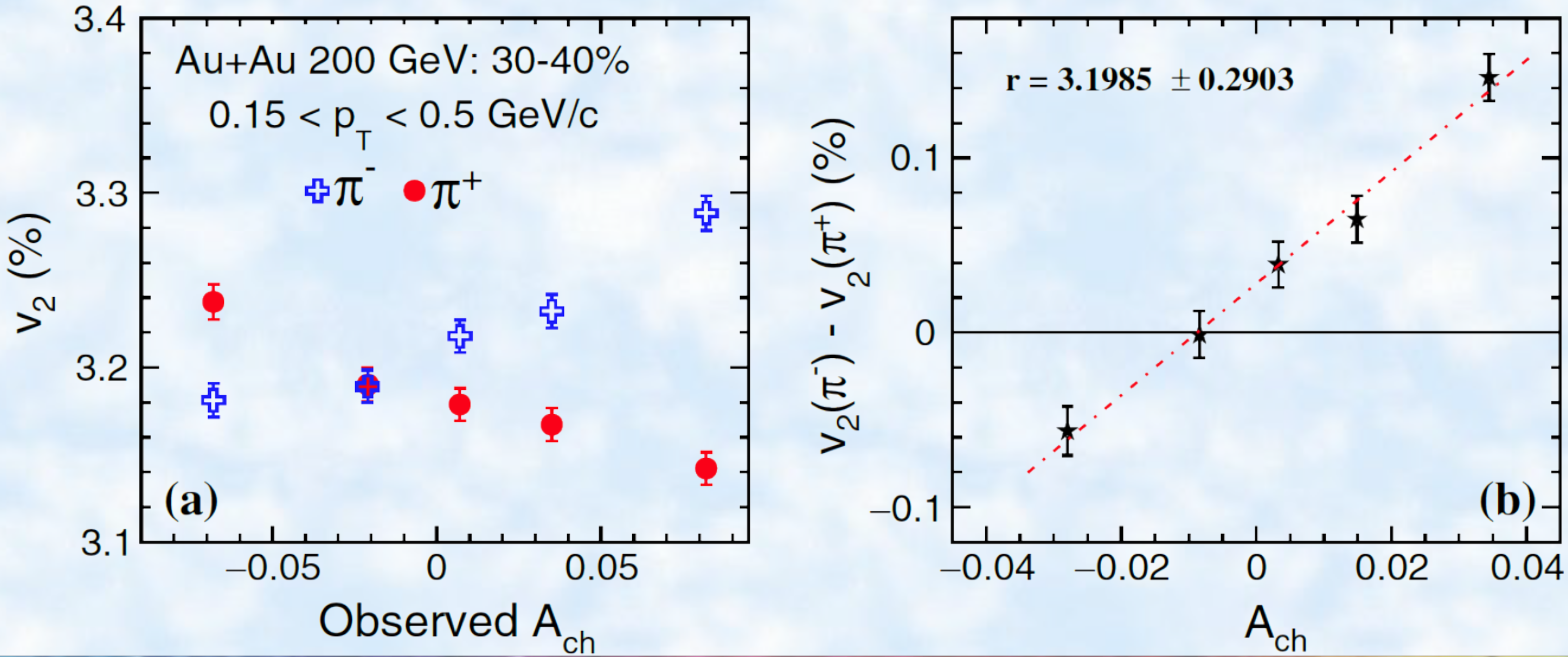
[Burnier, Kharzeev, Liao, Yee, PRL **107**, 052303 (2011)]

$$\frac{dN_\pm}{d\phi} \approx \overline{N}_\pm\left[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)\right]$$



[H. Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]



