



Collective modes in chiral (pseudo)relativistic matter

Igor Shovkovy
Arizona State University





CHIRAL MATTER

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma} \cdot \vec{p}\right) \Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \Psi = \text{sign}(p_0) \gamma^5 \Psi$$

For particles ($p_0 > 0$): chirality = helicity

For antiparticles ($p_0 < 0$): chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime

– High temperature: $T \gg m$

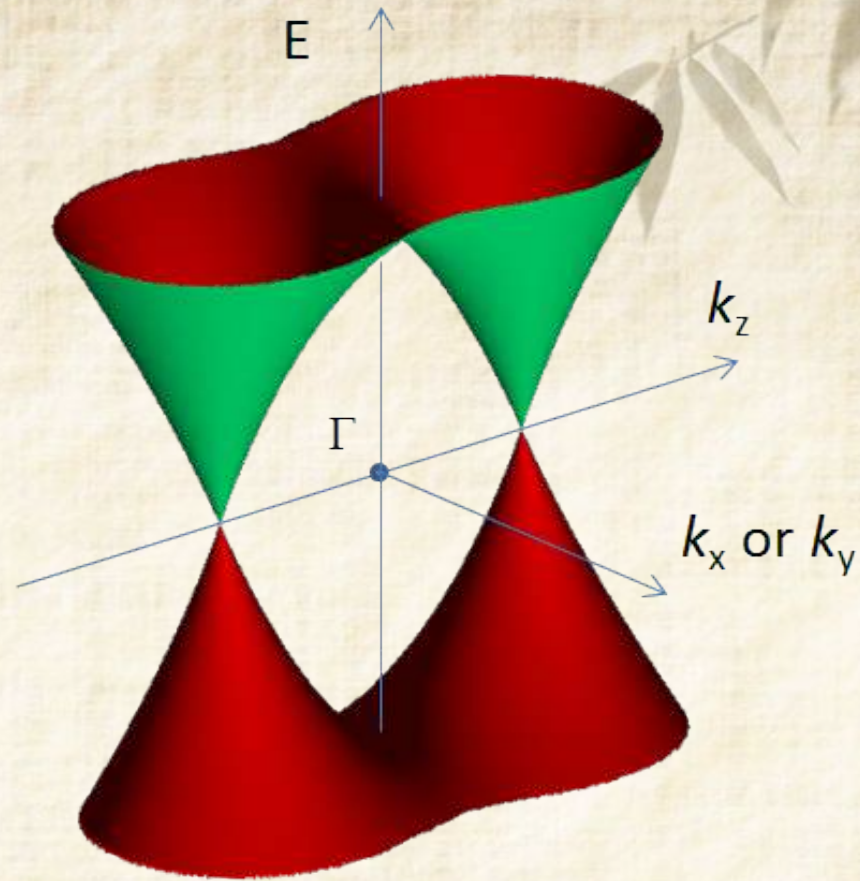
– High density: $\mu \gg m$

- Matter made of chiral fermions with $n_L \neq n_R$
- Unlike the electric charge ($n_R + n_L$), the chiral charge ($n_R - n_L$) is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c}$$

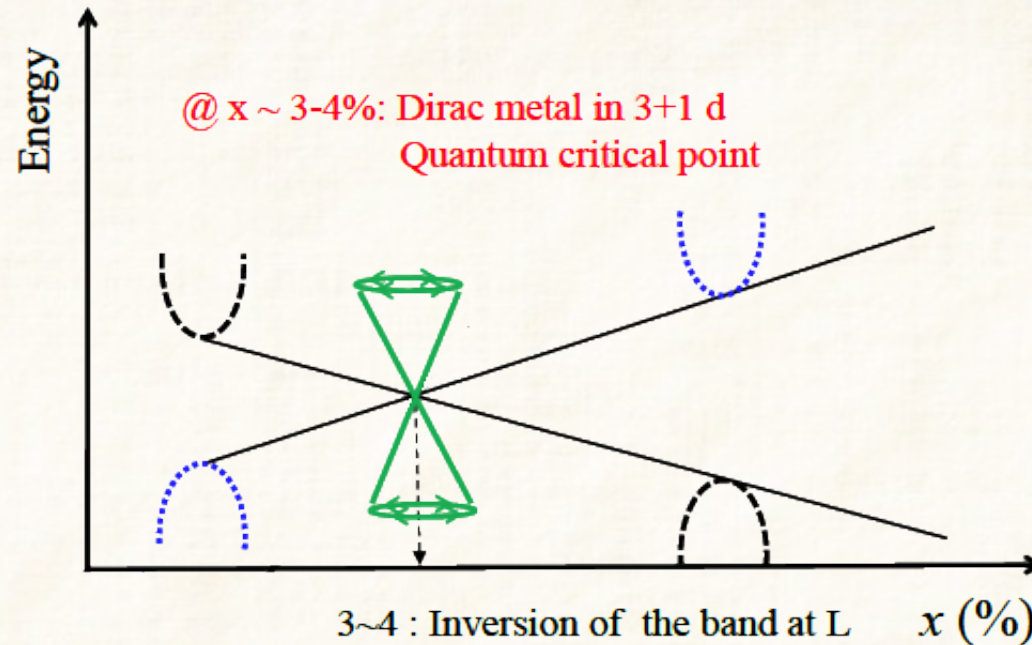
- The chiral symmetry is anomalous in quantum theory



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL MATERIALS

- Solid state materials with Dirac quasiparticles:
 - $\text{Bi}_{1-x}\text{Sb}_x$ alloy



- “New” 3D Dirac materials (ARPES):
 - Na_3Bi (Potassium bismuthide) [Liu et al., Science **343**, 864 (2014)]
 - Cd_3As_2 (Cadmium arsenide) [Neupane et al., Nature Commun. **5**, 3786 (2014)]
[Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

Dirac materials

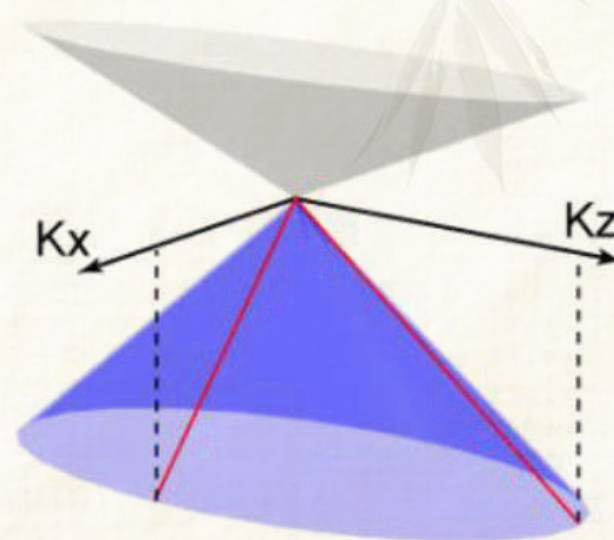
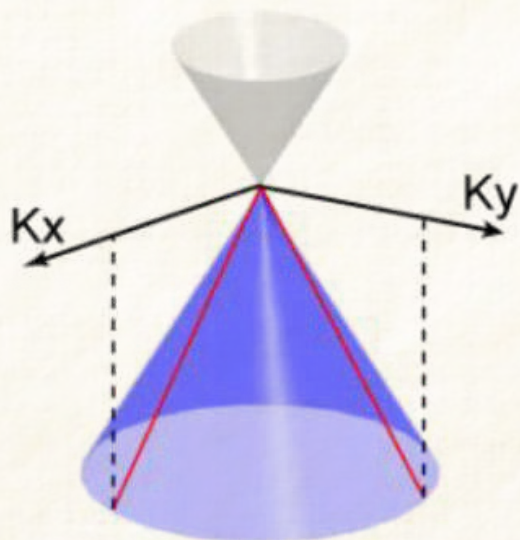
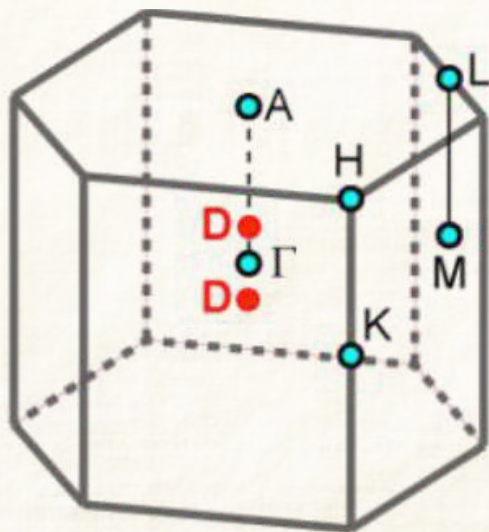
- $\text{Bi}_{1-x}\text{Sb}_x$ alloy (at $x \approx 4\%$)
- Na_3Bi
- Cd_3As_2
- ZrTe_5

[Z. K. Liu et al., Science **343**, 864 (2014)]

[M. Neupane et al., Nature Commun. **5**, 3786 (2014)]

[S. Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

[X. Li et al., Nature Physics **12**, 550 (2016)]



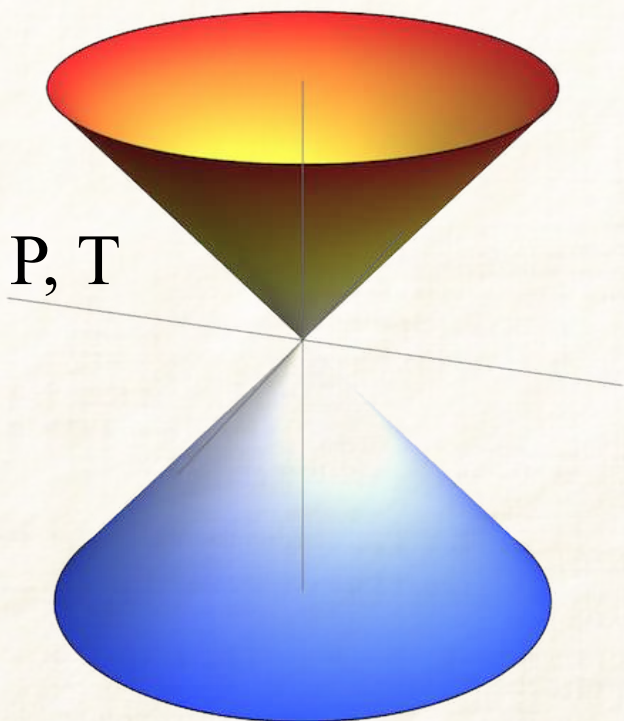
$$E = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + v_z^2 k_z^2}, \quad v_x \approx v_y \approx 3.74 \times 10^5 \text{ m/s}, \quad v_z \approx 2.89 \times 10^4 \text{ m/s}$$

