Institut für Theoretische Physik
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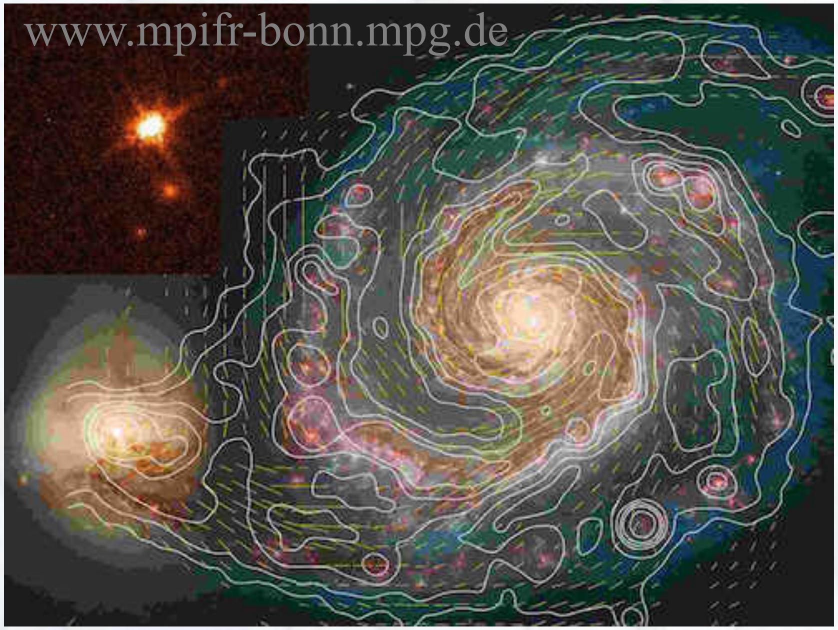
Transport properties of anomalous chiral plasmas

Igor Shovkovy
Arizona State University

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, Phys. Rev. D93, 105028 (2016)]

PUZZLE: MAGNETIC FIELD IN VOIDS

Magnetic field in Universe



- Current galactic magnetic fields ~ 10^{-6} G
- Intergalactic magnetic fields ~ 10^{-15} G
- Problem of magnetogenesis in Early Universe
- Perhaps during the electro-weak phase transition
  - **10^{20} to 10^{24} G (~ 1 GeV to 100 GeV)**

Limits on magnetic fields

[Neronov & Vovk, Science **328**, 74 (2010)]