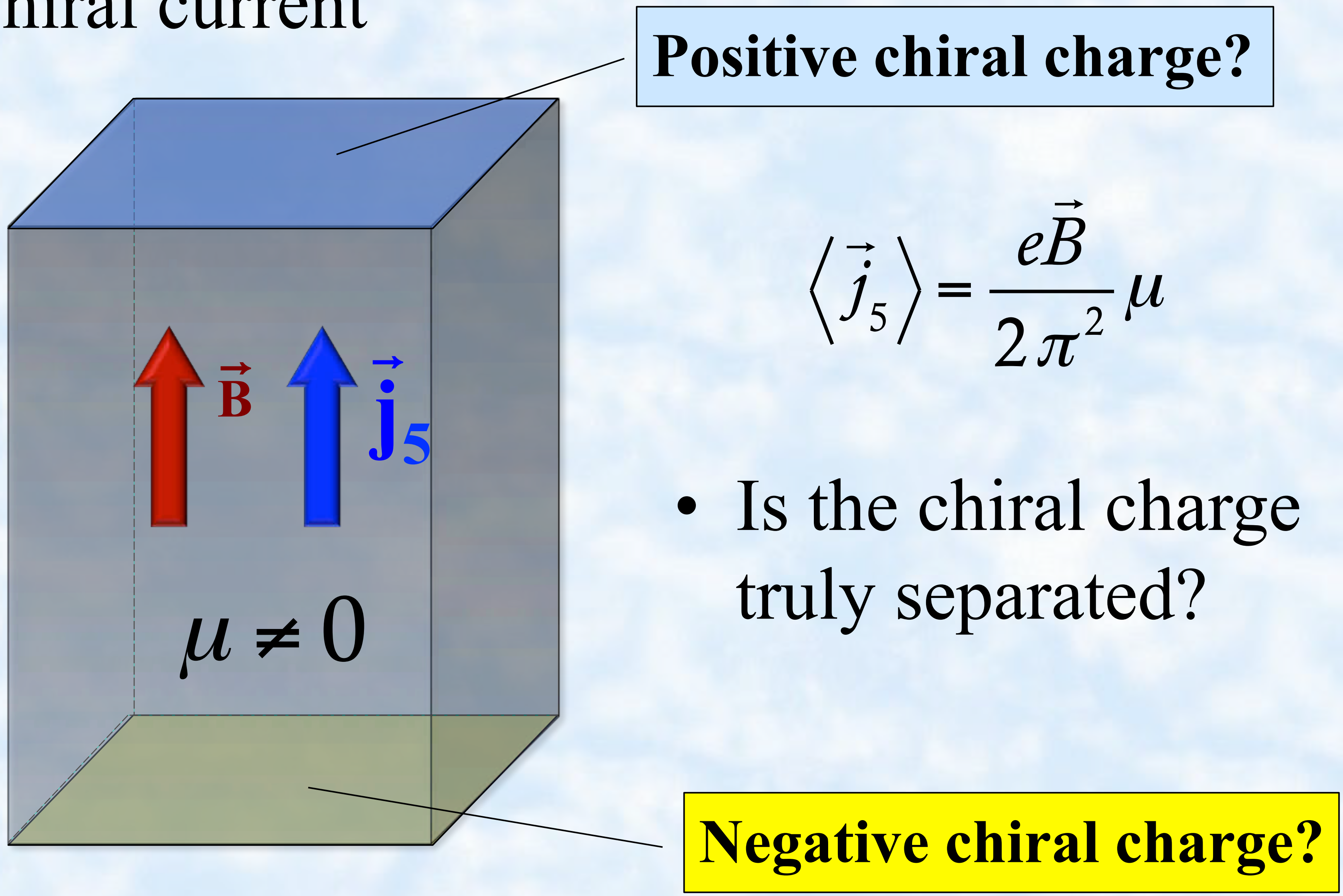
# FURTHER DEVELOPMENTS

- Effect of finite size

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

# Any effects of finite size?

- Magnetic field + electric chemical potential = chiral current



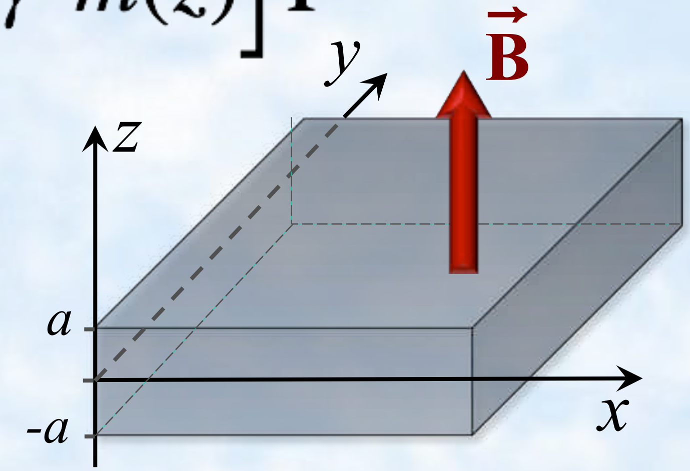

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

- Is the chiral charge truly separated?