Dirac vs. Weyl materials

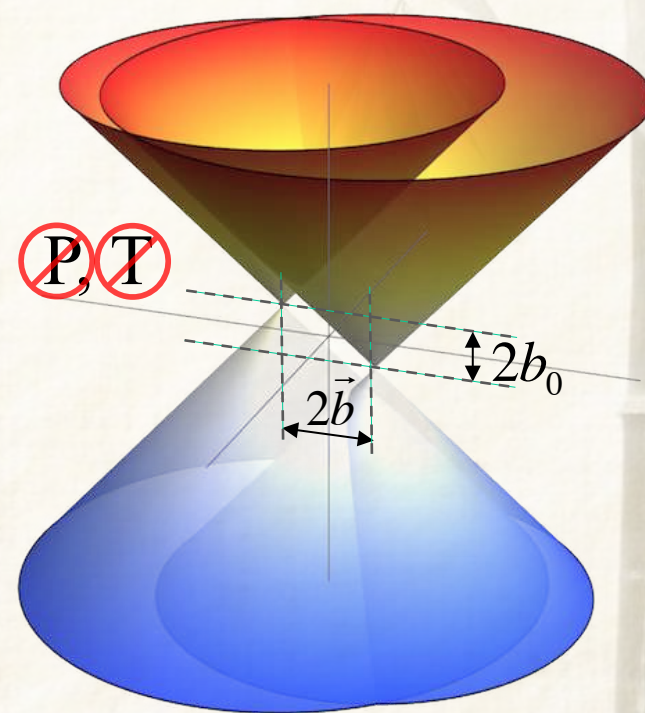
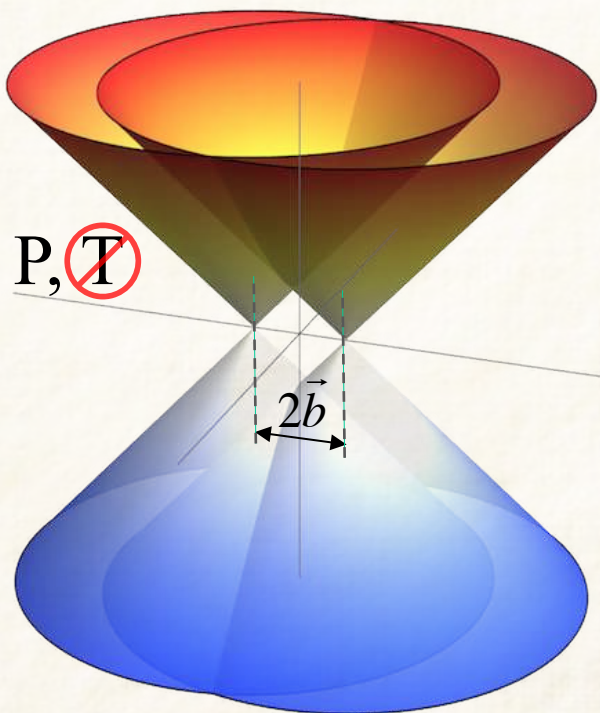
- Low-energy Hamiltonian of a Dirac/Weyl material

$$H = \int d^3\mathbf{r} \bar{\psi} \left[-iv_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - \overset{\text{T}}{\circlearrowleft} (\vec{b} \cdot \vec{\gamma}) \gamma^5 + \overset{\text{P}}{\circlearrowleft} b_0 \gamma^0 \gamma^5 \right] \psi$$

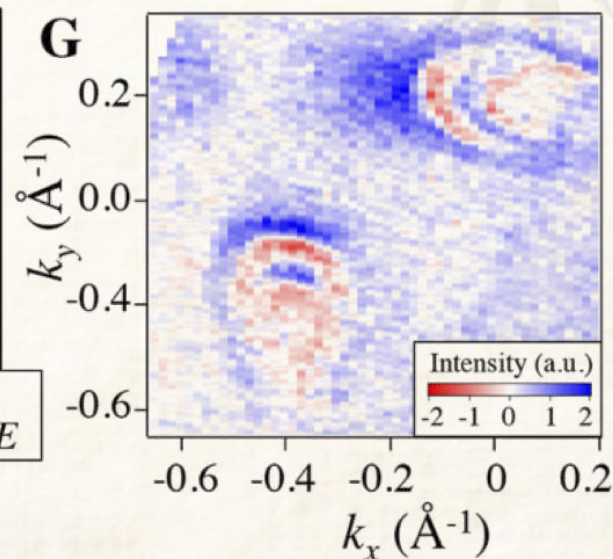
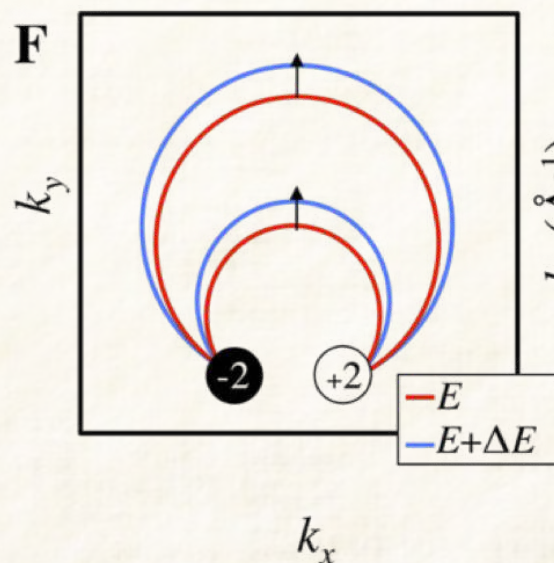
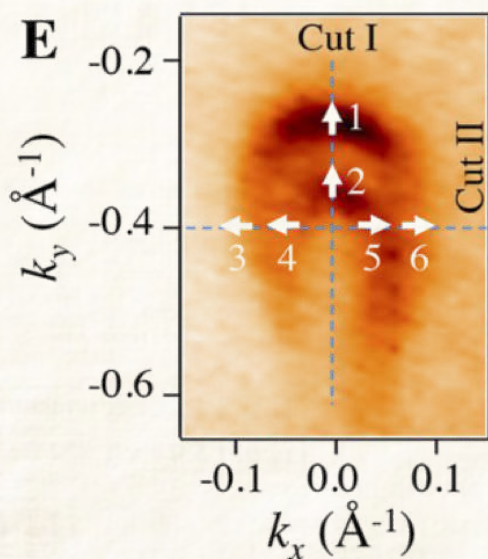
Dirac



Weyl



- TaAs (tantalum arsenide) [S.-Y. Xu et al., Science 349, 613 (2015)]
[B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
- NbAs (niobium arsenide) [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
- TaP (tantalum phosphide) [S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
- NbP (niobium phosphide) [I. Belopolski et al. arXiv:1509.07465]
- WTe₂ (tungsten telluride) [F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]



Low-energy Hamiltonian

- The Hamiltonian for massless Dirac fermions is given by

$$H_D = \begin{pmatrix} v_F(\vec{k} \cdot \vec{\sigma}) & 0 \\ 0 & -v_F(\vec{k} \cdot \vec{\sigma}) \end{pmatrix}$$

- This can be viewed as a combination of two Weyl fermions

$$H_\lambda = \lambda v_F(\vec{k} \cdot \vec{\sigma})$$

where $\lambda = \pm 1$ is a chirality

The Weyl energy eigenstates are given by

$$\psi_k^\lambda = \frac{1}{\sqrt{2\epsilon_k k_\perp}} \begin{pmatrix} \sqrt{\epsilon_k + \lambda k_z} k_- \\ \lambda \sqrt{\epsilon_k - \lambda k_z} k_\perp \end{pmatrix}$$

They describe particles of energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$

The mapping $k \rightarrow \psi_k^\lambda$ has a nontrivial topology

- Consider evolution from $\psi_{\mathbf{k}}$ to $\psi_{\mathbf{k}+\delta\mathbf{k}}$:

$$\langle \psi_{\mathbf{k}} | \psi_{\mathbf{k}+\delta\mathbf{k}} \rangle \approx 1 + \delta\mathbf{k} \cdot \langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle \approx e^{i\mathbf{a}_{\mathbf{k}} \cdot \delta\mathbf{k}}$$

where $\mathbf{a}_{\mathbf{k}} = -i\langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle$ is the Berry connection

- The Berry curvature is defined as follows:

$$\mathbf{\Omega}_{\mathbf{k}} = \nabla_{\mathbf{k}} \times \mathbf{a}_{\mathbf{k}}$$

- Note the similarity with gauge fields, but $\mathbf{a}_{\mathbf{k}}$ and $\mathbf{\Omega}_{\mathbf{k}}$ are defined in the momentum space
- It is convenient to define the Chern number (flux of $\mathbf{\Omega}_{\mathbf{k}}$)

$$C = \frac{1}{2\pi} \oint \mathbf{\Omega}_{\mathbf{k}} \cdot d\mathbf{S}_{\mathbf{k}}$$

- A nonzero (integer) Chern number indicates a nonzero (topological) charge inside the k -volume surrounded by the closed surface (Gauss's law)

Gauge theory	Berry effects
Local at coordinate space	Local at momentum space
Gauge field \vec{A}	Berry connection \vec{a}
Magnetic field $\vec{B} = \vec{\nabla}_r \times \vec{A}$	Berry curvature $\vec{\Omega} = \vec{\nabla}_k \times \vec{a}$
Aharonov-Bohm phase $\oint d\vec{r} \vec{A}(\vec{r})$	Berry phase $\oint d\vec{k} \vec{a}(\vec{k})$
Magnetic charge (Dirac monopole) $\int d\vec{r} (\vec{\nabla}_r \cdot \vec{B}) = const$	Berry monopole $\int d\vec{k} (\vec{\nabla}_k \cdot \vec{\Omega}) = const$

