ARIZONA STATE UNIVERSITY

EXTREME QCD
JUN 27, 2017
Pisa, Italy

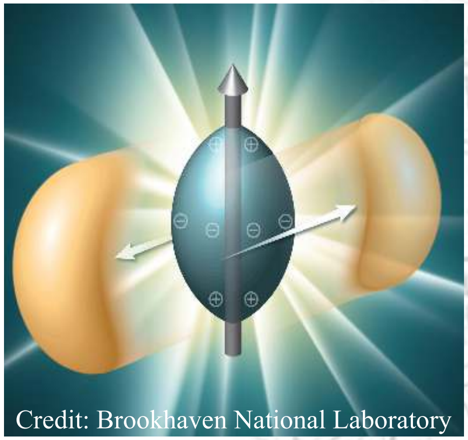
Chiral effects in strong magnetic backgrounds:
from QCD to condensed matter physics

Igor Shovkovy
Arizona State University

QCD in magnetic Fields

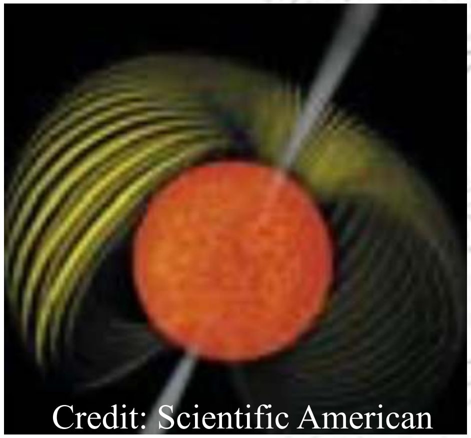
- Relativistic *heavy-ion collisions* produce strong magnetic fields

10^{18} - 10^{19} Gauss $(\sqrt{|eB|} \sim 100\ \text{MeV})$


Credit: Brookhaven National Laboratory

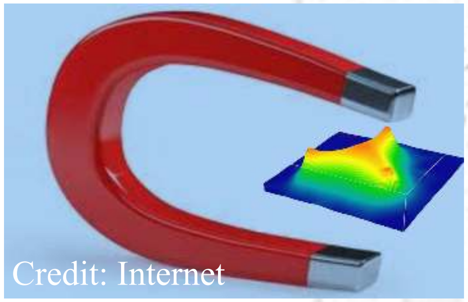
- Quark matter may form inside *magnetars*

10^{14} - 10^{16} Gauss $(\sqrt{|eB|} \sim 1\ \text{MeV to } 10\ \text{MeV})$


Credit: Scientific American

- Strong magnetic field is an instructive *theoretical tool* to study confined gauge theories such as QCD

$\gtrsim 10^{19}$ Gauss $(\sqrt{|eB|} \gtrsim 100\ \text{MeV to } 10\ \text{MeV})$


Credit: Internet

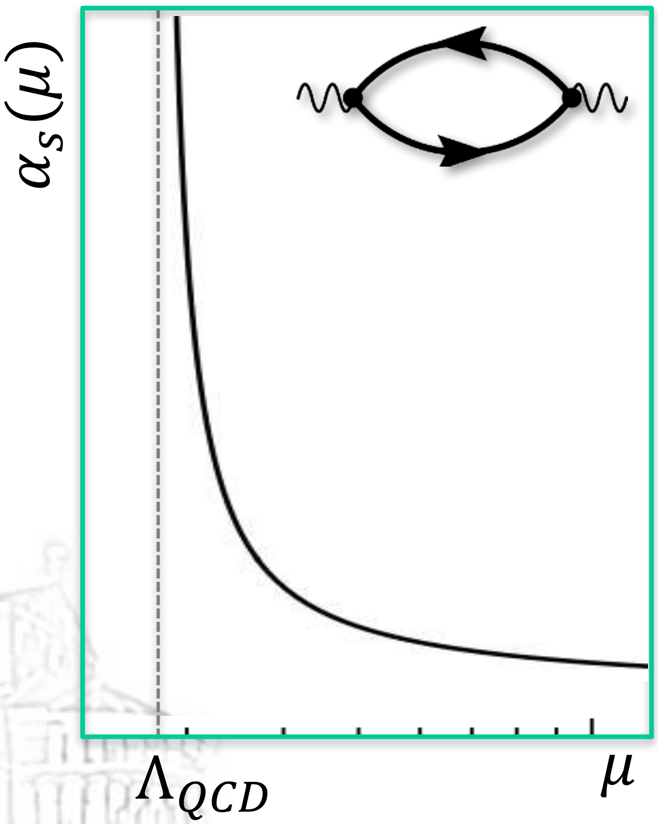
QCD in magnetic field

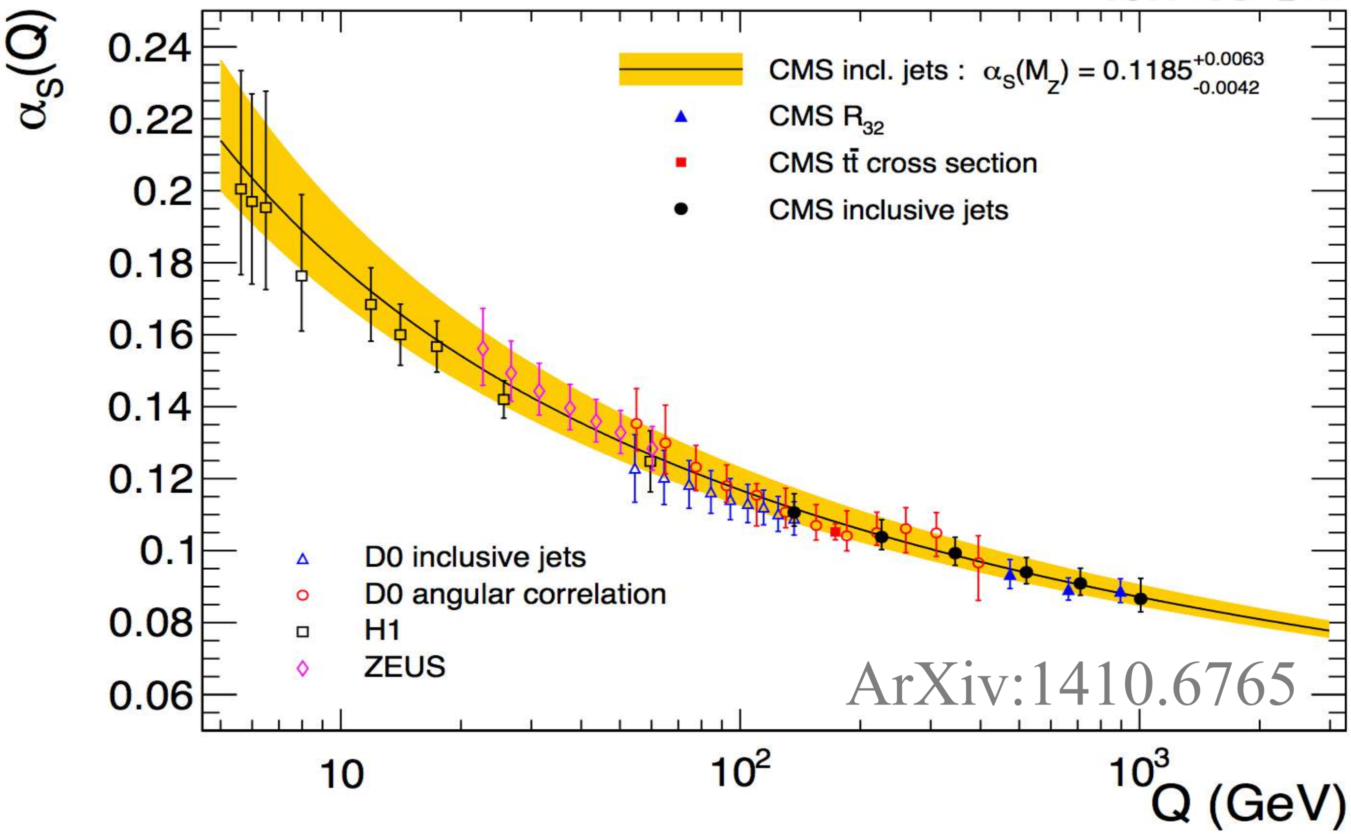
- QCD is strongly coupled & nonperturbative
- There are theoretical tools that provide insight
  - High-energy (weak-coupling) expansion
  - Large N_c expansion
  - High temperature limit ($T \gg \Lambda_{QCD}$)
  - High density limit ($\mu \gg \Lambda_{QCD}$)
  - Lattice QCD
- Strong magnetic field B is yet another tool
  - it probes physics at short distances $\ell \sim 1/\sqrt{|eB|}$

Running coupling & confinment

- Coupling constant in QCD runs with the energy scale,

$$\frac{1}{\alpha_s(\mu)} \simeq b \ln \frac{\mu^2}{\Lambda_{QCD}^2}\,, \quad \text{where} \quad b = \frac{11\,N_c - 2N_f}{12\pi}$$



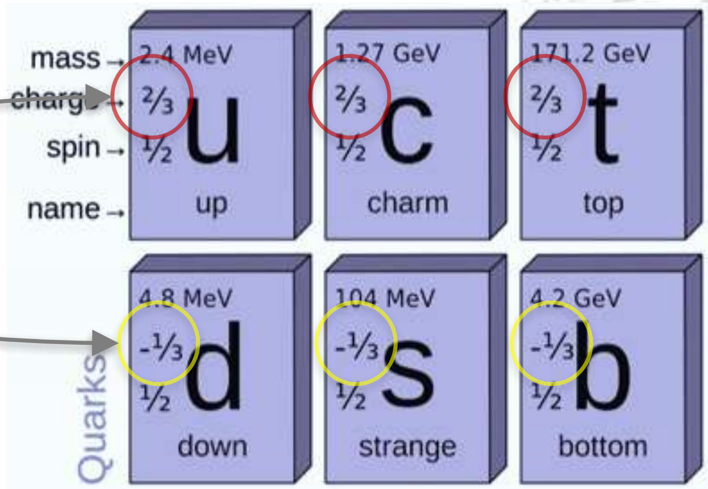


- The question is: What happens in a strong magnetic field?

QCD ground state at $\vec{B} \to \infty$

- Lagrangian density of QCD in an external magnetic field

$$\mathcal{L} = -\frac{1}{2} F_A^{\mu\nu} F_{\mu\nu}^A + \bar{\psi}_f (i\gamma^\mu D_\mu) \psi_f$$



where $D_\mu = \partial_\mu + igA_\mu^A \lambda^A/2 + ie_f A_\mu^{\text{ext}}$

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + gf^{ABC} A_\mu^B A_\nu^C$$

- The global chiral symmetry of the model

$$\underbrace{SU_L(N_u)\times SU_R(N_u)}_{\text{chiral symmetry of up-flavors}}\times\underbrace{SU_L(N_d)\times SU_R(N_d)}_{\text{chiral symmetry of down-flavors}}\times\underbrace{U_A^{(-)}(1)}_{\text{anomaly-free combination of } U_A^{(u)}(1) \text{ and } U_A^{(d)}(1)}$$