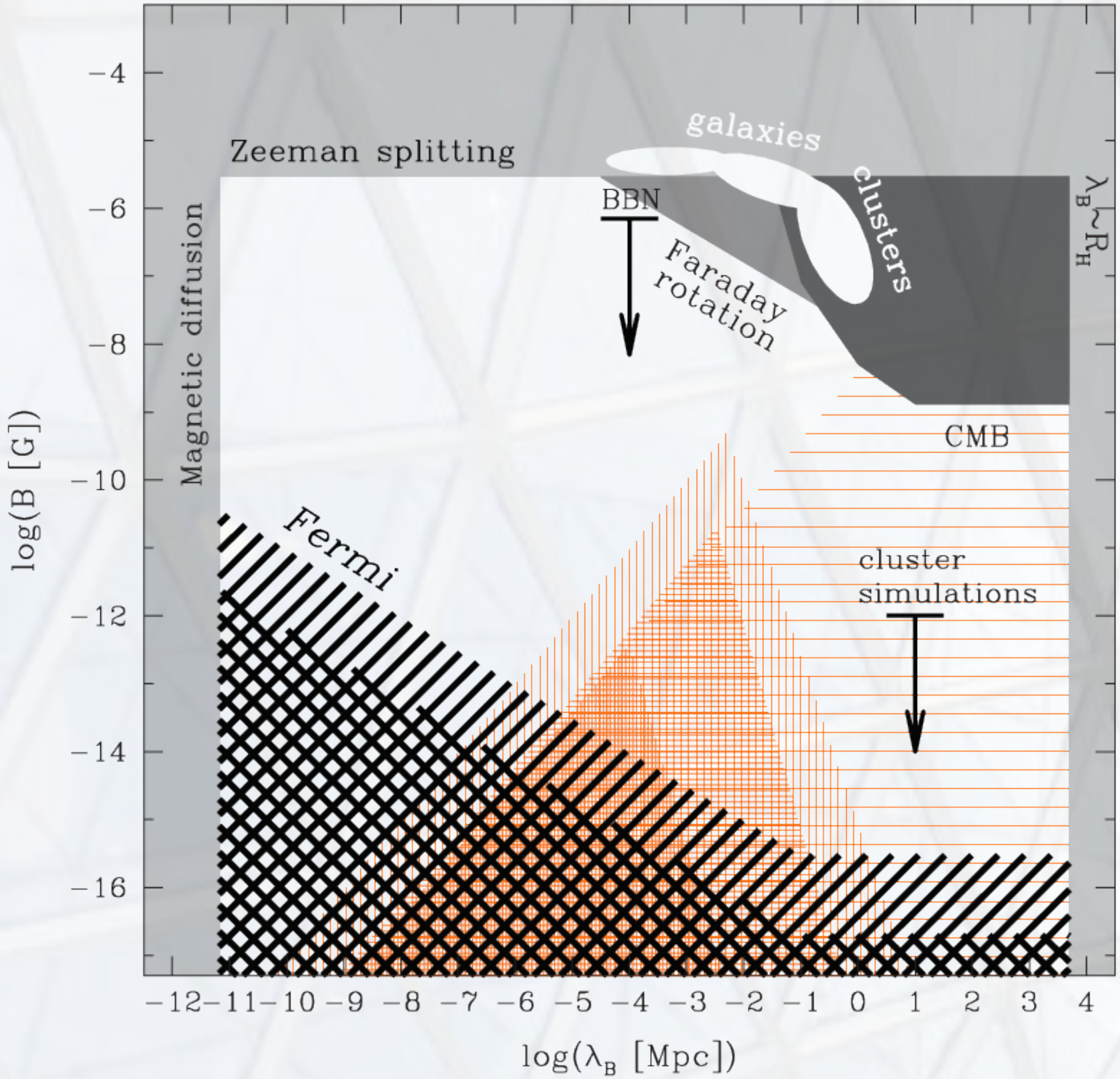


Fig. 2. Light, medium, and dark gray: known observational bounds on the strength and correlation length of EGMF, summarized in (*25*). The bound from Big Bang nucleosynthesis (BBN) is from (*2*). The black hatched region shows the lower bound on the EGMF derived from observations of 1ES 0347-121 (cross-hatching) and 1ES 0229+200 (single diagonal hatching) in this paper. Orange hatched regions show the allowed ranges of *B* and λ_B for magnetic fields generated at the epoch of inflation (horizontal hatching), the electroweak phase transition (dense vertical hatching), QCD phase transition (medium vertical hatching), and epoch of recombination (light vertical hatching) (*25*). White ellipses show the range of measured magnetic field strengths and correlation lengths in galaxies and galaxy clusters.

Magnetic field/helicity

- Magnetic helicity evolution (inverse cascade) in the Early Universe

$$\nabla \times \vec{B} = \frac{4\pi}{c}\left(\sigma \vec{E} + C_5 \mu_5 \vec{B}\right) + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = 4\pi \rho \, , \qquad \nabla \cdot \vec{B} = 0$$

[Vilenkin, Phys. Rev. D22, 3080 (1980)]
[Joyce & Shaposhnikov, astro-ph/9703005]
[Giovannini & Shaposhnikov, hep-ph/9710234]

- For specific helicity eigenmodes:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma}\left(C_5 \mu_5 - \frac{ck}{4\pi}\right) B_k$$

[Boyarsky et al., arXiv:1109.3350]
[Tashiro et al., arXiv:1206.5549]
[Manuel et al., arXiv:1501.07608]
[Buividovich et al., arXiv:1509.02076]
[Hirono et al., arXiv:1509.07790]

Feedback on μ_5(t)

- Constraint of the total helicity conservation

$$h_{\text{fermion}} = h_0 - h_{\text{gauge}}$$

- Common "homogeneous" approximation:

$$n_5(\vec{x},t) \approx \left\langle n_5(\vec{\mathbf{x}},t) \right\rangle_{\text{space}}, \quad \mu_5(t) \approx \frac{3c^3}{T^2} \left\langle n_5(\vec{\mathbf{x}},t) \right\rangle_{\text{space}}$$