- In the case of Weyl fermions,

$$\mathbf{\Omega}_k = \lambda \frac{\vec{k}}{2k^3}$$

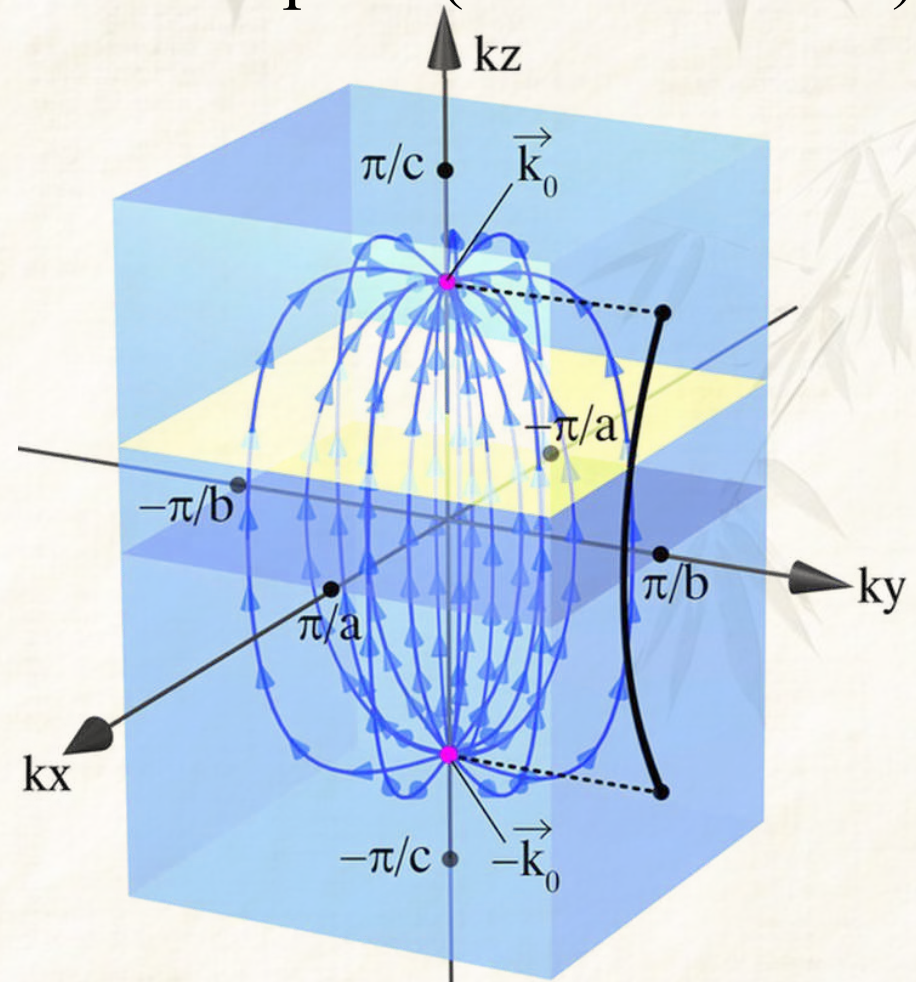
(Note: this looks like a field of a monopole at $\vec{k} = 0$)

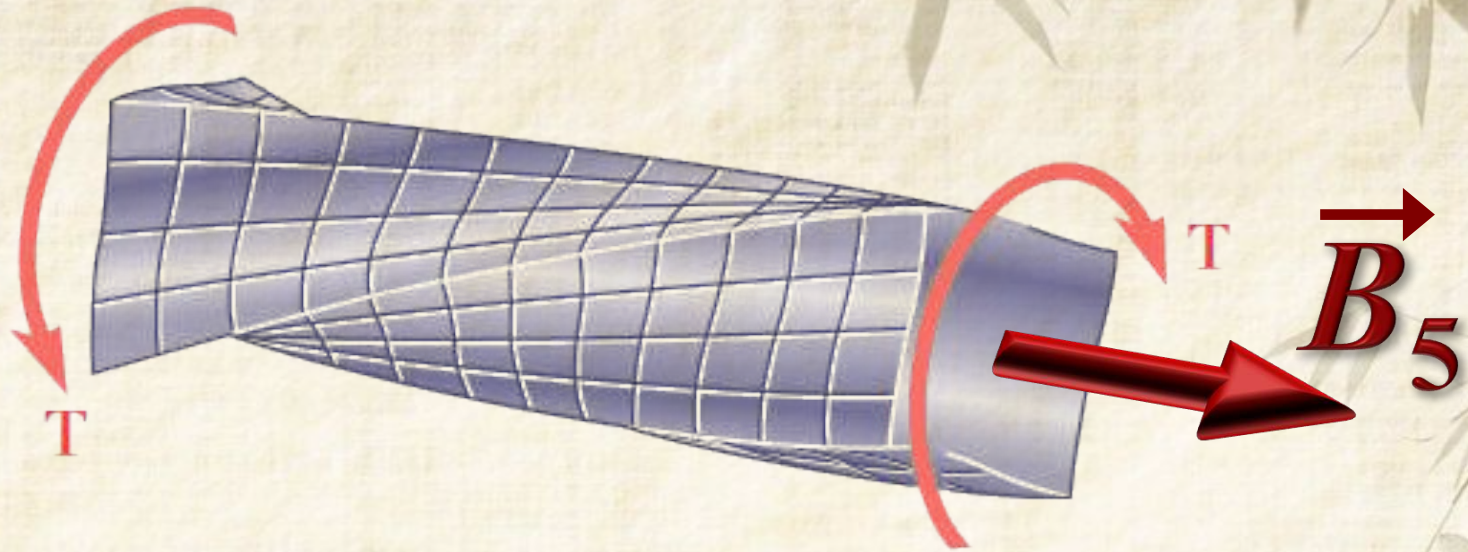
- Let us calculate the total flux of $\mathbf{\Omega}_k$ -field through the spherical surface of radius K with the center at $\vec{k} = 0$

$$C = \frac{1}{2\pi} \oint \lambda \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin \theta d\theta d\varphi = \lambda = \pm 1$$

- Thus, the electronic structure of massless Weyl fermions is characterized by a topological monopole at $\vec{k} = 0$
- Is the Berry monopole just a mathematical curiosity?
- Are there any observable consequences?

- In solid state physics, the momentum space (Brillouin zone) is compact
- A closed surface around a single Weyl node is also a closed surface (of opposite orientation) around the rest of the Brillouin zone
- Flipping surface orientation changes the sign of the flux
- There must be an opposite charge somewhere in the rest of the zone
- Thus, Weyl fermions come in pairs of opposite chirality
[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]





PSEUDO-ELECTROMAGNETIC FIELDS

[Zubkov, Annals Phys. **360**, 655 (2015)]

[Cortijo, Ferreira, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]

[Grushin, Venderbos, Vishwanath, Ilan, arXiv:1607.04268]

[Cortijo, Kharzeev, Landsteiner, Vozmediano, arXiv:1607.03491]

[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

- Strains affect low-energy quasiparticles in Weyl materials

$$H = \int d^3\mathbf{r} \bar{\psi} \left[-iv_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - (\vec{b} + \vec{A}_5) \cdot \vec{\gamma} \gamma^5 + (b_0 + A_{5,0}) \gamma^0 \gamma^5 \right] \psi$$

where the components of the chiral gauge fields are

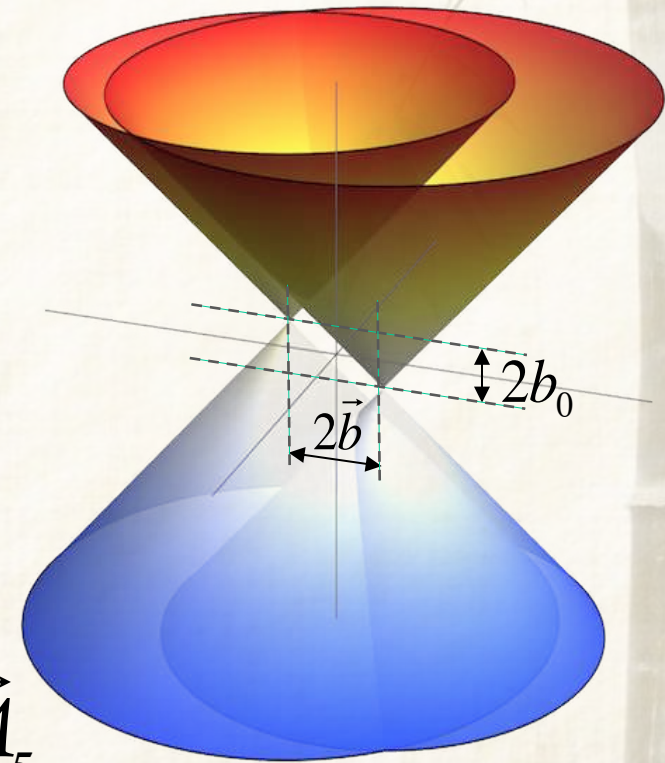
$$A_{5,0} \propto b_0 |\vec{b}| \partial_{||} u_{||}$$

$$A_{5,\perp} \propto |\vec{b}| \partial_{||} u_{\perp}$$

$$A_{5,||} \propto \alpha |\vec{b}|^2 \partial_{||} u_{||} + \beta \sum_i \partial_i u_i$$

The associated pseudo-EM fields are

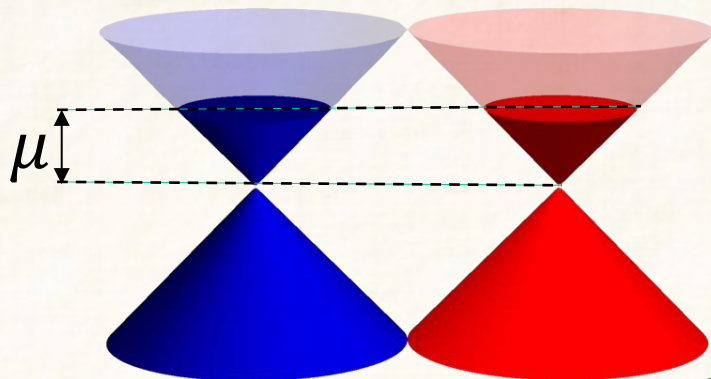
$$\vec{B}_5 = \vec{\nabla} \times \vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla} A_0 - \partial_t \vec{A}_5$$