- Quark masses $m_u \neq m_d \neq 0$ break the symmetry down to

$$SU_V(N_u)\times SU_V(N_d)$$

QCD at $|eB| \gg \Lambda_{QCD}^2$

- Energy scales in the problem at hand

confined gluodynamics, glueballs

Magnetic catalysis in weakly coupled QCD and strong B-field, strong gluon screening

pure (anisotropic) gluodynamics, all massive quarks decoupled,

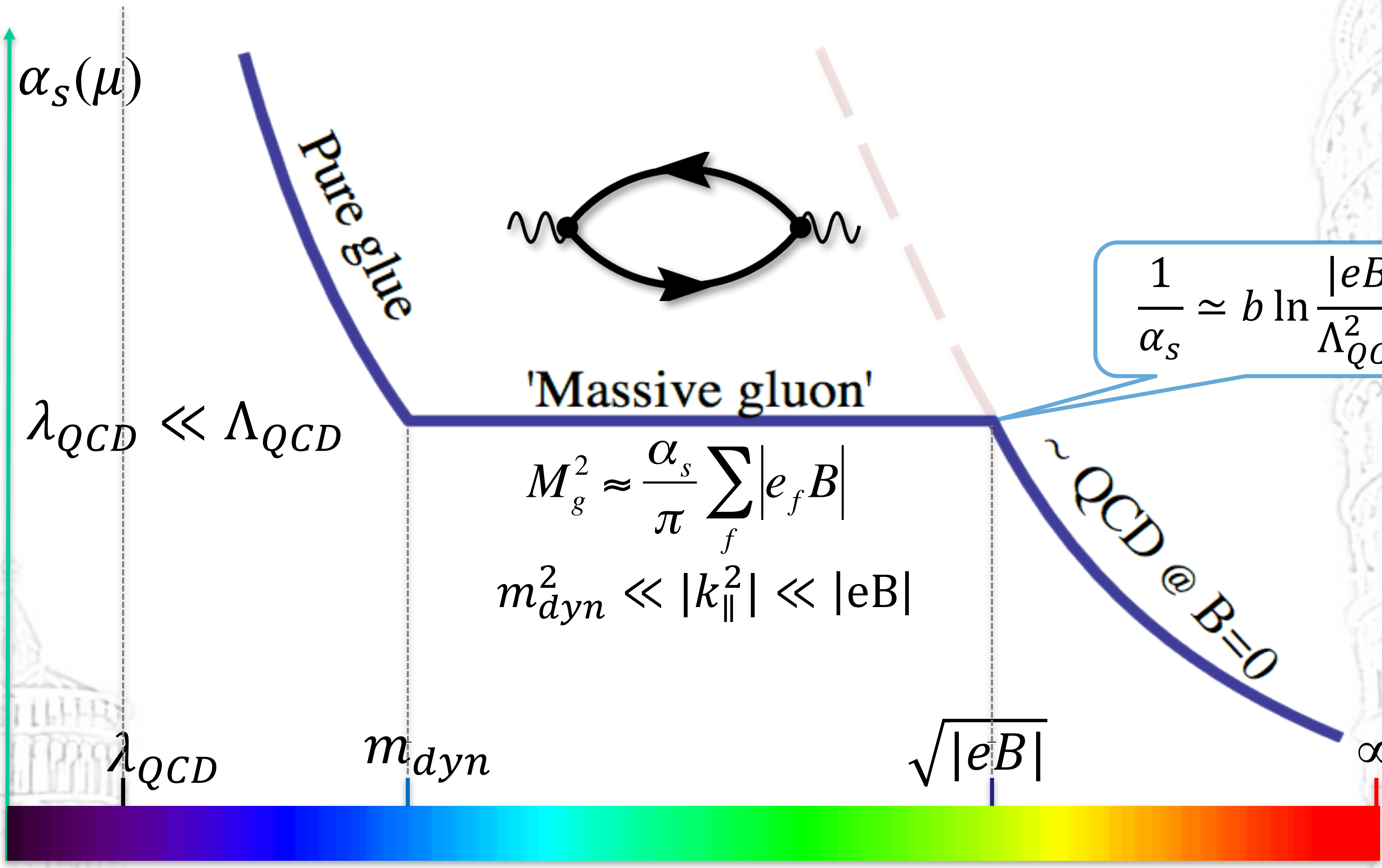
$$\frac{1}{\alpha_s(\mu)} \simeq b_0 \ln \frac{\mu^2}{\Lambda_{QCD}^2}$$

deep-UV region with asymptotic freedom and weak B-field

$$\frac{1}{\alpha_s(\mu)} \simeq b \ln \frac{\mu^2}{\Lambda_{QCD}^2}$$

Running α_s in QCD at strong B

- In deep UV region, α_s is not affected by B-field



Schwinger-Dyson equation

- The general form of the equation is similar to that in QED

$$G^{-1}(x,y) = G_0^{-1}(x,y) + 4\pi\alpha_s\,\gamma^\mu T^A\,G(x,y)\,\gamma^\nu T^B\,\mathcal{D}_{\mu\nu}^{AB}(y-x)$$

Note: $G^{-1}(x,y)$ and $G(x,y)$ have same Schwinger phases!

- Screening effects are included via the polarization function in the strong field limit ($\sqrt{|eB|} \gg \Lambda_{QCD}$)

$$\mathcal{P}^{AB,\mu\nu} \simeq \frac{\alpha_s}{6\pi}\delta^{AB}\left(k_\parallel^\mu k_\parallel^\nu - k_\parallel^2 g_\parallel^{\mu\nu}\right)\sum_{q=1}^{N_f}\frac{|e_q B|}{m_q^2}, \quad \text{for } |k_\parallel^2| \ll m_q^2$$

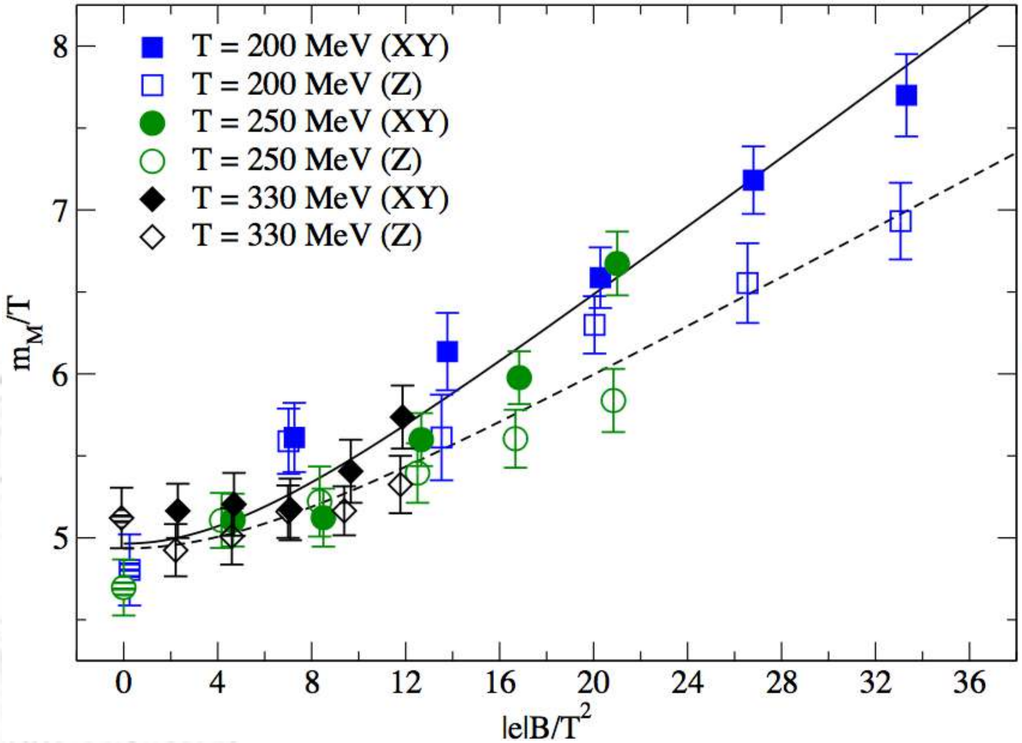
$$\boxed{\mathcal{P}^{AB,\mu\nu} \simeq -\frac{\alpha_s}{\pi}\delta^{AB}\left(k_\parallel^\mu k_\parallel^\nu - k_\parallel^2 g_\parallel^{\mu\nu}\right)\sum_{q=1}^{N_f}\frac{|e_q B|}{k_\parallel^2}, \quad \text{for } m_q^2 \ll |k_\parallel^2| \ll |eB|}$$

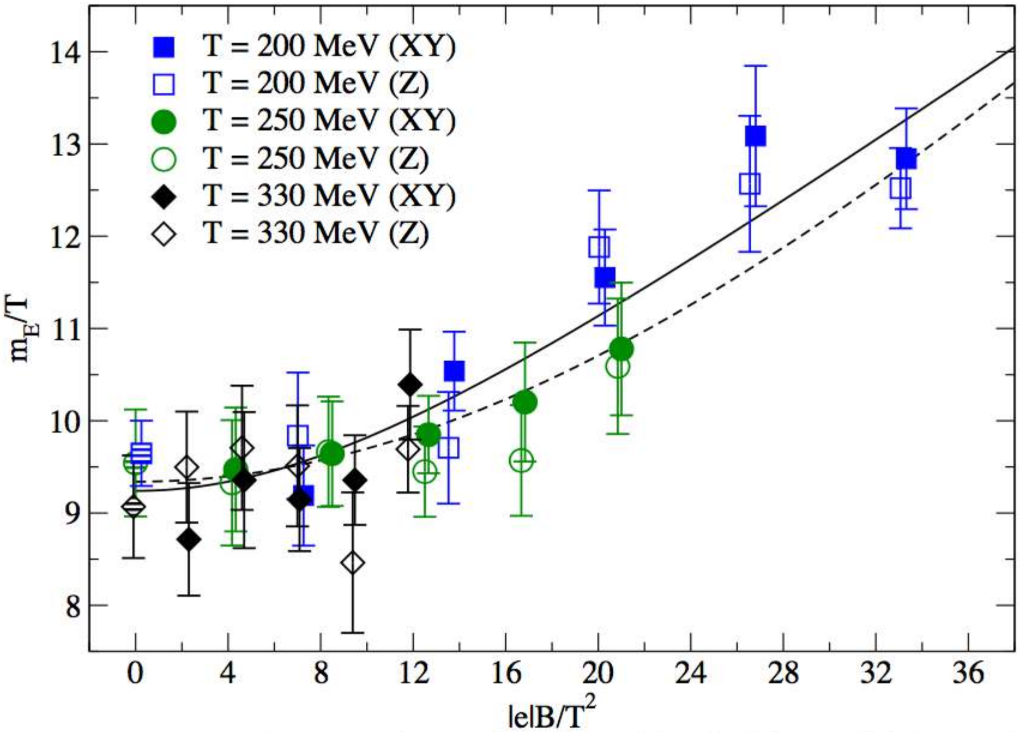
Screening masses: lattice

- Electric and magnetic screening masses on the lattice grow with the field [Bonati et al., Phys. Rev. D 95, 074515 (2017)]

$$\frac{m_E^d}{T} = a_E^d \left[1 + c_{1;E}^d \frac{|e|B}{T^2} \operatorname{atan}\left(\frac{c_{2;E}^d}{c_{1;E}^d} \frac{|e|B}{T^2} \right) \right]$$

(and a similar expression for the magnetic one)





Expression for dynamical mass

- In the region $m_{dyn}^2 \ll |k_\parallel^2| \ll |eB|$, which is most relevant for the fermion-pairing dynamics, the gluon has a "mass"

$$M_g^2 \simeq \frac{\alpha_s}{\pi}\sum_f |e_f B| = \frac{\alpha_s}{3\pi}(2N_u + N_d)\,|eB|$$

- *Rigorous* SD analysis (with higher-order diagrams under control) can be performed by using a special non-local gauge for the gluon propagator

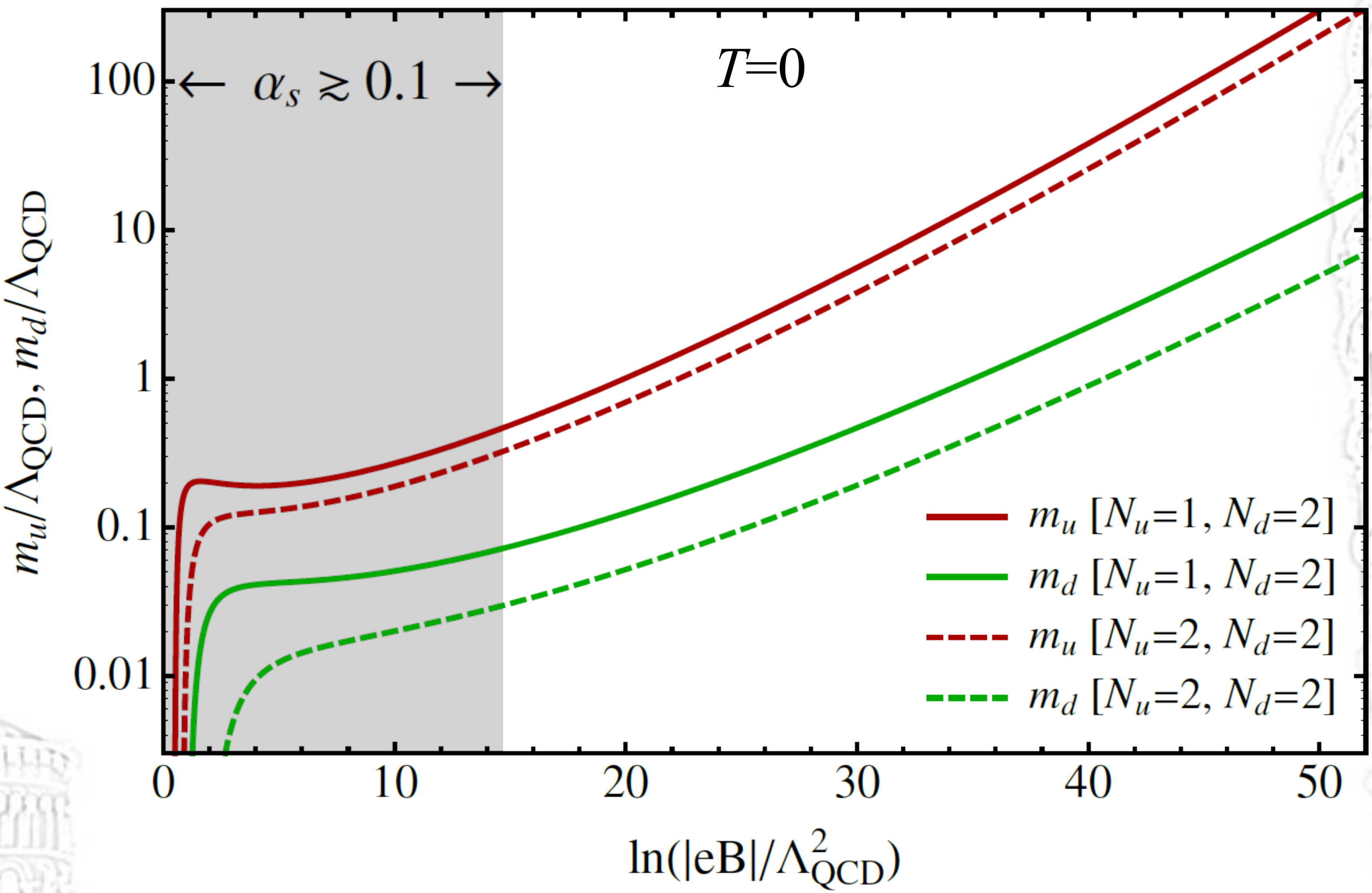
- The final result reads,

$$m_q^2 \simeq 2C_1|e_q B|\left(c_q\alpha_s\right)^{2/3}\exp\left[-\frac{4N_c\pi}{\alpha_s(N_c^2-1)\ln(C_2/c_q\alpha_s)}\right]$$

