ARIZONA STATE UNIVERSITY

CHIRAL MATTER
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RIKEN

Chiral matter in magnetic field

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CHIRAL MATTER

from quarks to Dirac semimetals

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iTHES interdisciplinary Theoretical Science Research Group

RIKEN NiSHiNA CENTER | RIKEN Nishina Center for Accelerator-Based Science

CEMS RIKEN Center for Emergent Matter Science

CHIRAL MATTER

Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{\boldsymbol{p}}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{\boldsymbol{p}}}{|\vec{\boldsymbol{p}}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

For particles $(p_0 > 0)$: chirality = helicity

For antiparticles $(p_0 < 0)$: chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

Chiral matter

- Matter made of chiral fermions with $n_{\mathrm{L}} \neq n_{\mathrm{R}}$
- Unlike the electric charge $(n_{\mathrm{R}} + n_{\mathrm{L}})$, the chiral charge $(n_{\mathrm{R}} - n_{\mathrm{L}})$ is **not** conserved

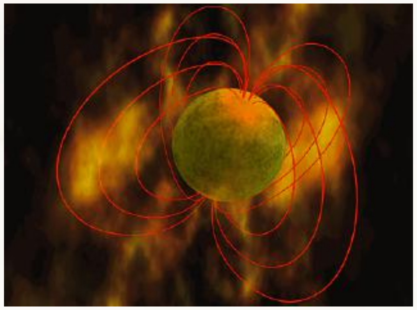
$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{\boldsymbol{j}} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{\boldsymbol{j}}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral symmetry is anomalous in quantum theory

Magnetic Fields

- Strong magnetic fields are common inside compact stars
  - **10^{10} to 10^{15} Gauss**



- In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  - **10^{18} to 10^{19} Gauss**

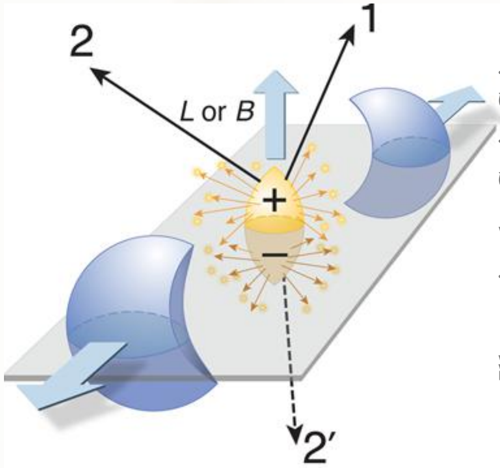
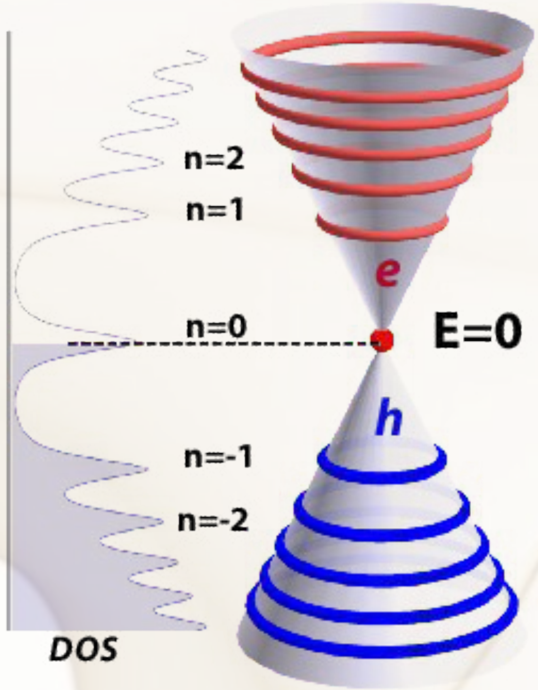


Illustration by Carin Cain

- Early Universe
  - **up to 10^{24} Gauss**
- High Magnetic Field Laboratory
  - **4.5×10^5 Gauss**



ROLE OF MAGNETIC FIELD

Landau spectrum at B≠0

- Dirac equation with massless fermions

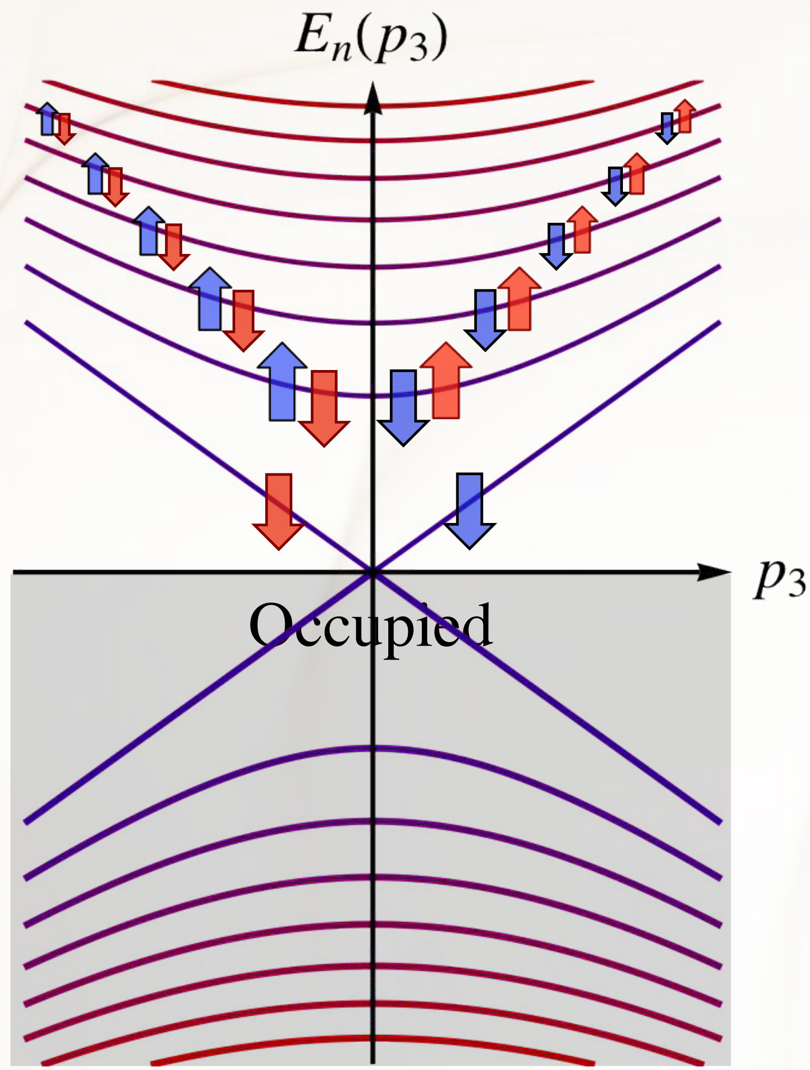
$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum

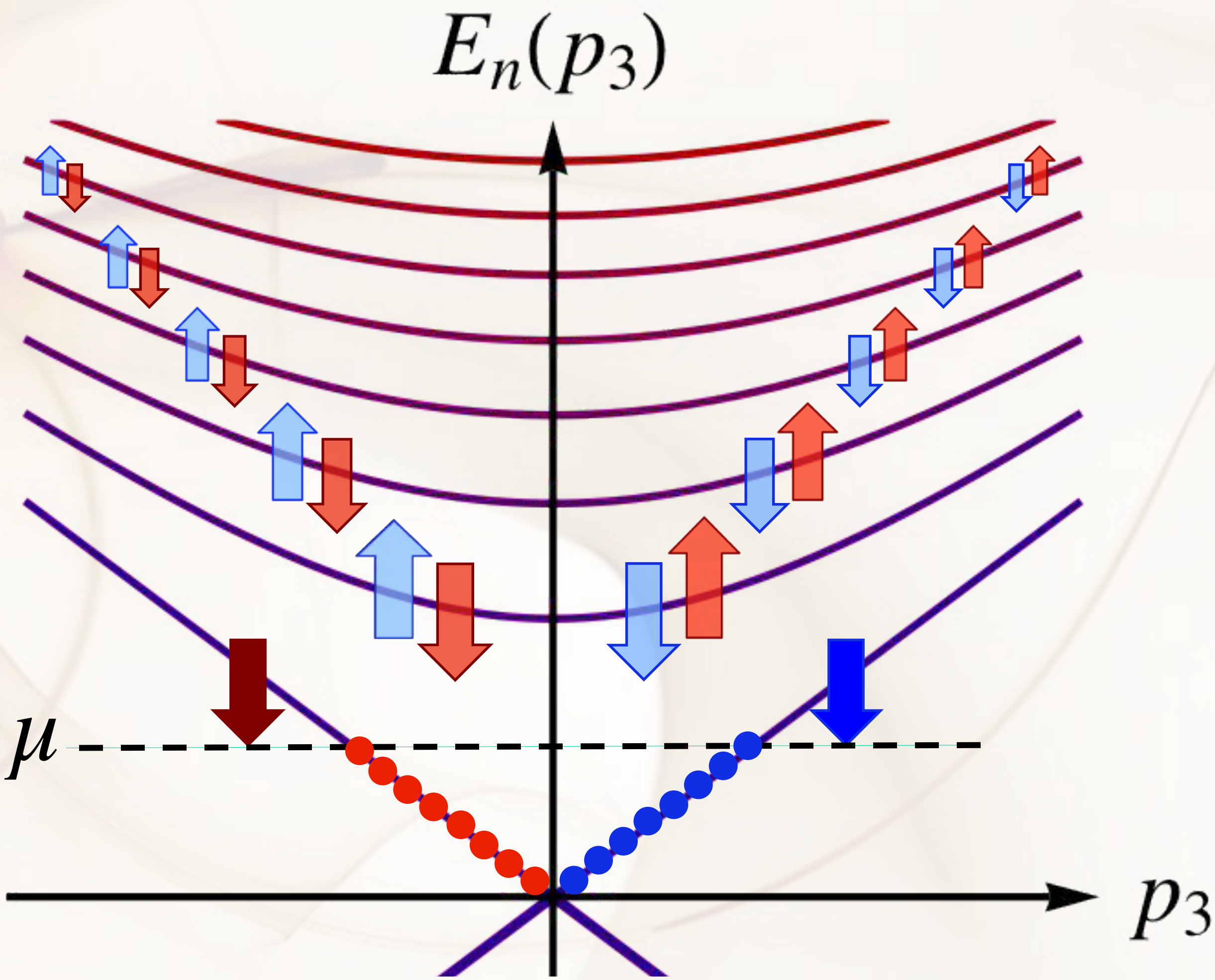
$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

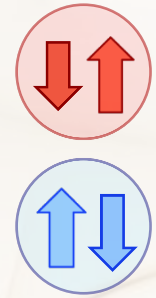
where

$$n = \underbrace{s}_{s=\pm 1/2 \text{ (spin)}} + \underbrace{k + \frac{1}{2}}_{k=0,\,1,\,2,\ldots \text{ (orbital)}}$$



Chiral separation effect ($\mu \neq 0$)





— **Right-handed**

— **Left-handed**

Spin (s=↓) **polarized** LLL:

- $p_3<0$ states are R-handed
- $p_3>0$ states are L-handed

This leads to CSE:

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

CSE vs. anomaly

- Slowly changing electric/chemical potential

$$\mu(z) = e\Phi(z) \quad \Rightarrow \quad eE_z = -\partial_z\left(e\Phi\right) = -\partial_z \mu$$

- From the anomaly relation,

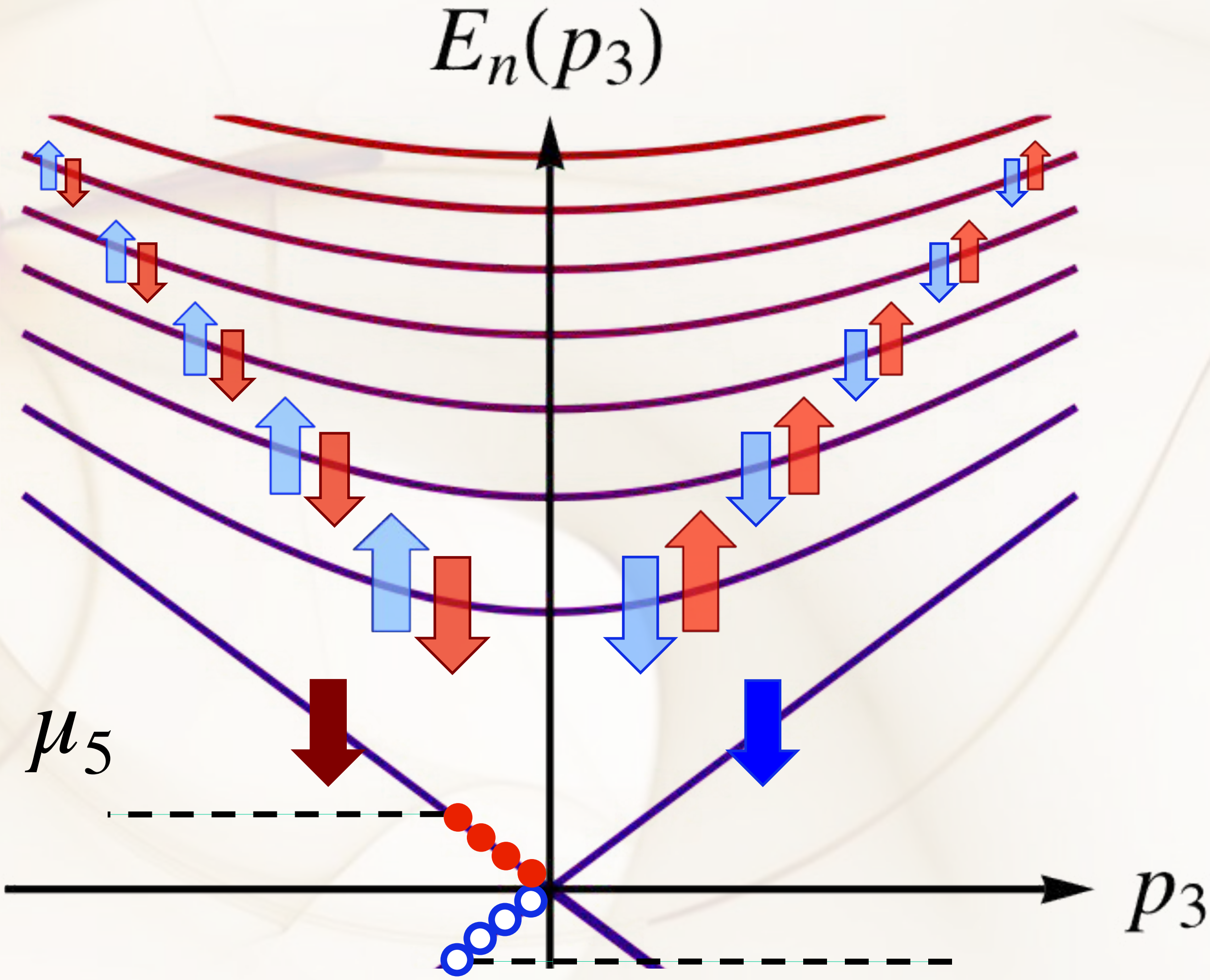
$$\partial_z j_5^3 = \frac{e^2}{2\pi^2} B_z E_z = -\frac{e^2}{2\pi^2} B_z \partial_z \mu$$

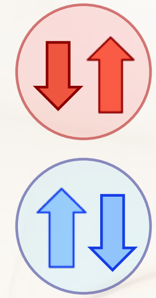
- Suggesting that for massless fermions,