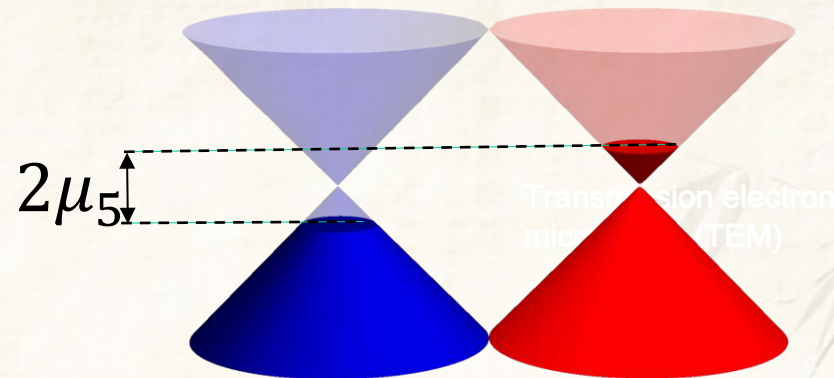
Equilibrium vs. nonequilibrium

$$\mu_\lambda = \mu + \lambda\mu_5$$

1) $\mu_5 = b_0 = 0$

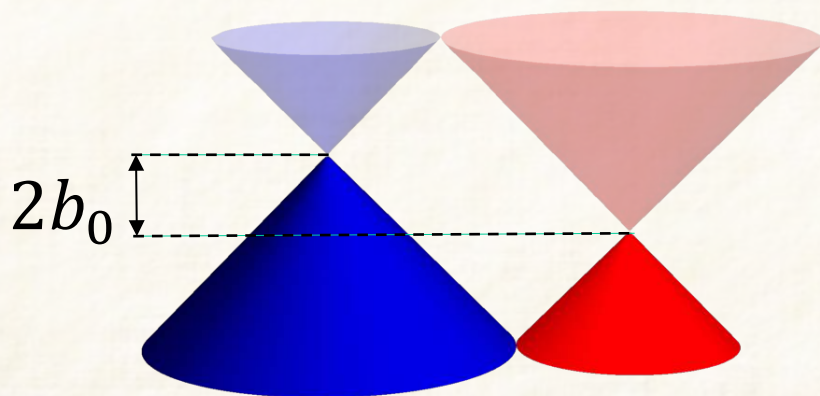


2) $\mu = b_0 = 0$

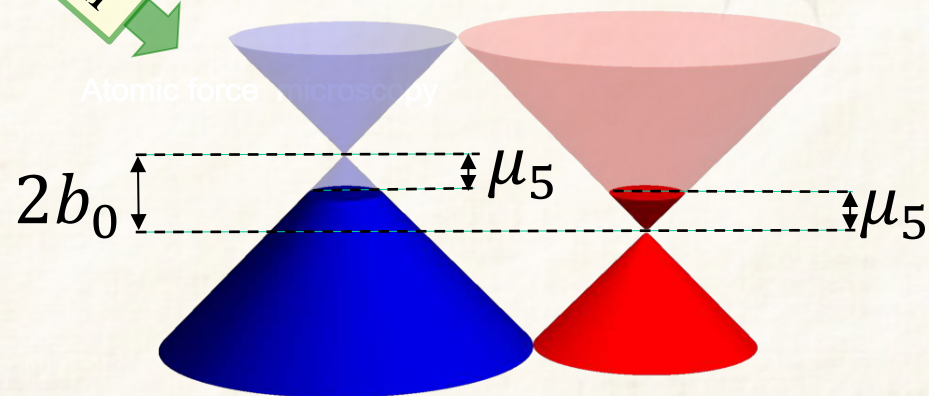


Equilibrium

3) $\mu = \mu_5 = 0$



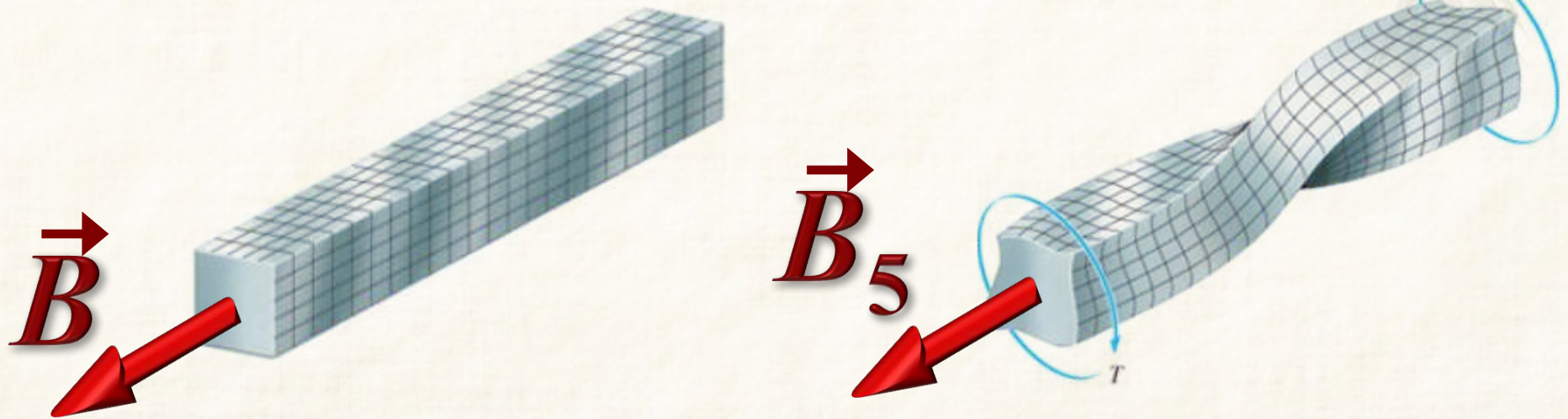
4) $\mu_5 = b_0, \mu = 0$



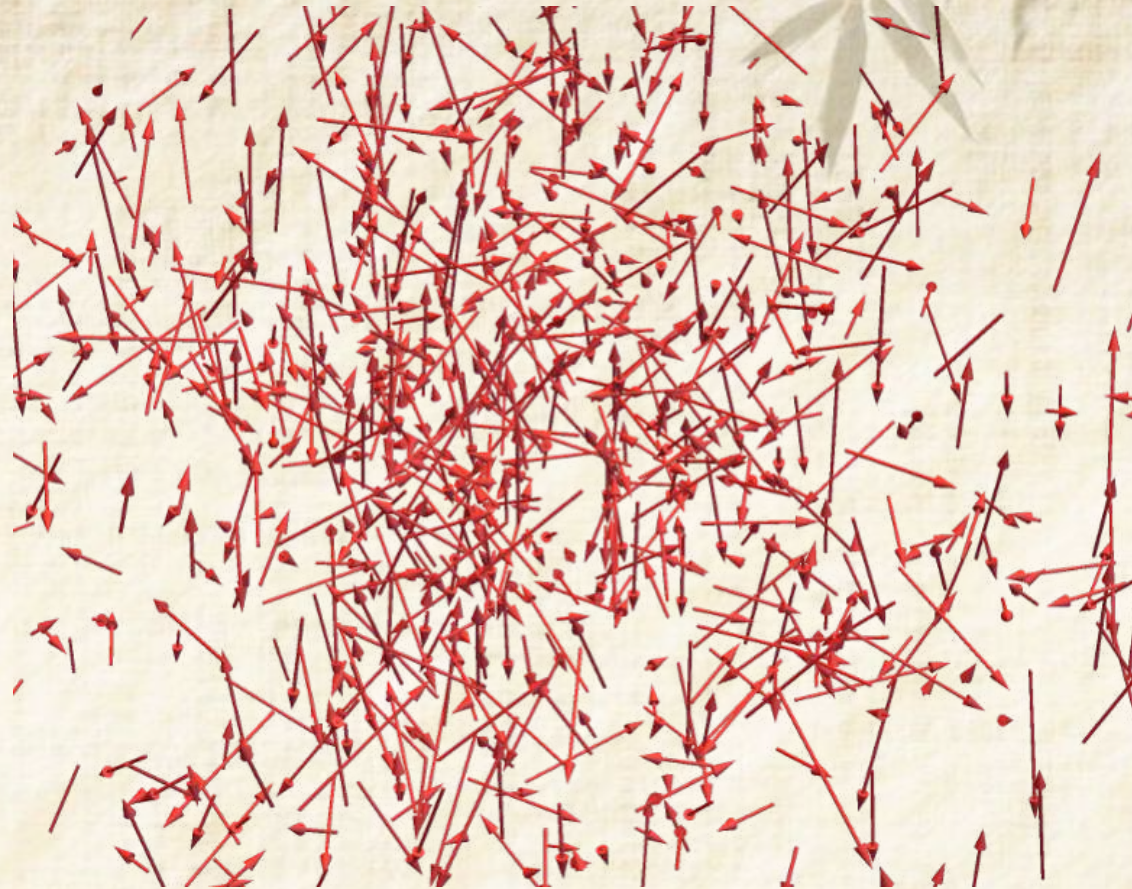
- Any qualitative properties of Weyl materials directly sensitive to b_0 and \vec{b} ?
- Some proposals:
 - Anomalous Hall effect
 - Anomalous Alfvén waves
 - Strain/torsion induced CME
 - Strain/torsion induced quantum oscillations
 - Strain/torsion dependent resistance
 - etc.
- Spectrum of chiral (pseudo-)magnetic plasmons
 - [Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
 - [Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

General question

- What are the properties of plasmons in magnetized chiral material with $b_0 \neq 0$ and $\vec{b} \neq 0$?
- Chiral matter ($\mu_R \neq \mu_L$)
 - This is the case in equilibrium when $b_0 \neq 0$ ($\mu_5 = -eb_0$)
- Magnetic or pseudomagnetic field is present



- In general, $\mathbf{E}_\lambda = \mathbf{E} + \lambda \mathbf{E}_5$ and $\mathbf{B}_\lambda = \mathbf{B} + \lambda \mathbf{B}_5$



MODEL & METHOD

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

- Kinetic equation: [Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v} \times \mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda \cdot \mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda \right] \cdot \nabla_{\mathbf{p}} f_\lambda}{1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \boldsymbol{\Omega}_\lambda)} + \frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda \times \boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v} \cdot \boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda \right] \cdot \nabla_{\mathbf{r}} f_\lambda}{1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\nabla_{\mathbf{r}}\epsilon_{\mathbf{p}}$, $\mathbf{v} = \nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}$,

$$\epsilon_{\mathbf{p}} = v_{Fp} \left[1 - \frac{e}{c}(\mathbf{B}_\lambda \cdot \boldsymbol{\Omega}_\lambda) \right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar \frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

