Chiral Fluids
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Hydrodynamic modes in charged chiral plasmas with vorticity

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Open problems and opportunities in chiral fluids

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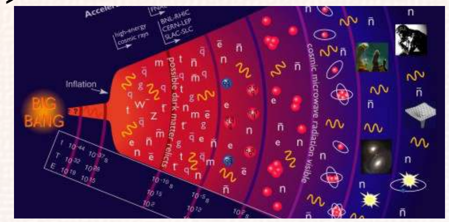
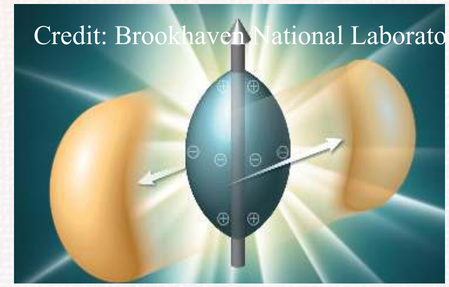
Chiral plasmas $(\vec{B}, \vec{\omega})$

- **Early Universe**, e.g.,

  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]



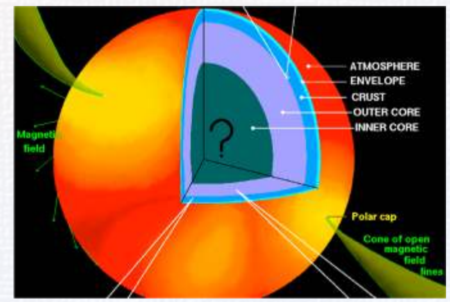


- **Heavy-ion collisions**, e.g.,

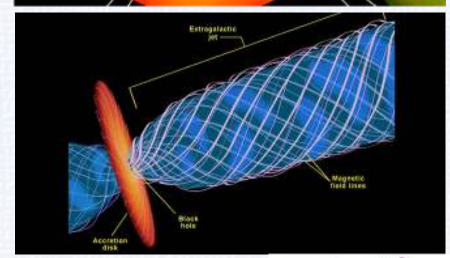
  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

- **Super-dense matter in compact stars**, e.g.,

  [Yamamoto, Phys.Rev. D93, 065017 (2016)]

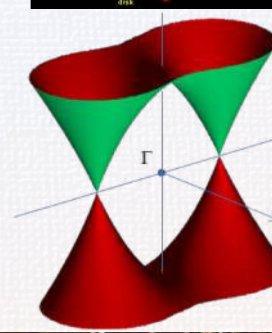


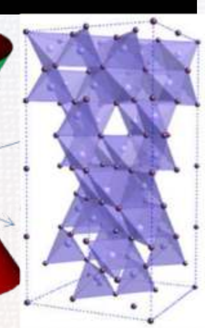
- **Ultra-relativistic jets from black holes**



- **Dirac/Weyl (semi-)metals**, e.g.,

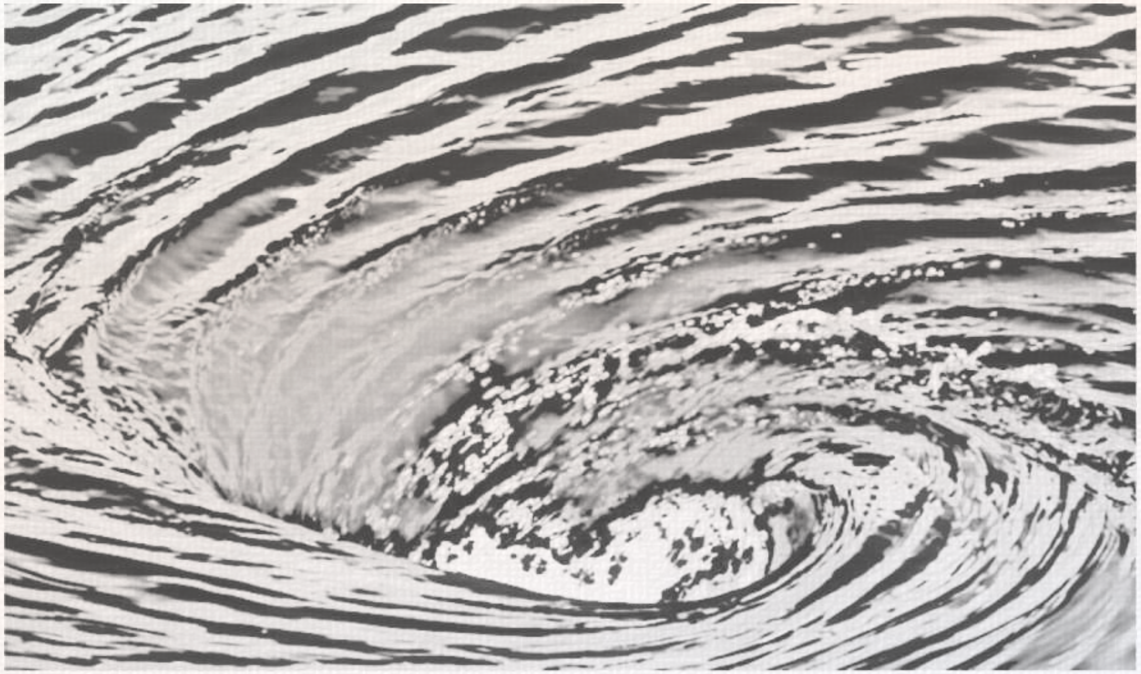
  [Li et. al. Nature Phys. 12, 550 (2016)]





- **Superfluid ³He-A**, e.g.,

  [Volovik, JETP Lett. 105, 34 (2017)]