# CSE in finite system

- Model of Dirac semimetal with a slab geometry

$$H = \int d^3 r \Psi^+ \left[ v_F \vec{\alpha} \cdot \left( -i\vec{\nabla} + e\vec{A} \right) + \gamma^0 m(z) \right] \Psi$$




where $\vec{A} = (0, Bx, 0)$ and

$$m(z) = M\,\theta(z^2 - a^2) + m\theta(a^2 - z^2),$$

with vacuum band gap: $M \to \infty$ (broken chiral symmetry)

- Boundary conditions:

$$\Psi_{\text{bulk}}(\vec{r}_\perp, a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, a) \quad \text{and} \quad \Psi_{\text{bulk}}(\vec{r}_\perp, -a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, -a)$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

# Wave functions

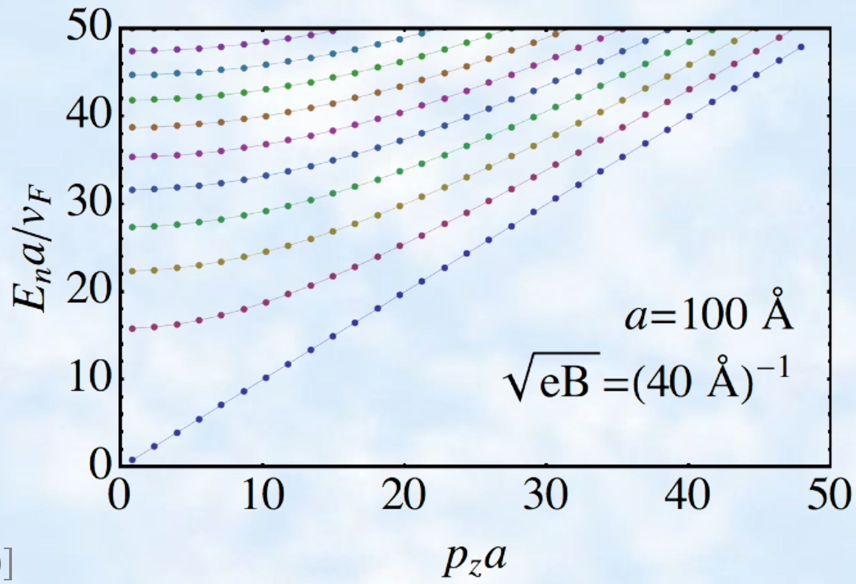
- Wave functions are standing waves, e.g.,

$$\text{LLL}: \quad \Psi_{\text{slab},n=0} = C_0 e^{-\frac{1}{2}(x/l+p_y l)^2} e^{i(p_y y + p_z a)} \begin{pmatrix} 0 \\ \dfrac{v_F p_z \cos\big(p_z(z-a)\big) - (m+iE_0)\sin\big(p_z(z-a)\big)}{im + v_F p_z - E_0} \\ 0 \\ -i\dfrac{v_F p_z \cos\big(p_z(z-a)\big) - (m-iE_0)\sin\big(p_z(z-a)\big)}{im + v_F p_z - E_0} \end{pmatrix}$$

where the wave vector $p_z$ is determined by the spectral equation

$$v_F p_z \cos\big(2a\, p_z\big) + m \sin\big(2a\, p_z\big) = 0$$

$$\Rightarrow \quad p_{z,k}^{(m)} \approx \frac{\pi(2k-1)}{4a} + \frac{2m}{\pi v_F (2k-1)} + \ldots$$