- The definitions of density and current are

$$\rho_\lambda = e \int \frac{d^3 p}{(2\pi\hbar)^3} \left[1 + \frac{e}{c} (\mathbf{B}_\lambda \cdot \boldsymbol{\Omega}_\lambda) \right] f_\lambda,$$

$$\mathbf{j}_\lambda = e \int \frac{d^3 p}{(2\pi\hbar)^3} \left[\mathbf{v} + \frac{e}{c} (\mathbf{v} \cdot \boldsymbol{\Omega}_\lambda) \mathbf{B}_\lambda + e (\tilde{\mathbf{E}}_\lambda \times \boldsymbol{\Omega}_\lambda) \right] f_\lambda$$

$$+ e \nabla \times \int \frac{d^3 p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}} \boldsymbol{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2 \hbar^2 c} \left[(\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5) \right] \quad \checkmark$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2 \hbar^2 c} \left[(\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B}) \right] \quad \times$$

- Additional Bardeen-Zumino term is needed,

$$\delta j^\mu = \frac{e^3}{4\pi^2 \hbar^2 c} \epsilon^{\mu\nu\rho\lambda} A_\nu^5 F_{\rho\lambda}$$

- In components,

$$\delta\rho = \frac{e^3}{2\pi^2 \hbar^2 c^2} (\mathbf{A}^5 \cdot \mathbf{B})$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2 \hbar^2 c} A_0^5 \mathbf{B} - \frac{e^3}{2\pi^2 \hbar^2 c} (\mathbf{A}^5 \times \mathbf{E})$$

- Its role and implications:

- Electric charge is conserved locally ($\partial_\mu J^\mu = 0$)
- Anomalous Hall effect is reproduced
- CME vanishes in equilibrium ($\mu_5 = -eb_0$)

We search for plane-wave solutions with

$$\mathbf{E}' = \mathbf{E} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}, \quad \mathbf{B}' = \mathbf{B} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

and the distribution function $f_\lambda = f_\lambda^{(\text{eq})} + \delta f_\lambda$,

where

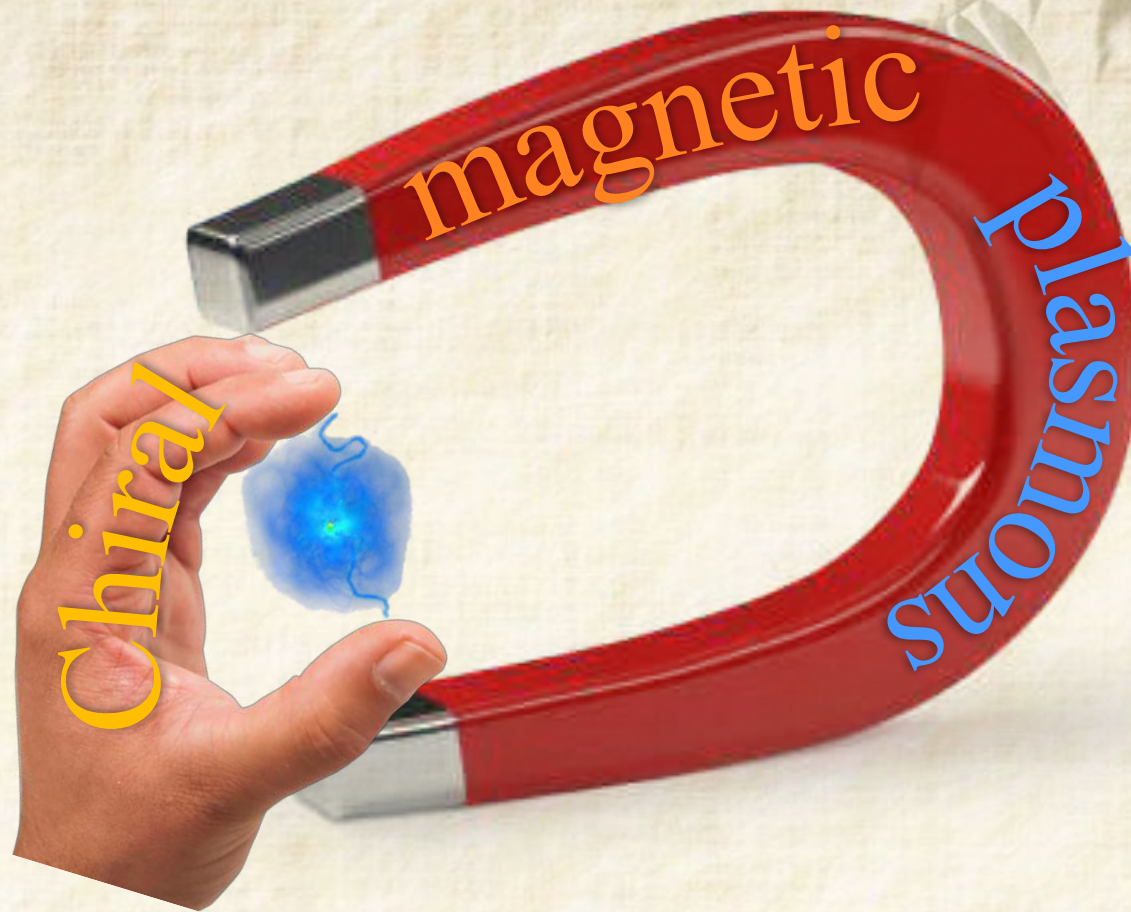
$$\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

The polarization vector & susceptibility tensor:

$$P^m = i \frac{J^{lm}}{\omega} = \chi^{mn} E'^n$$

The plasmon dispersion relations follow from

$$\det [(\omega^2 - c^2 k^2) \delta^{mn} + c^2 k^m k^n + 4\pi\omega^2 \chi^{mn}] = 0$$



RESULTS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ $k=0$:

$$\omega_l = \Omega_e, \quad \omega_{\text{tr}}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2} \left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right)}$$

and

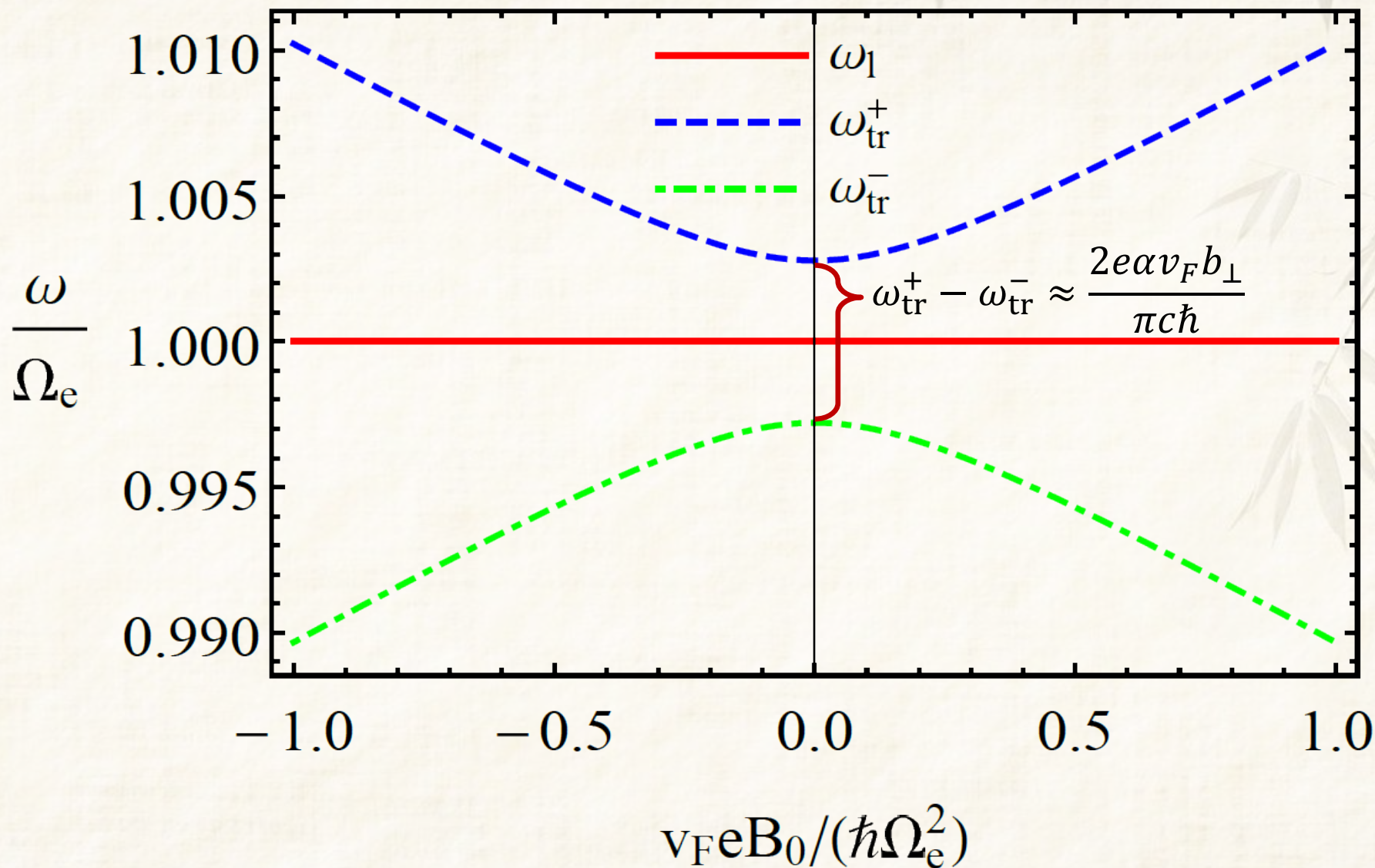
$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2} \left\{ 9\hbar^2 b_{\perp}^2 + \left[\frac{2v_F}{\Omega_e^2} (B_0\mu + B_{0,5}\mu_5) - 3\hbar b_{\parallel} - \frac{v_F\hbar^2}{4T} \sum_{\lambda=\pm} B_{0,\lambda} F \left(\frac{\mu_{\lambda}}{T} \right) \right]^2 \right\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

