XQCD
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Collective modes in chiral relativistic plasmas

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Chiral plasmas $(\vec{B}, \vec{\omega})$

- **Early Universe**, e.g.,

  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]

- **Heavy-ion collisions**, e.g.,

  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

- **Super-dense matter in compact stars**, e.g.,

  [Yamamoto, Phys.Rev. D93, 065017 (2016)]

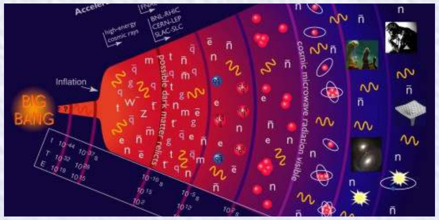
- **Ultra-relativistic jets from black holes**

- **Dirac/Weyl (semi-)metals**, e.g.,

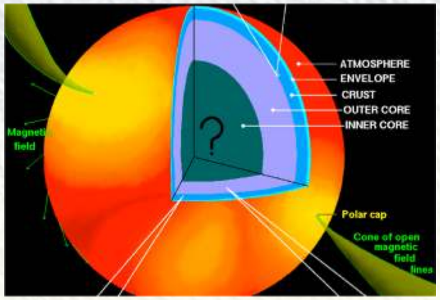
  [Li et. al. Nature Phys. 12, 550 (2016)]

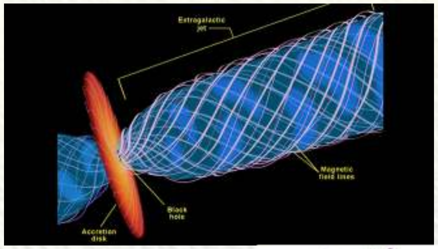
- **Superfluid ^{3}He-A**, e.g.,

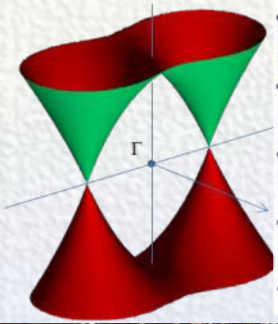
  [Volovik, JETP Lett. 105, 34 (2017)]

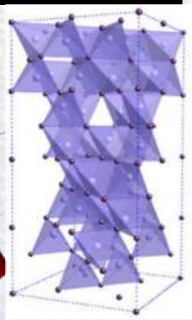


Credit: Brookhaven National Laboratory









Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\boldsymbol{\gamma}} \cdot \vec{\boldsymbol{p}}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma} \cdot \vec{\boldsymbol{p}}}{|\vec{\boldsymbol{p}}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

For particles $(p_0 > 0)$: chirality = helicity

For antiparticles $(p_0 < 0)$: chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