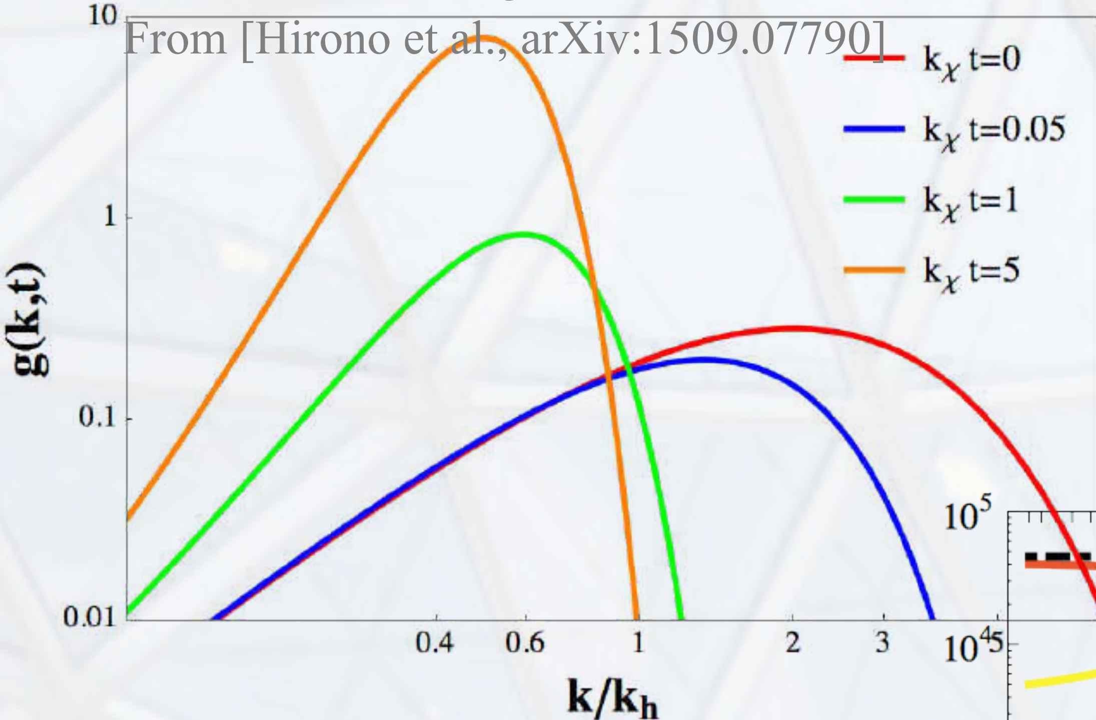
- In other words, the value of μ_5 remains constant on distance scales