CHIRAL HYDRODYNAMICS

Chiral hydrodynamics

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)] [Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = eF^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = nu^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

Anomalous contributions

- Currents included new non-dissipative terms:

$$j^\mu = nu^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{6\pi^2\hbar^2}, \quad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$

The Question

- **Can one derive chiral hydrodynamics from first principles?**
- Chiral kinetic theory (CKT) is a good starting point
- Note: CKT can be “derived” from field theory
- Original versions of CKT had several limitations:
  - No explicit Lorentz covariance
  - Collisions are tricky

Chiral kinetic theory

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

- Kinetic equation:

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$

$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}\,,\quad \mathbf{v} = \boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}\,,$

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\dfrac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Hydrodynamics from CKT

- Lorentz covariant formulation of CKT:

[Hidaka, Pu, Yang, Phys. Rev. D 95, 091901 (2017); Phys. Rev. D 97, 016004 (2018)]

$$\mathcal{D}_\mu W^\mu(p,x) = \delta(p^2) p \cdot C + \lambda \hbar e \tilde{F}^{\mu\nu} C_\mu p_\nu \delta'(p^2)$$

where $\mathcal{D}^\mu = \partial/\partial x^\mu - e F^{\mu\nu} \partial/\partial p^\nu$

$S^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta/(p\cdot u)$ is the spin tensor

C^μ is the collision operator

- Quasi-classical solution:

$$W^\mu(p,x) \equiv \underbrace{p^\mu \delta(p^2) f}_{\mathrm{O}(1)} + \underbrace{\lambda\hbar S^{\mu\nu}\delta(p^2)(D_\nu f - C_\nu) + \lambda \hbar e \tilde{F}^{\mu\nu} p_\nu \delta'(p^2) f}_{\mathrm{O}(\hbar)}$$

Approximations

- Relaxation time approximation:

$$\mathcal{D}_\mu W^\mu = -\frac{u_\mu (W^\mu - W^\mu_{\rm eq})}{\tau}$$

Note, the Wigner function W^μ is expressed in terms of distribution function, e.g.,

$$f_{\rm eq}(p,x) = \frac{1}{1+e^{\text{sign}(p_0)(\varepsilon_p - \mu_\lambda)/T}}$$

where $\mu_\lambda \equiv \mu + \lambda\mu_5$, $\quad \varepsilon_p = \underbrace{u_\mu p^\mu}_{\text{O}(1)} + \underbrace{\frac{\lambda\hbar}{2}\frac{p\cdot\omega}{p\cdot u}}_{\text{O}(\hbar)}$

and $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$ is the vorticity

Constitutive relations

- Conserved currents in CKT are moments of W^μ:

$$j^\mu = 2\sum_\lambda \int \frac{d^4p}{(2\pi)^3} W^\mu$$

$$j_5^\mu = 2\sum_\lambda \lambda \int \frac{d^4p}{(2\pi)^3} W^\mu$$

$$T^{\mu\nu} = \sum_\lambda \int \frac{d^4p}{(2\pi)^3} (W^\mu p^\nu + p^\mu W^\nu)$$

However, when using the CKT equation, we get

$$\begin{aligned} \partial_\mu j^\mu &= 0 \\ \partial_\mu j_5^\mu + \frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu &= 0 \\ \partial_\nu T^{\mu\nu} - eF^{\mu\nu} j_\nu &= 0 \end{aligned}$$

$\rightarrow$ 6 constraints, defining local equilibrium parameters T, μ, μ_5, u^μ

good

bad

1st-order dissipative hydro

- Dissipative terms (first-order):

$$\nu^\mu = \nu^\mu_{\text{eq}} + \frac{\tau}{3}\nabla^\mu n - \tau\dot{u}^\mu n + \sigma_E E^\mu$$

$$\nu^\mu_5 = \nu^\mu_{5,\text{eq}} + \frac{\tau}{3}\nabla^\mu n_5 - \tau\dot{u}^\mu n_5 + \sigma^5_E E^\mu$$

$$\pi^{\mu\nu} = \frac{8\tau\epsilon}{15}\Delta^{\mu\nu}_{\alpha\beta}(\partial^\alpha u^\beta)$$

where

$$\sigma_E = \tau e\frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{9\pi^2\hbar^3}$$

$$\sigma^5_E = \tau e\frac{2\mu\mu_5}{3\pi^2\hbar^3}$$

2^{nd}-order hydro (neutral)

- At this order, the constitutive relations are differential equations,

[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

$$\dot{\nu}^{\langle\mu\rangle} + \frac{\nu^\mu - \nu^\mu_{\rm eq}}{\tau} = -\dot{u}^\mu n + \frac{1}{3}\nabla^\mu n - \frac{n}{\epsilon + P}\Delta^{\mu\nu}\partial^\rho \pi_{\rho\nu} - \nu_\rho \omega^{\rho\mu} - (\partial\cdot u)\nu^\mu - \frac{9}{5}(\partial^{\langle\mu}u^{\rho\rangle})\nu_\rho + \ldots$$

$$\dot{\nu}_5^{\langle\mu\rangle} + \frac{\nu_5^\mu - \nu^\mu_{5,\rm eq}}{\tau} = -\dot{u}^\mu n_5 + \frac{1}{3}\nabla^\mu n_5 - \frac{n_5}{\epsilon + P}\Delta^{\mu\nu}\partial^\rho \pi_{\rho\nu} - \nu_{5,\rho} \omega^{\rho\mu} - (\partial\cdot u)\nu_5^\mu - \frac{9}{5}(\partial^{\langle\mu}u^{\rho\rangle})\nu_{5,\rho} + \ldots$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau} = -2h^{\langle\mu}\dot{u}^{\nu\rangle} + 2\pi_\rho^{\langle\mu}\omega^{\nu\rangle\rho} - \frac{10}{7}\pi_\rho^{\langle\mu}\sigma^{\nu\rangle\rho} - \frac{4}{3}\pi^{\mu\nu}\partial_\alpha u^\alpha + \frac{8}{15}(\partial^{\langle\mu}u^{\nu\rangle})\epsilon + \ldots$$