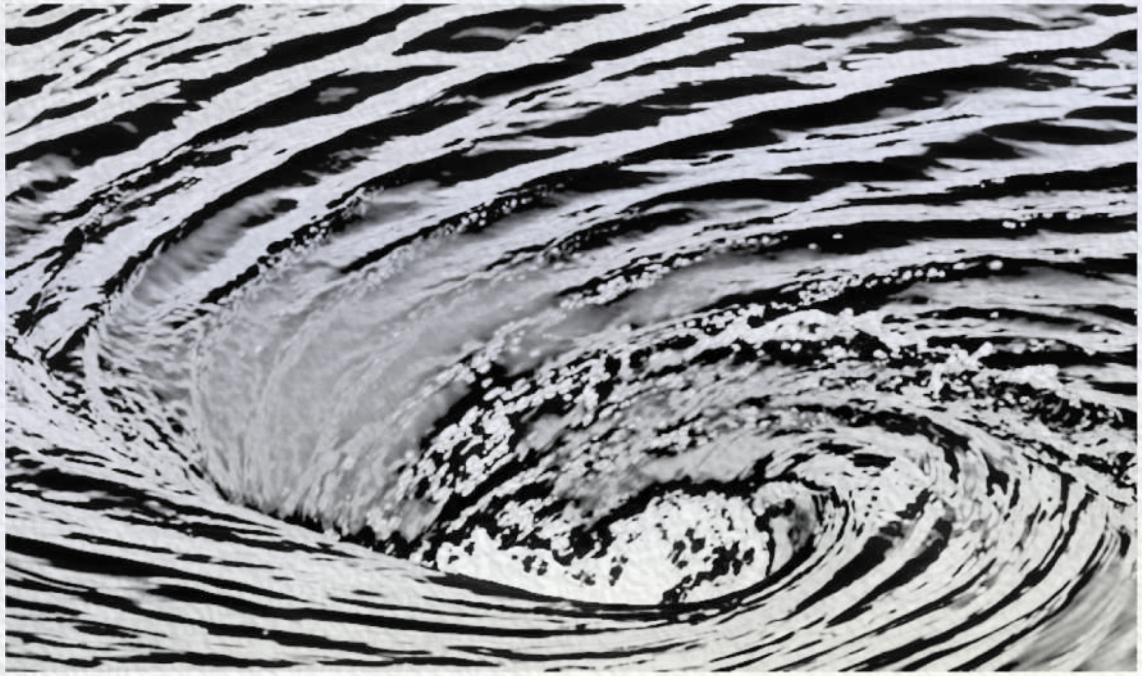
Chiral matter

- Matter made of chiral fermions with $n_{\rm L} \neq n_{\rm R}$
- Unlike the electric charge ($n_{\rm R} + n_{\rm L}$), the chiral charge ($n_{\rm R}$ - $n_{\rm L}$) is **not** conserved

$$\frac{\partial (n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{\boldsymbol{j}} = 0$$

$$\frac{\partial (n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{\boldsymbol{j}}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral symmetry is anomalous in quantum theory



CHIRAL HYDRODYNAMICS

Chiral hydrodynamics

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)] [Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = eF^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = n u^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

Anomalous contributions

- Currents included new non-dissipative terms:

$$j^\mu = n u^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2 T^2}{6\pi^2\hbar^2}, \quad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$

The Question

- **Can one derive chiral hydrodynamics from first principles?**
- Chiral kinetic theory (CKT) is a good starting point
- Note: CKT can be “derived” from field theory
- Original versions of CKT had several limitations:
  - No explicit Lorentz covariance
  - Collisions are tricky

Chiral kinetic theory

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

- Kinetic equation:

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$

$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}\,, \quad \mathbf{v} = \boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}\,,$

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Hydrodynamics from CKT

- Lorentz covariant formulation of CKT:

[Hidaka, Pu, Yang, Phys. Rev. D 95, 091901 (2017); Phys. Rev. D 97, 016004 (2018)]

$$\mathcal{D}_\mu W^\mu(p,x) = \delta(p^2) p\cdot C + \lambda\hbar e \tilde{F}^{\mu\nu} C_\mu p_\nu \delta'(p^2)$$

where $\mathcal{D}^\mu = \partial/\partial x^\mu - eF^{\mu\nu}\partial/\partial p^\nu$

$S^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta/(p\cdot u)$ is the spin tensor

C^μ is the collision operator

- Quasi-classical solution:

$$W^\mu(p,x) \equiv \underbrace{p^\mu \delta(p^2) f}_{O(1)} + \underbrace{\lambda\hbar S^{\mu\nu}\delta(p^2)(D_\nu f - C_\nu) + \lambda\hbar e \tilde{F}^{\mu\nu} p_\nu \delta'(p^2) f}_{O(\hbar)}$$