$$\Delta x \sim (k_{\text{crit}})^{-1} \sim (\mu_5)^{-1}$$

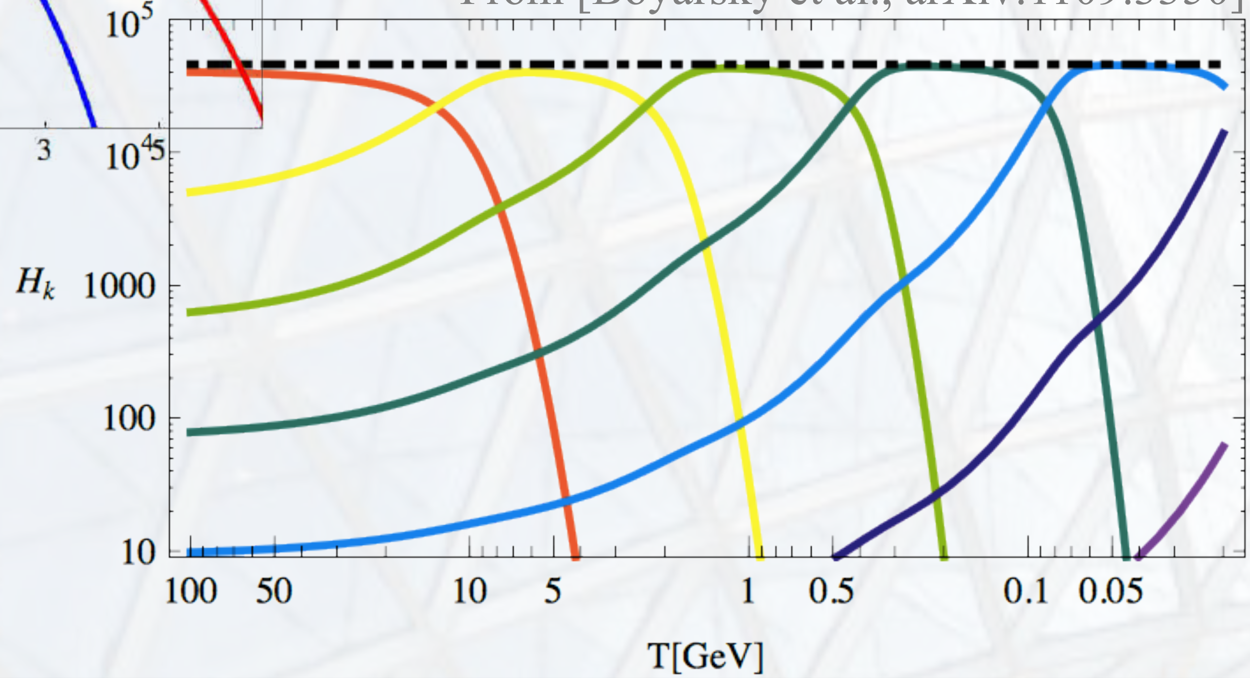
Magnetic field/helicity

- Magnetic helicity is transferred from short to long-wavelengths modes, while the value of μ_5 decreases

From [Hirono et al., arXiv:1509.07790]



From [Boyarsky et al., arXiv:1109.3350]



SOME PROBLEMS TO ADDRESS

- Role of inhomogeneities

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, Phys. Rev. D93, 105028 (2016)]

Open questions

- Will the cascade survive if there are variations of order $\delta\mu_5$ on distance scales $(k_{crit})^{-1}$?
- How large $\delta\mu_5$ can be tolerated?
- Will dynamical fluctuations of μ_5 stay under control?
- How to account for inhomogeneities in the anomalous Maxwell equations?

$$n_\lambda(\vec{x},t) = ? \qquad \vec{j}_\lambda(\vec{x},t) = ?$$

- How to obtain equations for $\mu(t,\mathbf{x})$ and $\mu_5(t,\mathbf{x})$?

Framework

- Chiral kinetic theory as a starting point:

$$\frac{\partial f_\lambda}{\partial t} + \frac{1}{1+\vec{\Omega}_\lambda \cdot \vec{B}}\left[\left(\vec{E} + \vec{v} \times \vec{B} + (\vec{E}\cdot \vec{B})\vec{\Omega}_\lambda\right)\cdot \frac{\partial f_\lambda}{\partial \vec{p}} + \left(\vec{v} + \vec{E} \times \vec{\Omega}_\lambda + (\vec{v}\cdot \vec{\Omega}_\lambda)\vec{B}\right)\cdot \frac{\partial f_\lambda}{\partial \vec{x}}\right] = I_{\mathrm{coll}}$$

where

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{cp - \mu_\lambda}{T}\right) + 1}$$

is an expansion in powers of e-m field & $\vec{\nabla}\mu_\lambda, \partial_t \mu_\lambda$

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, Phys. Rev. D93, 105028 (2016)]

1^{st} order solution

- Equation for $f_\lambda^{(1)}$:

$$\frac{D_\lambda}{T}\frac{\partial \mu_\lambda}{\partial t} - \frac{D_\lambda}{T}\vec{v}\cdot\left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}}\right) = -\frac{\delta f_\lambda^{(1)}}{\tau}$$

- Corresponding currents & densities

$$n_\lambda = \frac{\mu_\lambda\left(\mu_\lambda^2 + \pi^2 T^2\right)}{6\pi^2 c^3} - \frac{\tau\left(3\mu_\lambda^2 + \pi^2 T^2\right)}{6\pi^2 c^3}\frac{\partial \mu_\lambda}{\partial t}$$

$$\vec{j}_\lambda = \frac{\lambda e \mu_\lambda \vec{B}}{4\pi^2 c} + \frac{\tau\left(3\mu_\lambda^2 + \pi^2 T^2\right)}{18\pi^2 c}\left(e\vec{E} - \frac{\partial \mu}{\partial \vec{x}}\right)$$

- Continuity equation gives a constraint: $\frac{\partial \mu_\lambda}{\partial t} = 0$

2^{nd} order solution

- Equation for $f_\lambda^{(2)}$ is slightly more complicated…
- The currents & densities are

$$n_\lambda^{(2)} = \frac{\lambda e^2 \tau}{4\pi^2 c} \vec{B} \cdot \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right) - \frac{c^2 \tau^2}{3} \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \vec{\nabla} \cdot \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right)$$

$$- \frac{e\tau^2 \mu_\lambda}{3\pi^2 c} \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right) \cdot \frac{\partial \mu_\lambda}{\partial \vec{x}}$$

$$\vec{j}_\lambda^{(2)} = -\frac{ec^2 \tau^2}{3} \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \frac{\partial \vec{E}}{\partial t} - \frac{e\tau^2 \mu_\lambda}{6\pi^2} \vec{B} \times \left(e\vec{E} - \frac{\partial \mu_\lambda}{\partial \vec{x}} \right)$$