- Causality is Ok
- Stability is (probably) Ok

HYDRODYNAMIC MODES

Hydro modes in neutral plasma

- Sound waves

$$\Omega = \pm\frac{k_z}{\sqrt{3}} + \frac{3}{8}\hbar\bar{\omega}\frac{n_{5,\mathrm{eq}}}{\epsilon_{\mathrm{eq}}}k_z + \frac{2}{15}i\tau k_z^2$$

- Chiral vortical waves

$$\Omega = \hbar\bar{\omega}v_1 k_z - \frac{1}{3}i\tau k_z^2, \qquad \Omega = \hbar\bar{\omega}v_2 k_z - \frac{1}{3}i\tau k_z^2$$

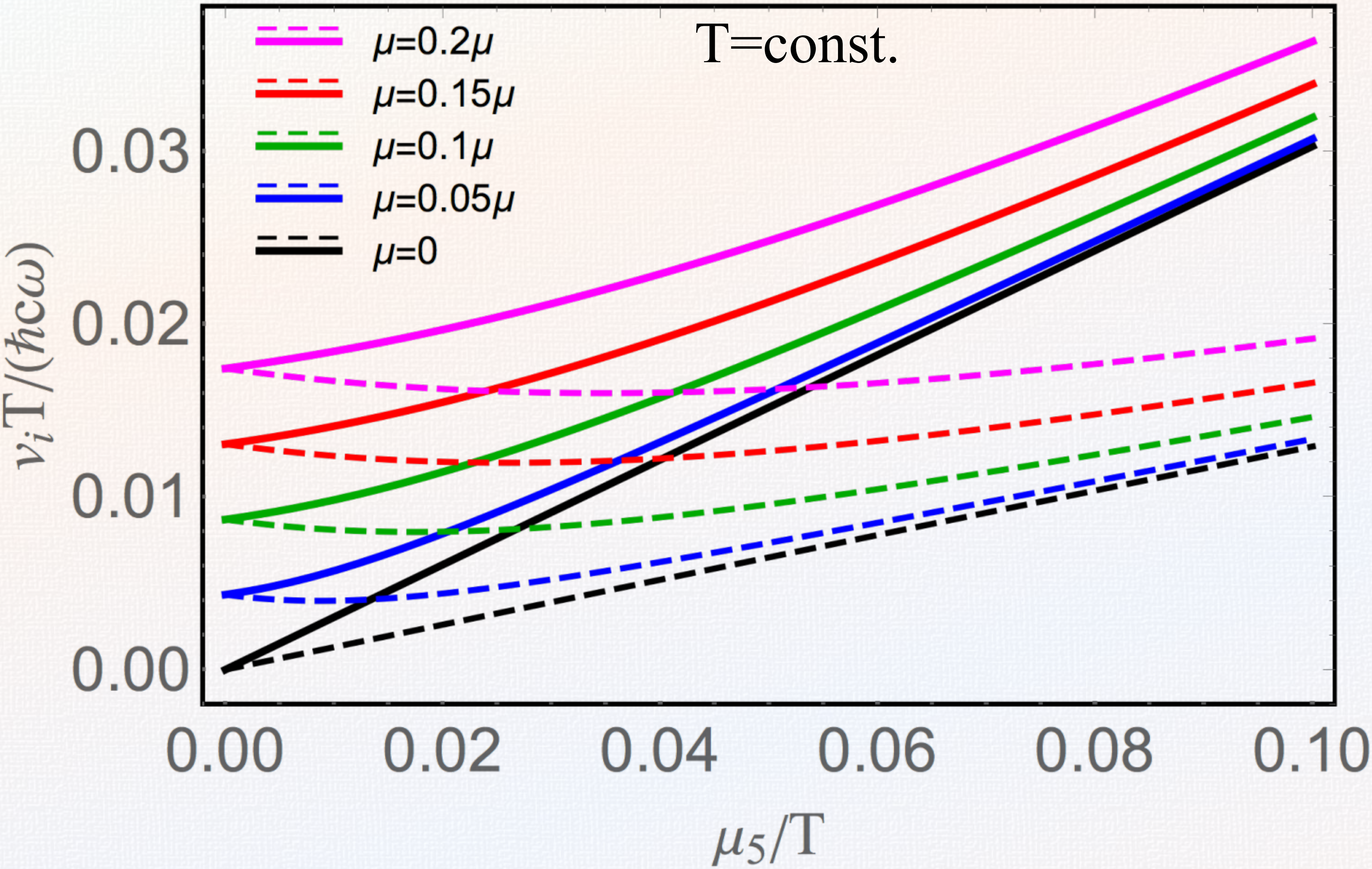
where $v_1 \neq v_2$ (along/against $\vec{\omega}$ direction)

- Oscillations of all thermodynamic parameters are important:

$$\delta\mu \neq 0,\ \delta\mu_5 \neq 0,\ \delta T \neq 0,\ \delta u^\mu \neq 0$$