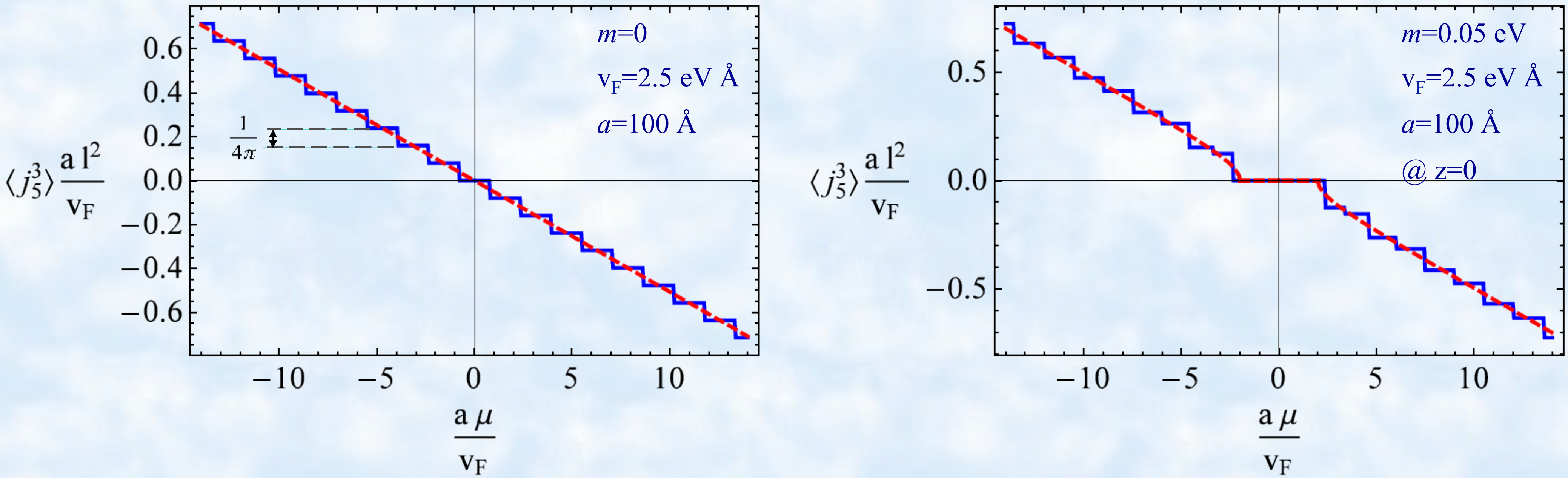

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

# Discretized CSE

- Only LLL contributes

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}v_F\,\mathrm{sign}(\mu)}{2\pi a}\sum_{p_z}\theta\left(\mu^2 - m^2 - v_z^2 p_z^2\right)\frac{\left(m^2 + v_z^2 p_z^2\right)\left[1-\cos\left(2z\,p_z\right)\cos\left(2a\,p_z\right)\right]}{2\left(m^2+v_z^2p_z^2\right)+mv_F/a}$$

- For $m \to 0$: $\langle \vec{j}_5 \rangle = -\frac{e\vec{B}v_F\,\mathrm{sign}(\mu)}{4\pi a}k_{\max}$, where $k_{\max} = \left[\frac{2a|\mu|}{\pi v_F} + \frac{1}{2}\right]$