- Continuity equations give should be enforced

Equations for chemical potentials

- Resulting equation of motion for μ_λ:

$$\frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(\frac{\partial \mu_\lambda}{\partial t}+\frac{e\tau c^2}{3}\vec{\nabla}\cdot\vec{E}_\lambda\right)+\frac{e\tau\mu_\lambda}{3\pi^2 c}\left(\vec{E}_\lambda\cdot\frac{\partial \mu_\lambda}{\partial \vec{x}}\right)=\frac{\lambda e^2}{4\pi^2 c}\left(\vec{E}_\lambda\cdot\vec{B}\right)$$

where $n_\lambda^{(0)}=\dfrac{\mu_\lambda^3+\pi^2T^2\mu_\lambda}{3\pi^2c^3}$ and $\vec{E}_\lambda=\vec{E}-\dfrac{1}{e}\dfrac{\partial \mu_\lambda}{\partial \vec{x}}$

The corresponding equations for the currents:

$$\vec{j}=\underbrace{\frac{e\mu_5\vec{B}}{2\pi^2 c}}_{\text{CME}}+\underbrace{\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{drift \& diffusion}}+\underbrace{\frac{e\tau^2\mu}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}_{\text{Hall type}}+\vec{j}_{\text{new}}$$

$$\vec{j}_5=\underbrace{\frac{e\mu\vec{B}}{2\pi^2 c}}_{\text{CSE}}-\underbrace{\frac{e\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{diffusion}}+\underbrace{\frac{2e\tau\mu_5\mu}{3\pi^2 c}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{CESE}}+\vec{j}_{5,\text{new}}$$

New types of currents

- New contribution to the electric current:

$$\vec{j}_{\text{new}} = -\underbrace{\frac{2\tau\mu\mu_5}{3\pi^2 c}\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{Chiral diffusion}} - \underbrace{\frac{e\tau^2\mu_5}{3\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right)}_{\text{Hall diffusion}} - \frac{e\tau^2\left(3\mu^2+3\mu_5^2+\pi^2T^2\right)}{9\pi^2 c}\frac{\partial\vec{E}}{\partial t}$$

- New contribution to the chiral current:

$$\vec{j}_{5,\text{new}} = -\overbrace{\frac{e\tau^2\mu}{3\pi^2}\left(\frac{\partial\mu_5}{\partial\vec{x}}\times\vec{B}\right)}^{\text{Chiral Hall diffusion}} + \overbrace{\frac{e\tau^2\mu_5}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}^{\text{Chiral Hall effect}} - \frac{2e\tau^2\mu\mu_5}{3\pi^2 c}\frac{\partial\vec{E}}{\partial t}$$

- There is also a term $\propto \frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu}{\partial\vec{x}}\right)$

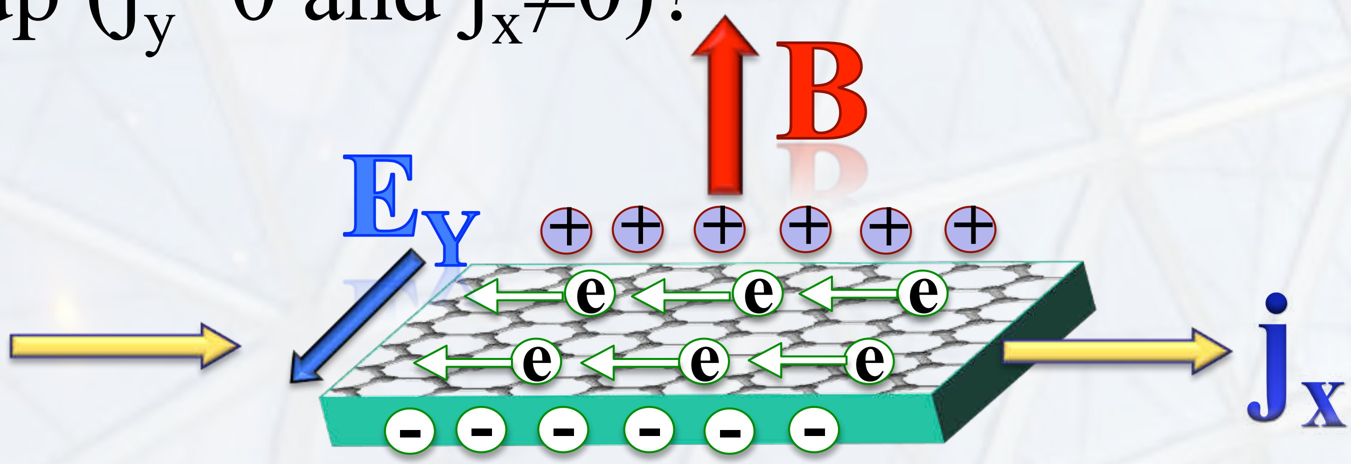
Question about Hall current

- Note that we had

$$\vec{j}^{\,(\mathrm{Hall})} = \frac{e^2 \tau^2 \mu}{3\pi^2} \vec{E} \times \vec{B}$$

- Why is this proportional to τ^2 ?