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

- There are radiative correction when gauge fields are dynamical [Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025025 (2013)]

Chiral magnetic effect ($\mu_5 \neq 0$)





Spin (s=↓) **polarized** LLL:

- $p_3<0$ states are R-handed electrons
- $p_3>0$ states are L-handed positrons

This leads to CME:

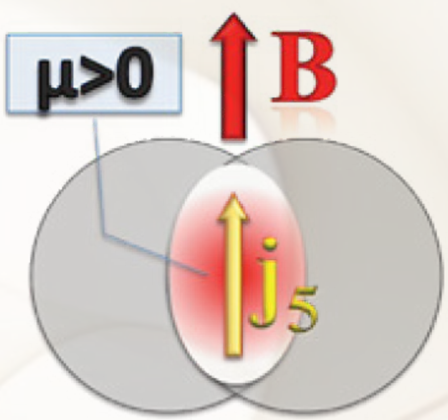
$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

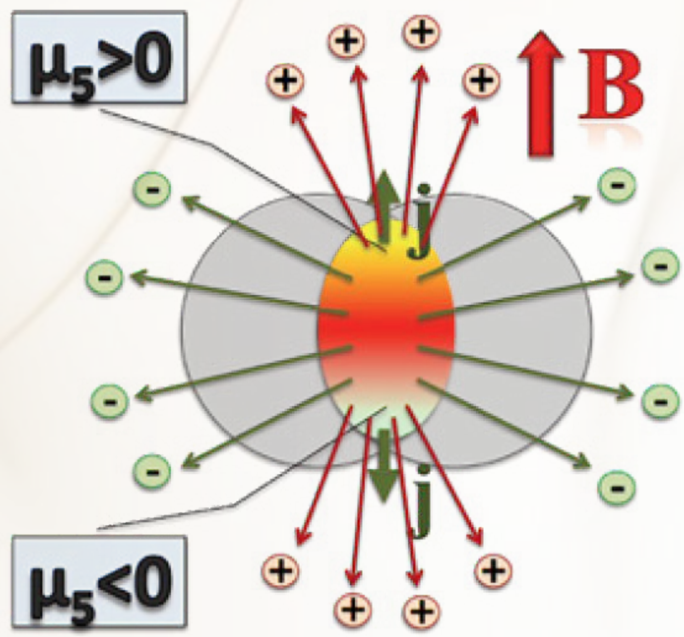
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CMW/Quadrupole CME

- Start from a small baryon density and B≠0

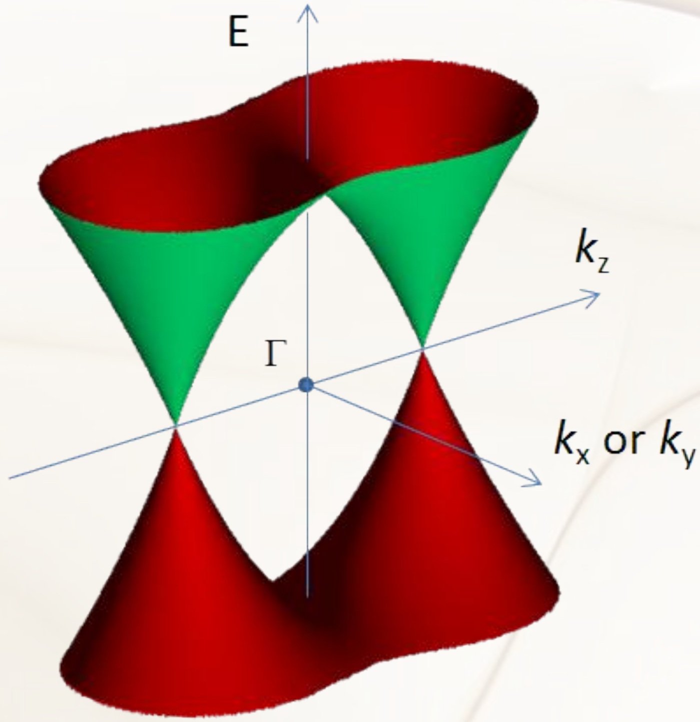
$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$





- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]



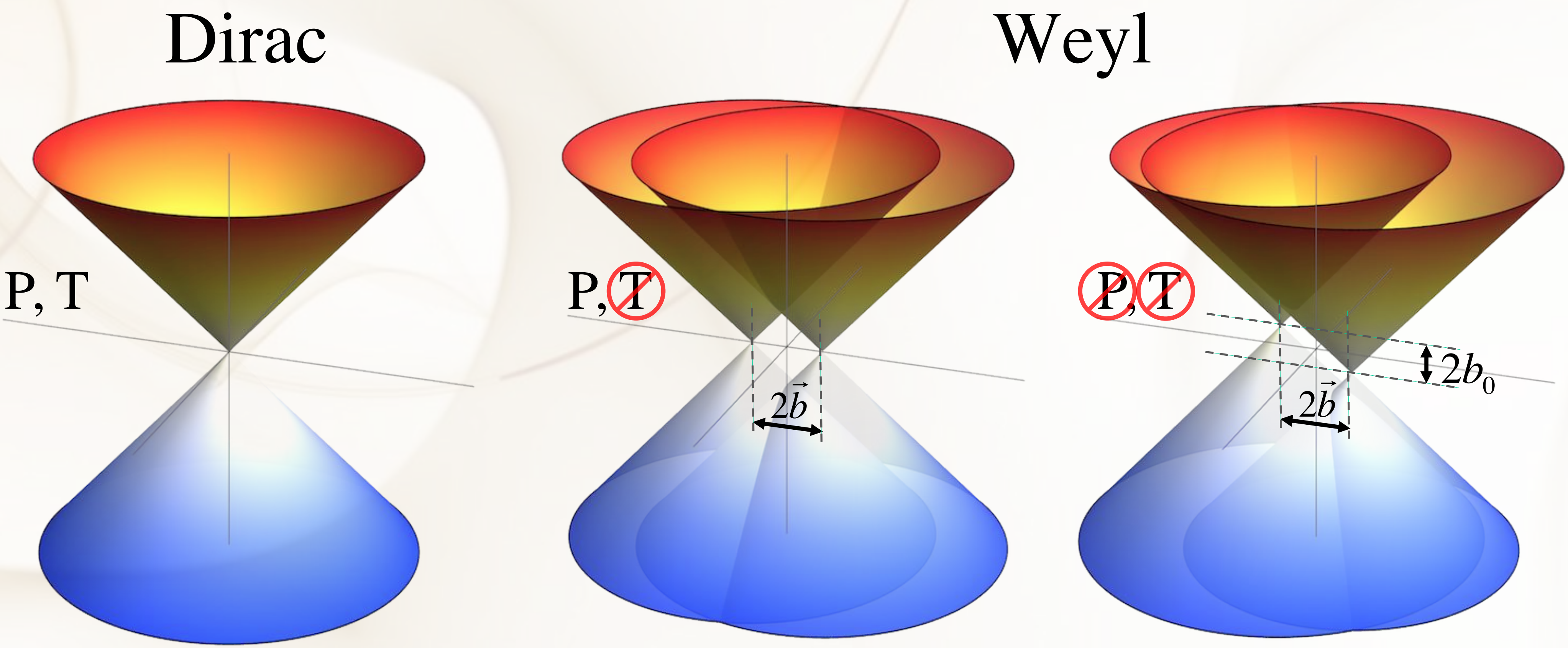
Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL MATERIALS

Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H=\int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5+b_0\gamma^0\gamma^5\right]\psi$$