where $C_1 \simeq C_2 \simeq 1$ and $c_q \simeq (2N_u + N_d)|e|/(6\pi|e_q|)$

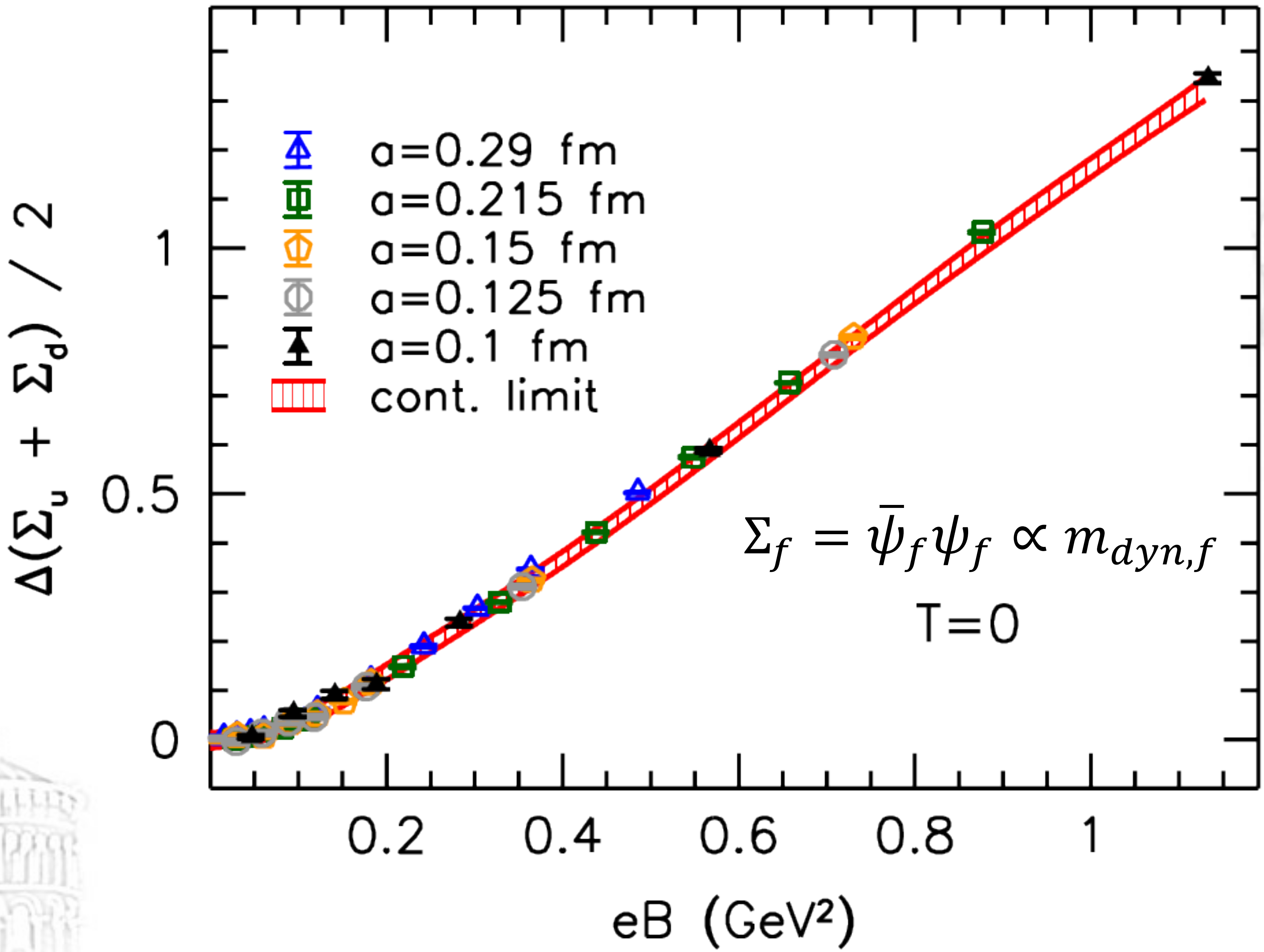
Quark mass vs. B

- Quantitatively, dynamical masses are ($\sqrt{|eB|} \gg \Lambda_{\mathrm{QCD}}$)



[Miransky & Shovkovy, Phys. Rev. D **66** (2002) 045006]

Chiral condensate in lattice QCD



[Bali et al., Phys. Rev. D**86**, 071502 (2012)]

Nambu-Goldstone bosons (pions)

- Original global chiral symmetry

$$SU_L(N_u)\times SU_R(N_u)\times SU_L(N_d)\times SU_R(N_d)\times U_A^{(-)}(1)$$

breaks down to

$$SU_V(N_u)\times SU_V(N_d)$$

- Thus, there should be $(N_u^2 + N_d^2 - 1)$ massless NG bosons
- The unitary pion fields can be written in terms of the coset space generators

$$\Sigma_u \equiv \exp\left(i \sum_{A=1}^{N_u^2-1} \lambda^A \pi_u^A / f_u\right),\ \Sigma_d \equiv \exp\left(i \sum_{A=1}^{N_d^2-1} \lambda^A \pi_d^A / f_d\right)$$

and $\tilde{\Sigma} \equiv \exp\left(i\sqrt{2}\tilde{\pi}/\tilde{f}\right)$

- In a very strong magnetic field another light pseudo-NG boson, associated with anomalous $U_A(1)$, may appear

Low-energy action for NG bosons

- The low-energy effective action should have the form

$$\mathcal{L}_{NG} \simeq \frac{f_u^2}{4} \mathrm{tr}\left(\mathrm{g}_{\parallel}^{\mu\nu}\partial_\mu \Sigma_u \partial_\nu \Sigma_u^\dagger + v_u^2 \mathrm{g}_{\perp}^{\mu\nu}\partial_\mu \Sigma_u \partial_\nu \Sigma_u^\dagger\right) + \cdots$$

- The pion decay constants are defined by

$$i\left\langle 0\left|\bar{\psi}\gamma^\mu\gamma^5\frac{\lambda^A}{2}\psi\right|\pi^B(P)\right\rangle = P^\mu f_\pi \delta^{AB} = -i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\left(\gamma^\mu\gamma^5\frac{\lambda^A}{2}\chi_q^B(k,P)\right)$$

where $P^\mu = \left(P^0, v_\perp^2 \vec{P}_\perp, P^3\right)$

- The Bethe-Salpeter wave function looks like in QED in LLL approximation. So, we find that $v_\perp^2 \approx 0$, and

$$f_q^2 = 4N_c\int\frac{d^2k_\perp d^2k_\parallel}{(2\pi)^4}\exp\left(-\frac{k_\perp^2}{|e_qB|}\right)\frac{m_q^2}{(k_\parallel^2+m_q^2)^2}$$

which can be easily calculated, giving

$$f_u^2 = \frac{N_c|eB|}{6\pi^2} \quad \text{and} \quad f_d^2 = \frac{N_c|eB|}{12\pi^2}$$

Far IR region, $|k_\parallel^2| \lesssim m_{dyn}^2$

- Massive quarks decouple from the low-energy dynamics

- Gluons are the only "light" degrees of freedom
- Assuming that $\Lambda_{QCD}^2 \ll m_{dyn}^2$, the gluodynamics has a semi-perturbative region, $|k_\parallel^2| \lesssim m_{dyn}^2$, where

$$\frac{1}{\tilde{\alpha}_s(\mu)} - \frac{1}{\alpha_s} \simeq b_0 \ln \frac{\mu^2}{m_{dyn}^2}$$

here $b_0 = \frac{11\, N_c}{12\pi}$ and $\frac{1}{\alpha_s} \simeq b \ln \frac{|eB|}{\Lambda_{QCD}^2}$ (Recall: $b = \frac{11\, N_c - 2N_f}{12\pi}$)

- Then, we find that the new confinement scale where $\tilde{\alpha}_s = \infty$:

$$-b \ln \frac{|eB|}{\Lambda_{QCD}^2} \simeq b_0 \ln \frac{\lambda_{QCD}^2}{m_{dyn}^2} \quad \Rightarrow \quad \lambda_{QCD} = m_{dyn} \left(\frac{\Lambda_{QCD}}{\sqrt{|eB|}} \right)^{b/b_0}$$

Gluodynamics in far IR

- Quadratic part of low-energy effective action for gluons

$$\mathcal{L}^{(2)}_{\text{glue,eff}} = -\frac{1}{2}\sum_{A=1}^{N_c^2-1} A^A_\mu(-k)\left[g^{\mu\nu}k^2 - k^\mu k^\nu + \kappa\left(g_\parallel^{\mu\nu}k_\parallel^2 - k_\parallel^\mu k_\parallel^\nu\right)\right]A^A_\nu(k)$$

where the susceptibility κ is extracted from the polarization tensor $\mathcal{P}^{AB}_{\mu\nu}$ in the region $\left|k_\parallel^2\right| \ll m_{dyn}^2$, i.e.,

$$\kappa = \frac{\alpha_s}{6\pi}\sum_{q=1}^{N_f}\frac{|e_q B|}{m_q^2} = \frac{1}{12C_1\pi}\sum_{q=1}^{N_f}\left(\frac{\alpha_s}{c_q^2}\right)^{1/3}\exp\left(\frac{4N_c\pi}{\alpha_s(N_c^2-1)\ln(C_2/c_q\alpha_s)}\right) \gg 1$$

- The requirement of gauge invariance allows to write down the complete expression for the gluon action

$$\mathcal{L}_{\text{glue,eff}} \simeq \frac{1}{2}\sum_{A=1}^{N_c^2-1}\left(\mathbf{E}^A_\perp\cdot\mathbf{E}^A_\perp + \epsilon E_3^A E_3^A - \mathbf{B}^A_\perp\cdot\mathbf{B}^A_\perp - B_3^A B_3^A\right)$$