- What do you observe in the usual experimental setup ($j_y=0$ and $j_x \neq 0$)?



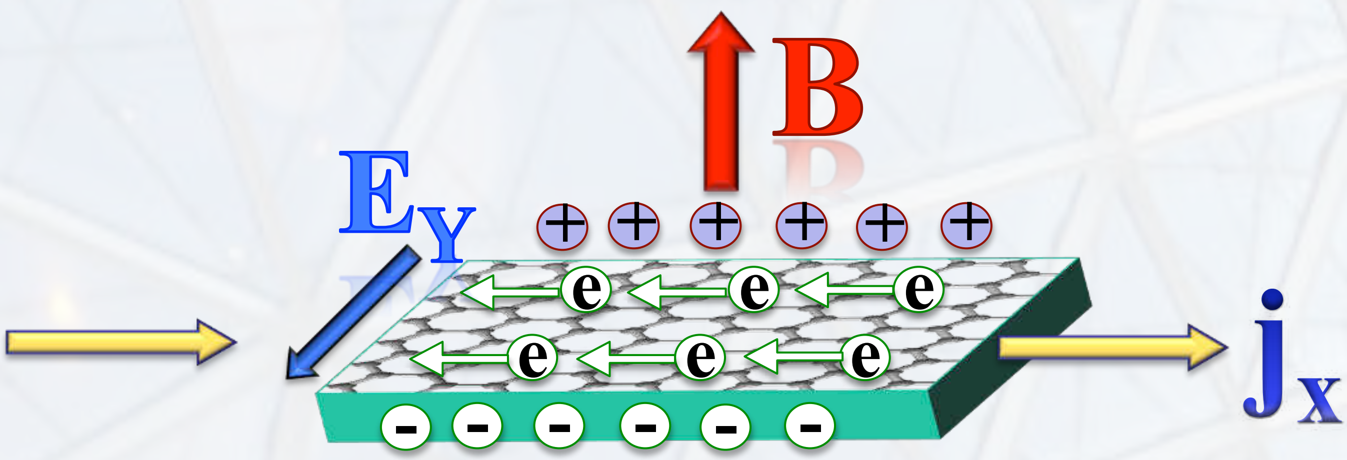
Question about Hall current

- Enforcing $j_y = 0$ gives

$$a\tau E_y = b\tau^2 E_x B_z$$

Then, in the approximation used,

$$j_x = a\tau E_x + b\tau^2 E_y B_z = \frac{(a\tau)^2}{b\tau^2 B_z} E_y + b\tau^2 E_y B_z \approx \frac{a^2}{b\,B_z} E_y$$