Approximations

- Relaxation time approximation:

$$\mathcal{D}_\mu W^\mu = -\frac{u_\mu(W^\mu - W^\mu_{\text{eq}})}{\tau}$$

Note, the Wigner function W^μ is expressed in terms of distribution function, e.g.,

$$f_{\text{eq}}(p,x) = \frac{1}{1 + e^{\text{sign}(p_0)(\varepsilon_p - \mu_\lambda)/T}}$$

where $\mu_\lambda \equiv \mu + \lambda\mu_5$, $\quad \varepsilon_p = \underbrace{u_\mu p^\mu}_{O(1)} + \underbrace{\frac{\lambda\hbar}{2}\frac{p\cdot\omega}{p\cdot u}}_{O(\hbar)}$

and $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$ is the vorticity

Constitutive relations

- Conserved currents in CKT are moments of W^μ:

$$j^\mu = 2\sum_\lambda \int \frac{d^4p}{(2\pi)^3} W^\mu$$

$$j_5^\mu = 2\sum_\lambda \lambda \int \frac{d^4p}{(2\pi)^3} W^\mu$$

$$T^{\mu\nu} = \sum_\lambda \int \frac{d^4p}{(2\pi)^3} (W^\mu p^\nu + p^\mu W^\nu)$$

However, when using the CKT equation, we get

$$\partial_\mu j^\mu = -\frac{1}{\tau}(n - n_{\text{eq}})$$

$$\partial_\mu j_5^\mu + \frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu = -\frac{1}{\tau}(n_5 - n_{5,\text{eq}})$$

$$\underbrace{\partial_\nu T^{\mu\nu} - eF^{\mu\nu} j_\nu}_{\text{good}} = \underbrace{-\frac{u^\mu}{\tau}(\epsilon - \epsilon_{\text{eq}} + \dots) - \frac{1}{\tau}(h^\mu - h^\mu_{\text{eq}} + \dots)}_{\text{bad}}$$

Constitutive relations

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However, when using the CKT equation, we get

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu + \frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu = 0$$

$$\partial_\nu T^{\mu\nu} - eF^{\mu\nu} j_\nu = 0$$

$\rightarrow$ 6 constraints, defining local equilibrium parameters T, μ, μ_5, u^μ

good (equations) — bad (constraints)

1^{st}-order dissipative hydro

- Dissipative terms (first-order):

$$\nu^\mu = \nu^\mu_{\rm eq} + \frac{\tau}{3}\nabla^\mu n - \tau\dot{u}^\mu n + \sigma_E E^\mu$$

$$\nu^\mu_5 = \nu^\mu_{5,\rm eq} + \frac{\tau}{3}\nabla^\mu n_5 - \tau\dot{u}^\mu n_5 + \sigma^5_E E^\mu$$

$$\pi^{\mu\nu} = \frac{8\tau\epsilon}{15}\Delta^{\mu\nu}_{\alpha\beta}(\partial^\alpha u^\beta)$$

where

$$\sigma_E = \tau e\frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{9\pi^2\hbar^3}$$

$$\sigma^5_E = \tau e\frac{2\mu\mu_5}{3\pi^2\hbar^3}$$

2^{nd}-order hydro (neutral)

- At this order, the constitutive relations are differential equations,

[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

$$\dot{\nu}^{\langle\mu\rangle} + \frac{\nu^\mu - \nu^\mu_{\text{eq}}}{\tau} = -\dot{u}^\mu n + \frac{1}{3}\nabla^\mu n - \frac{n}{\epsilon + P}\Delta^{\mu\nu}\partial^\rho \pi_{\rho\nu} - \nu_\rho \omega^{\rho\mu} - (\partial\cdot u)\nu^\mu - \frac{9}{5}(\partial^{\langle\mu}u^{\rho\rangle})\nu_\rho + \ldots$$

$$\dot{\nu}_5^{\langle\mu\rangle} + \frac{\nu^\mu_5 - \nu^\mu_{5,\text{eq}}}{\tau} = -\dot{u}^\mu n_5 + \frac{1}{3}\nabla^\mu n_5 - \frac{n_5}{\epsilon + P}\Delta^{\mu\nu}\partial^\rho \pi_{\rho\nu} - \nu_{5,\rho} \omega^{\rho\mu} - (\partial\cdot u)\nu^\mu_5 - \frac{9}{5}(\partial^{\langle\mu}u^{\rho\rangle})\nu_{5,\rho} + \ldots$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau} = -2h^{\langle\mu}\dot{u}^{\nu\rangle} + 2\pi^{\langle\mu}_\rho \omega^{\nu\rangle\rho} - \frac{10}{7}\pi^{\langle\mu}_\rho \sigma^{\nu\rangle\rho} - \frac{4}{3}\pi^{\mu\nu}\partial_\alpha u^\alpha + \frac{8}{15}(\partial^{\langle\mu}u^{\nu\rangle})\epsilon + \ldots$$