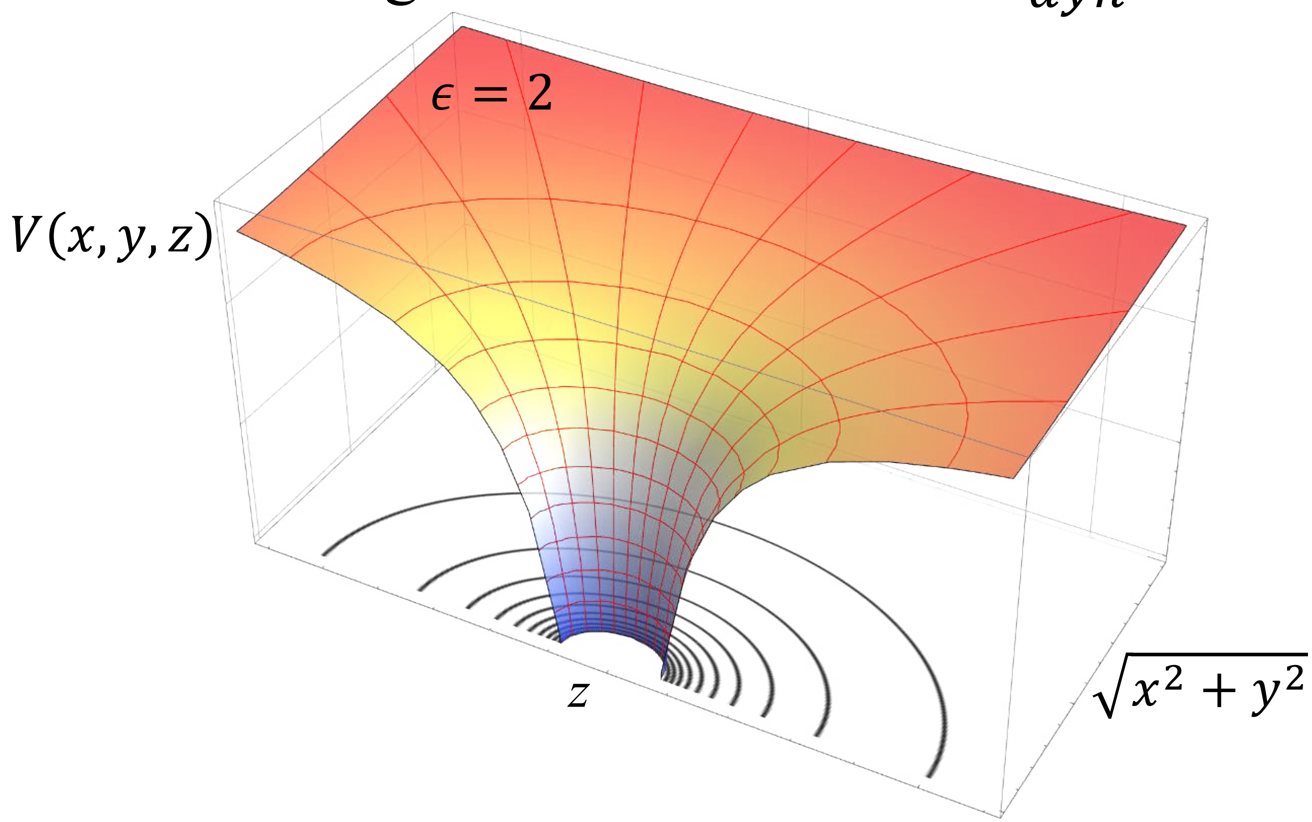
where $\epsilon = 1 + \kappa$ is a chromo-dielectric constant (note $\epsilon \gg 1$), $E_i^A = F_{0i}^A$ and $B_i^A = {}^1/_2\,\varepsilon_{ijk}F_{jk}^A$ are chromo-fields

Effective potential

- By using the guidance from an analogous *anisotropic* QED, the static potential between a pair of quarks should be given by

$$V(x,y,z) = -\frac{g_s^2}{4\pi\sqrt{z^2+\epsilon(x^2+y^2)}}$$

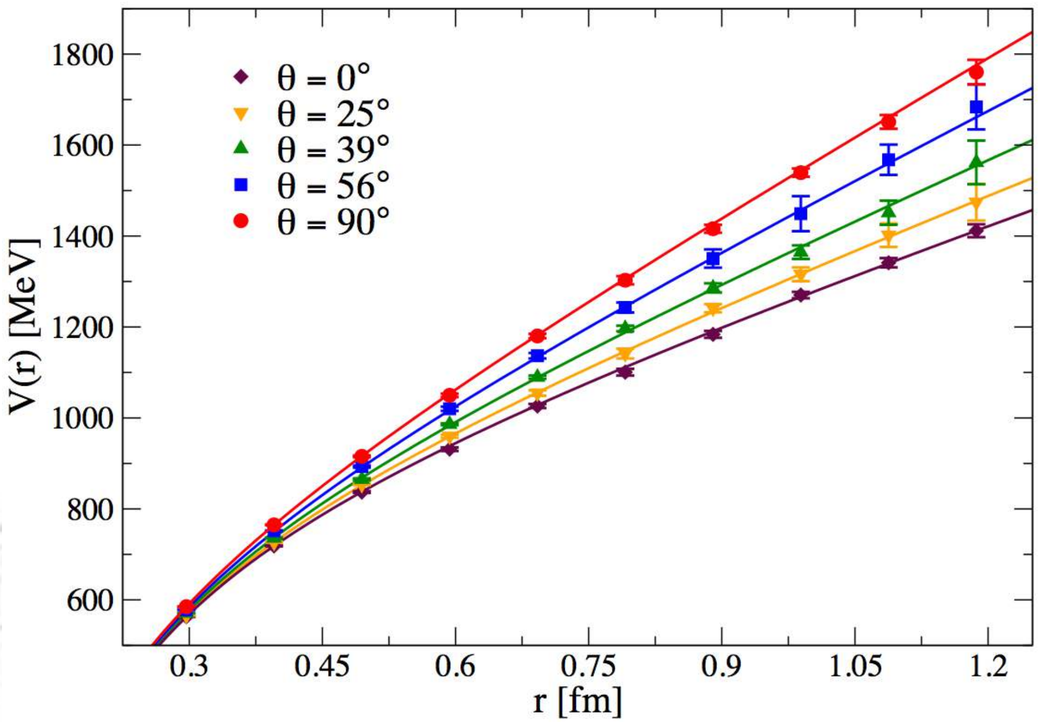
which is valid for a range of distance scales $m_{dyn}^{-1} \lesssim r \lesssim \lambda_{QCD}^{-1}$

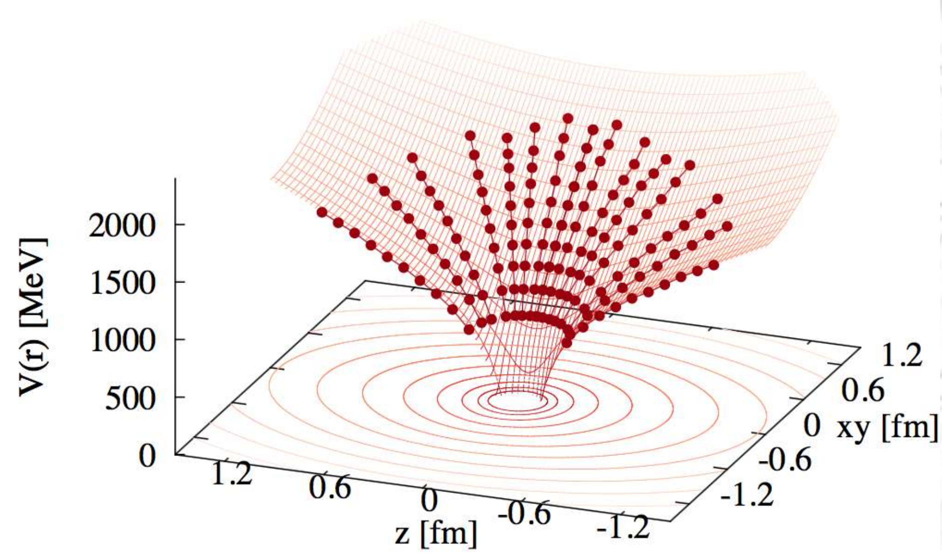


Anisotropy in detail

- The dependence of the potential as a function of angle θ between $\vec{B}$ and $q\bar{q}$ orientation [Bonati et al., Phys. Rev. D 94, 094007 (2016)]

$$V(r,\theta;B) = -\frac{\alpha(\theta;B)}{r} + \sigma(\theta;B)r + V_0(\theta;B)$$

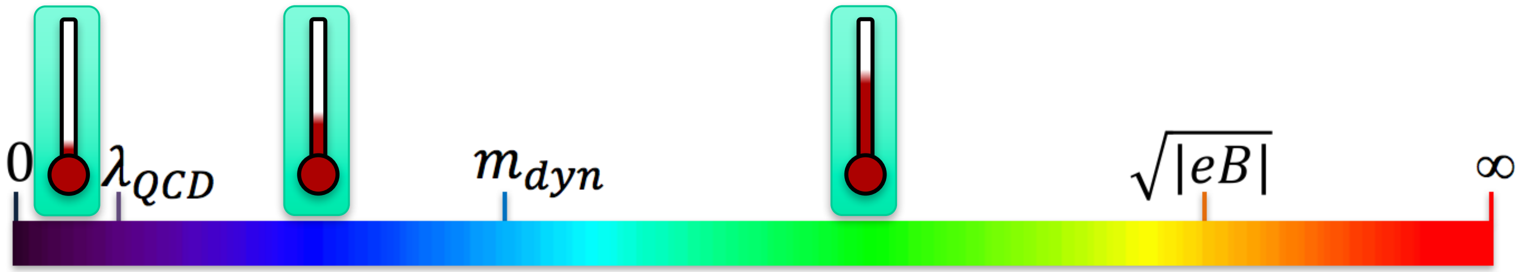




- With increasing angle θ, the string tension increases

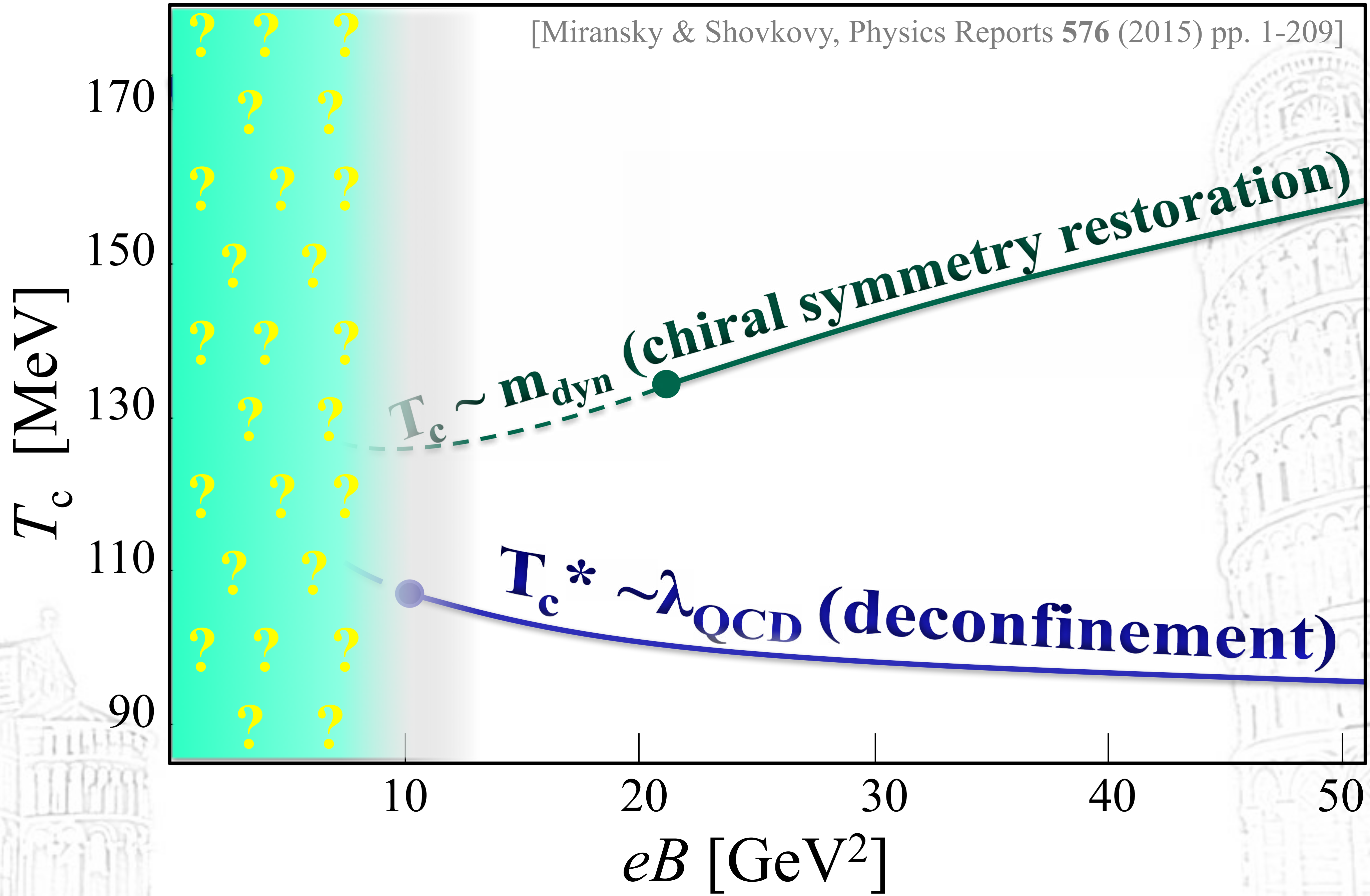
Nonzero temperature

- What to expect at nonzero temperature (in strong B limit)?