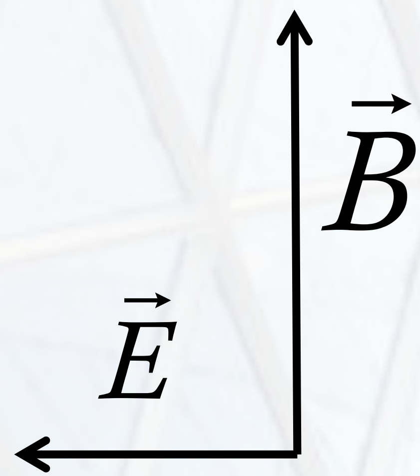


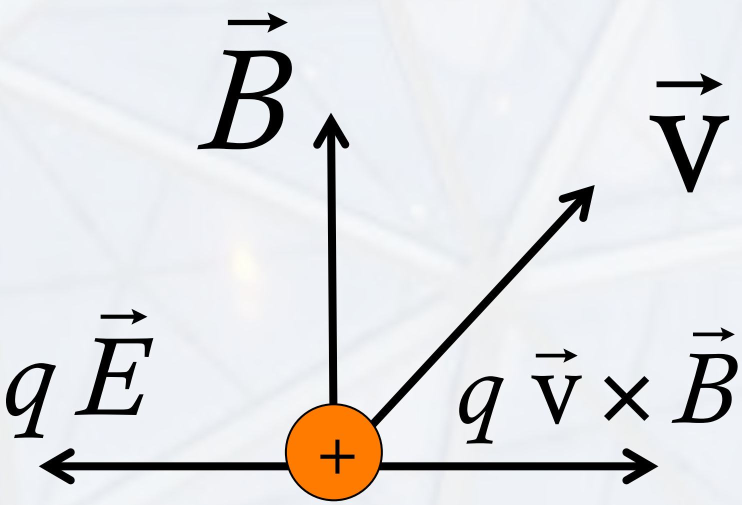
Plasma drift

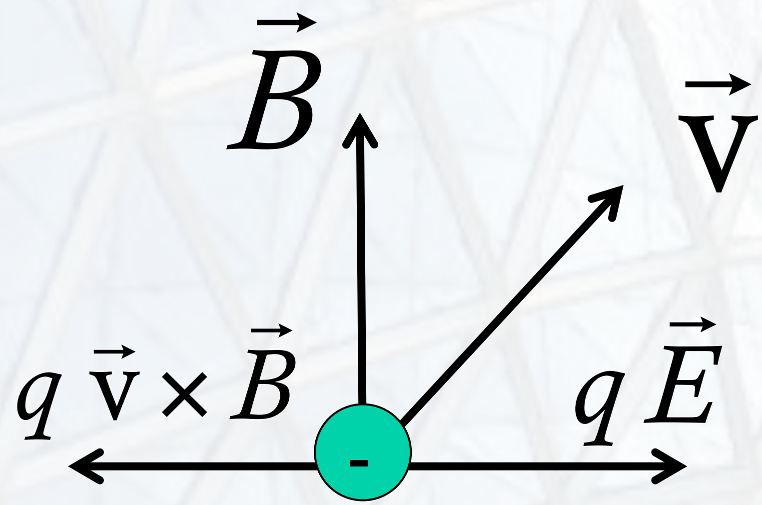
Plasma as a whole experiences a non-dissipative drift. Can this be included in $f_\lambda^{(0)}$?

Why should plasma drift?

Consider $\vec{E} \perp \vec{B}$ (with $E < B$):