- Causality is Ok
- Stability is (probably) Ok

Collective modes: neutral plasma

- Sound waves

$$\Omega = \pm \frac{k_z}{\sqrt{3}} + \frac{3}{8}\hbar\bar{\omega}\frac{n_{5,\mathrm{eq}}}{\epsilon_{\mathrm{eq}}}k_z + \frac{2}{15}i\tau k_z^2$$

- Chiral vortical waves

$$\Omega = \hbar\bar{\omega}v_1 k_z - \frac{1}{3}i\tau k_z^2, \qquad \Omega = \hbar\bar{\omega}v_2 k_z - \frac{1}{3}i\tau k_z^2$$

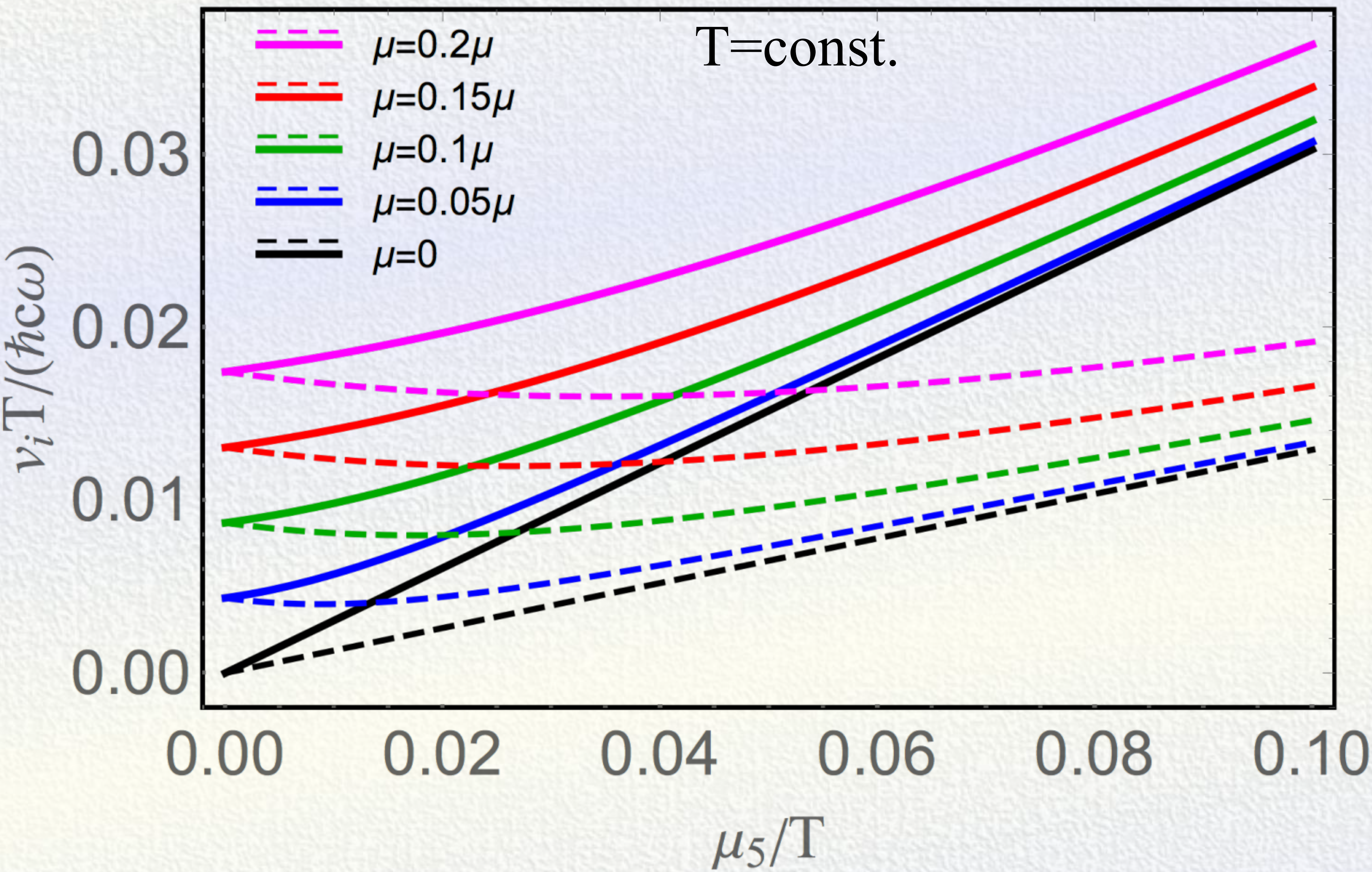
where $v_1 \neq v_2$ (along/against $\vec{\omega}$ direction)

