ARIZONA STATE UNIVERSITY

Hadron & Nuclear Physics
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KEK

Physics of strong magnetic field

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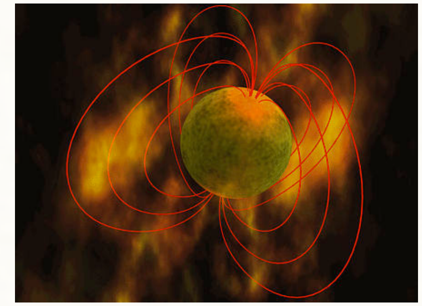
Range of topics

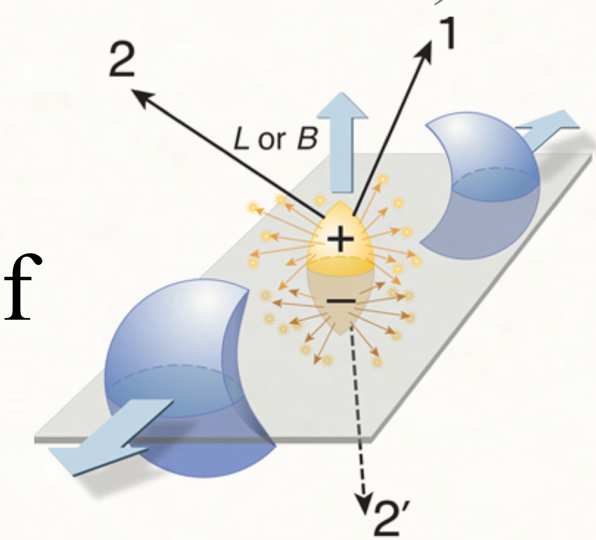
- What is covered in this talk
  - QCD vacuum in a super-strong magnetic field
  - Chiral effects in magnetized QCD plasma
- What is not covered in this talk
  - Thermodynamic properties of magnetized QCD plasma [talk by Andreas Schaefer]
  - Transport properties of magnetized QCD plasma [talk by Koichi Hattori]
  - Meson spectra in strong magnetic field [talk by Kei Suzuki]
  - Emission spectra from magnetized QCD plasma [...]

Why magnetic field?

- Magnetized hadronic and quark matter may exist inside compact stars
  - **10^{10} to 10^{16} G** **(10 keV to 10 MeV)**



- Magnetized quark-gluon plasma is produced in heavy-ion collisions (although $\Delta t \approx 10^{-24}$ s)
  - **10^{18} to 10^{19} G** **(~ 100 MeV)**



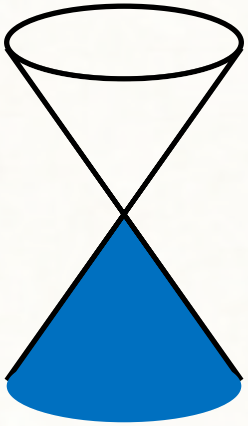
- Magnetic field is a useful probe of nonperturbative QCD properties
  - **10^{21} G** **(~ 2 GeV, or ~ 0.1 fm)**

Strong B-field in QCD

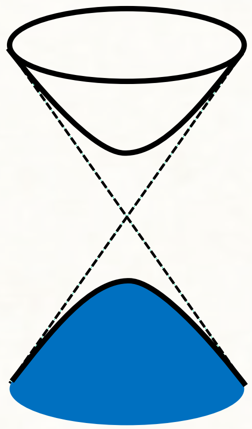
- QCD is strongly coupled & nonperturbative
- Theoretical tools that provide insight:
  - Lattice QCD
  - High-energy (weak-coupling) expansion
  - Large N_c expansion
  - High temperature limit ($T \gg \Lambda_{QCD}$)
  - High density limit ($\mu \gg \Lambda_{QCD}$)
- Strong magnetic field B is yet another tool
  - it allows to probe short distances $\ell \sim 1/\sqrt{|eB|}$
  - it induces gluon screening effects, $M_g^2 \propto \alpha_s |eB|$

QCD vacuum and *B*-field

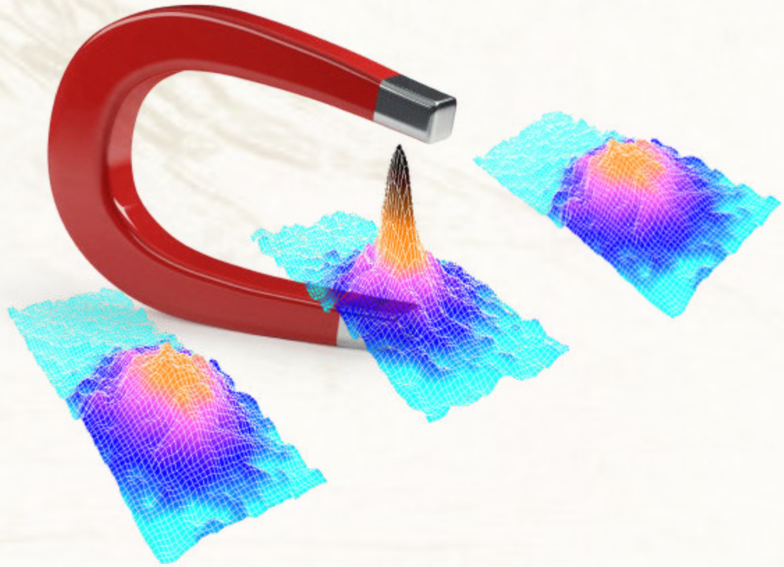
- In the chiral limit, the perturbative QCD vacuum is a semimetal
  - no energy gap and *zero* density of states @ E=0



- Because of strong coupling, the QCD ground state becomes an insulator
  - A particle-hole condensate forms
  - A dynamical gap (mass) opens in the spectrum



- Perturbative QCD vacuum in a magnetic field is more like a metal, i.e.,
  - no energy gap, but a *nonzero* density of states @ E=0



MAGNETIC CATALYSIS

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]
[Shovkovy, Lect. Notes Phys. 871 (2013) pp. 13-49]

Landau spectrum at B≠0

- Dirac equation with massless fermions

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla}+ie\vec{A}\right)\right]\Psi = 0$$

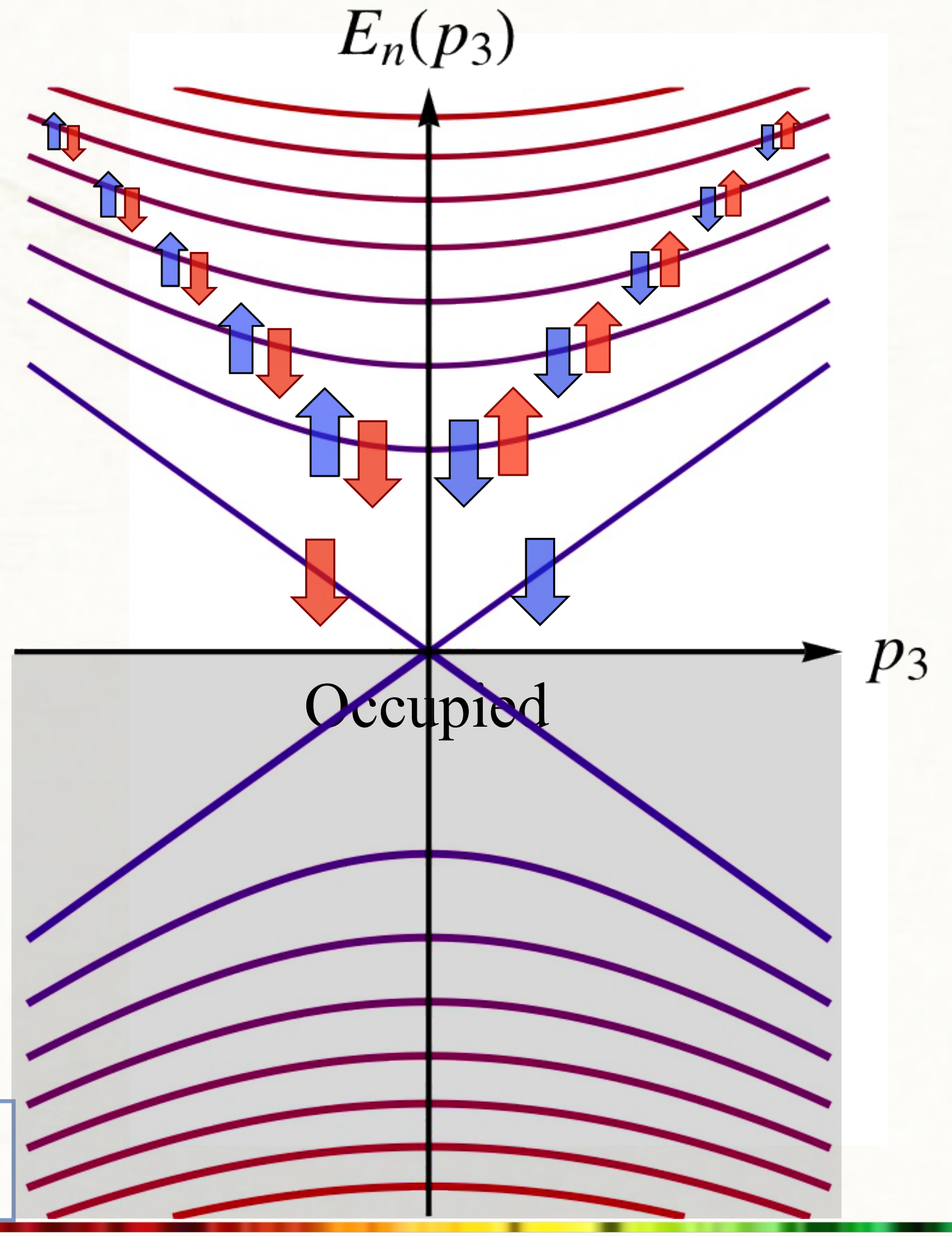
- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