Dirac materials

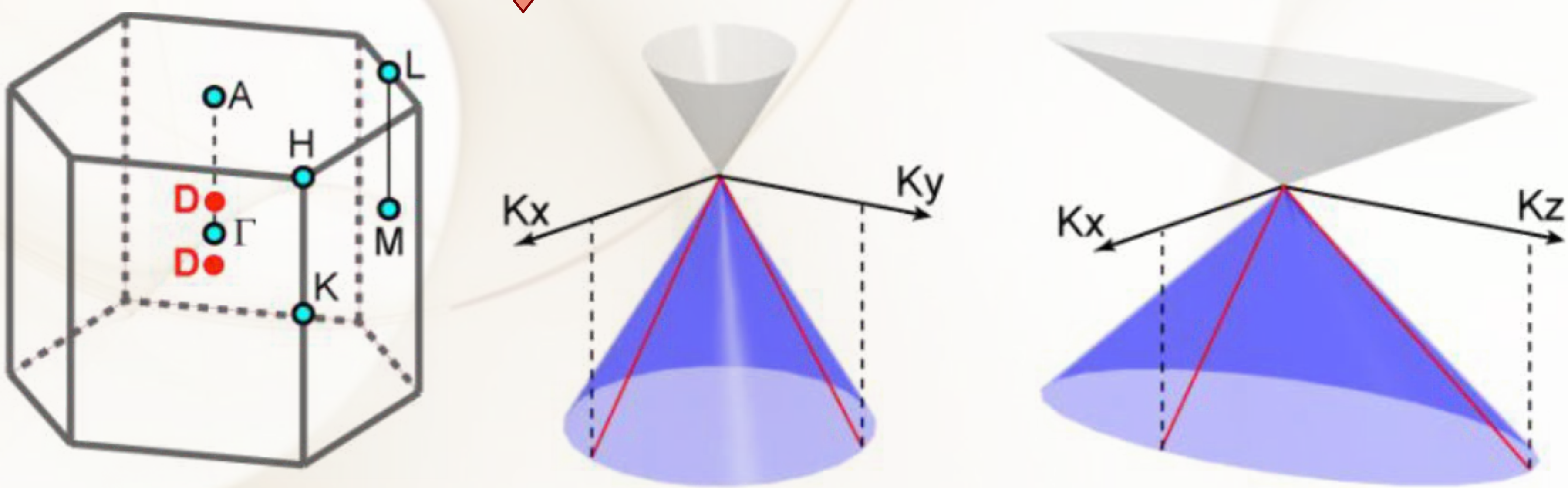
- $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
- Na_3Bi
- Cd_3As_2
- $ZrTe_5$

[Z. K. Liu et al., Science **343**, 864 (2014)]

[M. Neupane et al., Nature Commun. **5**, 3786 (2014)]

[S. Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

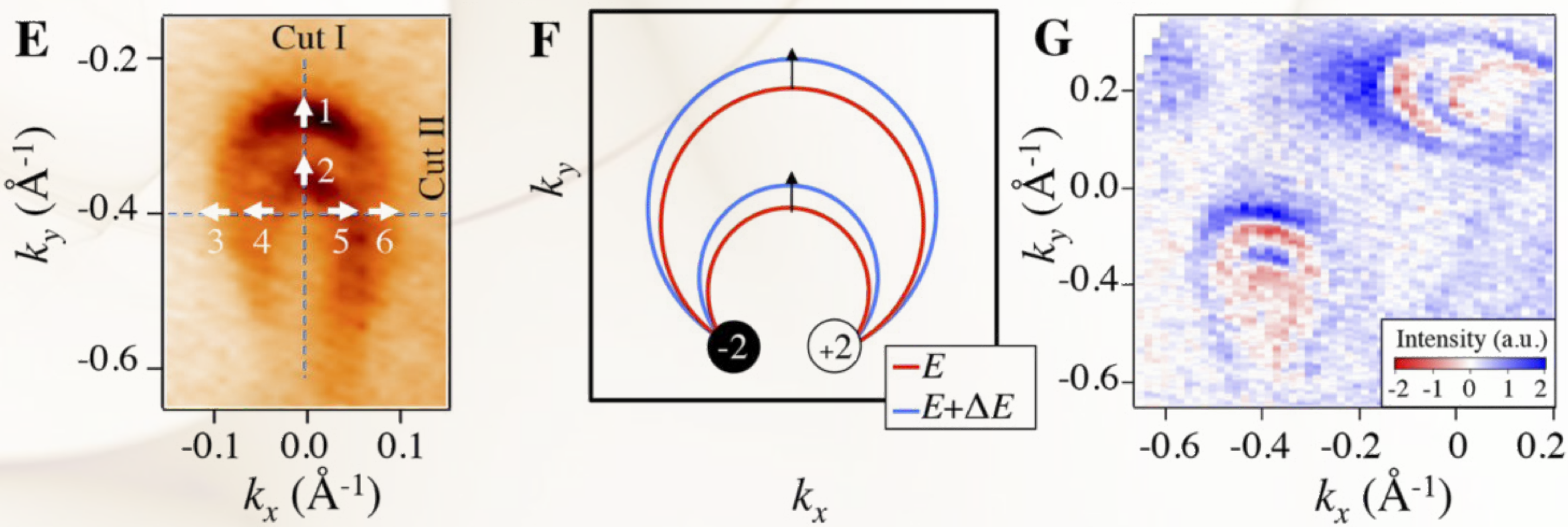
[X. Li et al., Nature Physics **12**, 550 (2016)]

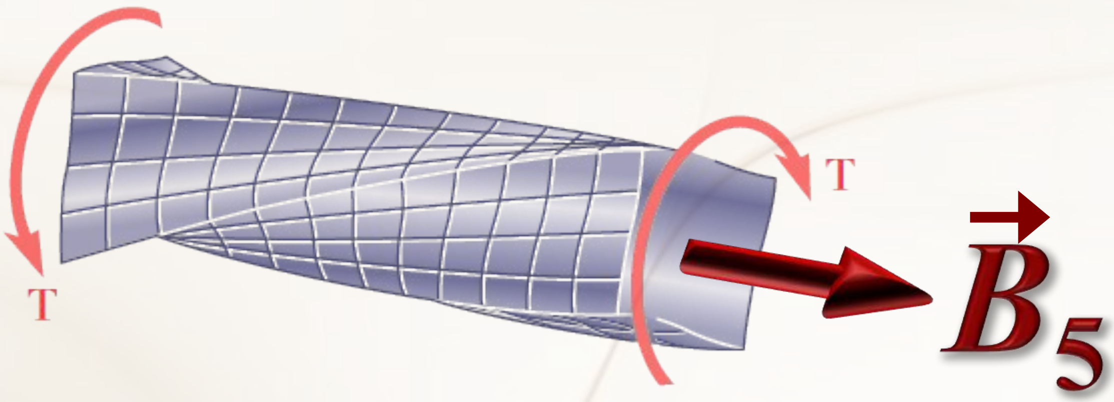


$$E = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + v_z^2 k_z^2}, \quad v_x \approx v_y \approx 3.74 \times 10^5 \, \mathrm{m/s}, \quad v_z \approx 2.89 \times 10^4 \, \mathrm{m/s}$$

Weyl materials

- TaAs (tantalum arsenide) [S.-Y. Xu et al., Science 349, 613 (2015)] [B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
- NbAs (niobium arsenide) [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
- TaP (tantalum phosphide) [S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
- NbP (niobium phosphide) [I. Belopolski et al. arXiv:1509.07465]
- WTe_2 (tungsten telluride) [F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]





PSEUDO-ELECTROMAGNETIC FIELDS

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]
[Grushin, Venderbos,Vishwanath, Ilan, arXiv:1607.04268]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, arXiv:1607.03491]
[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Strain in Weyl materials

- Strains affect low-energy quasiparticles in Weyl materials [see also talks by M. Franz & M. Vozmediano]