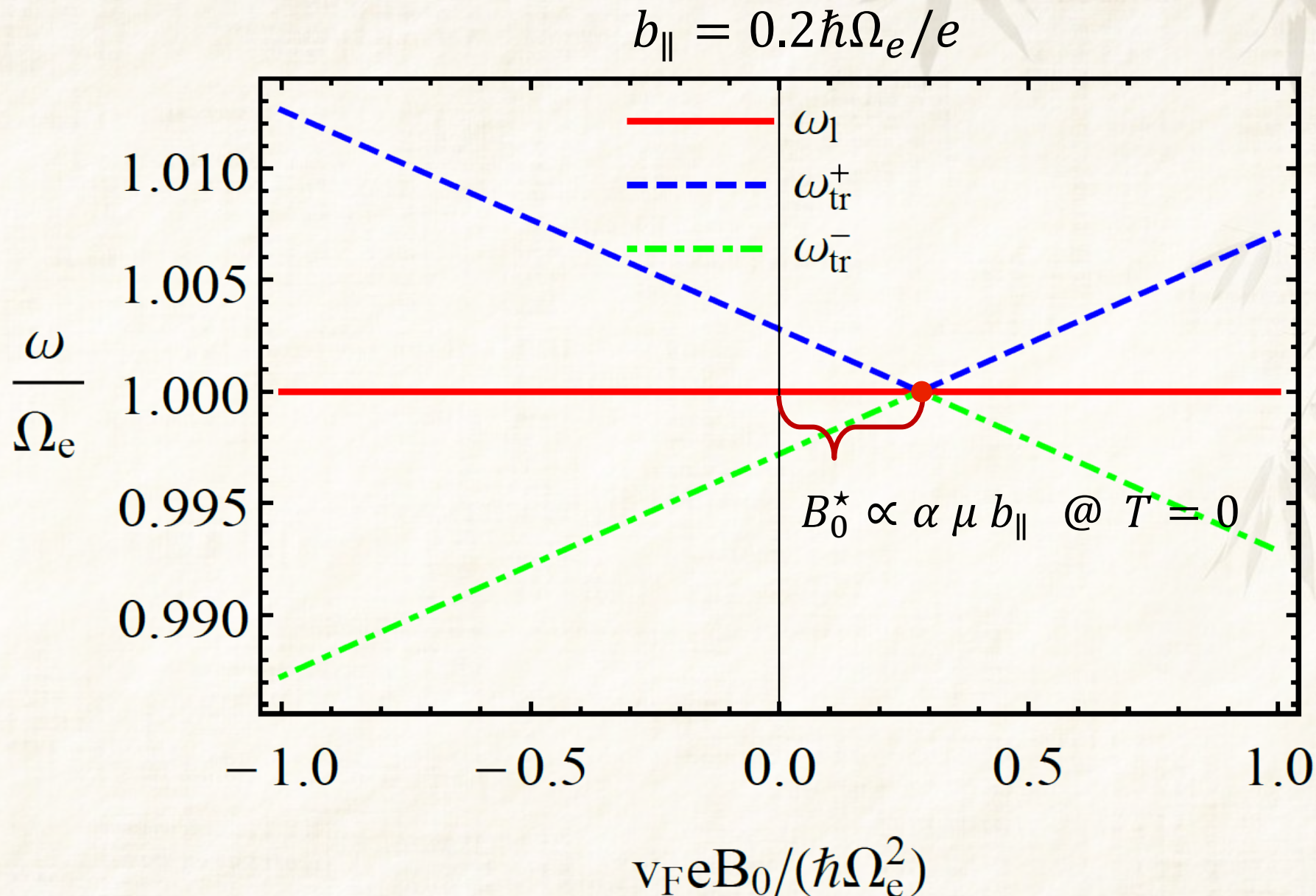
Plasmon frequencies, $\vec{B} \perp \vec{b}$

$$b_{\perp} = 0.2\hbar\Omega_e/e$$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

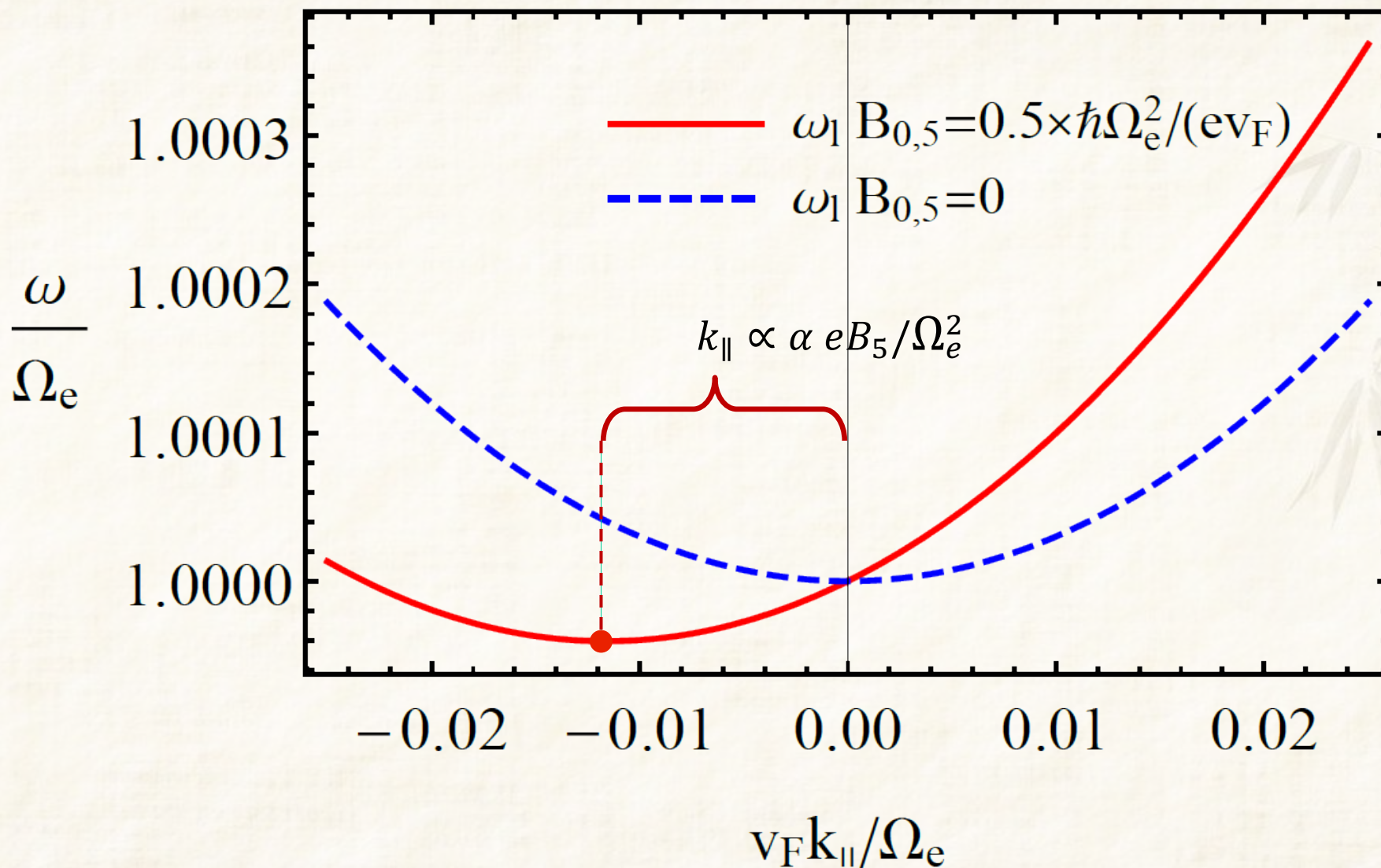
Plasmon frequencies, $\vec{B} \parallel \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmons with $\vec{k} \neq 0, \vec{k} \parallel \vec{B}_5$

- The longitudinal mode is sensitive to \vec{B}_5

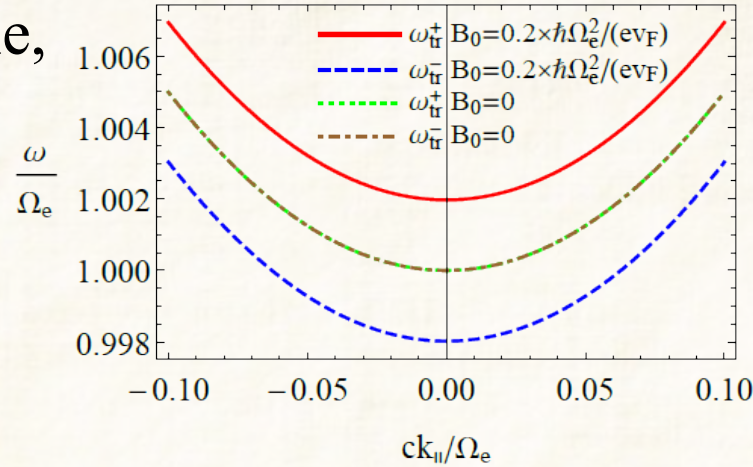


[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

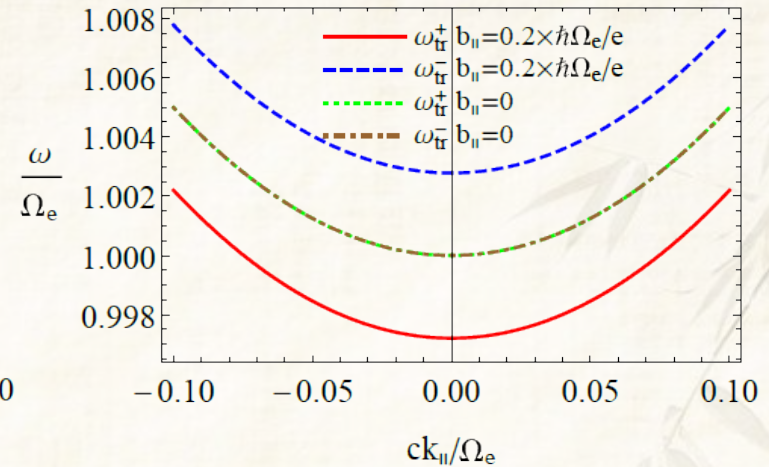
Plasmons with $\vec{k} \neq 0, \vec{k} \parallel \vec{B}, \vec{B}_5$

The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$, or (iii) $b_{\parallel} \neq 0$, or (iv) $b_{\perp} \neq 0$.