$s = \pm 1/2$ (spin)

where $n = s + k + \frac{1}{2}$

$k = 0, 1, 2, \ldots$ (orbital)

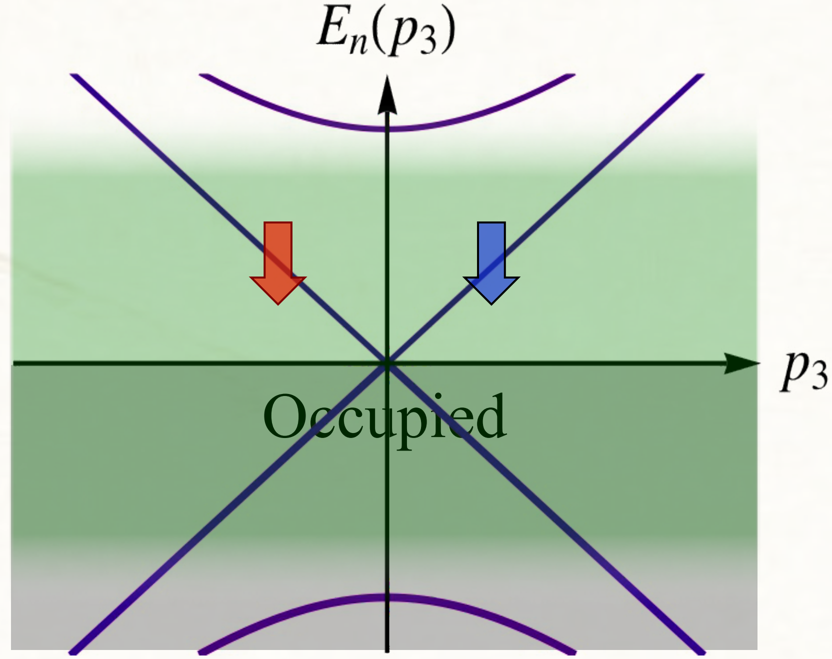


Dimensional reduction

- Low-energy is due to n=0 Landau level

$$n=0: \qquad E_0^{(3+1)}(p_3) = \pm p_3$$

$$\left(k=0, s=-\frac{1}{2}\right)$$



- This is (1+1)D spectrum!

- Propagator also looks like in (1+1)D:

$$S(p_\parallel) \approx i\, e^{-p_\perp^2 l^2} \frac{\hat{p}_\parallel + m}{\hat{p}_\parallel + m} \underbrace{\left(1 - i\gamma^1\gamma^2\right)}, \text{where } \hat{p}_\parallel = p_0\gamma^0 - p_3\gamma^3$$

$s = -1/2$ spin projector

Magnetic catalysis: idea

- Low-energy fermion dynamics is dimensionally reduced

$$D \to D-2$$

- Nonzero density of states at $E = 0$

$$\left.\frac{dn}{dE}\right|_{E\to 0} = \frac{|eB| N_f}{4\pi^2}$$

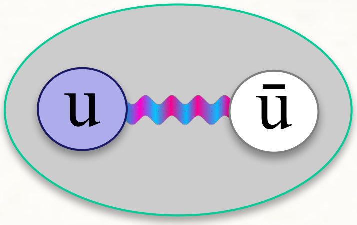
- Attractive interaction → symmetry breaking
  (reminiscent of superconductivity…)

[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994)]

[Shovkovy, Lect. Notes Phys. **871**, 13 (2013)]

Universality of magnetic catalysis

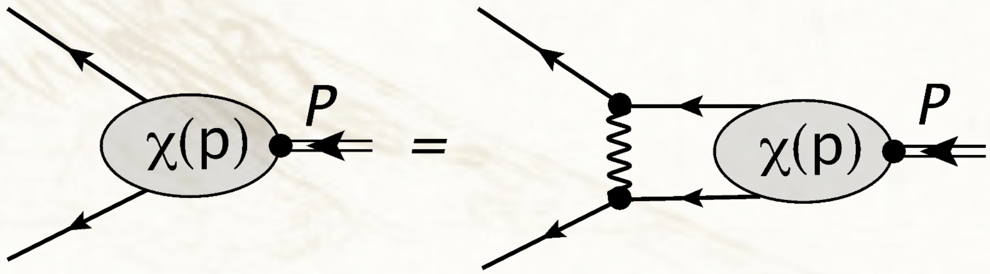
- **Input**
  - Spin-½ charged particles and B≠0
  - Attractive particle-antiparticle interaction
- **Output**
  - Dimensional reduction D→D-2 @ low energies
  - particle-antiparticle bound states form
  - Symmetry breaks down
  - Dynamical mass is generated



MAGNETIC CATALYSIS IN QCD

Bethe-Salpeter equation

- Particle-antiparticle pairing (pion channel)



- Massless bound state (pion)

$$\chi(r, P \to 0) \propto \Psi(r_{\parallel}), \quad \text{where} \quad r_{\parallel} = (it, z)$$

and

$$\left[-\nabla^2_{r_{\parallel}} + m^2_{\text{dyn}} + V(r_{\parallel})\right]\Psi(r_{\parallel}) = 0$$

i.e., a Schrödinger equation for a bound state with

$$E_{\text{b}} = -m^2_{\text{dyn}}$$

[Gusynin, Miransky, Shovkovy, Phys. Rev. **66** (2002) 045006]

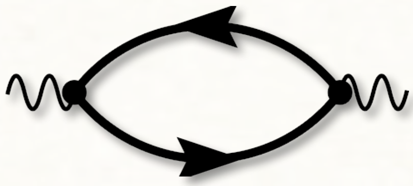
Magmetic catalysis in QCD

- Screening effects in gluon interaction

$$\Pi^{\mu\nu} \cong -\frac{\alpha_s}{\pi}\left(k_{\parallel}^{\mu}k_{\parallel}^{\mu} - g_{\parallel}^{\mu\nu}k_{\parallel}^{2}\right)\sum_f \frac{|e_f B|}{k_{\parallel}^2} \quad \text{for} \quad m_f^2 \ll |k_{\parallel}^2| \ll |eB|$$

which gives an effective gluon mass

$$M_g^2 \cong \frac{\alpha_s}{\pi}\sum_f |e_f B|$$



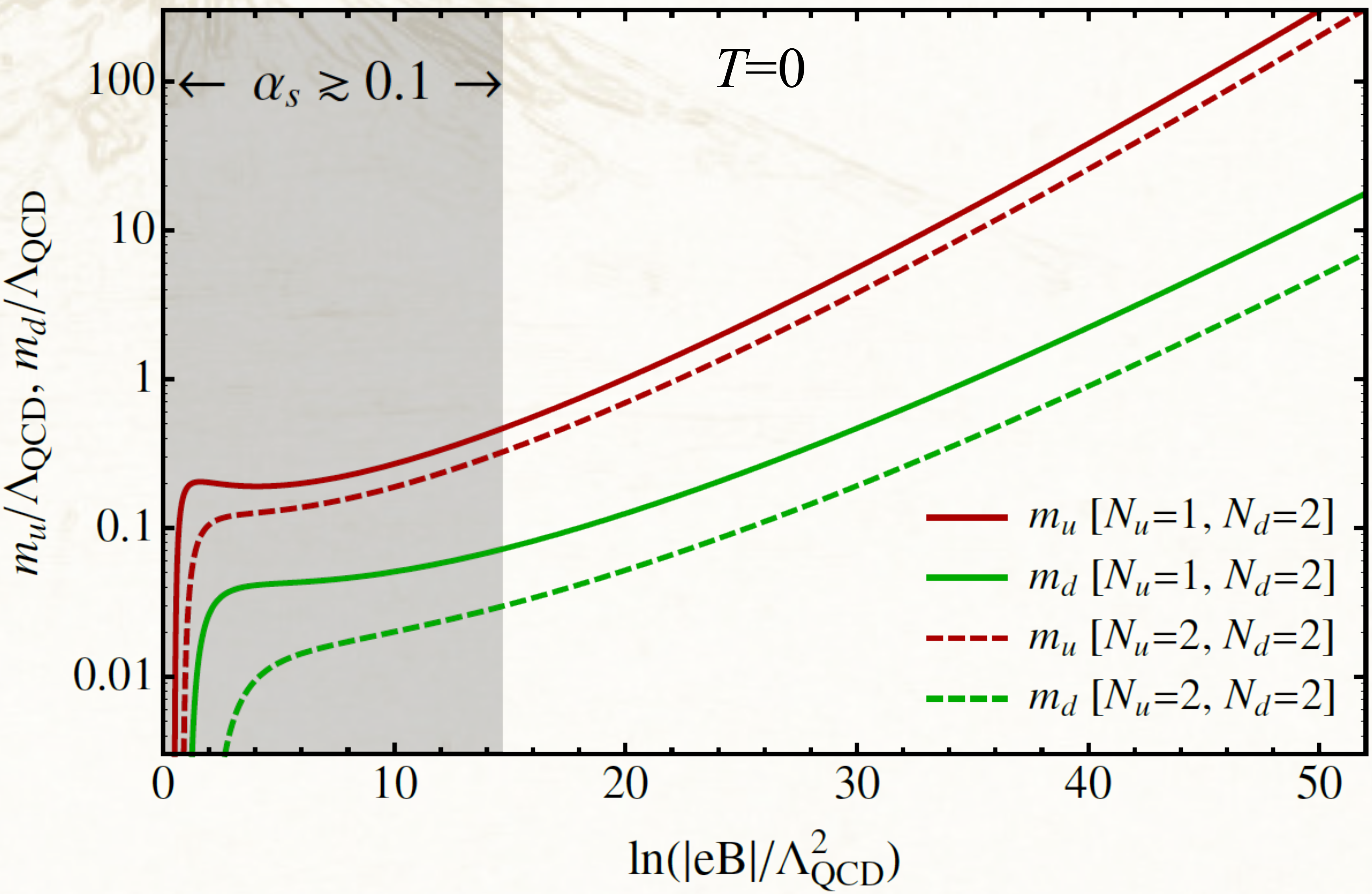
- The resulting dynamical quark mass (T=0)

$$m_f^2 \propto |e_f B| \alpha_s^{2/3} \exp\left(-\frac{4\pi N_c}{\alpha_s (N_c^2 - 1)\ln(C/\alpha_s)}\right)$$