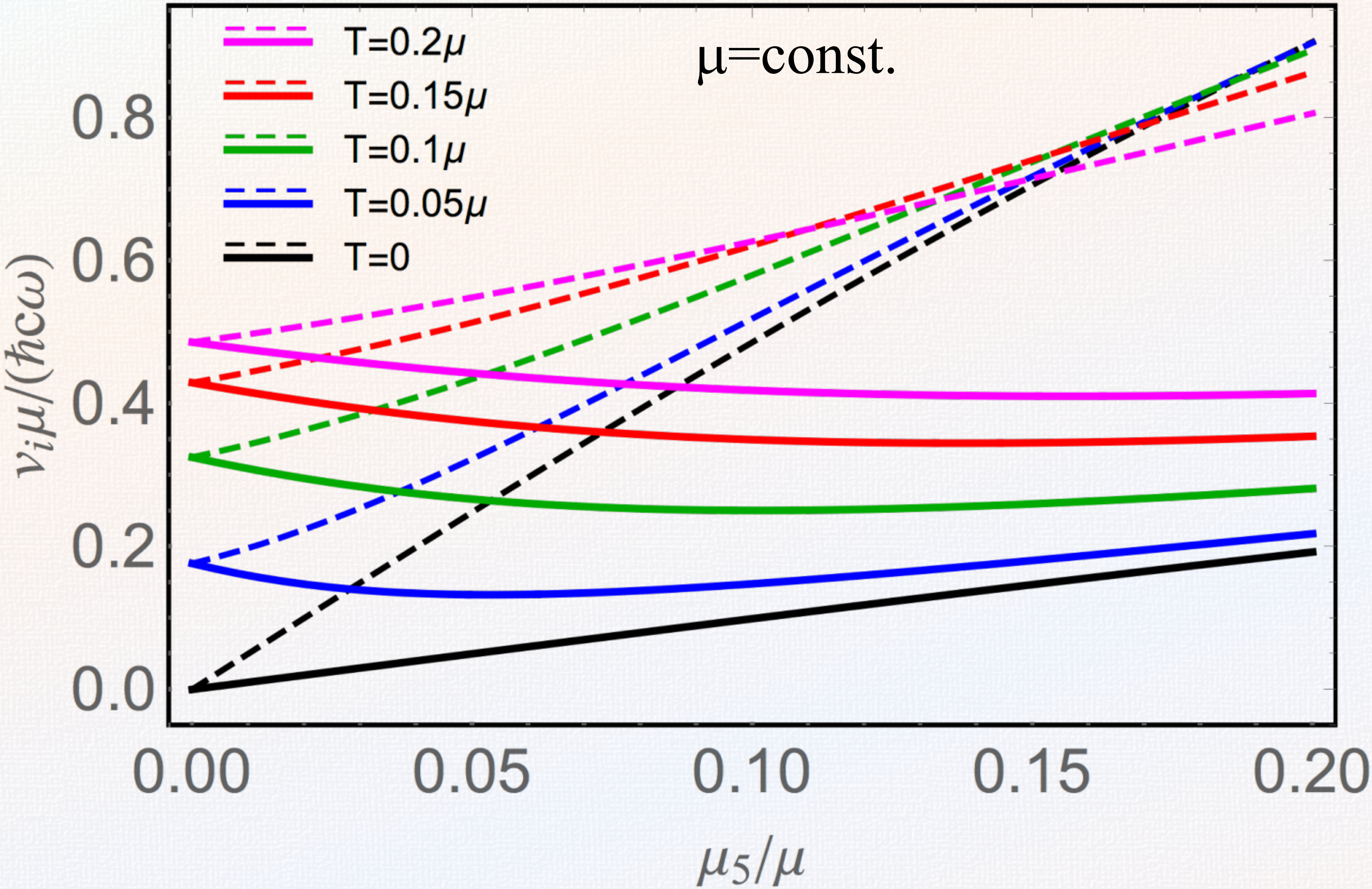
[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

CVW velocities @ high T



[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

CVW velocities @ large μ

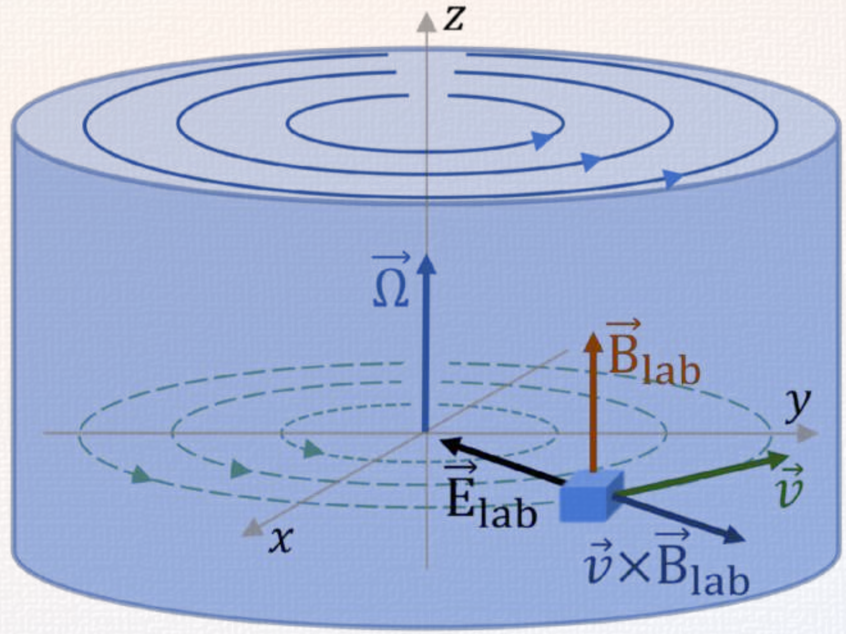


[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

Rotating charged plasma

- Fluid velocity

$$\bar{u}^{\nu} = \gamma \begin{pmatrix} 1 \\ -\Omega y \\ \Omega x \\ 0 \end{pmatrix}$$



where $\gamma = 1/\sqrt{1-(\Omega\, r)^2}$

Vorticity: $\bar{\omega}^{\mu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{u}_{\nu}\partial_{\alpha}\bar{u}_{\beta} = \gamma^2\Omega\delta_3^{\mu}$