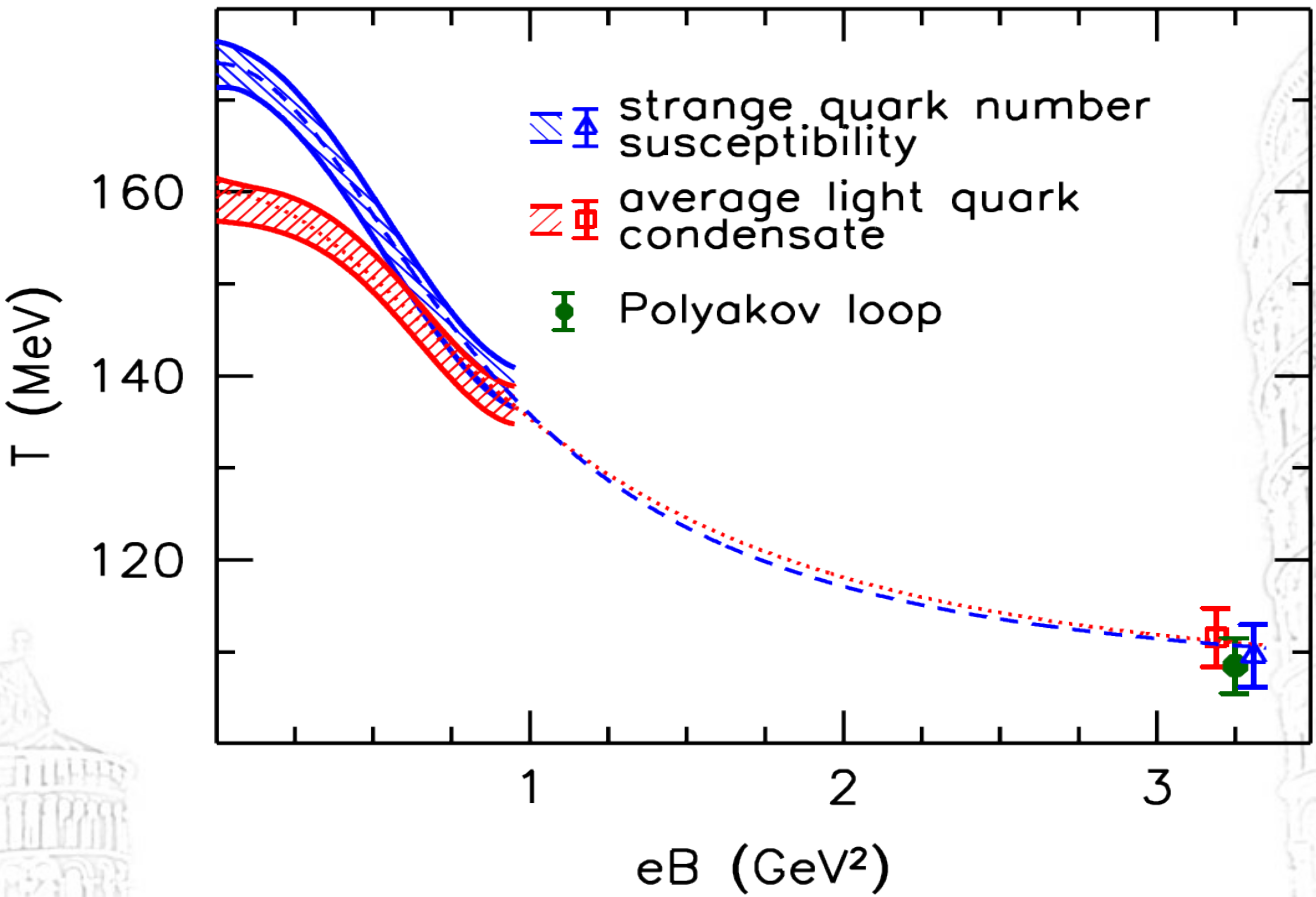


- Very low temperatures, $T \ll \lambda_{QCD}$
  - Ground state in not affected much
  - Color is confined, lowest energy states are glueballs
  - Chiral symmetry is broken ($T \ll \lambda_{QCD} \ll m_{dyn}$)
- Intermediate temperatures, $\lambda_{QCD} \ll T \ll m_{dyn}$
  - Color is deconfined; gluons are thermally populated
  - Chiral symmetry is still broken ($\lambda_{QCD} \ll T \ll m_{dyn}$)
- Moderately high temperatures, $m_{dyn} \ll T \ll \sqrt{|eB|}$
  - Chiral symmetry is restored ($m_{dyn} \ll T$)

Predicted phase diagram

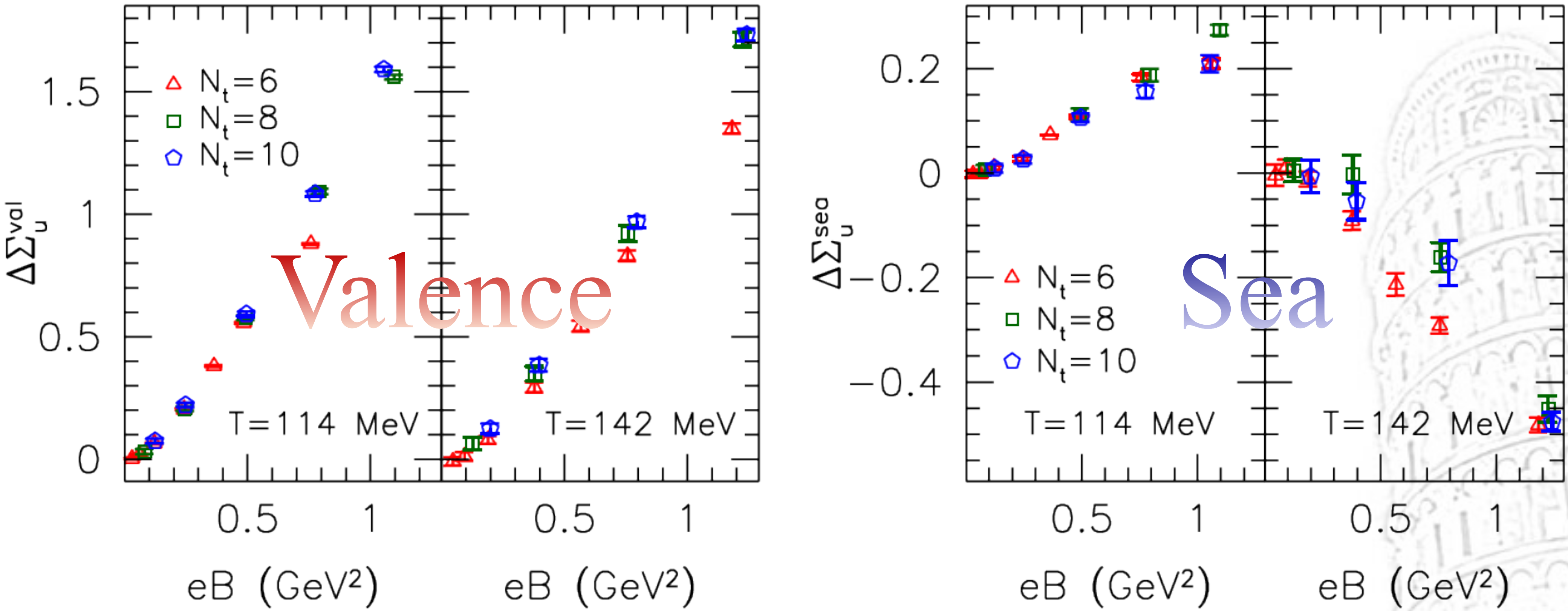


Dependence of T_c vs. B



[Bali et al., JHEP 02, 044 (29012)], [Bali et al., PRD**86**, 071502 (2012)], [G. Endrodi, JHEP 1507 (2015) 173]

Valence vs. sea

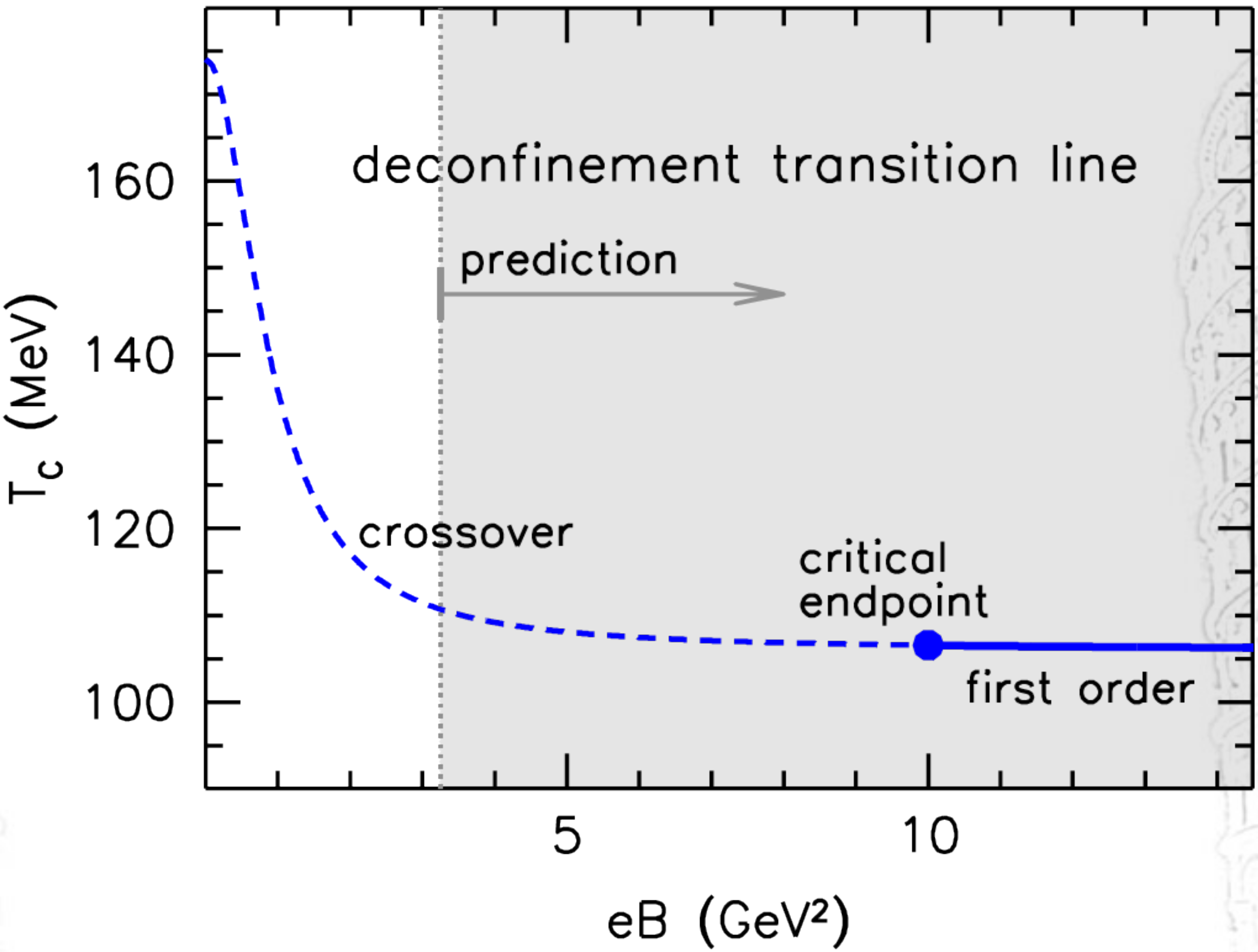


[Bruckmann, Endrodi, Kovacs, JHEP 04 (2013) 112]

- Gluon screening (?)
- Polyakov loops (?)