[Miransky, Shovkovy, Phys. Rev. D **66** (2002) 045006]

Quark mass vs. B

- Approximate dynamical mass ($\sqrt{|eB|} \gg \Lambda_{\rm QCD}$)



[Miransky & Shovkovy, Phys. Rev. D **66** (2002) 045006]

Pions

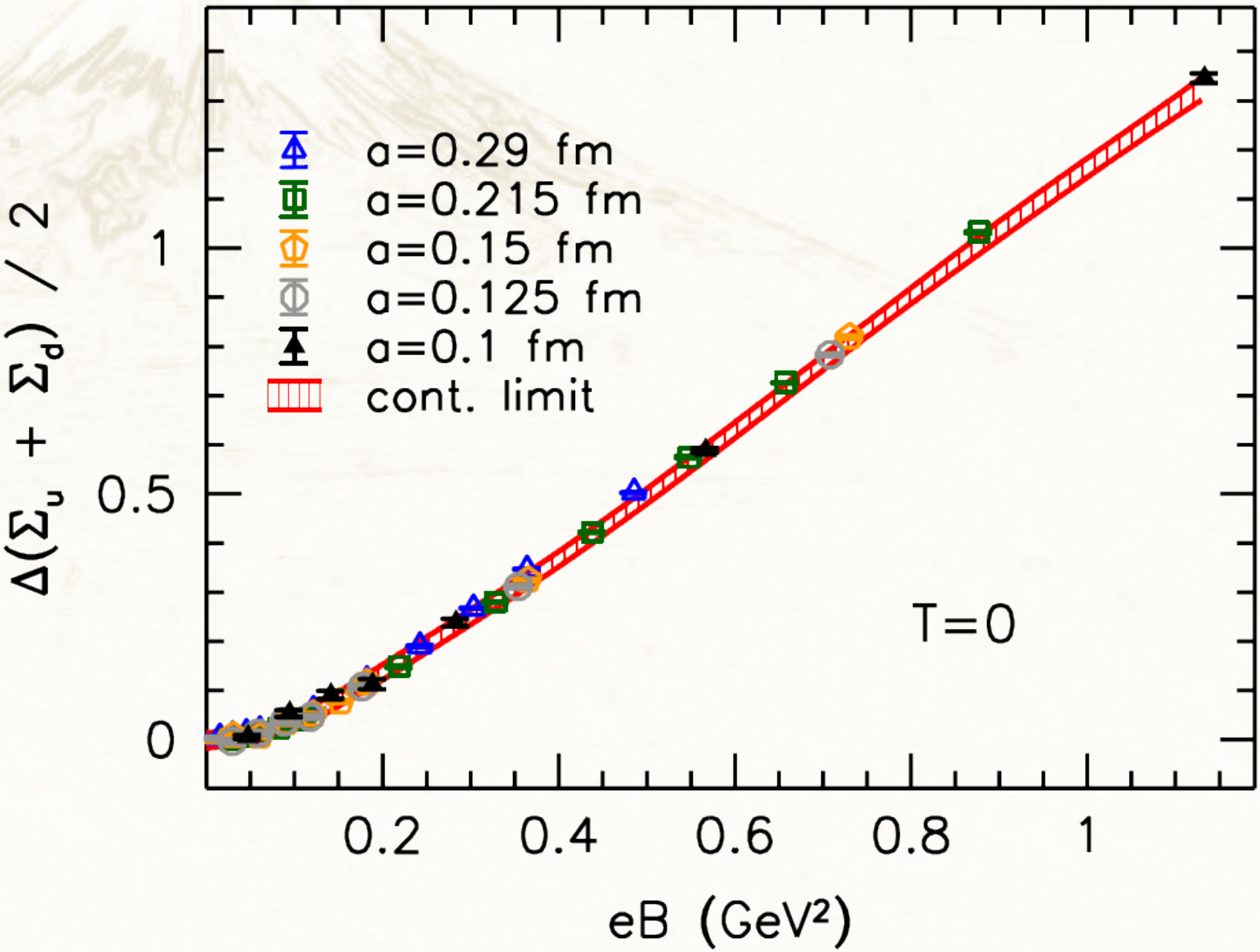
- Only neutral mesons ($N_u^2 + N_d^2 - 1$) remain the NG bosons at large B
- Pion/kaon decay constant: $f_\pi^2 \propto N_c|eB|$
- Pion dispersion relations are anisotropic:

$$v_\parallel \approx 1$$

$$v_\perp \approx \frac{m_d^2}{|eB|} ln \frac{|eB|}{m_d^2} \ll 1$$

[Miransky, Shovkovy, Phys. Rev. D **66** (2002) 045006]

Chiral condensate in lattice QCD



[Bali et al., Phys. Rev. D**86**, 071502 (2012)]

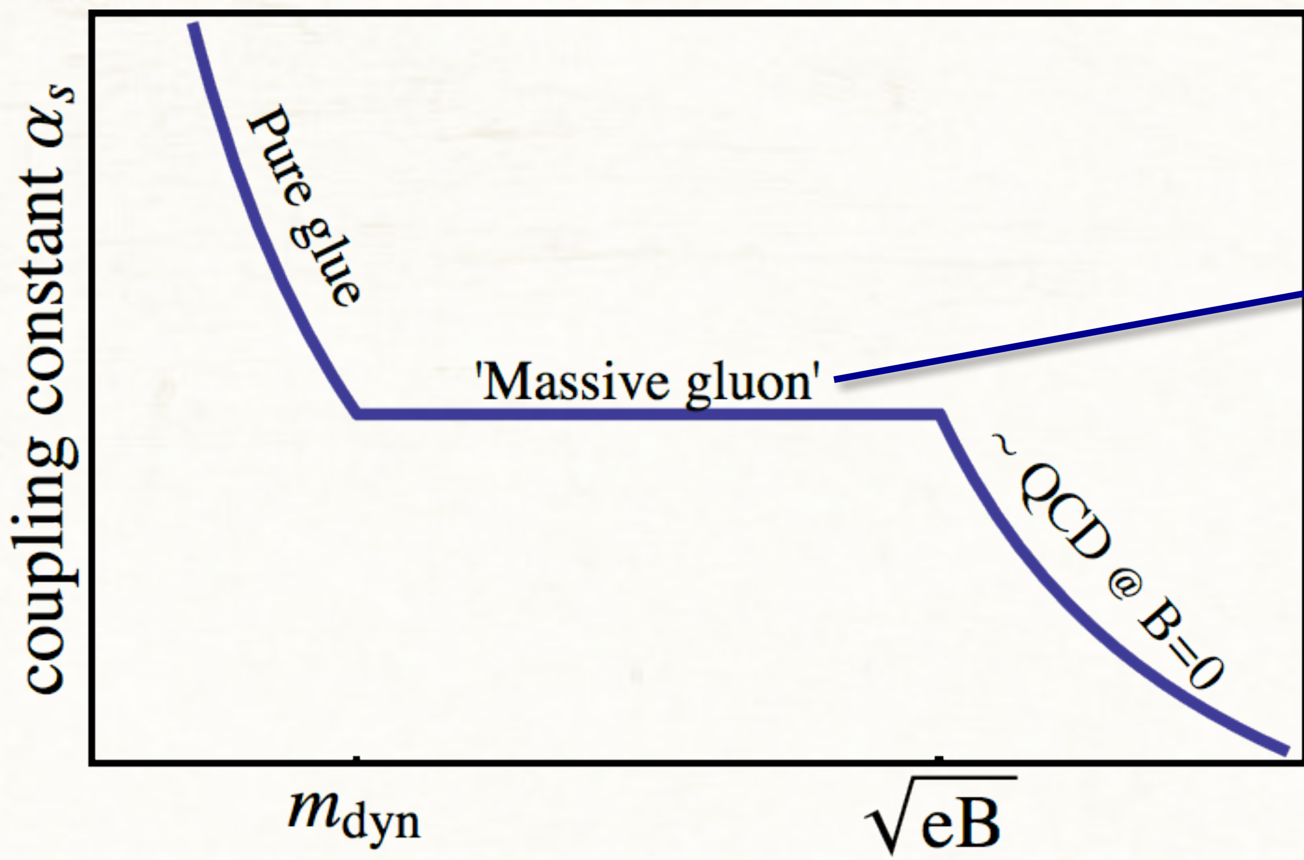
GLUON SECTOR: ANISOTROPIC CONFINEMENT

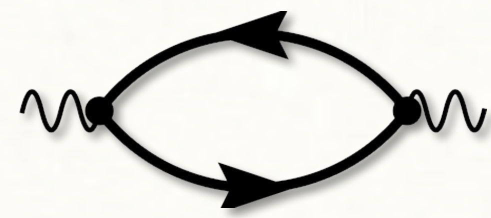
QCD in a magnetic field

- Weak coupling, (semi-)perturbative regime:

$$\sqrt{|eB|} >> \Lambda_{QCD}$$

- Running QCD coupling





$$m_f^2 << |k_\parallel^2| << \sqrt{|eB|}$$

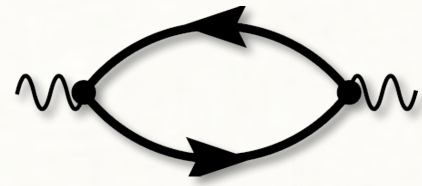
$$M_g^2 \approx \frac{\alpha_s}{\pi} \sum_f |e_f B|$$