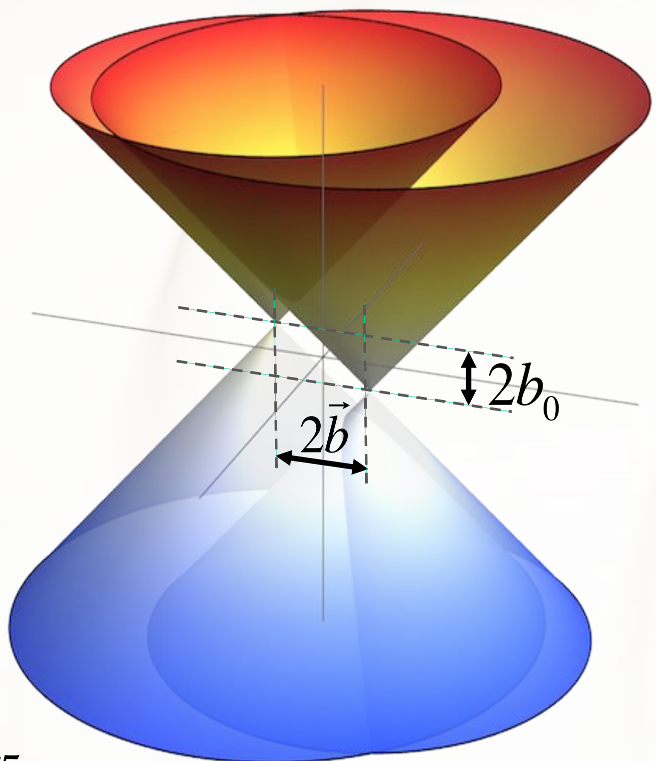
$$H = \int d^3\mathbf{r}\, \bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\,\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

where the components of the chiral gauge fields are

$$A_{5,0} \propto b_0 \left|\vec{b}\right| \partial_{||} u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right| \partial_{||} u_{\perp}$$

$$A_{5,||} \propto \alpha \left|\vec{b}\right|^2 \partial_{||} u_{||} + \beta \sum_i \partial_i u_i$$



The associated pseudo-EM fields are

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0 - \partial_t \vec{A}_5$$

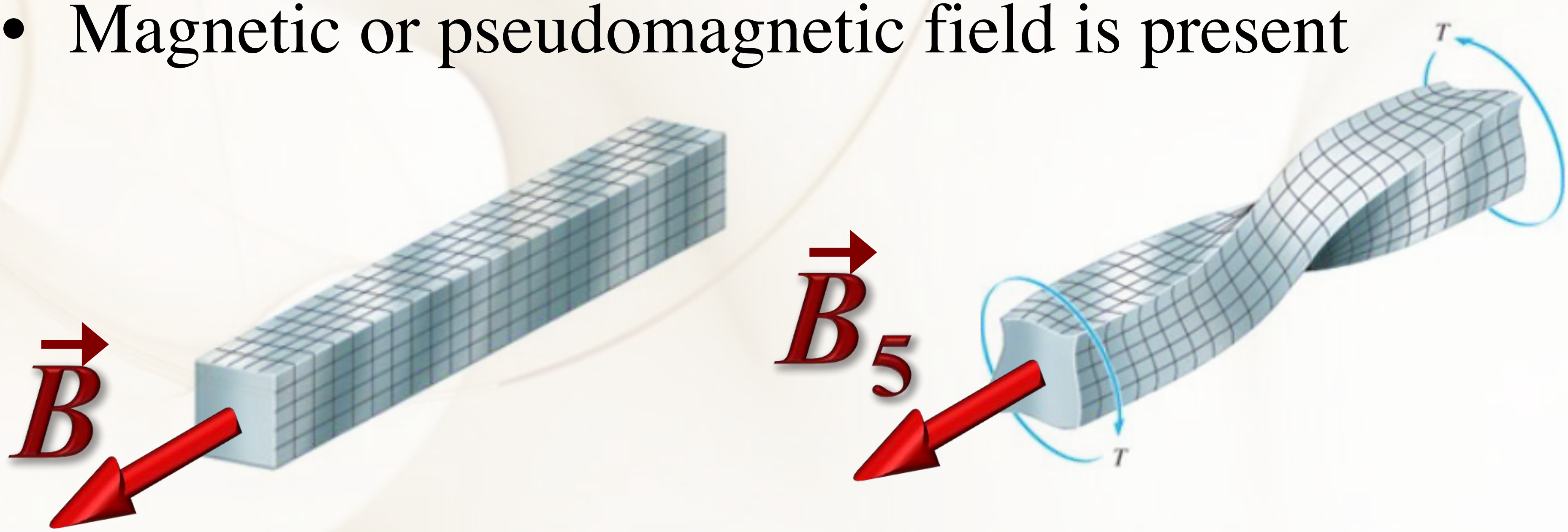
Chiral effects in Weyl materials

- Any qualitative properties of Weyl materials directly sensitive to b_0 and $\vec{b}$?
- Some proposals:
  - Anomalous Hall effect
  - Anomalous Alfven waves
  - Strain/torsion induced CME
  - Strain/torsion induced quantum oscillations
  - Strain/torsion dependent resistance
  - etc.
- Spectrum of chiral (pseudo-)magnetic plasmons

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

General question

- What are the properties of plasmons in magnetized chiral material with $b_0 \neq 0$ and $\vec{b} \neq 0$?
- Chiral matter ($\mu_R \neq \mu_L$)
  - This is the case in equilibrium when $b_0 \neq 0$ $(\mu_5 = -e\, b_0)$
- Magnetic or pseudomagnetic field is present



- In general, $\mathbf{E}_\lambda = \mathbf{E} + \lambda \mathbf{E}_5$ and $\mathbf{B}_\lambda = \mathbf{B} + \lambda \mathbf{B}_5$

MODEL & METHOD

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

Chiral kinetic theory

- Kinetic equation:

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$

$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}\,,\quad \mathbf{v} = \boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}\,,$

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

[see talks by Q. Wang & M. Stephanov]

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Current and chiral anomaly

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right]f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda)\right]f_\lambda$$

$$+e\mathbf{\nabla}\times\int \frac{d^3p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}}\mathbf{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t}+\mathbf{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B})+(\mathbf{E}_5\cdot\mathbf{B}_5)\Big]$$ ✔

$$\frac{\partial\rho}{\partial t}+\mathbf{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B}_5)+(\mathbf{E}_5\cdot\mathbf{B})\Big]$$ ✘

Consistent definition of current

- Additional Bardeen-Zumino term is needed,

$$\delta j^\mu = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda} A^5_\nu F_{\rho\lambda}$$

- In components,

$$\delta\rho = \frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{A}^5 \cdot \mathbf{B})$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c} A^5_0 \mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c}(\mathbf{A}^5 \times \mathbf{E})$$

- Its role and implications:
  - Electric charge is conserved locally ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)

Collective modes

We search for plane-wave solutions with

$$\mathbf{E}' = \mathbf{E}e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}, \quad \mathbf{B}' = \mathbf{B}e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

and the distribution function $f_\lambda = f_\lambda^{(\mathrm{eq})} + \delta f_\lambda$,

where

$$\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

The polarization vector & susceptibility tensor:

$$P^m = i\frac{J'^m}{\omega} = \chi^{mn} E'^n$$

The plasmon dispersion relations follow from

$$\det\left[\left(\omega^2 - c^2k^2\right)\delta^{mn} + c^2k^m k^n + 4\pi\omega^2\chi^{mn}\right] = 0$$