- Oscillations of all thermodynamic parameters are important:

$$\delta\mu \neq 0,\ \delta\mu_5 \neq 0,\ \delta T \neq 0,\ \delta u^{\mu} \neq 0$$

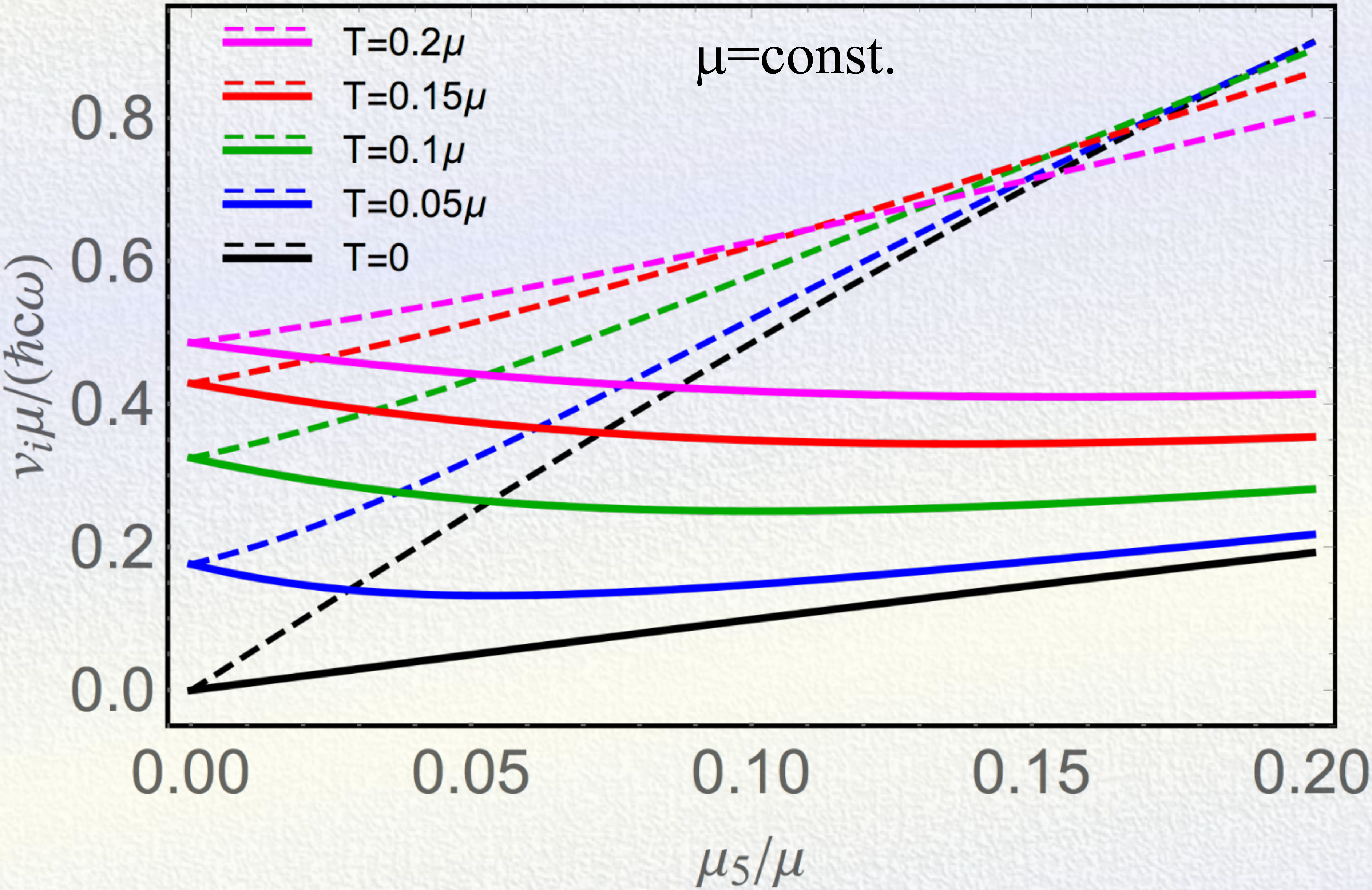
[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