[Miransky & Shovkovy, Phys. Rev. D 66 (2002) 045006]

Anisotropic confinement in QCD

- Low-energy gluodynamics ($p << m_{\text{dyn}}$):

$$L \approx \frac{1}{2}\sum_{a=1}^{N_c^2-1}\left(\vec{E}_\perp^a\cdot\vec{E}_\perp^a + \varepsilon E_z^a E_z^a - \vec{B}_\perp^a\cdot\vec{B}_\perp^a - B_z^a B_z^a\right)$$



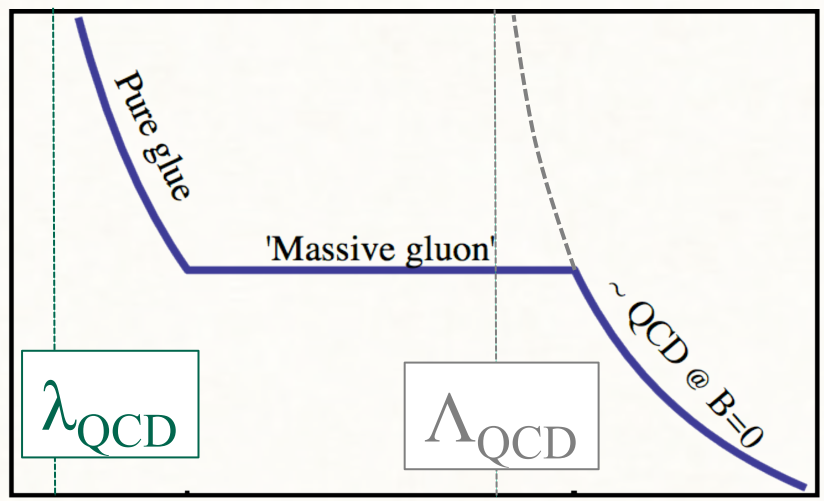
where the chromo-dielecric constant is given by

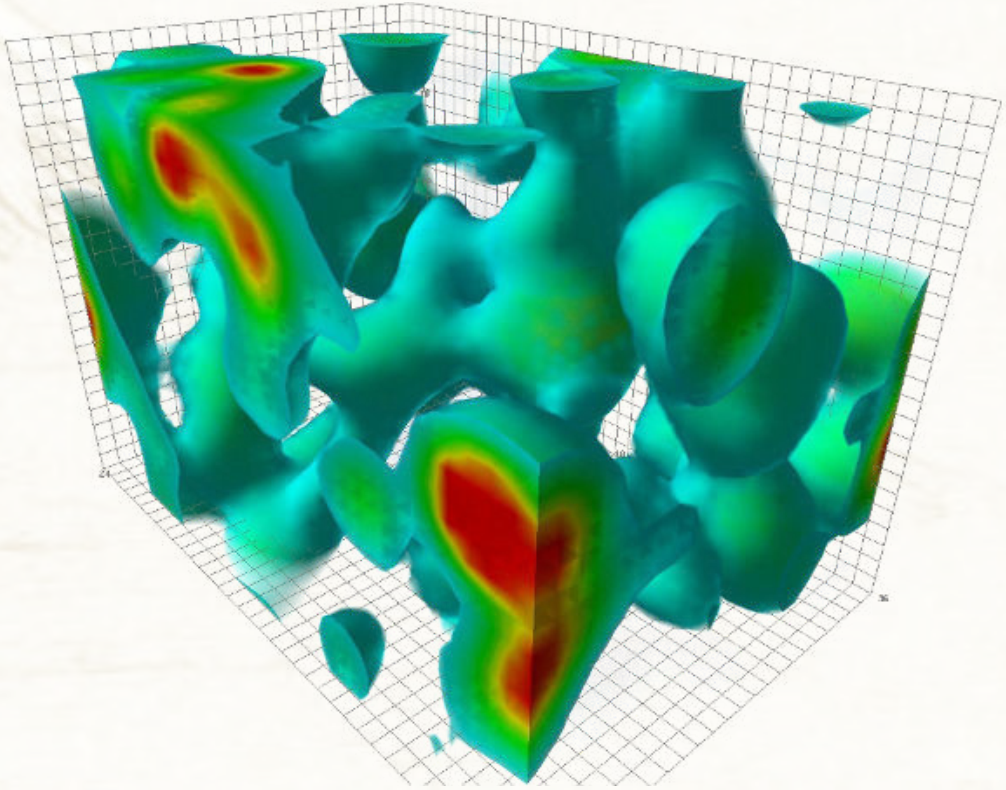
$$\varepsilon \approx 1 + \frac{\alpha_s}{6\pi}\sum_{f=1}^{N_f}\frac{|q_f B|}{m_f^2} >> 1$$

The new confinement scale is

$$\lambda_{\text{QCD}} \approx m_d\left(\frac{\Lambda_{\text{QCD}}}{\sqrt{|eB|}}\right)^{\frac{11N_c-2N_f}{11N_c}}$$

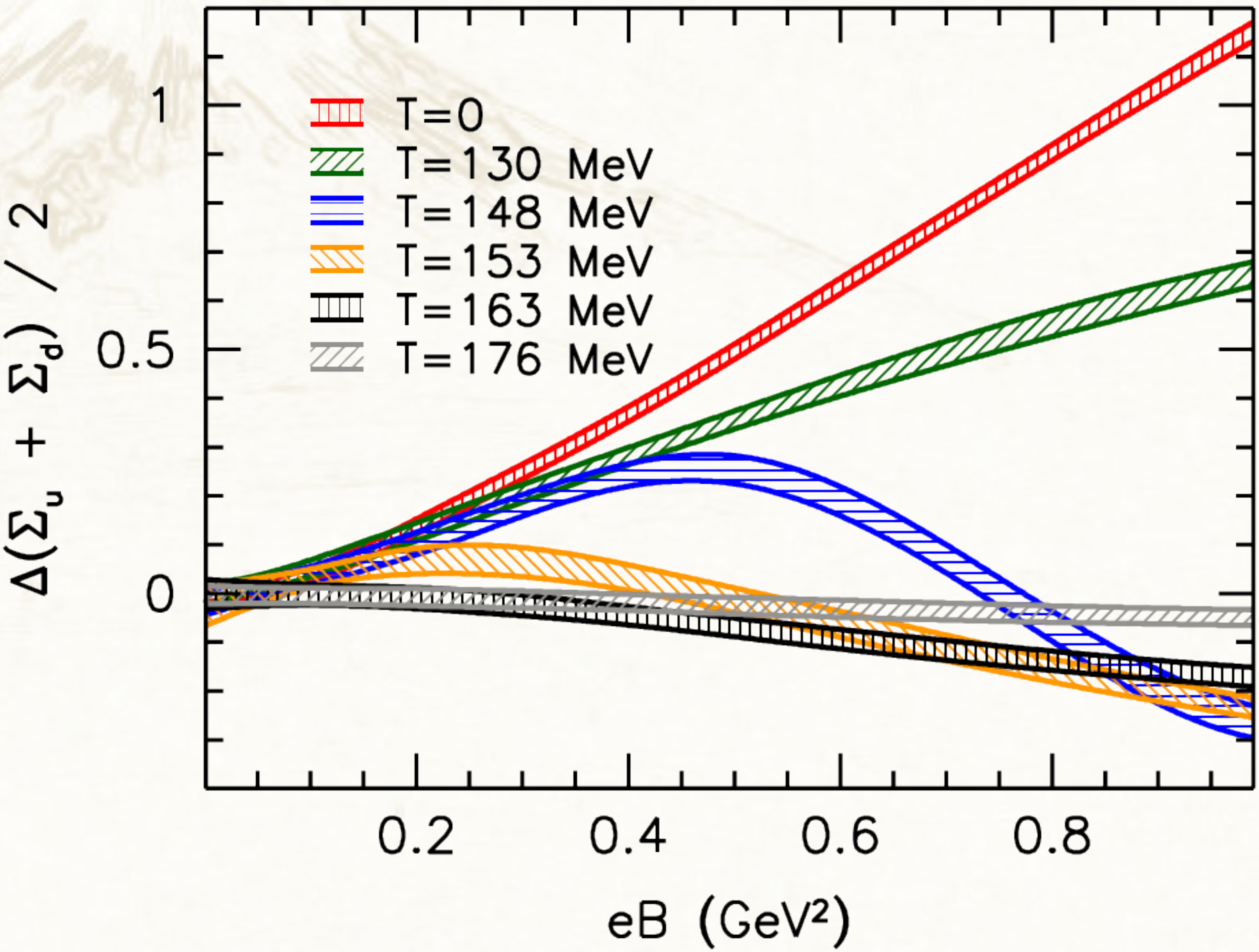
Note that $\lambda_{\text{QCD}} \ll \Lambda_{\text{QCD}}$