RESULTS

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

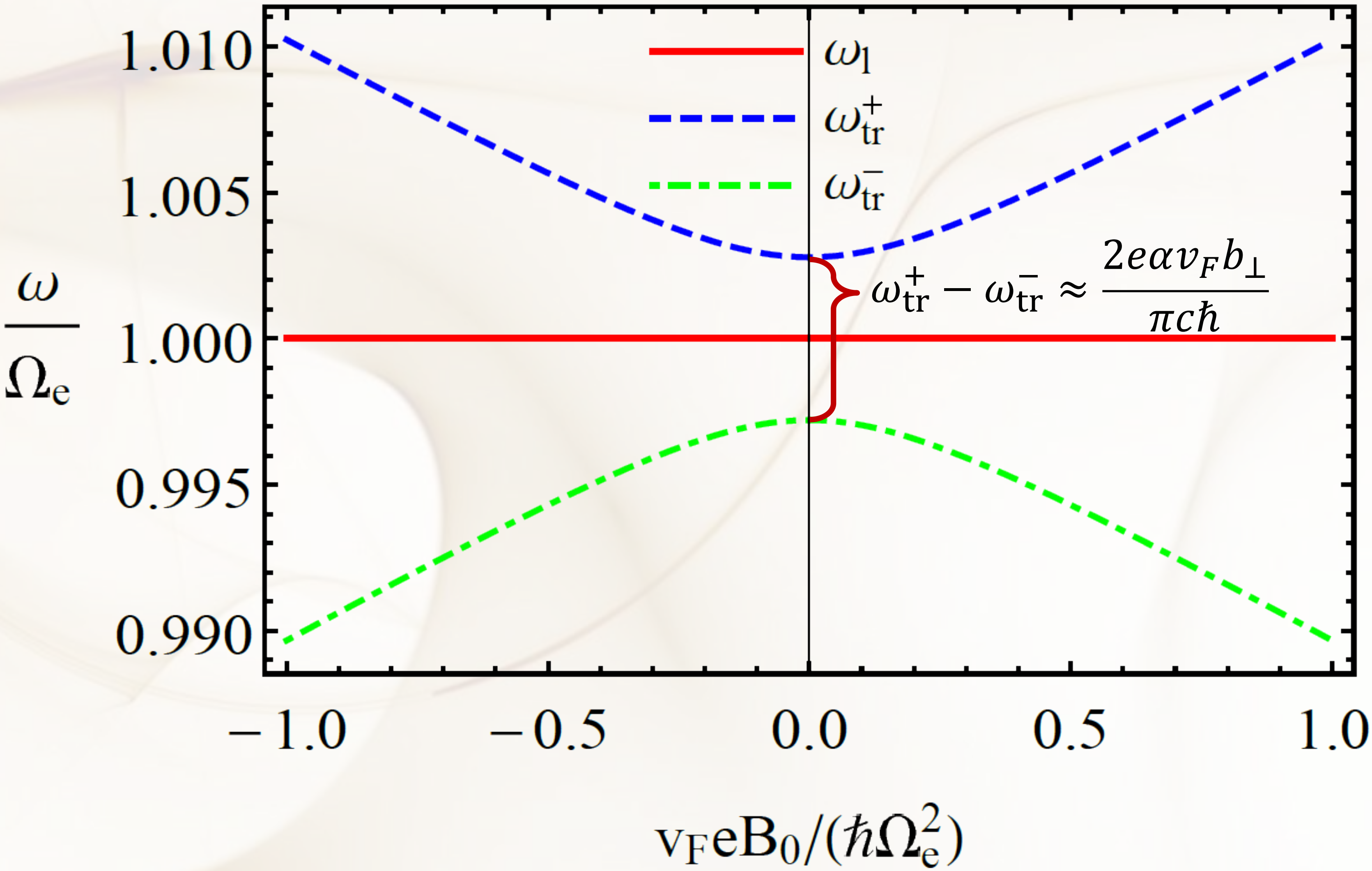
and

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\Bigg\{9\hbar^2 b_\perp^2 + \Bigg[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_{||} - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\Bigg]^2\Bigg\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

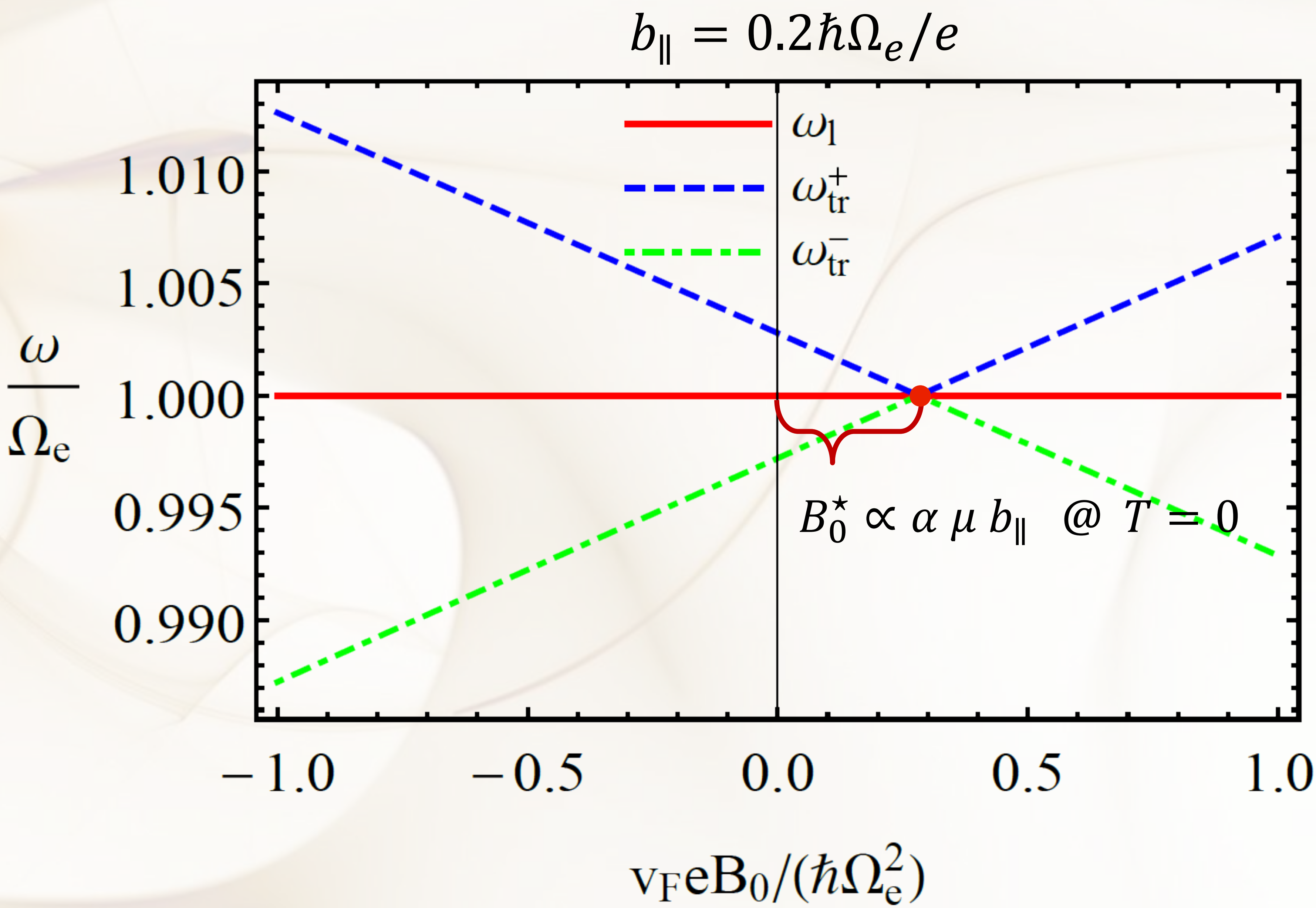
Plasmon frequencies, $\vec{B} \perp \vec{b}$