CVW velocities @ high T

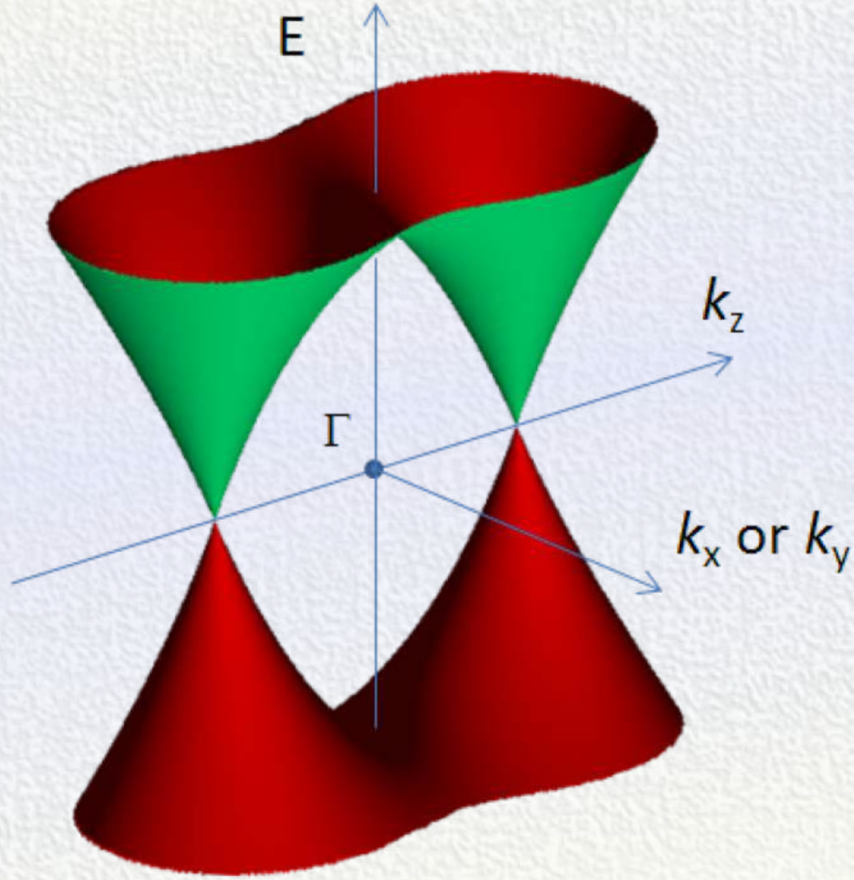


[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

CVW velocities @ large μ



[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL MATERIALS

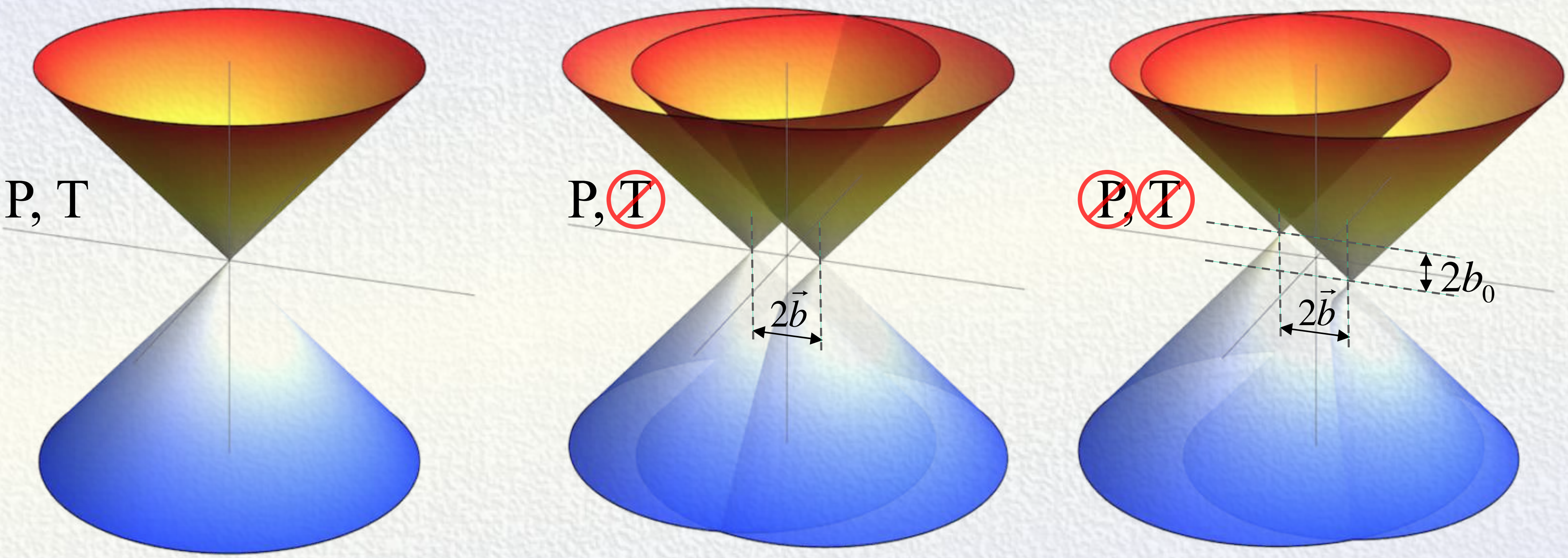
Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

T P

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5+b_0\gamma^0\gamma^5\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) **Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Strain in Weyl materials

- Strains in the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

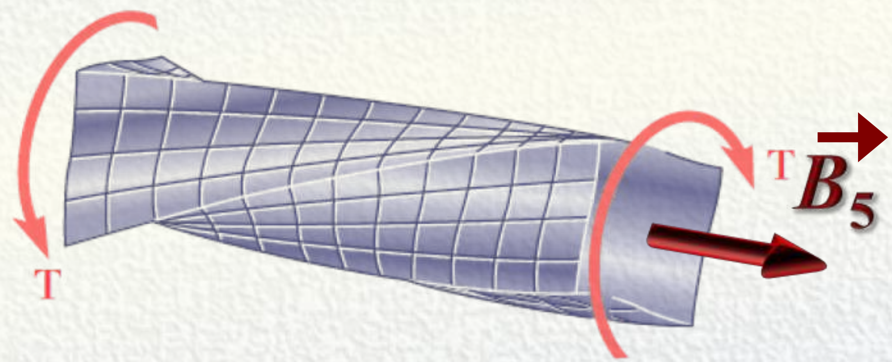
where the chiral gauge fields are

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
[Grushin, Venderbos,Vishwanath, Ilan, PRX **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