Credit: Centre for the Subatomic Structure of Matter, University of Adelaide

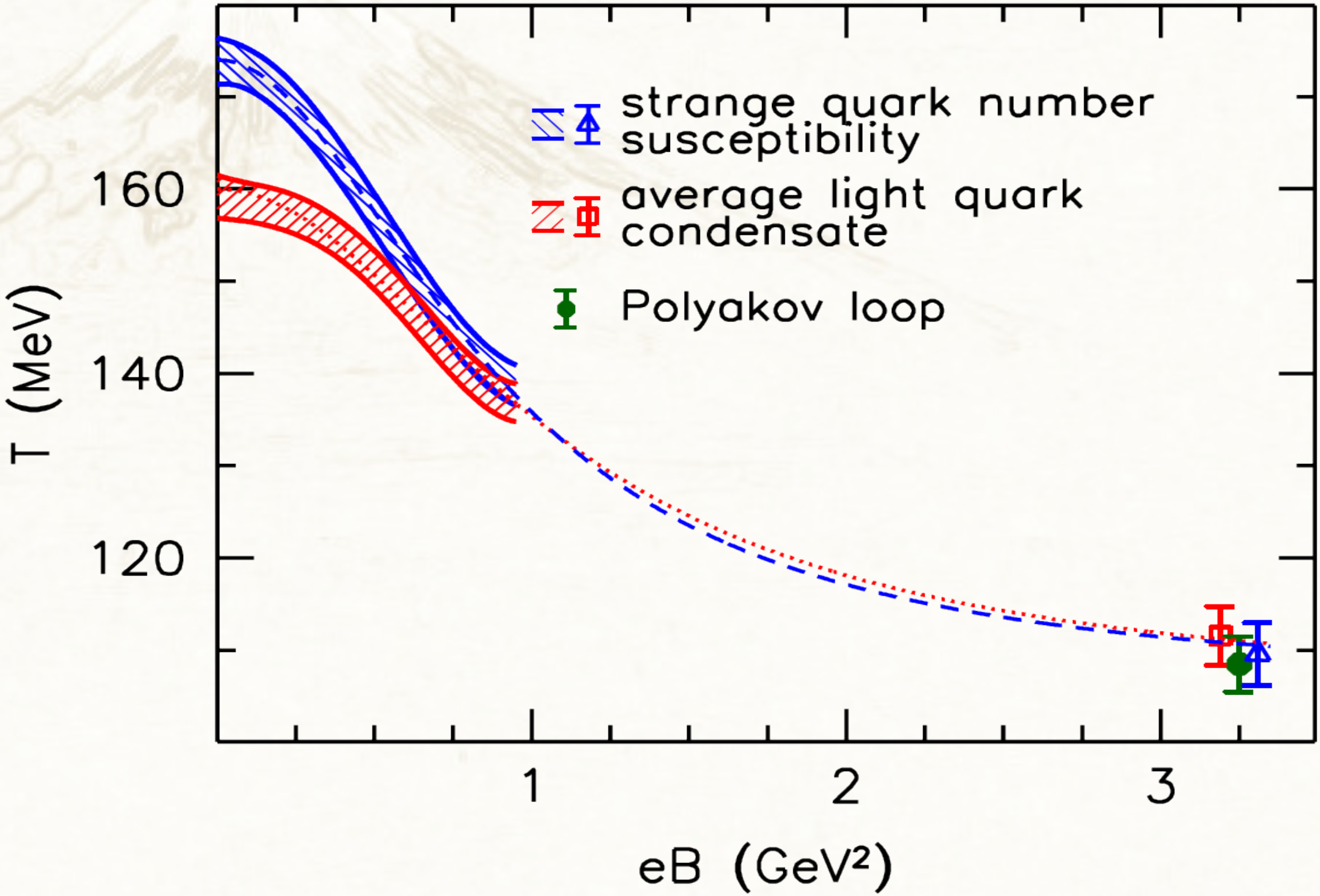
INVERSE MAGNETIC CATALYSIS IN QCD

(Inverse) Catalysis at T≠0?



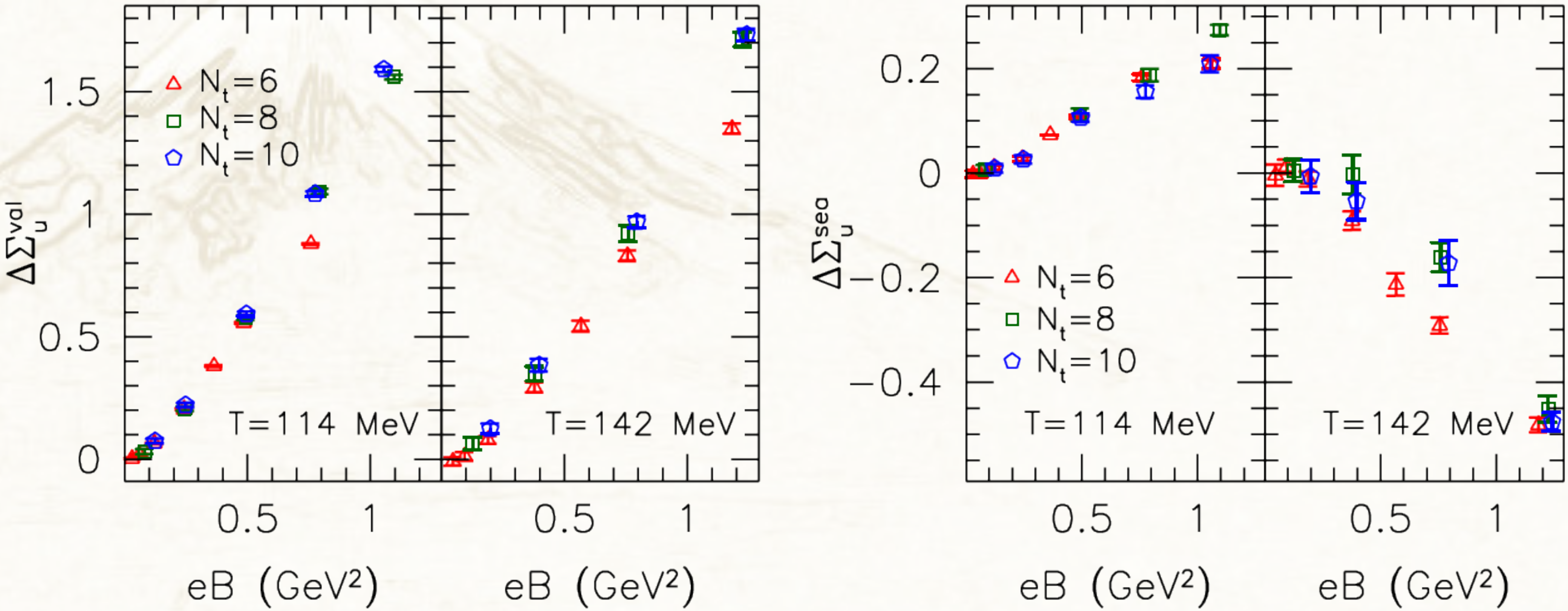
[Bali et al., Phys. Rev. D**86**, 071502 (2012)]

Inverse catalysis: T_c vs. B



[Bali et al., JHEP 02, 044 (29012)], [Bali et al., PRD**86**, 071502 (2012)], [G. Endrodi, JHEP 1507 (2015) 173]