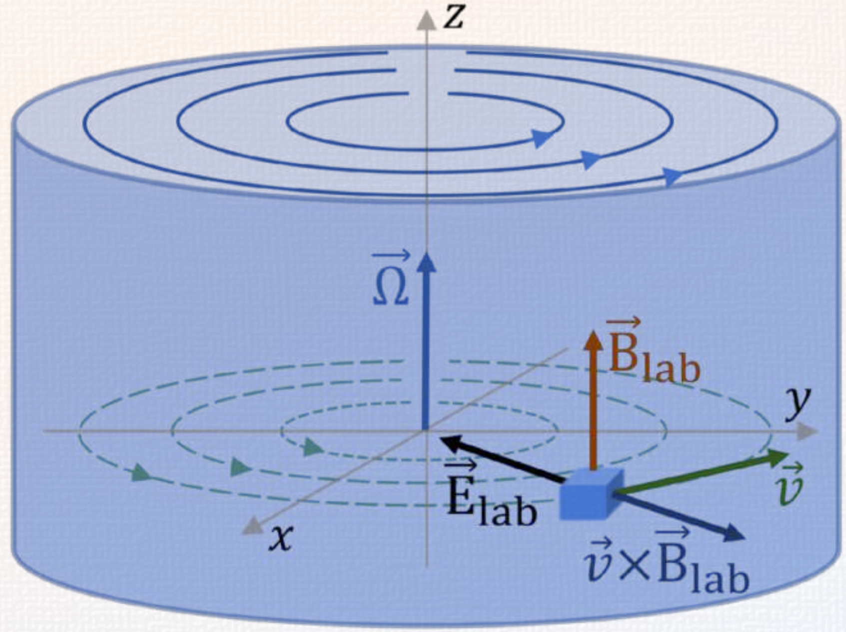
EM fields in lab frame: $\mathbf{B}_{\text{lab}} = \gamma B\hat{\mathbf{z}}$

$$\mathbf{E}_{\text{lab}} = -\gamma B\Omega\mathbf{r}_{\perp}$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Rotating charged plasma: equilibrium

- Maxwell equations:



$$\partial_\nu F^{\nu\mu} = enu^\mu + e\nu^\mu - en_{\text{bg}}u^\mu_{\text{bg}}$$

$$\partial_\nu \tilde{F}^{\nu\mu} = 0$$

where n_{bg} is the background

The solution is radially nonuniform:

$$B(r) = \gamma\left(B_0 - \frac{1}{2}en_{\text{bg}}\Omega r^2\right) \simeq B_0 - \frac{1}{2}en_{\text{bg}}\Omega r^2 + O\left(B_0 r^2\Omega^2\right)$$

$$en_{\text{eq}}(r) = \gamma^3\left(en_{\text{bg}} - 2B_0\Omega\right) \simeq en_{\text{bg}} - 2B_0\Omega + O\left(en_{\text{bg}}r^2\Omega^2\right)$$

(This is consistent with $\mu = \gamma\mu_0,\ \mu_5 = \gamma\mu_{5,0},\ T = \gamma T_0$.)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Linearized equations, etc.

- Small perturbations ($\Omega \to 0$):

$$\delta s(x) = e^{-ik_0 t + ik_z z + im\theta} \delta s(r)$$
$$\delta v^3(x) = e^{-ik_0 t + ik_z z + im\theta} \delta v^3(r)$$
$$\delta v_\pm(x) = e^{-ik_0 t + ik_z z + i(m\pm 1)\theta} \delta v_\pm(r)$$

where

$$\delta s(r) = \delta s\, J_m(k_\perp r), \quad \text{for} \quad s = \mu, \mu_5, T$$
$$\delta v^3(r) = \delta v^3\, J_m(k_\perp r), \quad \text{for} \quad v^3 = u^3, B^3, E^3$$
$$\delta v_\pm(r) = \delta v_\pm\, J_{m\pm 1}(k_\perp r), \quad \text{for} \quad v_\pm = u_\pm, B_\pm, E_\pm$$

- Boundary conditions: $\delta s(R) = 0,\ \delta v^3(R) = 0$

- Transverse wave vectors: $k_\perp^{(i)} = \alpha_{m,i}/R$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Hydro modes @ high T

- Sound waves ($T \gg \mu$):

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2 + m\Omega \left(\frac{2}{3} + \frac{5e^2\mu^2}{14\pi^2\hbar^3 k^2} \right)$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = m\Omega + sk_z \sqrt{\frac{45\mathcal{B}_\pm^2 \hbar^3}{7\pi^2 T^4} + \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm \mu}{14\pi^2 T^2 k} \right)^2} \pm k_z \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm \mu}{14\pi^2 T^2 k} \right)$$

which also has a small imaginary part (not shown)