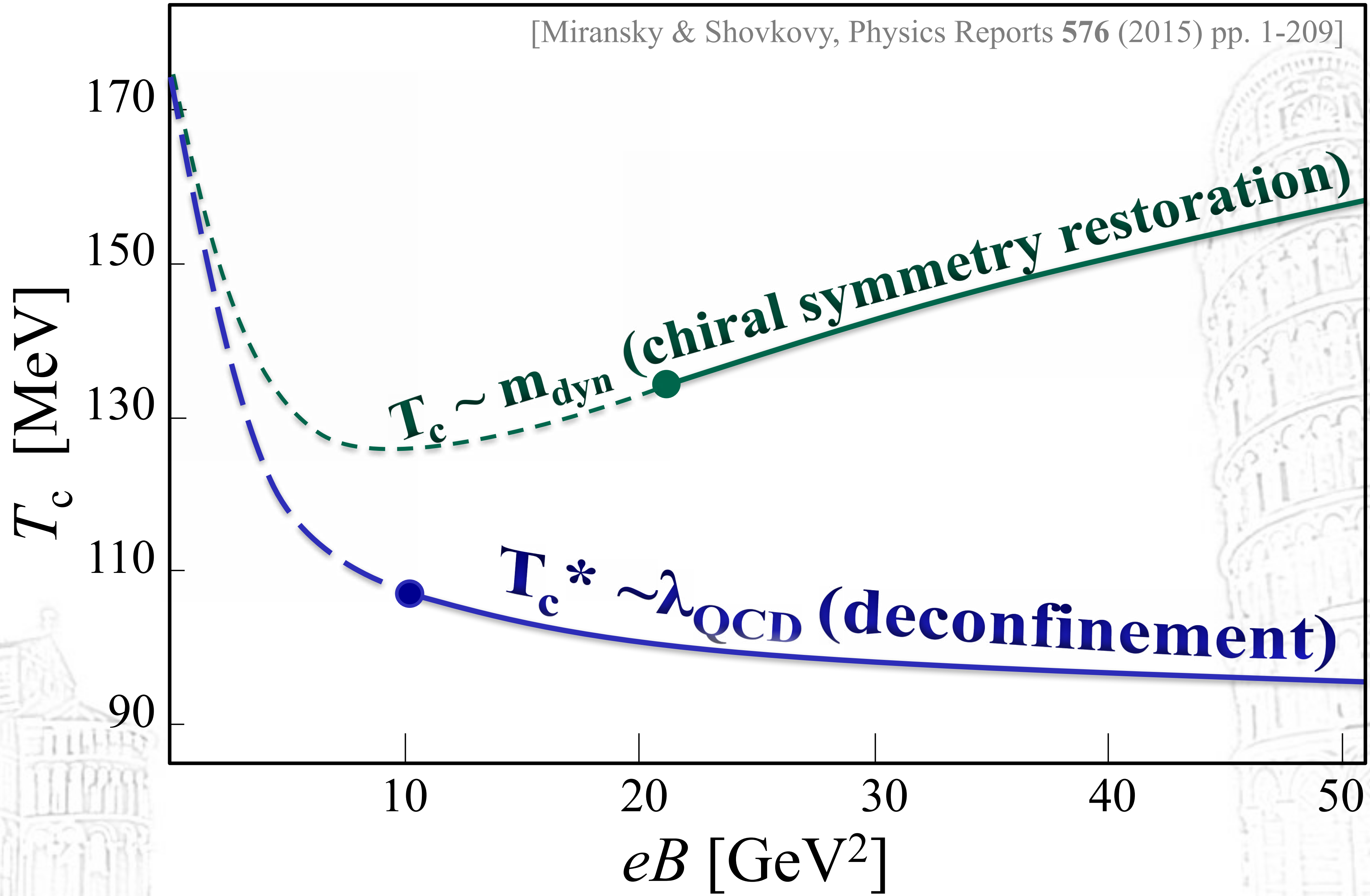
or, perhaps, something else (?)

Super-strong B: prediction



[Cohen & Yamamoto, PRD**89**, 054029 (2014)], [G. Endrodi, JHEP 1507 (2015) 173]

Predicted phase diagram



CHIRAL MATTER

Chiral matter

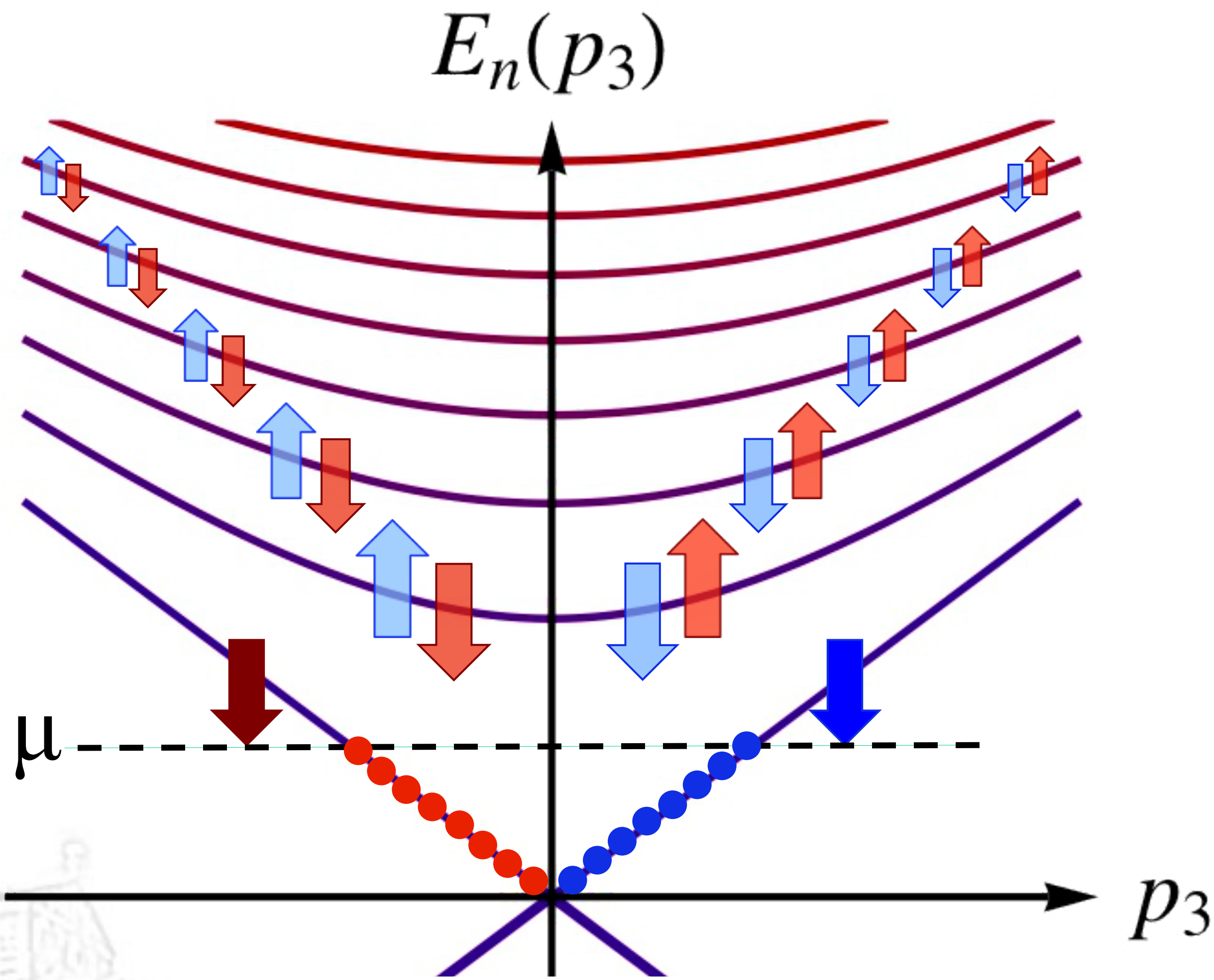
- Matter made of chiral fermions with $n_\mathrm{L} \neq n_\mathrm{R}$
- Unlike the electric charge ($n_\mathrm{R} + n_\mathrm{L}$), the chiral charge ($n_\mathrm{R}$ - n_L) is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral symmetry is anomalous in quantum theory

Chiral separation effect (μ≠0)



— **Right-handed**

— **Left-handed**

Spin (s=↓) **polarized** LLL:

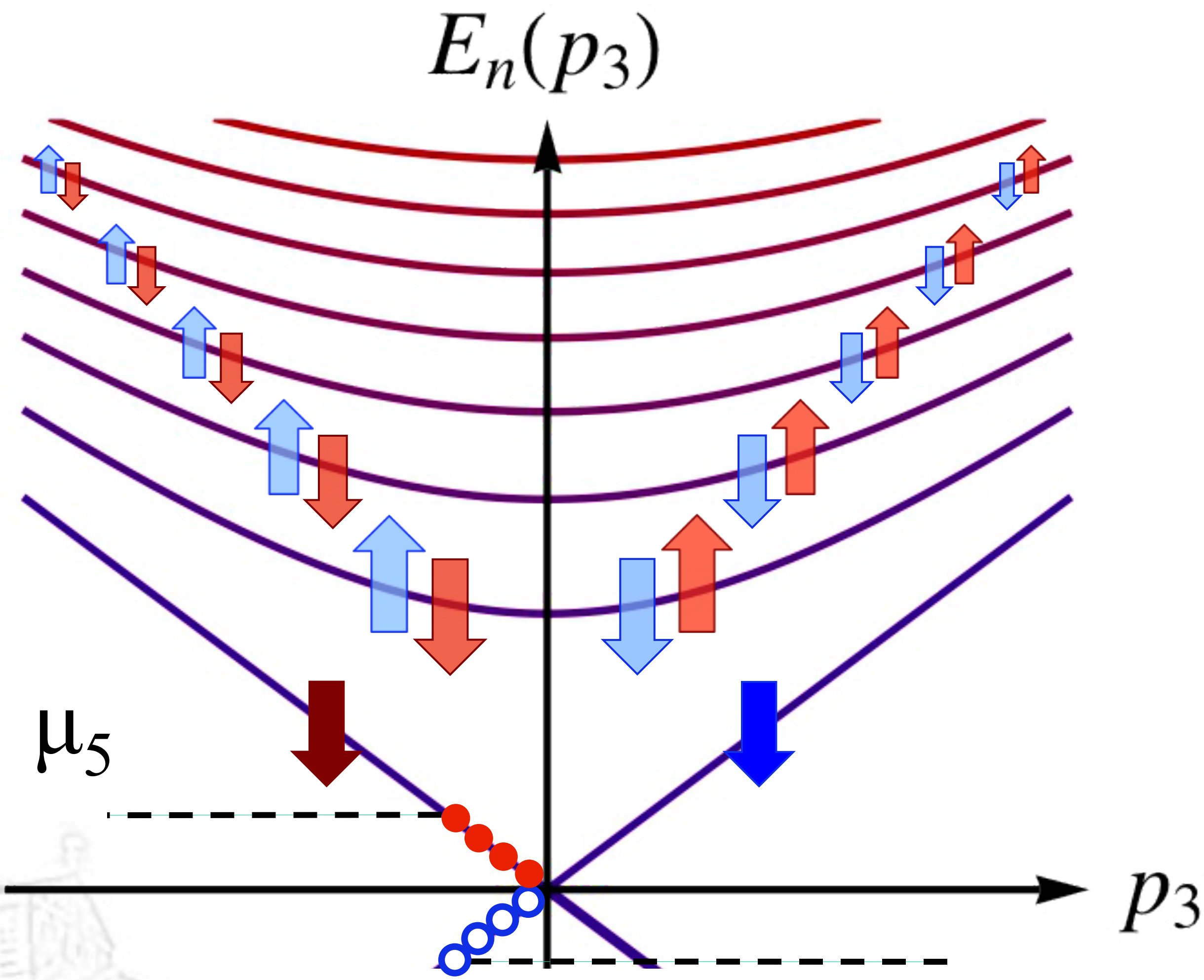
- $p_3<0$ states are R-handed
- $p_3>0$ states are L-handed

This leads to CSE:

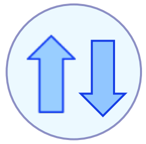
$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Chiral magnetic effect ($\mu_5 \neq 0$)



— **Right-handed**



— **Left-handed**

Spin (s=↓) **polarized** LLL:

- $p_3 < 0$ states are R-handed electrons
- $p_3 > 0$ states are L-handed positrons

This leads to CME:

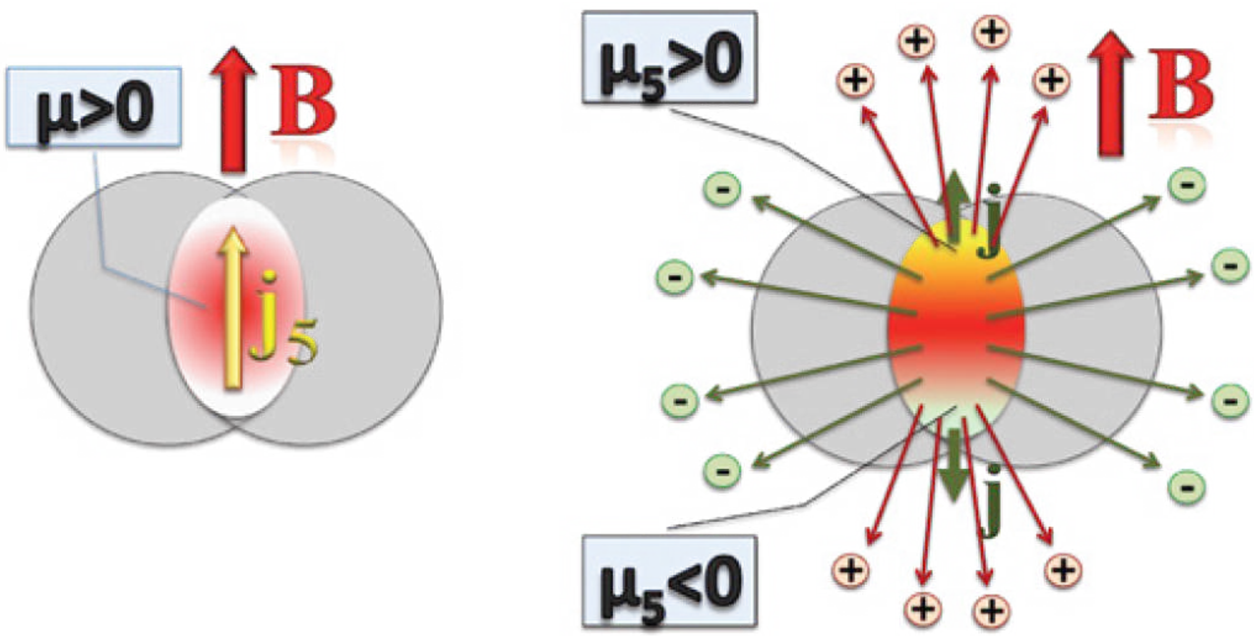
$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CMW/Quadrupole CME