Valence vs. sea

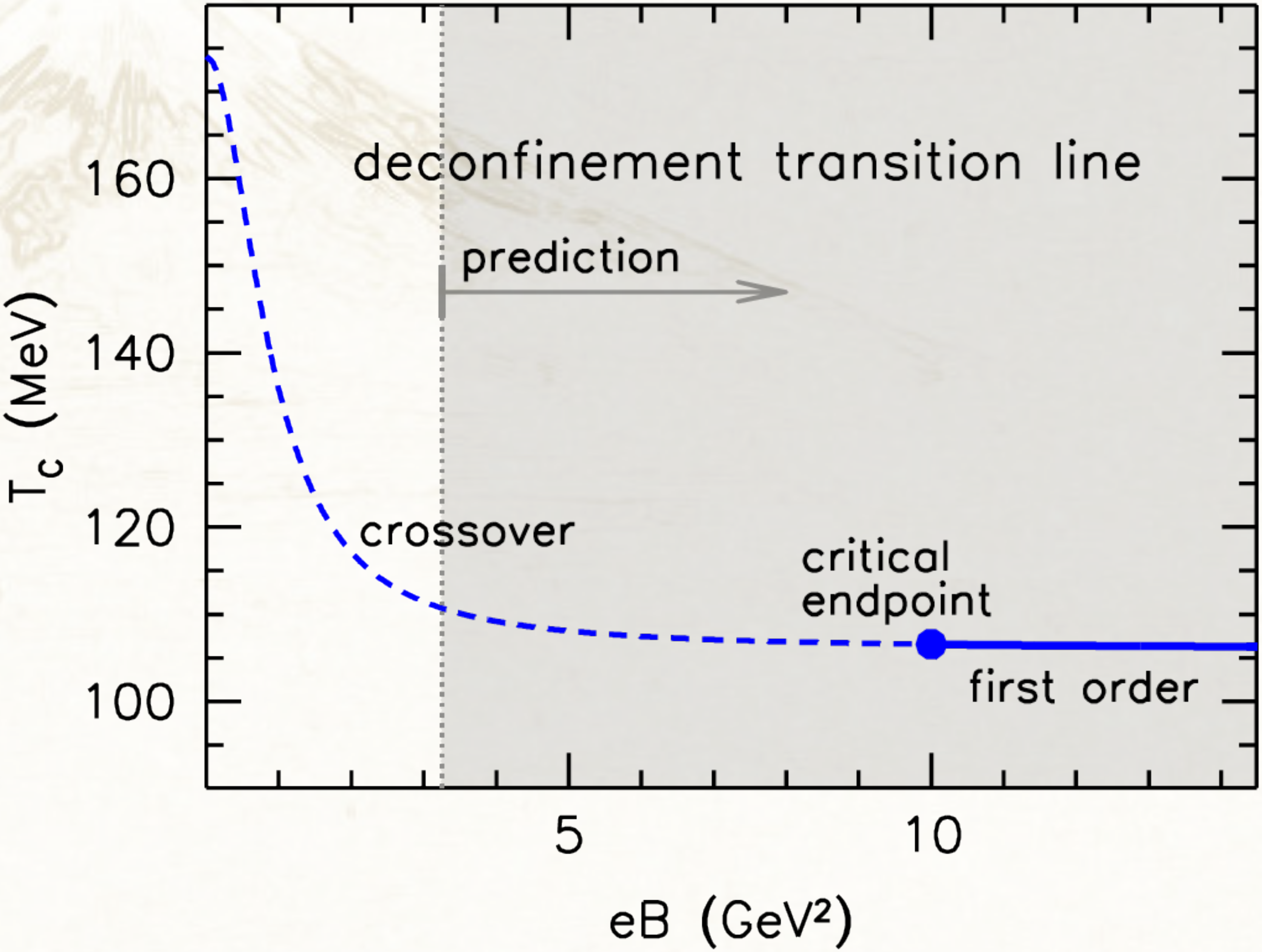


[Bruckmann, G. Endrodi, T. G. Kovacs, JHEP 04 (2013) 112]

- Gluon screening (?)
- Polyakov loops (?)

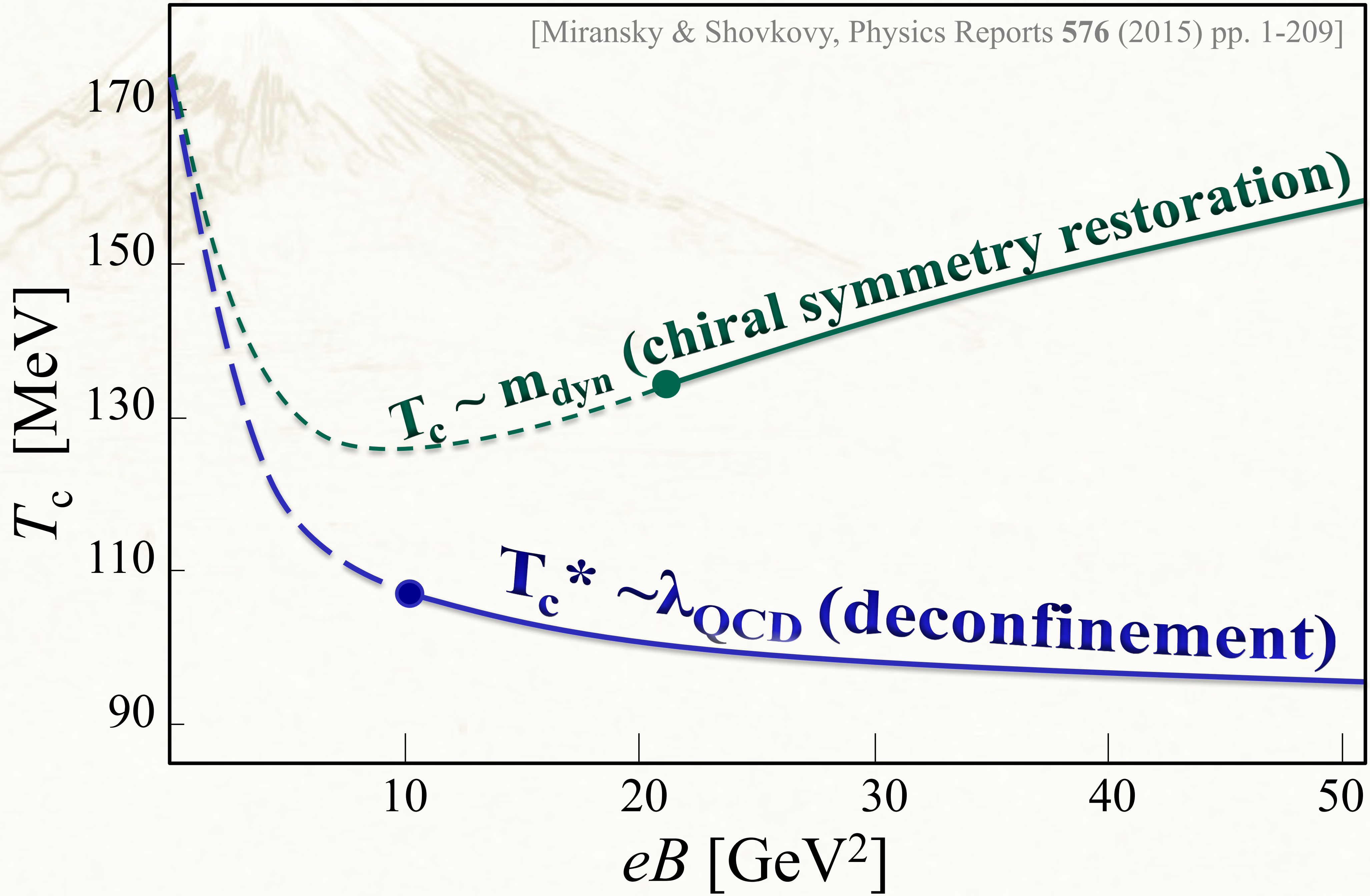
} or, perhaps, something else (?)

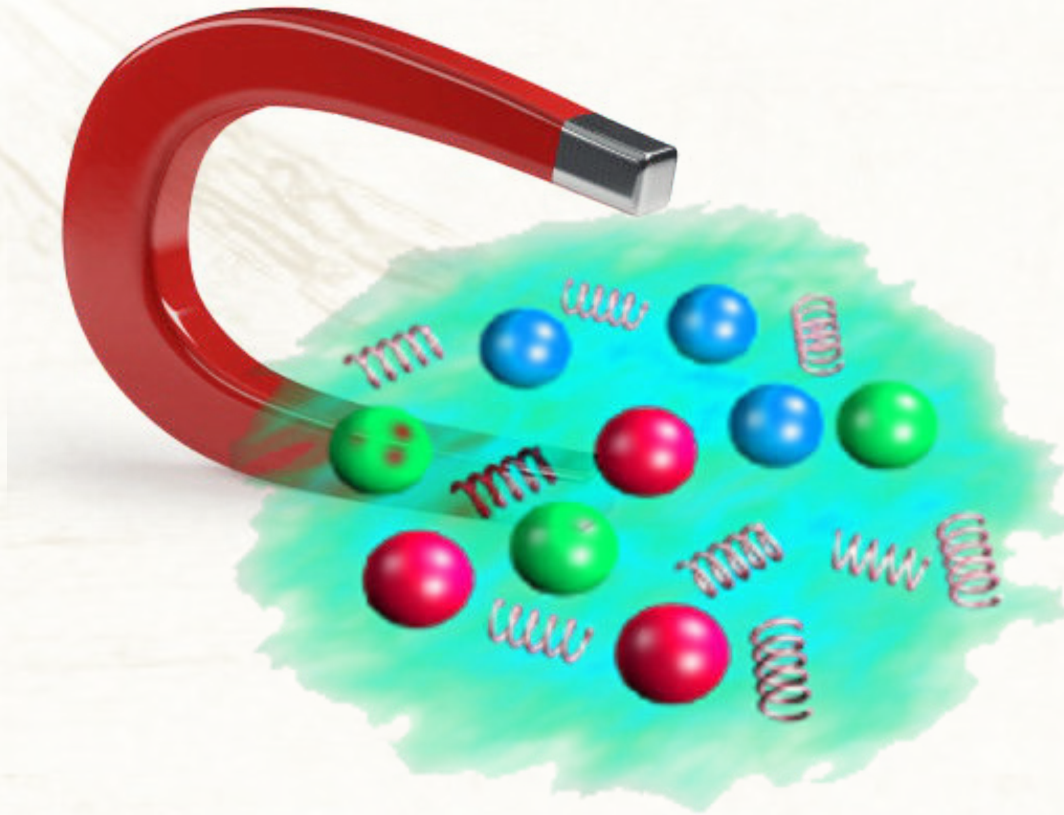
Super-strong *B*: prediction



[Cohen & Yamamoto, PRD**89**, 054029 (2014)], [G. Endrodi, JHEP 1507 (2015) 173]

Complete diagram: prediction





MAGNETIZED QCD MATTER

[Miransky & Shovkovy, Physics Reports **576** (2015) pp. 1-209]