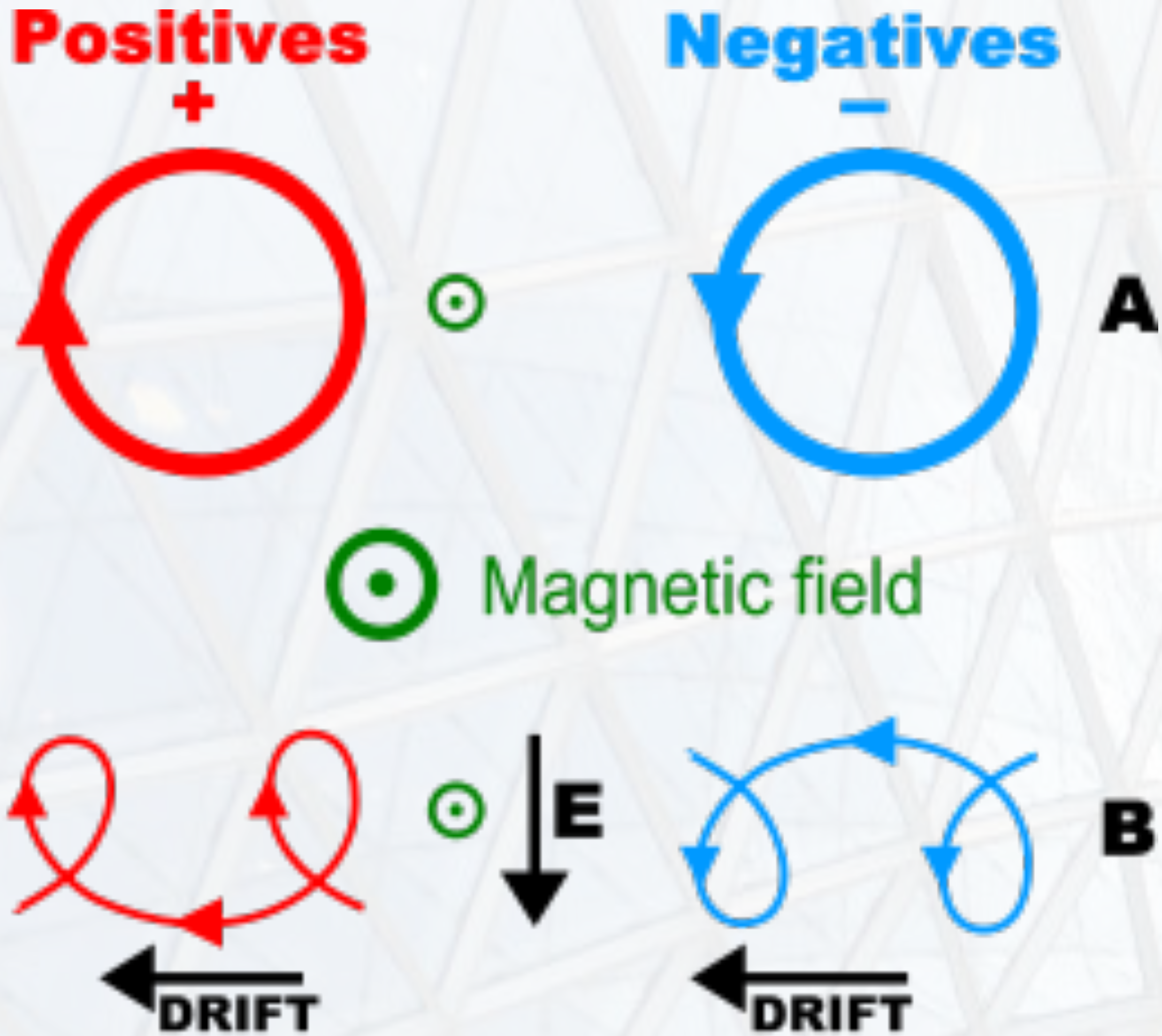
Frames of reference

Another viewpoint

$f_\lambda^{(0)}$

Drifting frame:

Lab frame:



$f_\lambda^{(0)}$ for magnetized plasma

- Consider a special case
  - Plasma consists of only e-m charged degrees of freedom
  - Fields so that $\vec{E} \perp \vec{B}$ (with $E < B$)
- No electric field in the drift frame
- Lab frame: use boosted Fermi-Dirac distribution

$$f_\lambda^{(\mathrm{lab})} = \frac{1}{\exp\left(\frac{c\,p - \vec{p}\cdot\vec{\mathrm{v}}_{\mathrm{drift}} - \mu_\lambda}{T}\right) + 1}$$

with

$$\vec{\mathrm{v}}_{\mathrm{drift}} = c\,\frac{\vec{E}\times\vec{B}}{B^2}$$

1^{st} Perturbation in drifting plasma

- Charge density

$$n_\lambda^{(\text{lab})} = n_\lambda^{(0)} - \tau \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(\frac{\partial \mu_\lambda}{\partial t} + \vec{v}_{\text{drift}} \cdot \nabla \mu_\lambda\right) + \tau n_\lambda^{(0)}\left(\nabla \cdot \vec{v}_{\text{drift}}\right) + \ldots$$

- Current density

$$\vec{j}_\lambda^{(\text{lab})} = c n_\lambda^{(0)} \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\lambda e \mu_\lambda}{4\pi^2 c}\vec{B} + \frac{\lambda e \mu_\lambda}{4\pi^2 c}\vec{E}_\perp \frac{\left(\vec{E}\cdot\vec{B}\right)}{E_\perp^2}\left(\frac{B}{2E_\perp} ln \frac{B+E_\perp}{B-E_\perp} - 1\right)$$

$$-\tau \frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda}\left(g_1\left(\frac{e\left(\vec{E}\cdot\vec{B}\right)}{B^2}\vec{B} - \nabla \mu_\lambda\right) + g_2 \vec{v}_{\text{drift}}\left(\vec{v}_{\text{drift}} \cdot \nabla \mu_\lambda\right) + g_3 \frac{\partial \mu_\lambda}{\partial t}\vec{v}_{\text{drift}}\right) + \ldots$$

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, Phys. Rev. D93, 105028 (2016)]

Drift in QGP plasma?

- In QGP, gluons play a profound role
  - Gluons are neutral and, thus, are not drifting
  - The zeroth approximation is the usual Fermi-Dirac distribution

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{c\,p - \mu_\lambda}{T}\right) + 1}$$

- Expansion in small e-m fields and gradients is well define

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

- Expansion of the 1st type (no drift) may be better

Summary

- Chiral plasmas are widespread
- Anomaly plays a role in such plasmas
- Anomalous effects are often triggered by a magnetic field
- Inhomogeneities can be accounted in a systematic way
- New transport phenomena are revealed
- Consequences of these effects in physical systems are still to be fully investigated