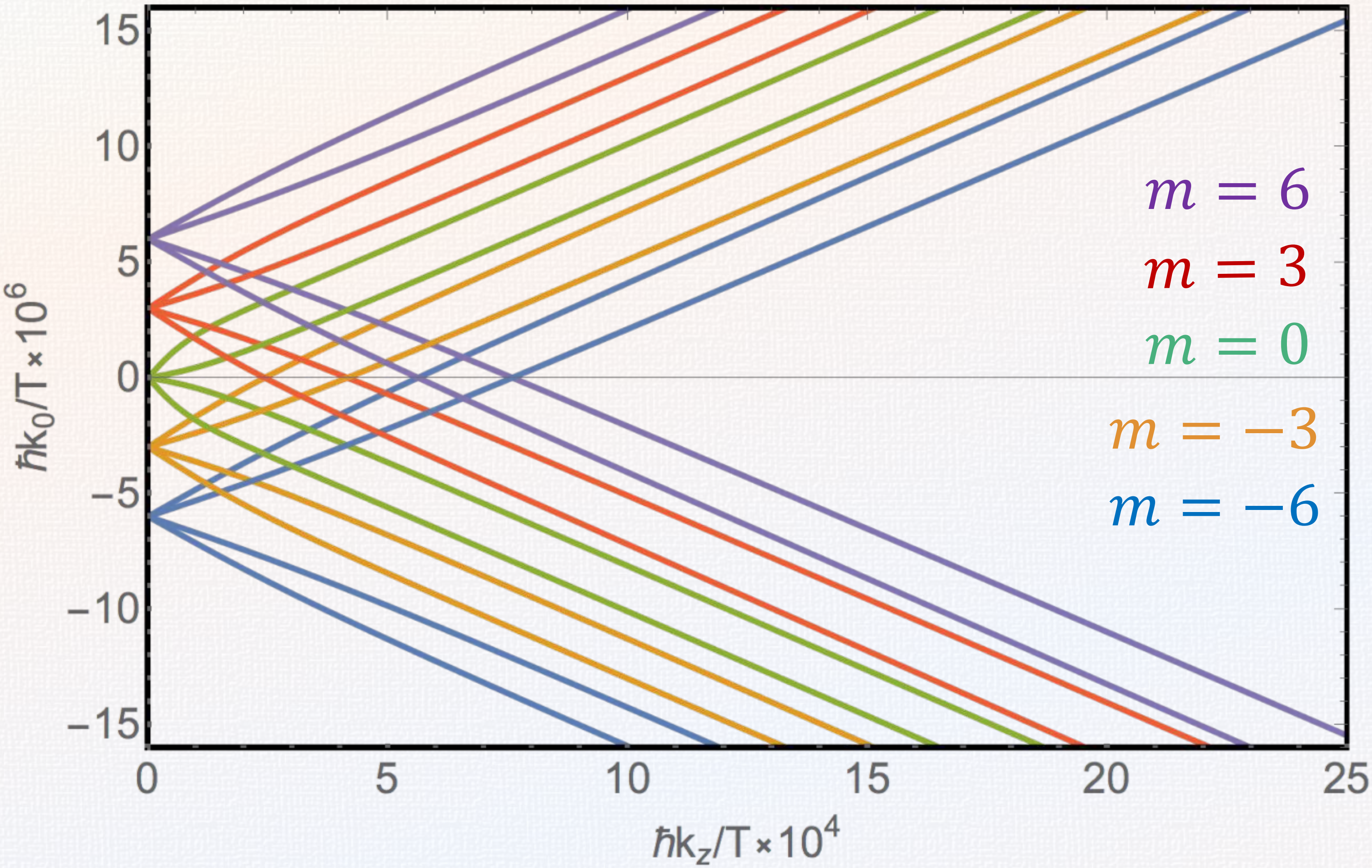
Here

$$\mathcal{B}_\pm = B - \frac{en_{\mathrm{eq}}\Omega}{6k_\perp^2} \left[2(m \pm 1)(m \pm 2) + k_\perp^2 R^2 \right]$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Alfven waves ($T \gg \mu$)

- Alfven waves with angular momenta $m = 0, \pm 3, \pm 6$



[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Hydro modes @ high μ

- Plasmon modes ($\mu \gg T$):

$$k_0^{(s)} = \pm \frac{e\mu}{\sqrt{3}\pi\hbar^{3/2}} + s\frac{e\mathcal{B}_s}{2\mu} - \frac{ie^2\tau T^2}{18\hbar^3} - \frac{1}{10}i\tau k^2$$

(not affected by Ω at linear order)

- Helicon ($\mu \gg T$):