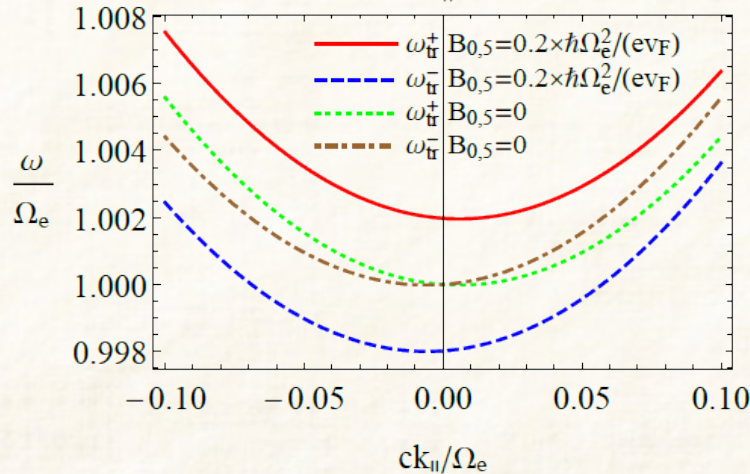
For example,



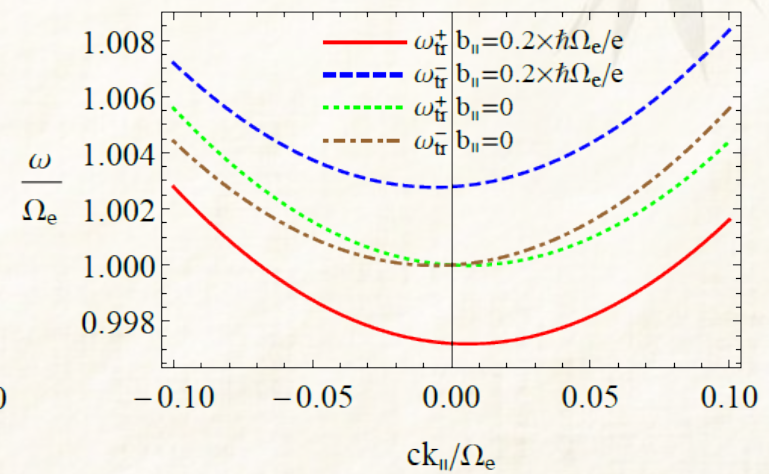
(a) $\mu = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e, \mu_5 = 0, B_{0,5} = 0, b_{\parallel} = 0$



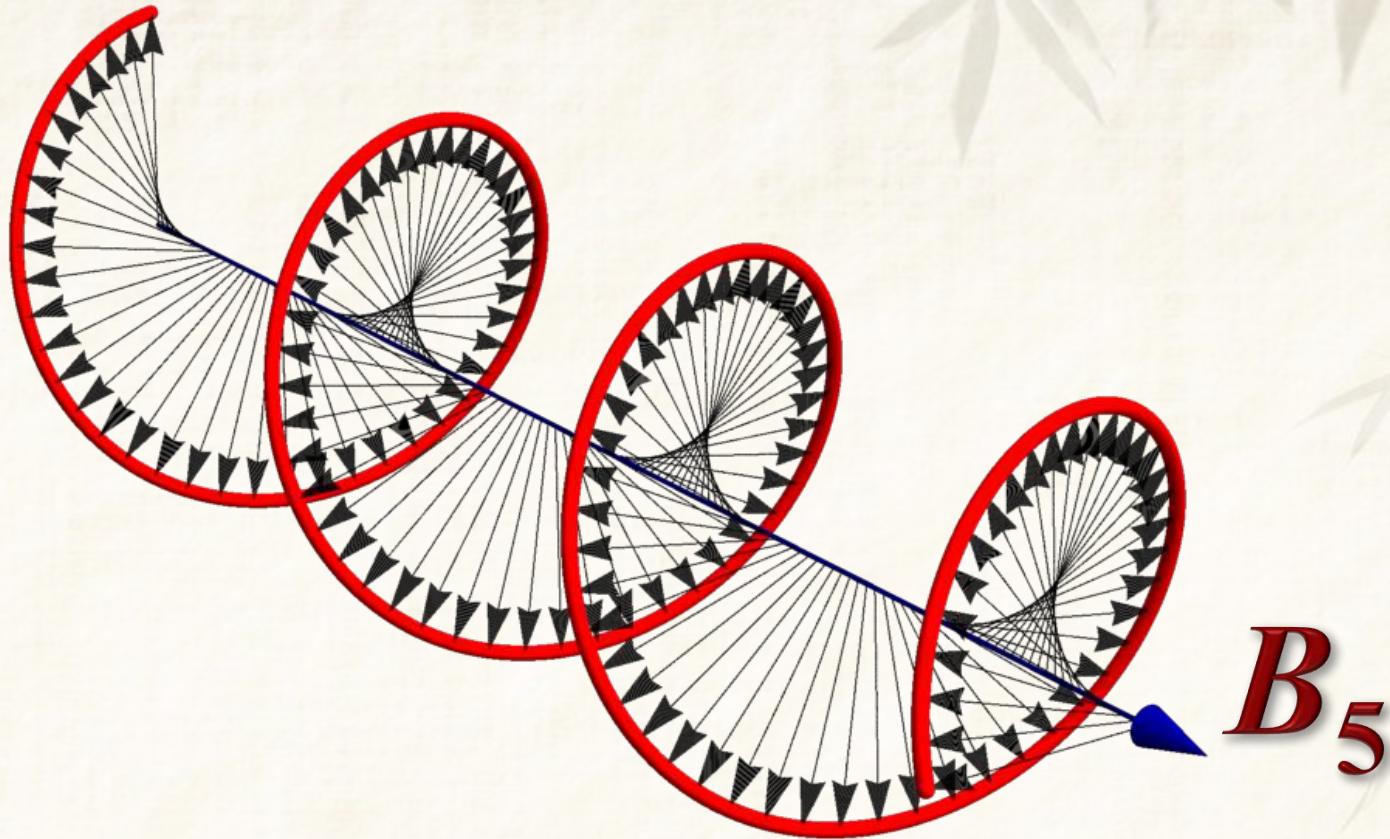
(c) $\mu = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e, \mu_5 = 0, B_0 = 0, B_{0,5} = 0$



(e) $\mu = 0, \mu_5 = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e, B_0 = 0, b_{\parallel} = 0$



(f) $\mu = 0, \mu_5 = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e, B_0 = 0, B_{0,5} = 0$



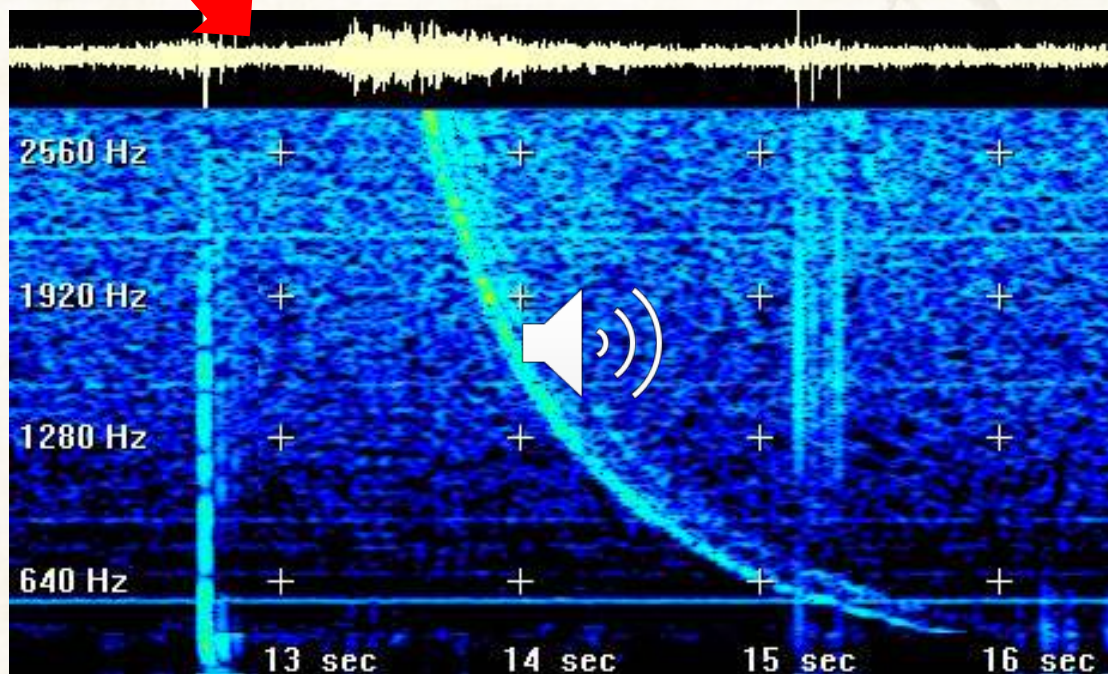
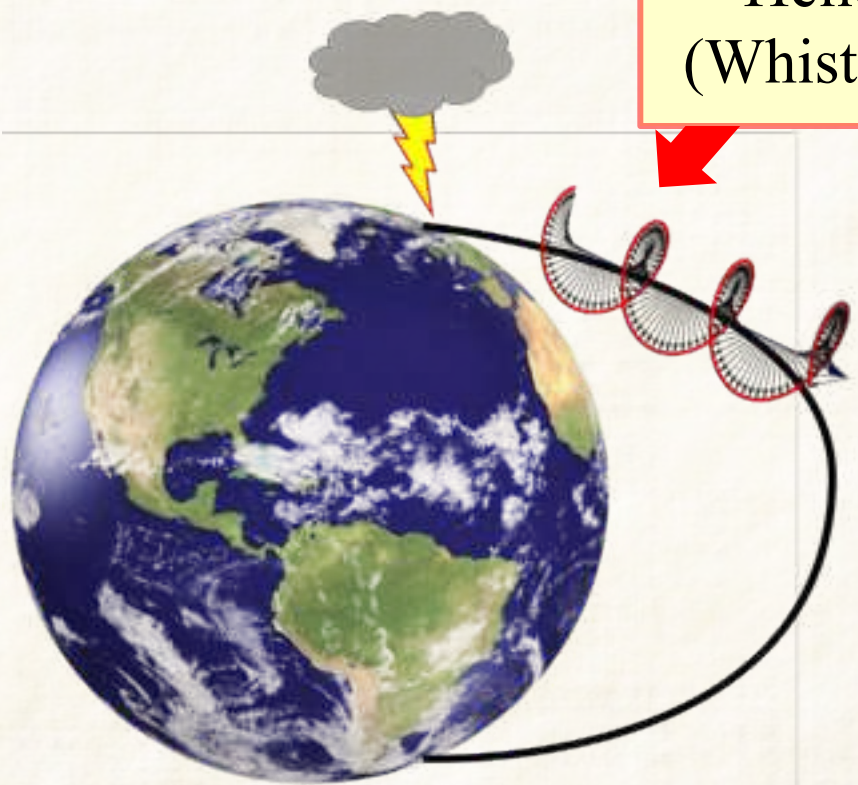
PSEUDOMAGNETIC HELICONS