- Start from a small baryon density and B≠0

$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Dirac & Weyl materials

- Na_3Bi

[Z. K. Liu et al., Science **343**, 864 (2014)]

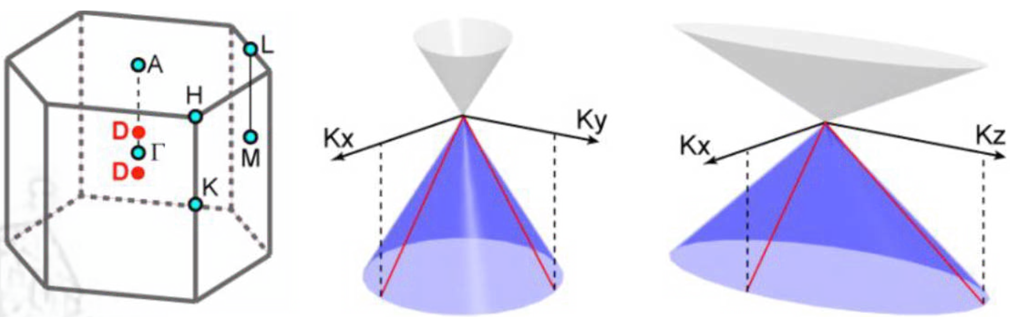
- Cd_3As_2

[M. Neupane et al., Nature Commun. **5**, 3786 (2014)]

[S. Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

- $ZrTe_5$

[X. Li et al., Nature Physics **12**, 550 (2016)]



- TaAs (tantalum arsenide)

[S.-Y. Xu et al., Science 349, 613 (2015)]

[B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]

- NbAs (niobium arsenide)

[S.-Y. Xu et al., Nature Physics 11, 748 (2015)]

- TaP (tantalum phosphide)

[S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]

- NbP (niobium phosphide)

[I. Belopolski et al. arXiv:1509.07465]

- WTe_2 (tungsten telluride)

[F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]

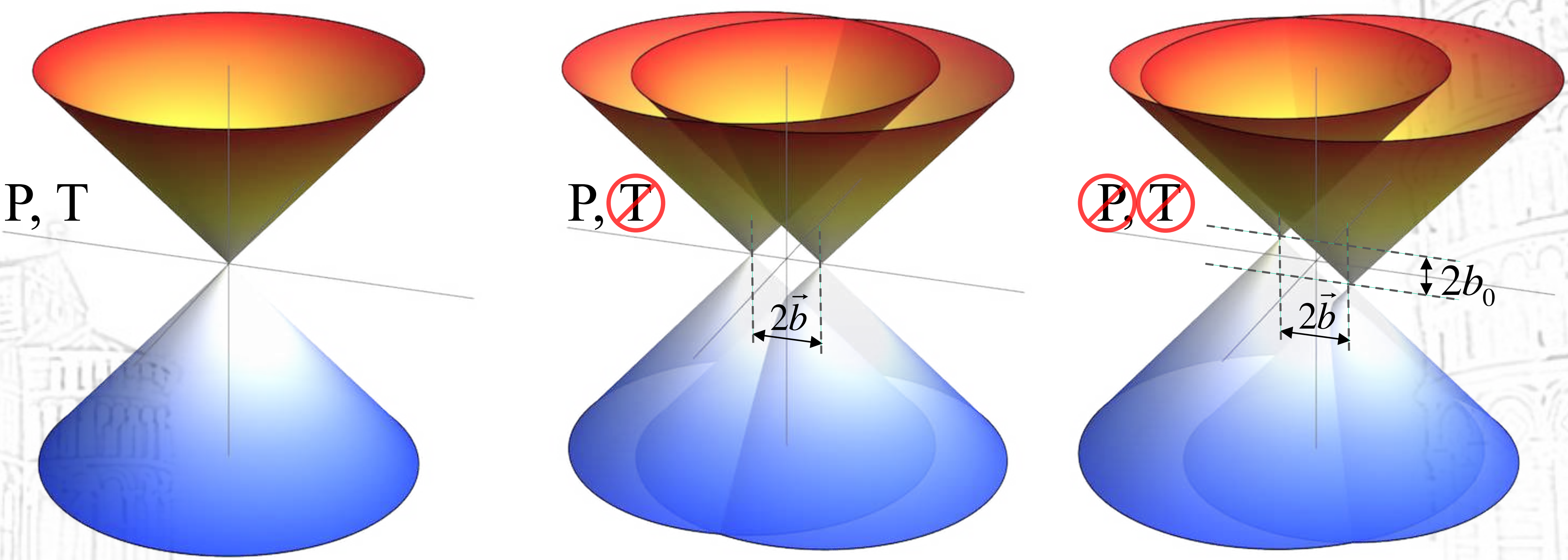
$$E=\sqrt{v_x^2k_x^2+v_y^2k_y^2+v_z^2k_z^2},\quad v_x\approx v_y\approx 3.74\times10^5\,\mathrm{m/s},\quad v_z\approx 2.89\times10^4\,\mathrm{m/s}$$

Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H=\int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\overbrace{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5}^{\cancel{T}}+\overbrace{b_0\gamma^0\gamma^5}^{\cancel{P}}\right]\psi$$

Dirac | Weyl



Strain in Weyl materials

- Strains affect low-energy quasiparticles in Weyl materials

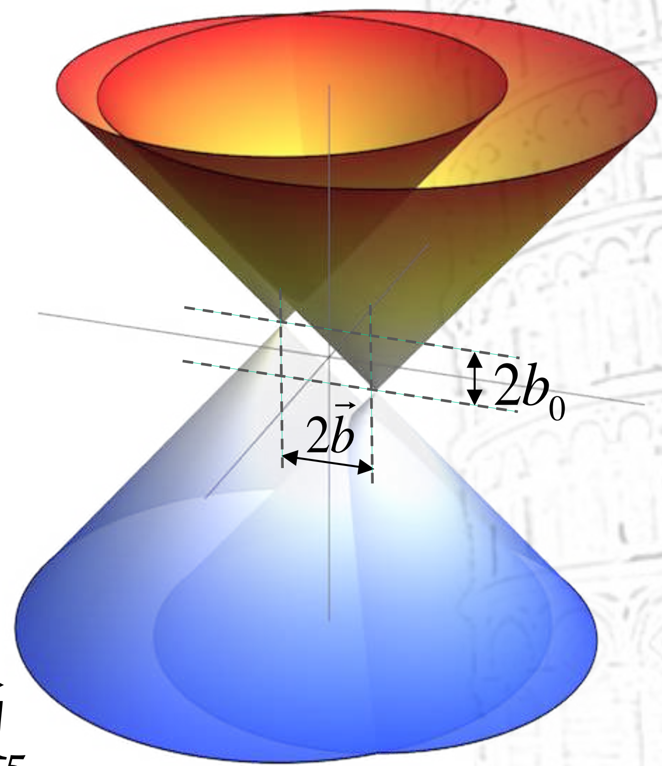
$$H=\int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

where the components of the chiral gauge fields are

$$A_{5,0}\propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

$$A_{5,\perp}\propto\left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||}\propto\alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$



The associated pseudo-EM fields are

$$\vec{B}_5=\vec{\nabla}\times\vec{A}_5\quad\text{and}\quad\vec{E}_5=-\vec{\nabla}A_0-\partial_t\vec{A}_5$$

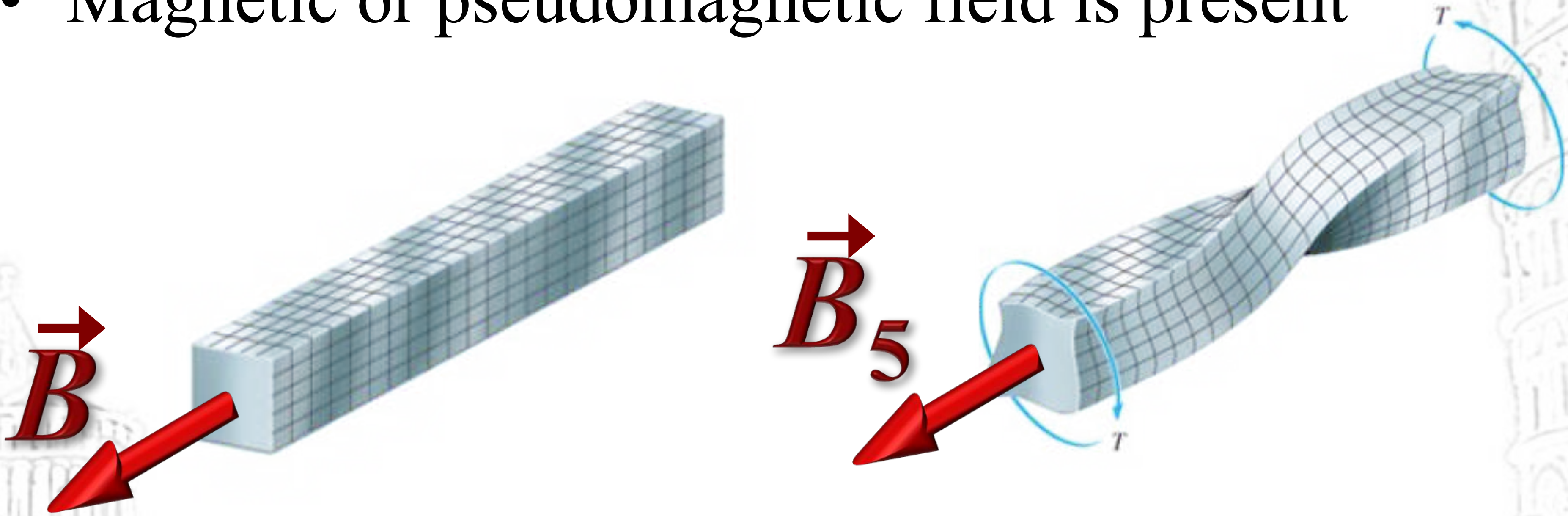
Chiral effects

- Any signature properties of Dirac/Weyl materials directly sensitive to chiral anomaly?
- Some proposals:
  - Anomalous Hall effect
  - Anomalous Alfven waves
  - Strain/torsion induced CME
  - Strain/torsion induced quantum oscillations
  - Strain/torsion dependent resistance
  - etc.
- Spectrum of chiral (pseudo-)magnetic plasmons

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

General question

- What are the properties of plasmons in magnetized chiral material with $b_0 \neq 0$ and $\vec{b} \neq 0$?
- Chiral matter ($\mu_R \neq \mu_L$)
  - This is the case in equilibrium when $b_0 \neq 0$ ($\mu_5 = -eb_0$)
- Magnetic or pseudomagnetic field is present



- In general, $\mathbf{E}_\lambda = \mathbf{E} + \lambda \mathbf{E}_5$ and $\mathbf{B}_\lambda = \mathbf{B} + \lambda \mathbf{B}_5$

Chiral kinetic theory

- Kinetic equation:

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$
$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}\,, \qquad \mathbf{v} = \boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}\,,$

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Current and chiral anomaly

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right] f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda)\right] f_\lambda$$

$$+e\boldsymbol{\nabla}\times\int \frac{d^3p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}}\boldsymbol{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B})+(\mathbf{E}_5\cdot\mathbf{B}_5)\right] \quad ✔$$

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B}_5)+(\mathbf{E}_5\cdot\mathbf{B})\right] \quad ✘$$

Consistent definition of current

- Additional Bardeen-Zumino term is needed,

$$\delta j^{\mu} = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda} A^5_{\nu} F_{\rho\lambda}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

- In components,

$$\delta\rho = \frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{A}^5 \cdot \mathbf{B})$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c} A^5_0 \mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c}(\mathbf{A}^5 \times \mathbf{E})$$

- Its role and implications:
  - Electric charge is conserved locally ($\partial_{\mu} J^{\mu} = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)