$b_\perp = 0.2\hbar\Omega_e/e$



[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

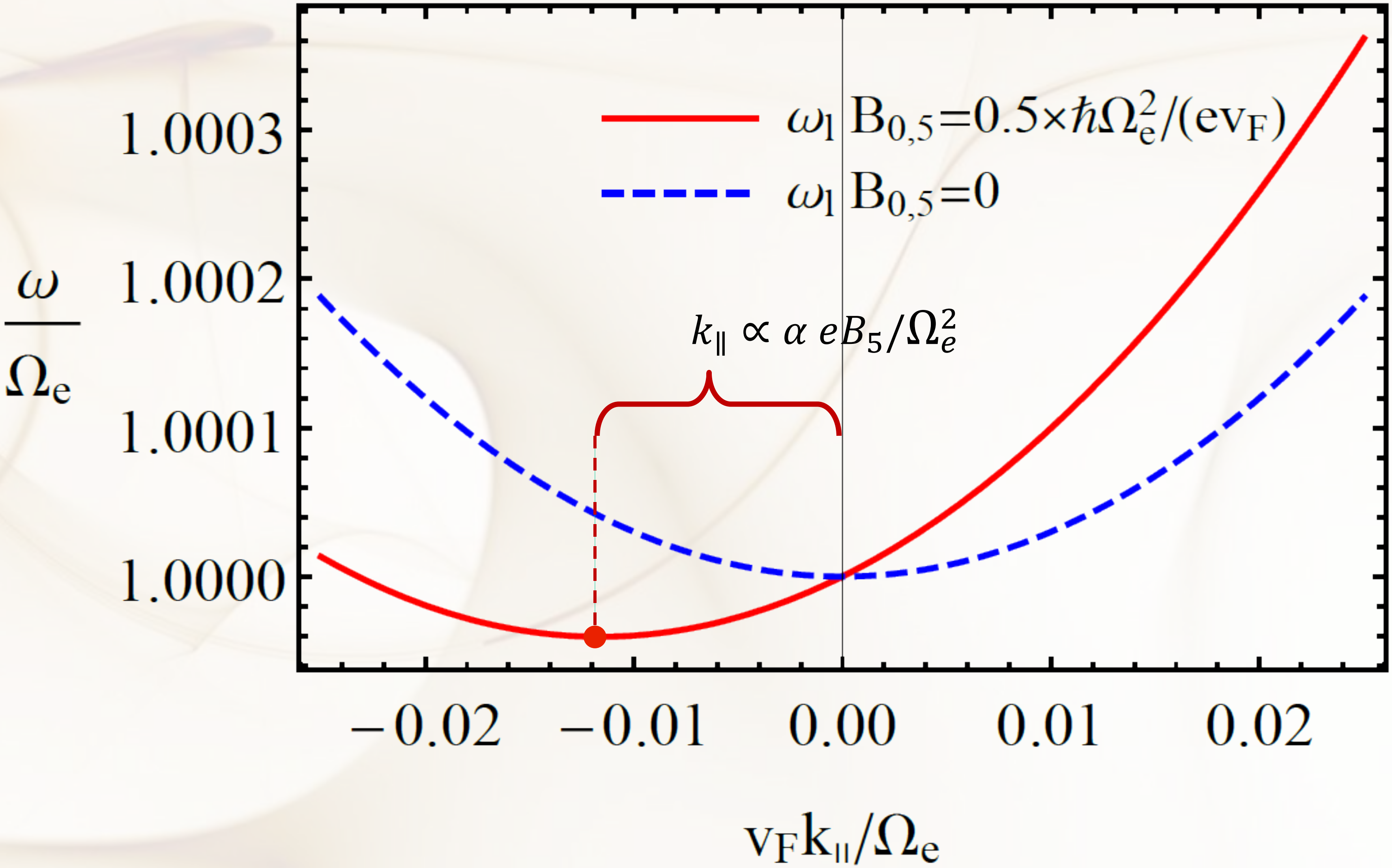
Plasmon frequencies, $\vec{B} \parallel \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]

Plasmons with $\vec{k} \neq 0, \vec{k} \parallel \vec{B}, \vec{B}_5$

- The longitudinal mode is sensitive to $\vec{B}_5$

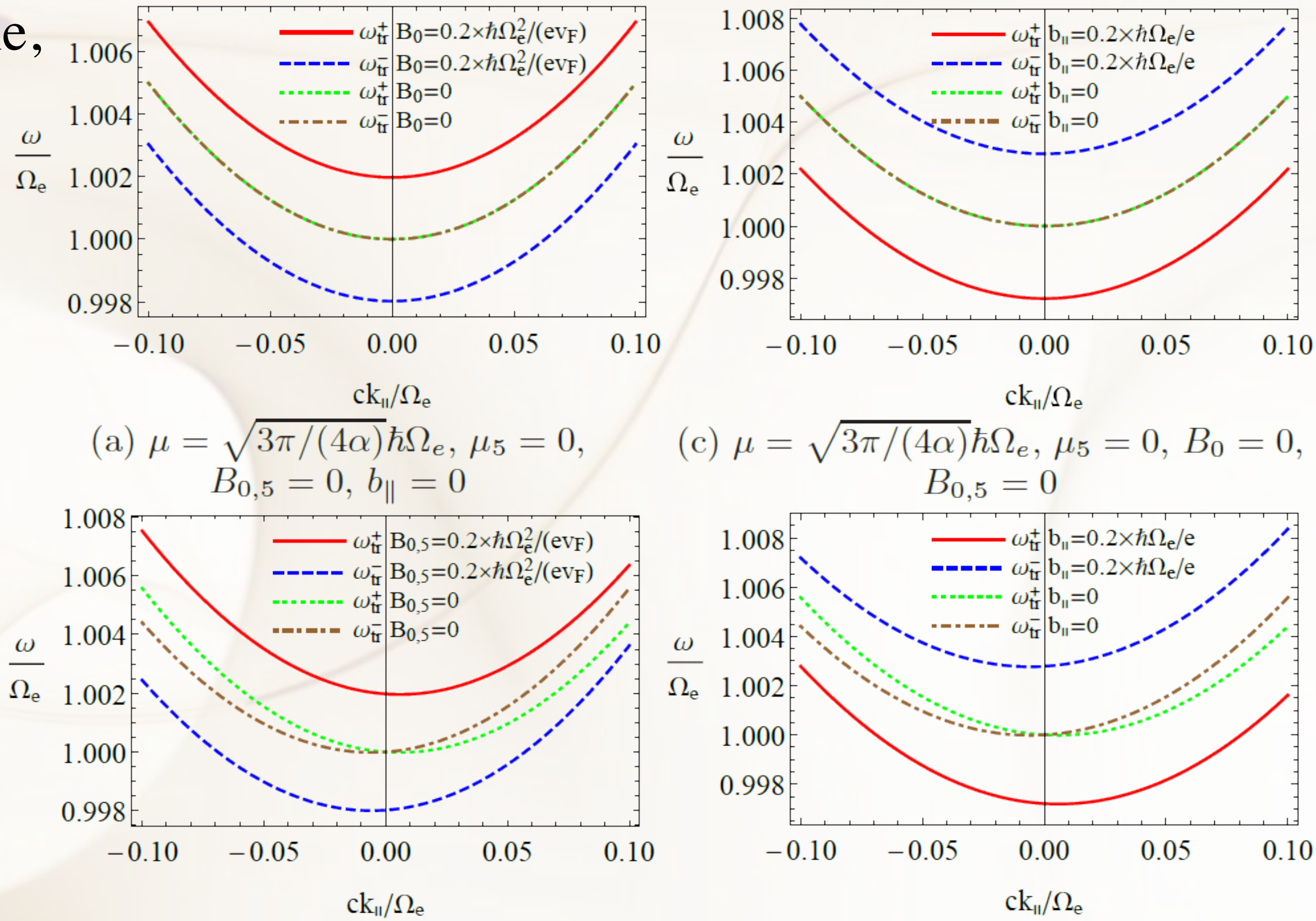


[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1610.07625]

Plasmons with $\vec{k} \neq 0, \vec{k} \parallel \vec{B}, \vec{B}_5$

The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$, or (iii) $b_{\parallel} \neq 0$, or (iv) $b_{\perp} \neq 0$.