$$A_{5,0} \propto b_0 \left|\vec{b}\right| \partial_{||} u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right| \partial_{||} u_{\perp}$$

$$A_{5,||} \propto \alpha \left|\vec{b}\right|^2 \partial_{||} u_{||} + \beta \sum_i \partial_i u_i$$



leading to the pseudo-EM fields

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0 - \partial_t\vec{A}_5$$

Current and chiral anomaly

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right]f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda)\right]f_\lambda$$
$$+e\mathbf{\nabla}\times\int\frac{d^3p}{(2\pi\hbar)^3}f_\lambda\epsilon_{\mathbf{p}}\mathbf{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t}+\mathbf{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B})+(\mathbf{E}_5\cdot\mathbf{B}_5)\Big] \quad ✔$$

$$\frac{\partial\rho}{\partial t}+\mathbf{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B}_5)+(\mathbf{E}_5\cdot\mathbf{B})\Big] \quad ✘$$

Consistent definition of current

- Additional Bardeen-Zumino term is needed,

$$\delta j^\mu = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda} A^5_\nu F_{\rho\lambda}$$

- i.e.,

[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]

$$\delta\rho = \frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{A}^5\cdot\mathbf{B})$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c} A^5_0\mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c}(\mathbf{A}^5\times\mathbf{E})$$

- Its role and implications:
  - Electric charge is conserved locally ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)

Hydrodynamics in Weyl metals

The Euler equation

$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right)=-en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_{\mathbf{r}})$$

The energy conservation

$$\partial_t\epsilon=-\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_{\mathbf{r}})$$

plus the Maxwell equations that include the Chern-Simons (Bardeen-Zumino) terms, i.e.,

$$\rho_{\rm CS}=-\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\rm CS}=-\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **97**, 121105(R) (2018)]

Rich spectrum of hydro modes

- Magneto-acoustic wave ($\rho = 0$):

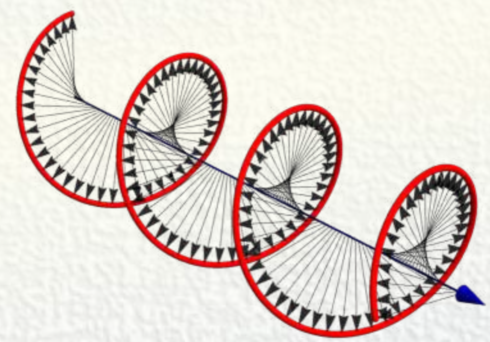
$$\omega_{\mathrm{s},\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau}\sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)}\left[2|\mathbf{k}|^2 B_0^2 - (\mathbf{k}\cdot\mathbf{B}_0)\right]}{3w_0}}$$

- *Gapped* chiral magnetic wave ($\rho = 0$):

$$\omega_{\mathrm{gCMW},\pm} = \pm\frac{eB_0\sqrt{3v_F^3\left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_\parallel^2\right)}}{2\pi^2 T_0^2 c\sqrt{\varepsilon_e \hbar}}$$

- Helicons ($\rho \neq 0$):

$$\omega_{\mathrm{h},\pm} \approx \mp\frac{c k_\parallel^2 B_0}{4\pi\mu_m\rho_0} - \frac{i}{\tau}\frac{c^2 k_\parallel^2 w_0}{4\pi\mu_m v_F^2 \rho_0^2} + O(k_\parallel^3)$$



- New anomalous Hall waves at $\vec{b} \neq 0$, etc.

[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv:1802.10110]

Summary

- Lorentz covariant form of consistent chiral kinetic (CKT) theory is formulated for (pseudo-)relativistic plasmas
- Topological terms play a critical role in Dirac and Weyl (semi-)metals
- Dissipative chiral hydrodynamics is derived from CKT
- Anomalous terms also affect dissipative dynamics
- Hydrodynamic modes in (pseudo-)relativistic plasmas with vorticity/magnetic field are studied