Chiral matter

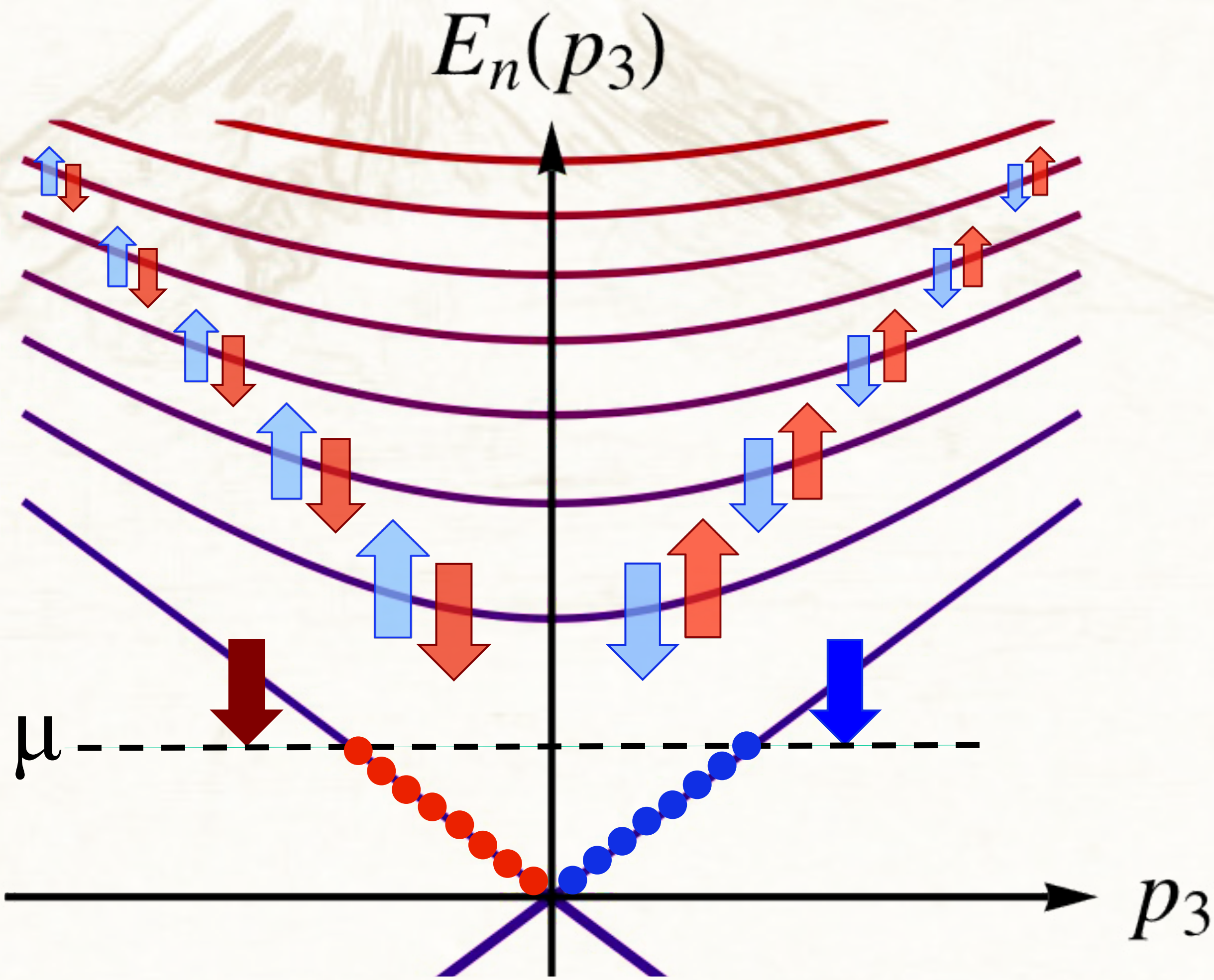
- Matter made of chiral fermions with $n_\mathrm{L} \neq n_\mathrm{R}$
- Unlike the electric charge $(n_\mathrm{R} + n_\mathrm{L})$, the chiral charge $(n_\mathrm{R} - n_\mathrm{L})$ is **not** conserved

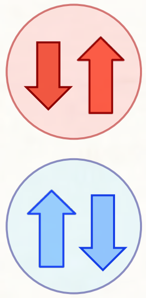
$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla}\cdot\vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}}\cdot\vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral symmetry is anomalous in quantum theory

Chiral separation effect ($\mu \neq 0$)





— **Right-handed**

— **Left-handed**

Spin (s=↓) **polarized** LLL:

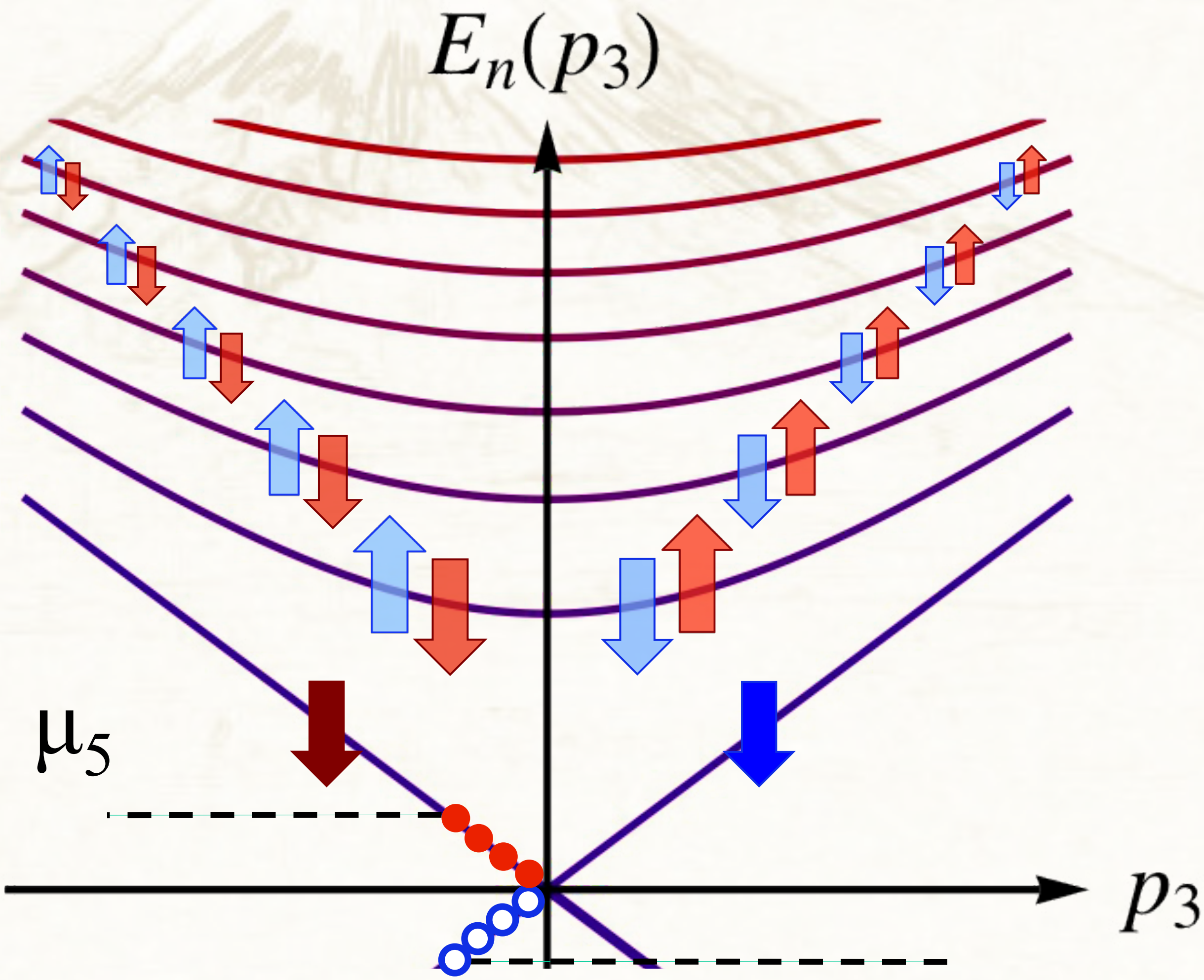
- $p_3<0$ states are R-handed
- $p_3>0$ states are L-handed

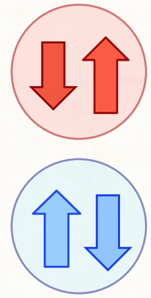
This leads to CSE:

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Chiral magnetic effect ($\mu_5 \neq 0$)





— **Right-handed**

— **Left-handed**

Spin (s=↓) **polarized** LLL:

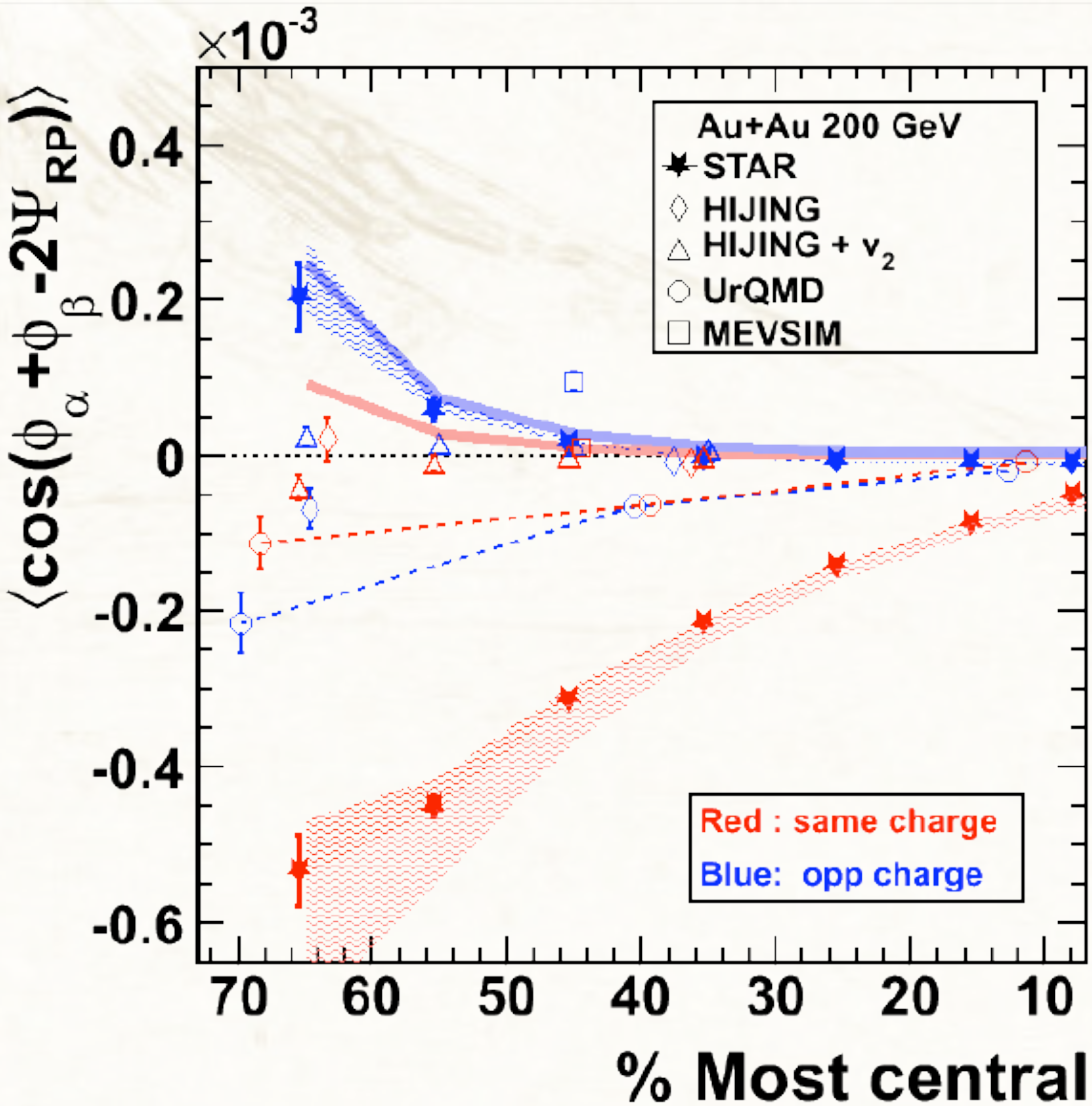
- $p_3<0$ states are R-handed electrons
- $p_3>0$ states are L-handed positrons

This leads to CME:

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Experimental evidence

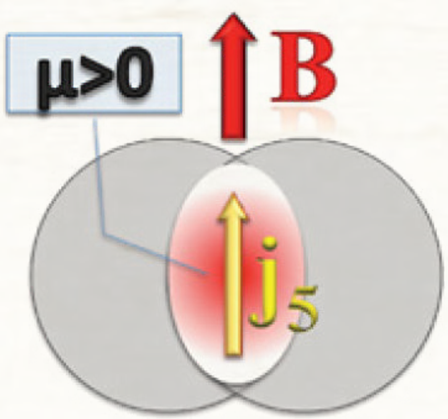


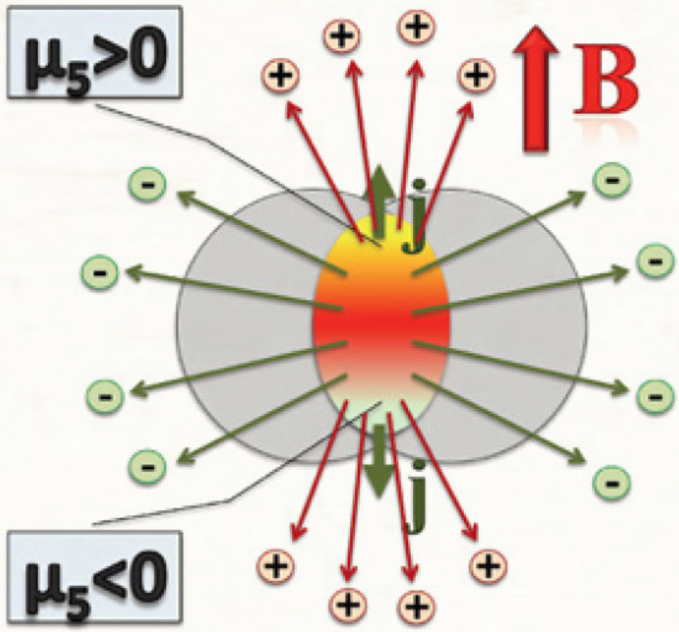
[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. **103**, 251601 (2009)]
[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C **81**, 054908 (2010)]
[Adamczyk et al. (STAR Collaboration), Phys. Rev. C **88**, 064911 (2013)]

CMW/Quadrupole CME

- Start from a small baryon density and B≠0

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$





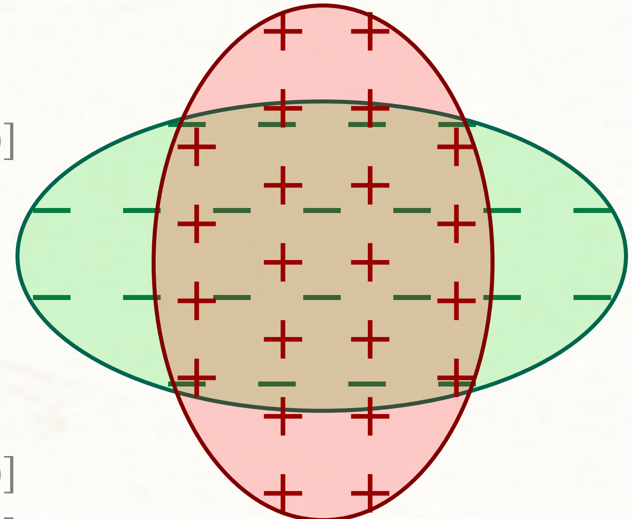
- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Experimental evidence

- Elliptic flows of π^+ and π^- depend on charge asymmetry:

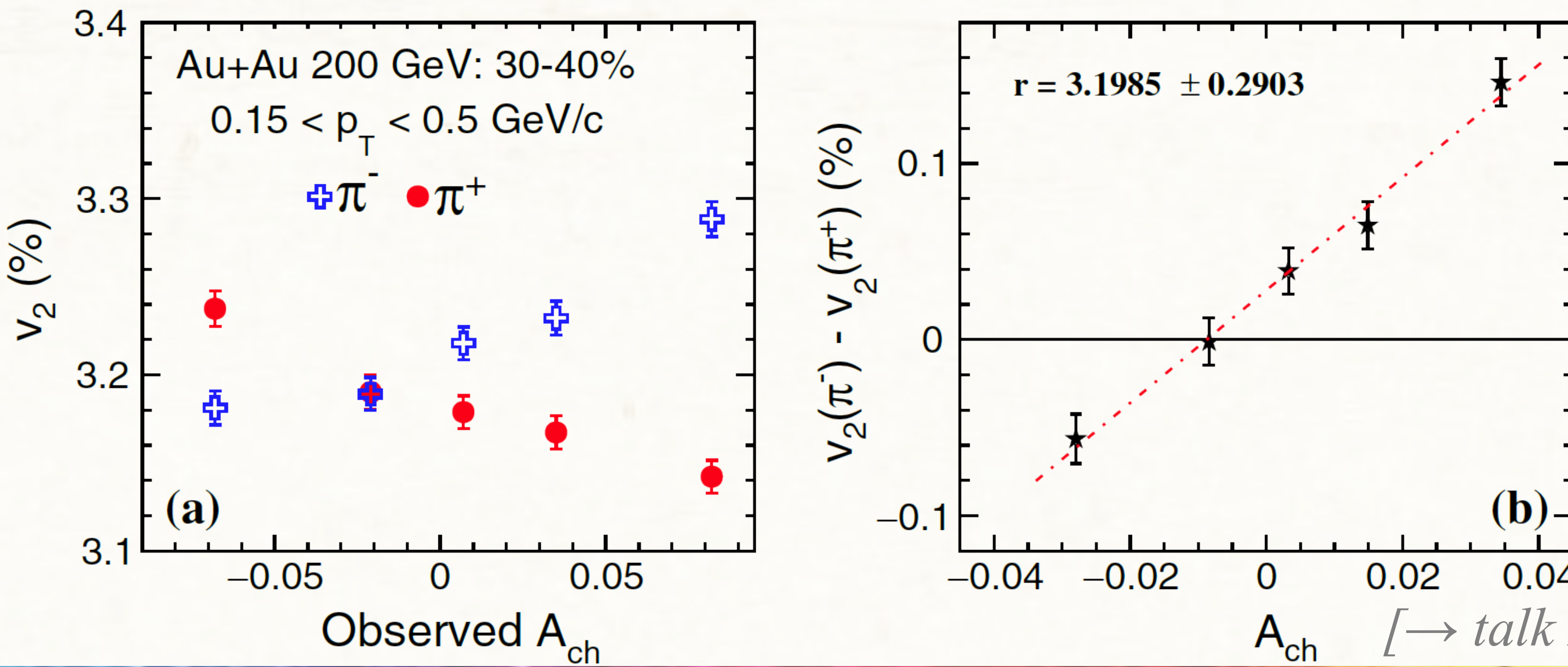
[Burnier, Kharzeev, Liao, Yee, PRL **107**, 052303 (2011)]



$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm \left[1 + 2v_2 \cos(2\phi) \mp A_\pm r \cos(2\phi) \right]$$

[H. Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]



[→ *talk Ron Belmont*]

Summary

- Magnetism profoundly affects chiral properties of QCD
- $T=0$ & $\mu=0$: Magnetic catalysis (lattice)
- $T\neq 0$ & $\mu=0$: Inverse magnetic catalysis (lattice)
- $T=0$ & $\mu\neq 0$: Chiral shift (compact stars)
- $T\neq 0$ & $\mu\neq 0$: CME, CSE, … (relativistic HIC)