For example,



(a) $\mu = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ \mu_5 = 0,\ B_{0,5} = 0,\ b_{\parallel} = 0$

(c) $\mu = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ \mu_5 = 0,\ B_0 = 0,\ B_{0,5} = 0$

(e) $\mu = 0,\ \mu_5 = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ B_0 = 0,\ b_{\parallel} = 0$

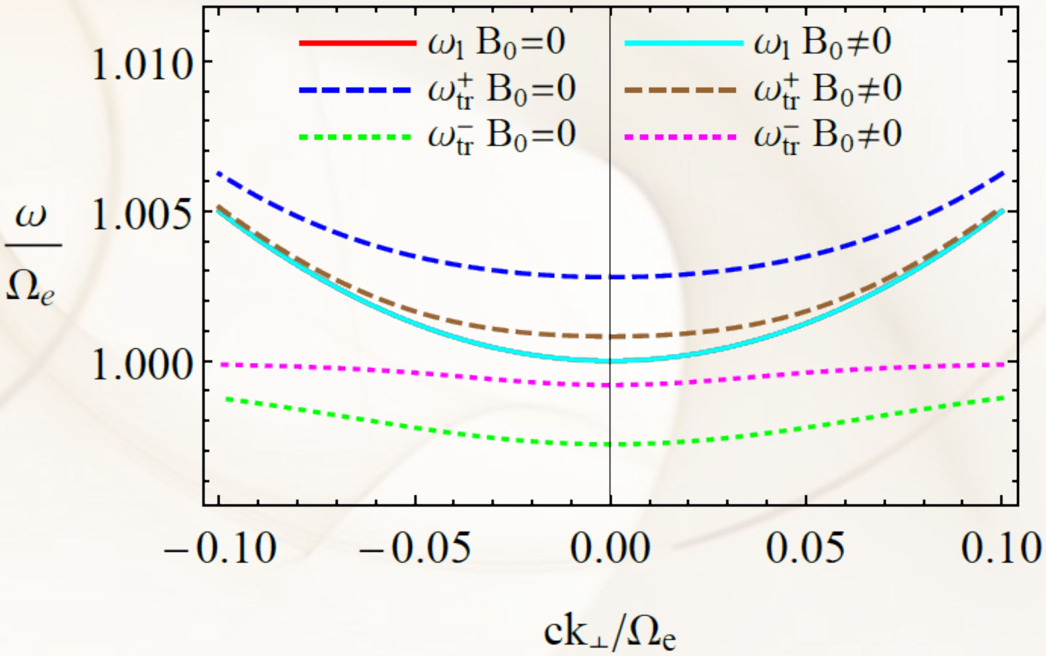
(f) $\mu = 0,\ \mu_5 = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ B_0 = 0,\ B_{0,5} = 0$

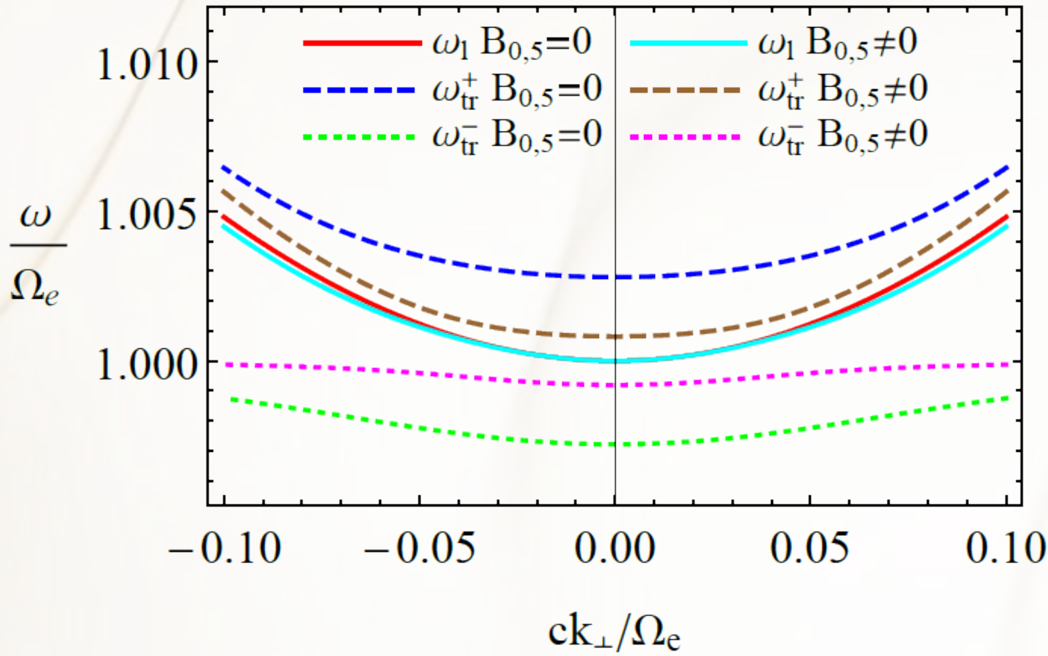
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

Plasmons with $\vec{k} \neq 0, \vec{k} \perp \vec{B}, \vec{B}_5$

The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$

$$b_{\parallel} = 0.2\hbar\Omega_e/e$$



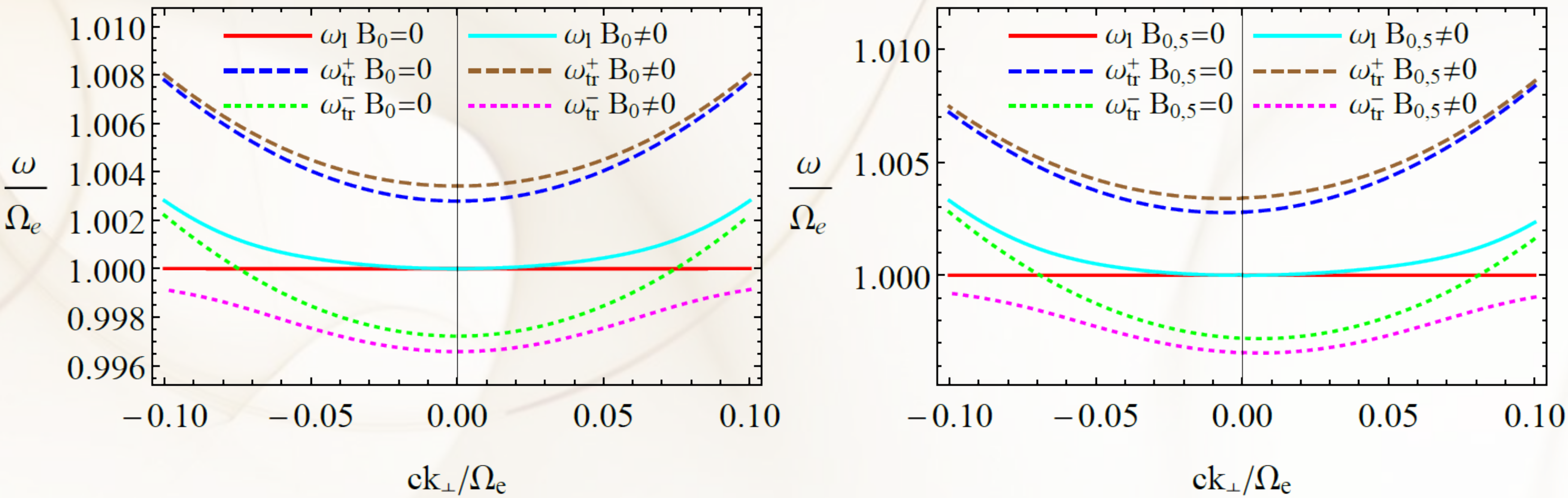


[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

Plasmons with $\vec{k} \neq 0, \vec{k} \perp \vec{B}, \vec{B}_5$

The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$

$$b_\perp = 0.2\hbar\Omega_e/e$$



[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1611.05470]

Summary

- Consistent chiral kinetic theory is needed
- Chiral magnetic plasmons (χMPs) are sensitive to local charge (non-)conservation
- Properties of χMPs carry information about b_0 and $\vec{b}$
- χMPs are not only due to the oscillation of *electric* charge, but also *chiral* charge