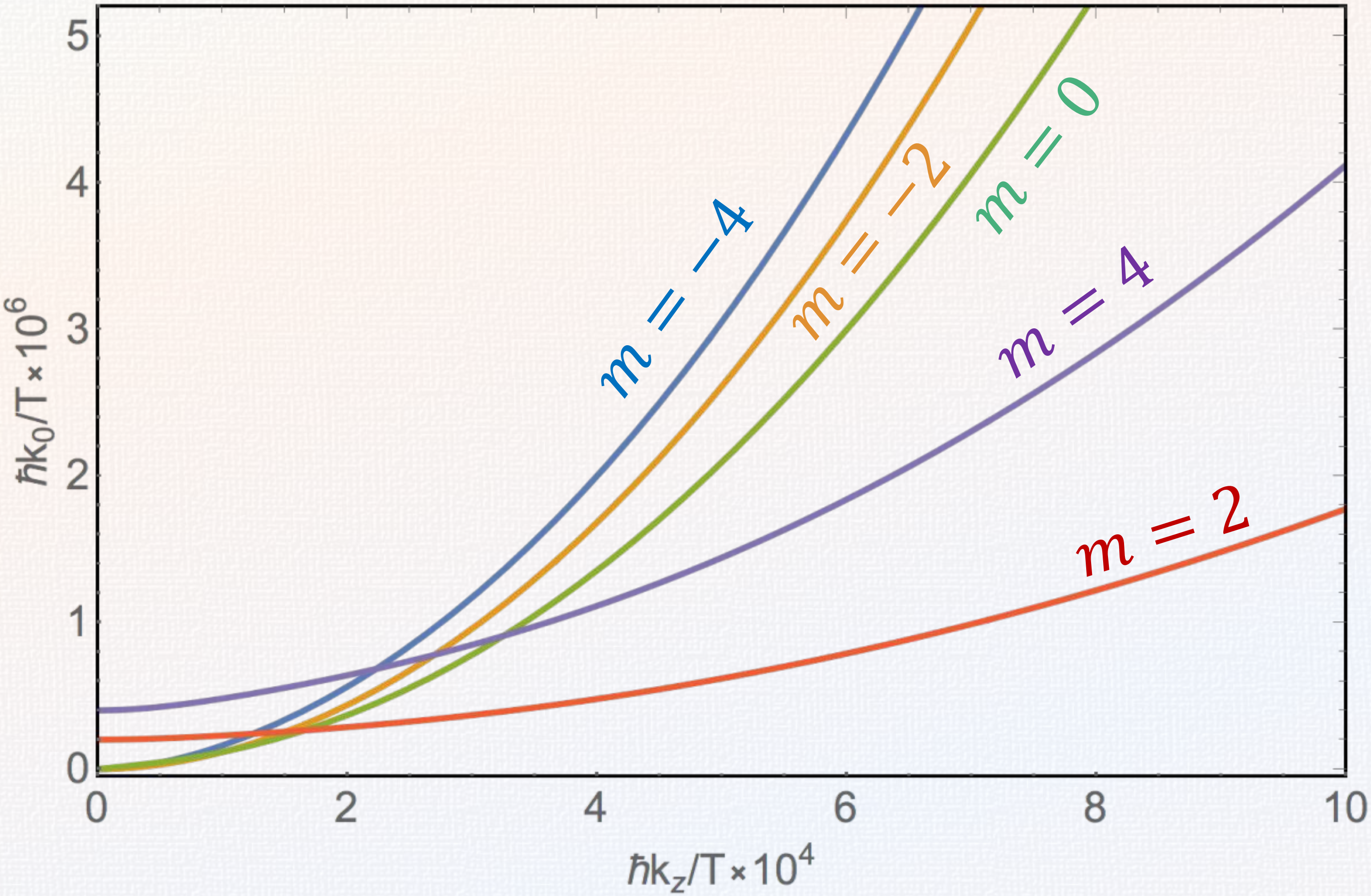
$$k_0 = m\Omega\left(\frac{1}{2} - \frac{k_z^2}{k_\perp^2}\right) + s\sqrt{\frac{m^2\Omega^2}{4} + \frac{9\pi^4(\mathcal{B}_+ + \mathcal{B}_-)^2 k^2 k_z^2 \hbar^5}{4\mu^6}}$$

which also has a small imaginary part (not shown)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

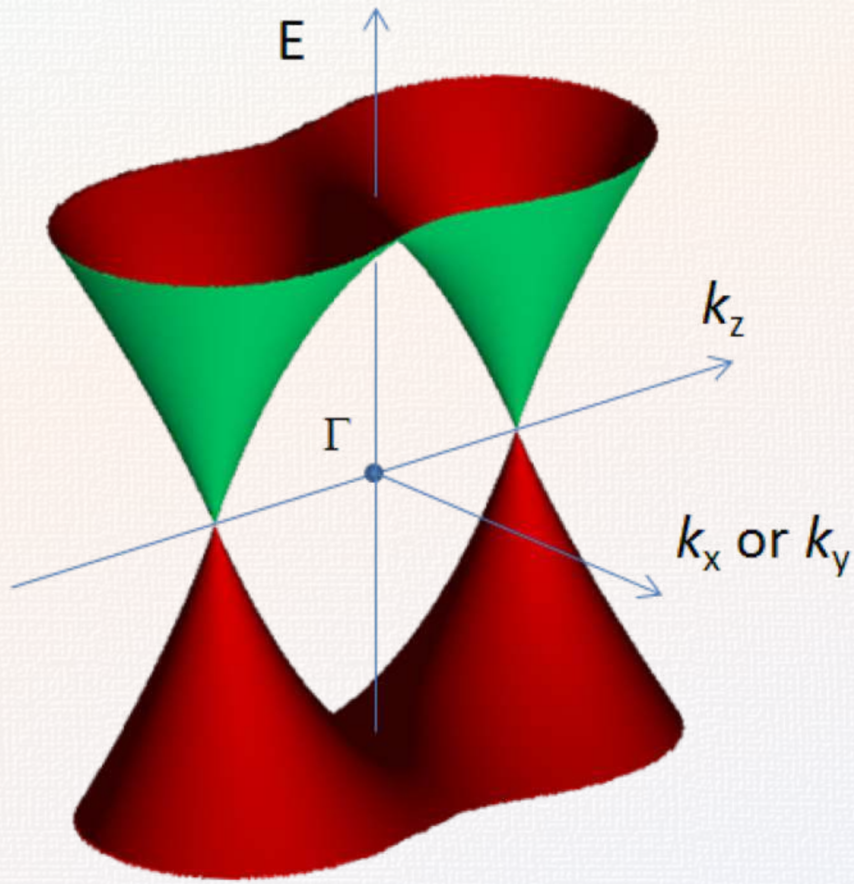
Helicons ($\mu \gg T$)

- Helicons with angular momenta $m = 0, \pm 2, \pm 4$:



- Note that the modes are gapless for all $m \leq 0$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

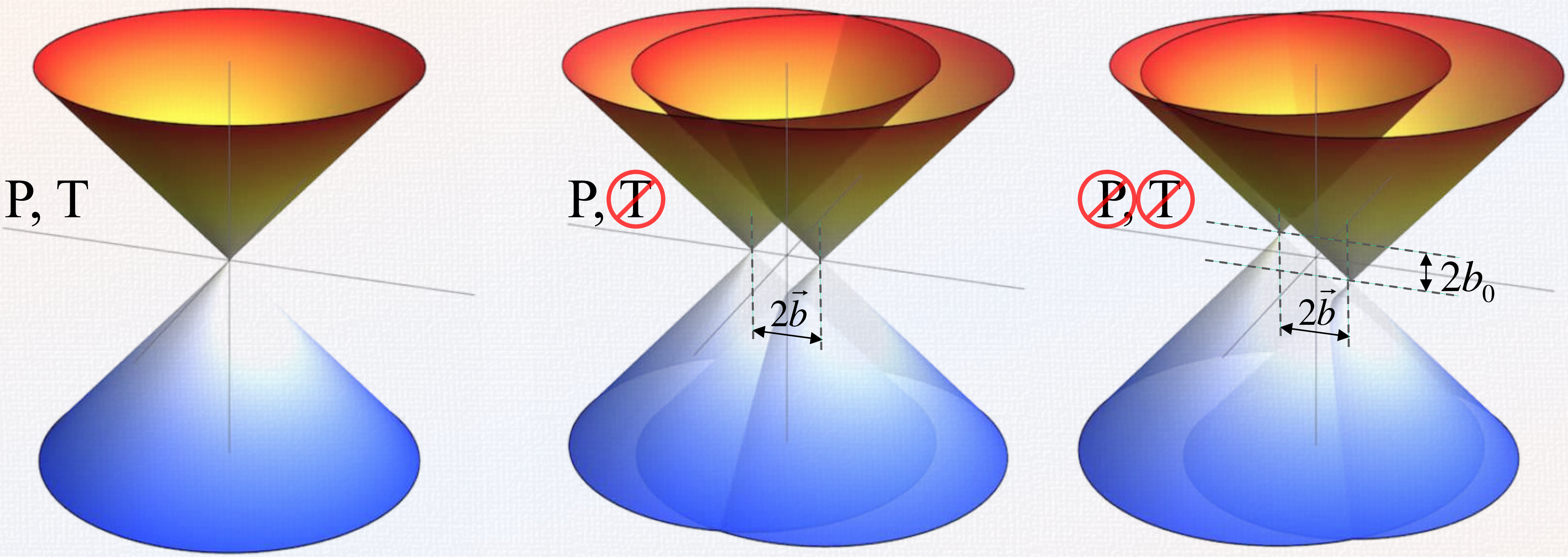
DIRAC & WEYL MATERIALS

Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \overset{\cancel{T}}{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5} + \overset{\cancel{P}}{b_0\gamma^0\gamma^5}\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) **Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Hydrodynamics in Weyl metals

The Euler equation

$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right)=-en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_{\mathbf{r}})$$

The energy conservation

$$\partial_t\epsilon=-\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_{\mathbf{r}})$$

plus the Maxwell equations that include the Chern-Simons (Bardeen-Zumino) terms, i.e.,

$$\rho_{\text{CS}}=-\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\text{CS}}=-\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **97**, 121105(R) (2018)]

Rich spectrum of hydro modes

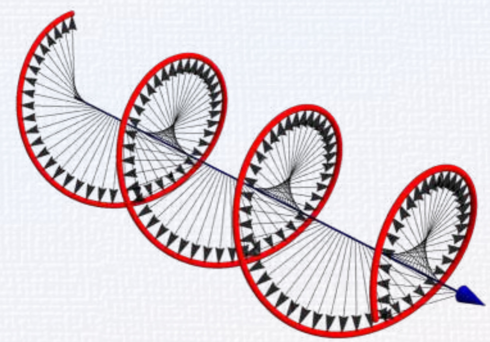
- Magneto-acoustic wave ($\rho = 0$):

$$\omega_{\mathrm{s},\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau}\sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)}\left[2|\mathbf{k}|^2 B_0^2 - (\mathbf{k}\cdot\mathbf{B}_0)\right]}{3w_0}}$$

- *Gapped* chiral magnetic wave ($\rho = 0$):

$$\omega_{\mathrm{gCMW},\pm} = \pm\frac{eB_0\sqrt{3v_F^3\left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_\parallel^2\right)}}{2\pi^2 T_0^2 c\sqrt{\varepsilon_e \hbar}}$$

- Helicons ($\rho \neq 0$):



$$\omega_{\mathrm{h},\pm} \approx \mp\frac{ck_\parallel^2 B_0}{4\pi\mu_m\rho_0} - \frac{i}{\tau}\frac{c^2 k_\parallel^2 w_0}{4\pi\mu_m v_F^2\rho_0^2} + O(k_\parallel^3)$$

- New anomalous Hall waves at $\vec{b} \neq 0$, etc.

[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Condens. Matter **30**, 275601 (2018)]

Summary

- Neutral chiral plasma has chiral vortical waves
  - Speeds for opposite direction waves differ @ $\mu_5 \neq 0$
- Equilibrium state of charged rotating plasma is radially nonuniform
- Propagating (not overdamped) hydro modes in charged rotating plasma are
  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density
- Dynamical electromagnetism plays a crucial role
- Many interesting hydro modes in Dirac/Weyl materials