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

Pseudo-magnetic helicons

- Usual helicons are transverse low-energy gapless excitations propagating along the background magnetic field \vec{B}_0 in uncompensated media (e.g., metals with different electron and hole densities)

Helicon in the ionosphere
(Whistler) and its spectrogram



- Helicon dispersion law at $T \rightarrow 0$:

$$\omega_h |_{B_{0,5} \rightarrow 0, \mu_5 \rightarrow 0} \stackrel{b_0 \rightarrow 0}{=} \frac{e B_0 c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu + 2 B_0 e^4 v_F^2 b_{\parallel}} + O(k^3)$$

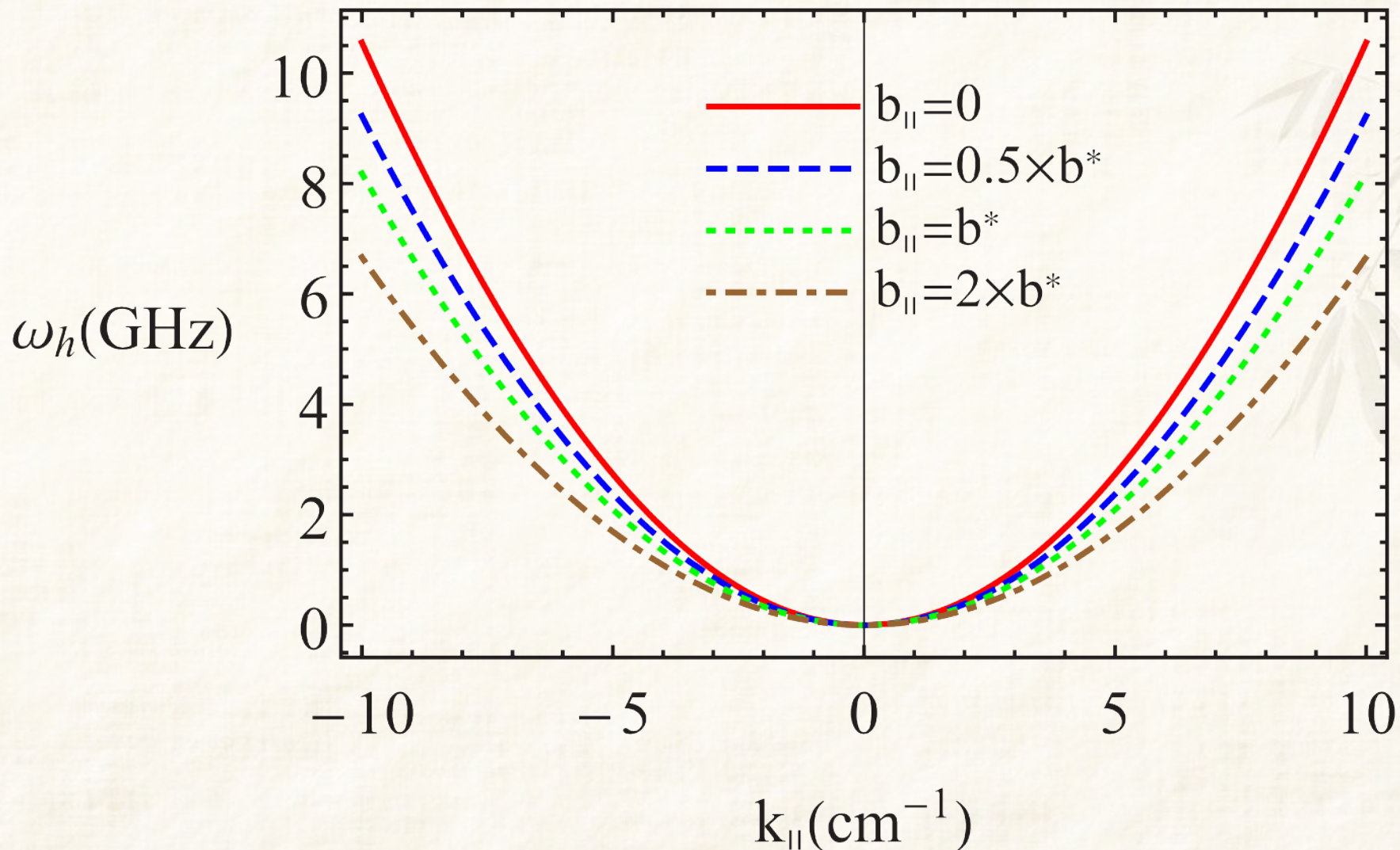
$$\omega_h |_{B_0 \rightarrow 0, \mu \rightarrow 0} \stackrel{b_0 \rightarrow -\mu_5/e}{=} \frac{e B_{0,5} c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu_5 + 2 B_{0,5} e^4 v_F^2 b_{\parallel}} + O(k^3)$$

- Properties:
 - Gapless electromagnetic wave propagates in metals **without magnetic field!**
 - Chiral shift modifies effective helicon mass
 - In the equilibrium regime $eb_0 = -\mu_5$, the linear in the wave vector term is **absent**

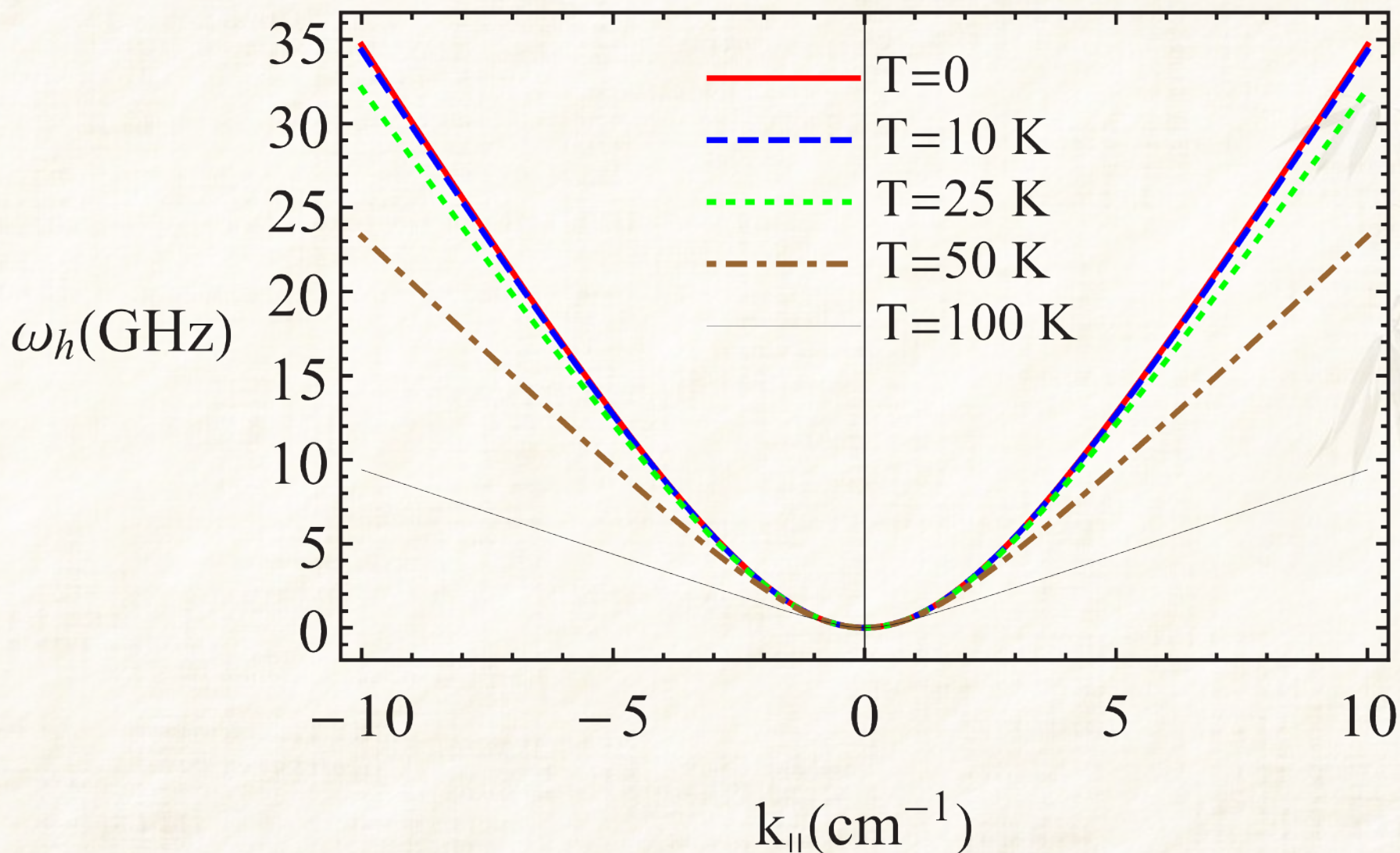
[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

Helicons at different b_{\parallel}

$$eb_0 = -\mu_5, B_{0,5} = 10^{-2}\text{T}, \mu_5 = 5 \text{ meV}, \mu = 0, eb^* = 0.3\pi\hbar v_F/c_3$$



$$eb_0 = -\mu_5, B_{0,5} = 10^{-2}T, b_{\parallel} = 0.5b^*, \mu_5 = 5 \text{ meV}, \mu = 0$$



- Consistent chiral kinetic theory is needed
- Chiral magnetic plasmons (χ MPs) are sensitive to local charge (non-)conservation
- Properties of χ MPs carry information about b_0 and \vec{b}
- χ MPs are not only due to the oscillation of *electric* charge, but also *chiral* charge
- New types of collective modes, pseudomagnetic helicons, may exist in Weyl materials