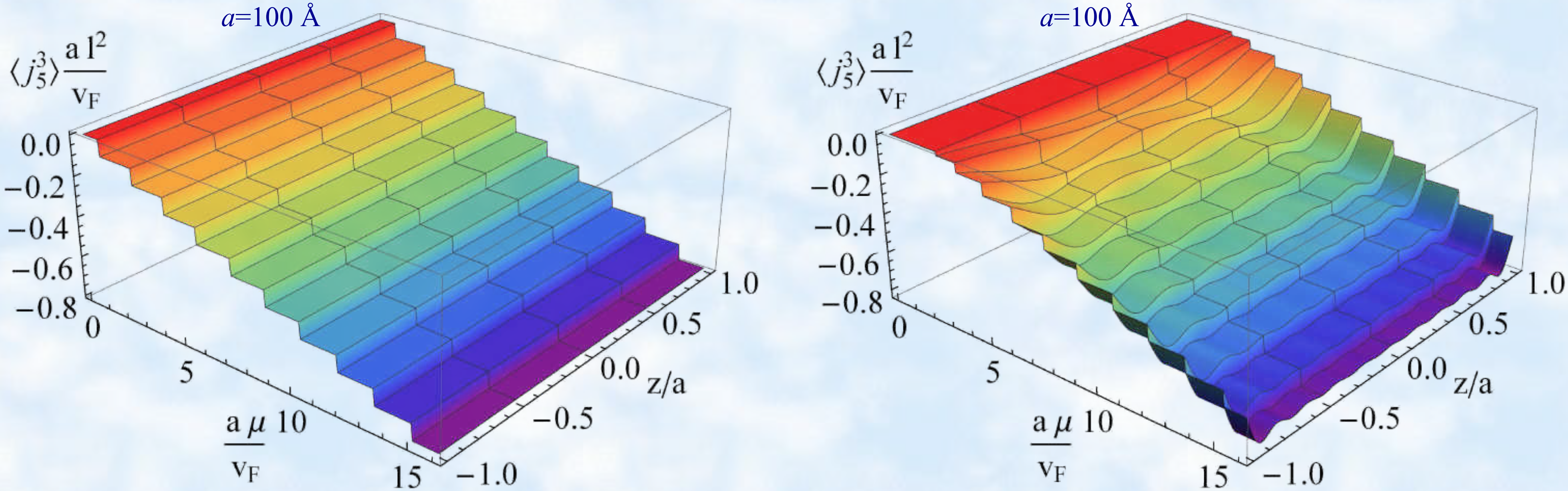

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

# Quantization of axial current

- Axial current density is non-uniform when $m \neq 0$

$m=0$
$v_F=2.5$ eV Å
$a=100$ Å

$m=0.05$ eV
$v_F=2.5$ eV Å
$a=100$ Å

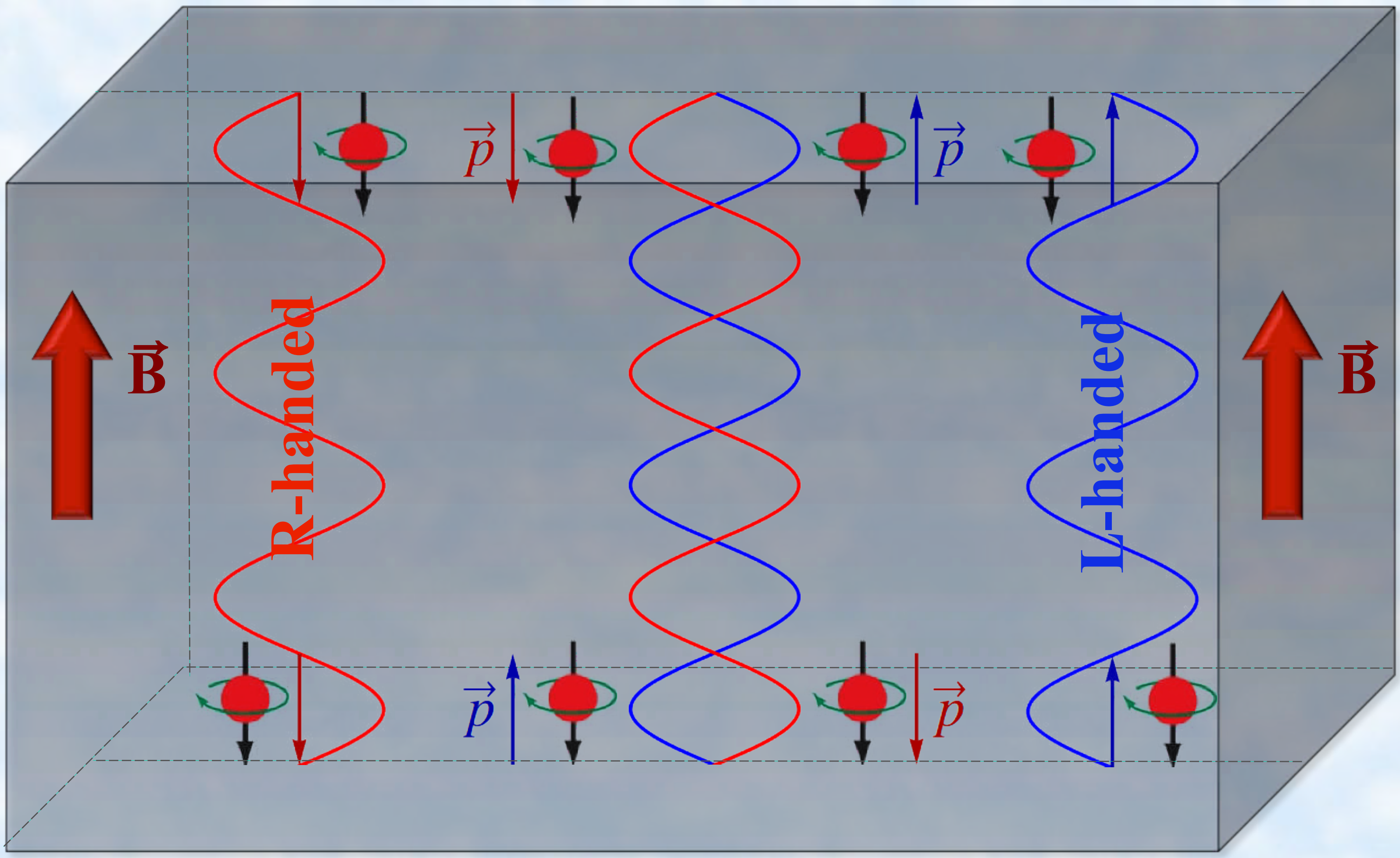



- Note that axial charge density vanishes: $\left\langle j_5^0 \right\rangle = 0$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

# Axial current as a standing wave?

- Recall that LLL is spin polarized




- A perfect chirality flip at the boundary

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

# Bottom line

- Chiral current in the CSE is discretized
- $m \neq 0$: chiral current density is non-uniform
- $m = 0$: chiral current density is uniform
- Chiral current is **not** necessarily connected with a "flow" of chiral charge
- Chiral current need **not** lead to chiral charge accumulation on the boundary
- CME is qualitatively different from CSE

# Summary

- Chiral plasmas have widespread applications
- Anomaly plays a profound role in such plasmas
- Many interesting chiral/anomalous effects are triggered by a magnetic field
- There are nontrivial effects due to a finite size
- Consequences of these effects in physical systems are still to be fully investigated