Collective modes

We search for plane-wave solutions with

$$\mathbf{E}' = \mathbf{E} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}, \quad \mathbf{B}' = \mathbf{B} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

and the distribution function $f_\lambda = f_\lambda^{(\mathrm{eq})} + \delta f_\lambda$,

where

$$\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

The polarization vector & susceptibility tensor:

$$P^m = i\frac{J'^m}{\omega} = \chi^{mn} E'^n$$

The plasmon dispersion relations follow from

$$\det\left[\left(\omega^2 - c^2 k^2\right)\delta^{mn} + c^2 k^m k^n + 4\pi\omega^2 \chi^{mn}\right] = 0$$

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

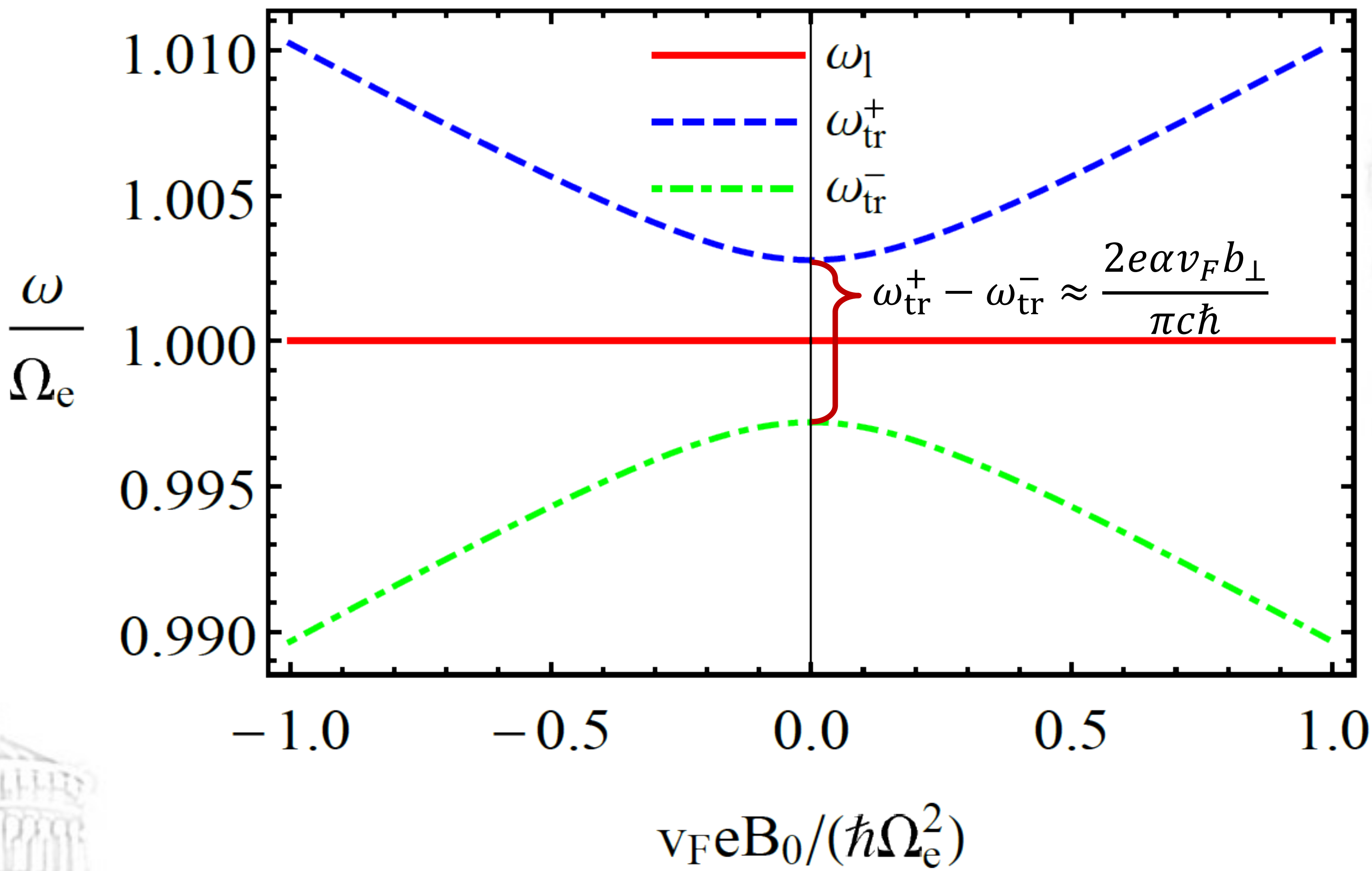
$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

and $$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\Bigg\{9\hbar^2 b_\perp^2 + \Bigg[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_{\parallel} - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\Bigg]^2\Bigg\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

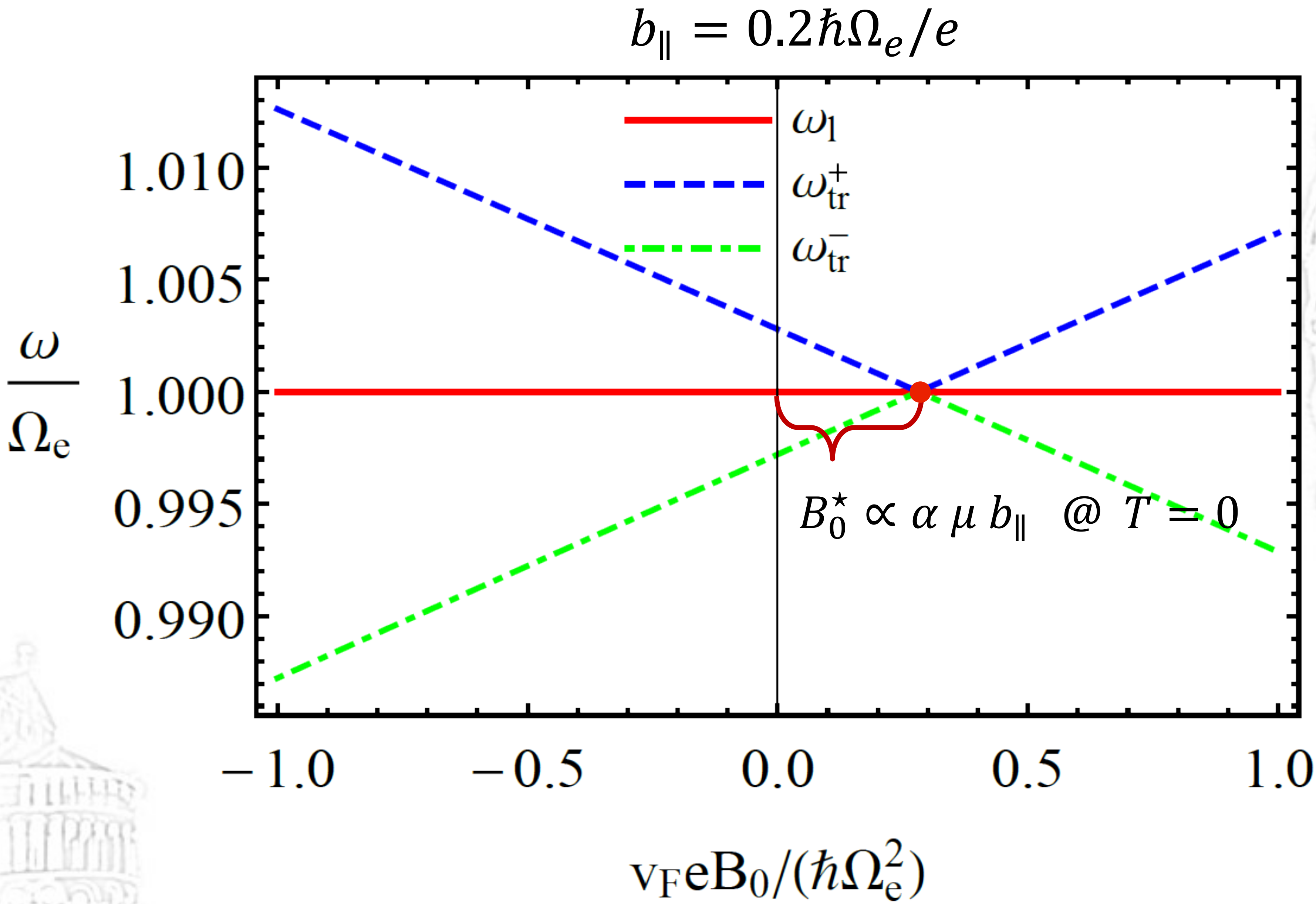
Plasmon frequencies, $\vec{B} \perp \vec{b}$

$b_\perp = 0.2\hbar\Omega_e/e$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

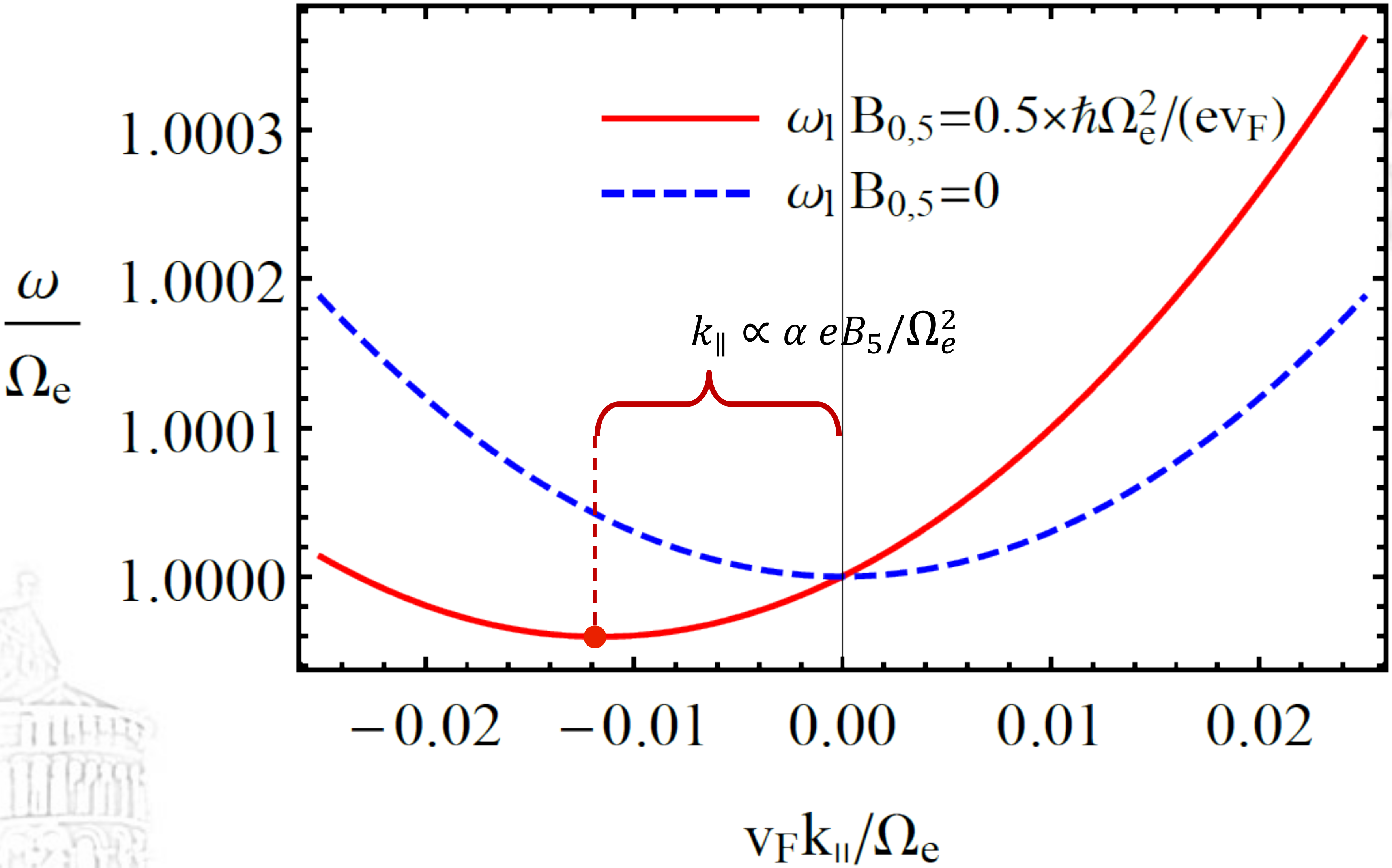
Plasmon frequencies, $\vec{B} \parallel \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmons with $\vec{k} \neq 0$, $\vec{k} \parallel \vec{B}, \vec{B}_5$

- The longitudinal mode is sensitive to $\vec{B}_5$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Summary

- Questions remain about the competition of magnetic catalysis and inverse magnetic catalysis
- Other properties need to be addressed on the lattice
  - Nucleon masses
  - Masses, spectra & decay constants of neutral pions
- Chiral anomalous effects can be tested in many branches of physics
  - Heavy-ion collisions (CME, CVE, CMW, CVE, etc.)
  - early Universe and compact stars (generation of large-scale helical magnetic fields)
  - Condensed matter physics (phase transitions, transport, collective modes, etc.)